Influence of Nonlinear Effects on Statistical Properties of the Radiation from SASE FEL

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Abstract

The paper presents analysis of statistical properties of the radiation from self amplified spontaneous emission (SASE) free electron laser operating in nonlinear mode. The present approach allows one to calculate the following statistical properties of the SASE FEL radiation: time and spectral field correlation functions, distribution of the fluctuations of the instantaneous radiation power, distribution of the energy in the electron bunch, distribution of the radiation energy after monochromator installed at the FEL amplifier exit and the radiation spectrum. It has been also obtained that statistics of the instantaneous radiation power from SASE FEL operating in the nonlinear regime changes significantly with respect to the linear regime. All numerical results presented in the paper have been calculated for the 70 nm SASE FEL at the TESLA Test Facility being under construction at DESY.

1. Introduction

It is expected that VUV and X-ray free electron lasers [1, 2] will be constructed in the nearest future. The projects of X-ray FELs are developed at SLAC and DESY [3, 4] and 6 nm SASE FEL is under construction at the TESLA Test Facility (TTF) at DESY [5, 6]. An appropriate design of the SASE FELs and planning of user's equipment and experiments depend strongly on the knowledge of the radiation amplification process in the SASE FEL and on the properties of the radiation.

At present there is significant progress in the description of the linear stage of SASE FEL operation in terms of statistical optics (time and spectral correlation functions, time of coherence, interval of spectral coherence, probability density functions of the instantaneous radiation power and of the finite-time integrals of the instantaneous power, probability density function of the radiation energy after the monochromator installed at the exit of SASE FEL) [7]. In particular, it has been shown that the radiation from SASE FEL operating in the linear regime possesses all the features corresponding to completely chaotic polarized radiation: the higher order correlation functions (time and spectral) are expressed via the first order correlation function, the probability density distribution of the instantaneous radiation power follows the negative exponential distribution and the probability density function of the finite-time integrals of the instantaneous power and of the energy after monochromator follows the gamma distribution.

Nevertheless, a reasonable question arises what are the features of the radiation from SASE FEL operating in the nonlinear mode and, in particular, at saturation. This question can be answered only on the base of the results obtained with nonlinear simulation codes. There exist several time-dependent simulation codes [7, 8, 9, 10, 11]. At present, only limited number of SASE FEL characteristics have been calculated. They are average output power [8], correlation time and variation of the energy in the radiation pulse [9]. There was an attempt to calculate radiation spectra, but the accuracy of calculation was not sufficient [11]. In this paper we present complete description of the nonlinear mode of SASE FEL radiation in terms of statistical optics. To be specific, we calculated all the numerical results for the 70 nm SASE FEL at the TESLA Test Facility being under construction at DESY [12]. Nevertheless, they can be simply scaled for calculation of other SASE FELs by applying similarity techniques (see, e.g. ref. [13, 14]).

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2. Description of SASE FEL in terms of statistical optics

In all existent time-dependent simulation codes the radiation pulse is simulated by discrete representation of the radiation field $E(z,t)$ with the step in longitudinal coordinate equal to radiation wavelength λ (see, e.g. refs. [8, 9, 10, 11]). Each simulation run starts from original statistical set of initial data and output results change from run to run. To obtain information about general properties of the output radiation one should use statistical methods. It has been shown in ref. [7] that description of SASE FEL in terms of statistical optics is adequate to the problem. In this paper we apply this approach for the description of the nonlinear mode of SASE FEL operation.

To calculate spectral characteristics of the radiation pulse we use Fourier transformation:

$$\bar{E}(z,\omega) = \int\limits_{-\infty}^{\infty} e^{i\omega t}[\tilde{E}(z,t)e^{-i\omega_0 t} + C.C.]dt \ . \qquad (1)$$

The first and the second order time and spectral correlation functions, $g_1(t,t')$, $g_2(t,t')$, $g_1(\omega,\omega')$ and $g_2(\omega,\omega')$, are calculated in accordance with the definitions:

$$\begin{aligned}
g_1(t-t') &= \frac{\langle \tilde{E}(t)\tilde{E}^*(t')\rangle}{\left[\langle|\tilde{E}(t)|^2\rangle\langle|\tilde{E}(t')|^2\rangle\right]^{1/2}} \ , \\
g_2(t-t') &= \frac{\langle|\tilde{E}(t)|^2|\tilde{E}(t')|^2\rangle}{\langle|\tilde{E}(t)|^2\rangle\langle|\tilde{E}(t')|^2\rangle} \ , \\
g_1(\omega,\omega') &= \frac{\langle \bar{E}(\omega)\bar{E}^*(\omega')\rangle}{\left[\langle|\bar{E}(\omega)|^2\rangle\langle|\bar{E}(\omega')|^2\rangle\right]^{1/2}} \ , \\
g_2(\omega,\omega') &= \frac{\langle|\bar{E}(\omega)|^2|\bar{E}(\omega')|^2\rangle}{\langle|\bar{E}(\omega)|^2\rangle\langle|\bar{E}(\omega')|^2\rangle} \ , \qquad (2)
\end{aligned}$$

where $\langle\ldots\rangle$ means shot-to-shot averaging. The coherence time τ_c [15] and interval of the spectral coherence $\Delta\omega_c$ [7] are calculated as follows:

$$\begin{aligned}
\tau_c &= \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2 \, d\tau \ , \\
\Delta\omega_c &= \int\limits_{-\infty}^{\infty} |g_1(\omega-\omega')|^2 \, d(\omega-\omega') \ . \qquad (3)
\end{aligned}$$

Normalized envelope of the radiation spectrum is reconstructed from the first order time correlation function as follows (see refs. [7, 16] for more detail):

$$G(\Delta\omega) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} d\tau g_1(\tau)\exp(-i\Delta\omega\tau) \ . \qquad (4)$$

To obtain output characteristics of the radiation from SASE FEL one should perform large number of simulation runs with time-dependent simulation code. The result of each run contains parameters of the output radiation (field and phase) stored in the boxes over the full length of the radiation pulse. At the next stage of numerical experiment the arrays of data should be handled to extract information on statistical properties of the radiation (see eq. (1) – (4)). Probability distribution functions of the instantaneous radiation power, of the finite-time integrals of the instantaneous power and of the radiation energy after the monochromator installed at the exit of SASE FEL are calculated by plotting histograms of large number of statistical data.

3. Results of numerical simulations

In this section we illustrate with numerical example the results of computer modelling of statistical properties of the radiation from SASE FEL operating in nonlinear regime. Simulations have been performed by means of 1-D time-dependent nonlinear simulation code [7, 10] with 100 statistically independent runs. Then the arrays of the output data have been handled with postprocessor codes to extract statistical properties of the SASE FEL radiation.

In Fig. 1a we present the dependence of the average radiation power on the reduced undulator length (solid line). Dotted line in this plot presents the output power for the steady-state FEL amplifier starting with the value of input power of 8 W (corresponding to the value of effective power of shot noise). It is seen that the main difference occurs at the nonlinear stage of amplification. It is seen that the output power of SASE FEL continues to grow due to the spectrum broadening (see ref. [7, 16]). Coherence time, τ_c, decreases drastically in the nonlinear regime (see Fig. 1b).

In Fig. 1c we present the results of calculation of the normalized rms deviation of the instantaneous fluctuations of the radiation power, $\sigma_{\mathrm{P}} = \langle(P-\langle P\rangle)^2\rangle^{1/2}/\langle P\rangle$, as function of the undulator length. One can see from this plot that at the linear stage of the SASE FEL operation the value of the deviation is equal to unity. It has been shown in refs. [7, 17] that in this case the probability density distribution of the radiation power is described by the negative exponential law. In the nonlinear mode of operation the deviation of the power fluctuations differs significantly ¿from unity. It indicates that the probability density function should differ from the negative exponential law. Fig. 2 presents the distributions of the instantaneous radiation power at different undulator lengths. It is seen that this distribution near the saturation point (upper right plot) differs cardinally from a negative exponential one. Nevertheless, in the deep nonlinear regime the power distribution tends again to the negative exponential one and the value of the rms deviation also tends to unity (see Fig. 2, lower right plot).

We performed numerical study of the fluctuations of the energy in the radiation pulse integrated over finite time. Fig. 1d presents the normalized rms deviation of the energy fluctuations in the radiation pulse as function of the undulator length. It is seen that the fluctuations achieve their maximum in the end of linear regime. The first local minimum corresponds to the saturation point. This is in a good agreement with Fig. 1c showing that relative fluctuations of the instantaneous radiation power achieve their minimum at the saturation point.

In Fig. 2 we present the results of calculations of the first and the second order time correlation functions. It is seen that there is significant difference with respect to the linear mode of operation (see ref. [17]). First, we obtain that relation $g_2(\tau) = 1 + |g_1(\tau)|^2$ does not takes place in the nonlinear regime. Second, the first order time correlation function begins to have two maxima at the increase of the undulator length (which is connected with the shape of the radiation spectrum (see ref. [7, 16] for more detail).

From practical point of view it is important to know characteristics of the output radiation from SASE FEL operating at saturation. Fig. 3a presents distribution of the instantaneous power at saturation point. In Fig. 3b we present the results of calculations of the first and the second order time correlation functions. In Fig. 3c we present the histogram of the probability density distribution of the radiation energy after the monochromator. The width of the monochromator is less than the interval of spectral coherence (3). It is seen from this plot that at a small window of the monochromator the fluctuations of the radiation energy follow the negative exponential law. We also performed calculations of the first and the second order spectral correlation functions for the radiation of SASE FEL operating at saturation (see Fig. 3d). Solid circles in this plot correspond to to the spectral correlation functions of the radiation pulse from SASE FEL operating in linear regime [7, 17]. With the accuracy of performed calculation we can state that spectral correlation functions are the same as those for the linear mode of the SASE FEL operation.

4. Conclusion

We have shown in this paper that description of SASE FEL radiation in terms of statistical optics (time and spectral correlation functions, time of coherence, interval of spectral coherence, probability density functions of the instantaneous radiation power and of the finite-time integrals of the instantaneous power, probability density function of the radiation energy after the monochromator installed at the exit of SASE FEL) is adequate for the problem. All these characteristics of SASE FEL operating in the nonlinear regime are presented in the paper. It has been also obtained that statistics of the instantaneous radiation power from SASE FEL operating in the nonlinear regime changes significantly with respect to the linear regime. Precise calculations of the time correlation functions have shown their complicated behaviour. In particular, it has been found that the relation $g_2(\tau) = 1 + |g_1(\tau)|^2$ is not fulfilled in the nonlinear regime. On the other hand, the spectral correlation functions and distribution of the radiation energy after monochromator calculated in the saturation point are the same as in the linear regime (see ref. [7]).

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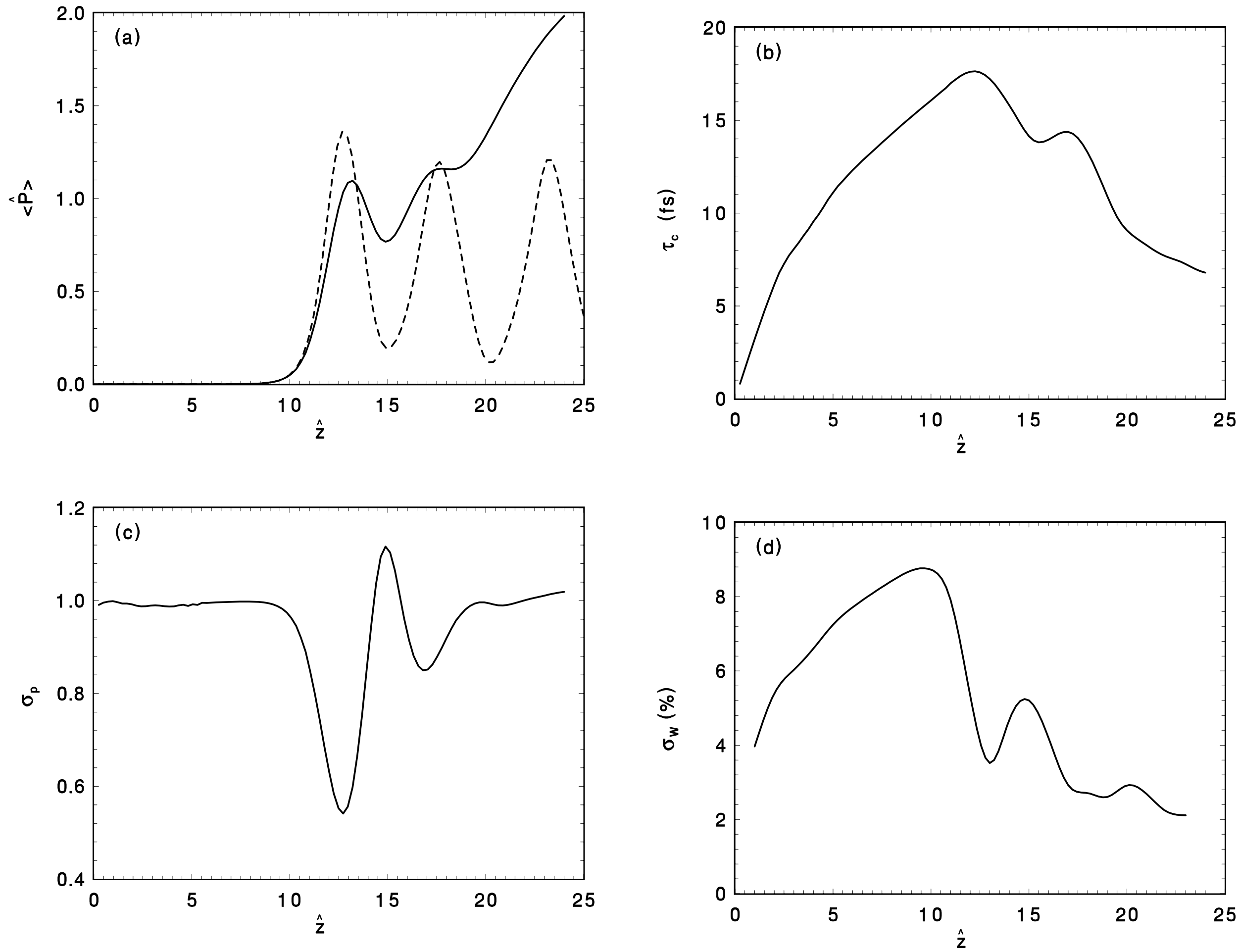

Fig. 1. Averaged characteristics of SASE FEL as function of undulator length: averaged output power (plot (a)), coherence time (plot (b)), normalized rms deviation of the instantaneous fluctuations of the radiation power (plot (c)) and normalized rms deviation of the energy fluctuations in the radiation pulse (plot (d)).

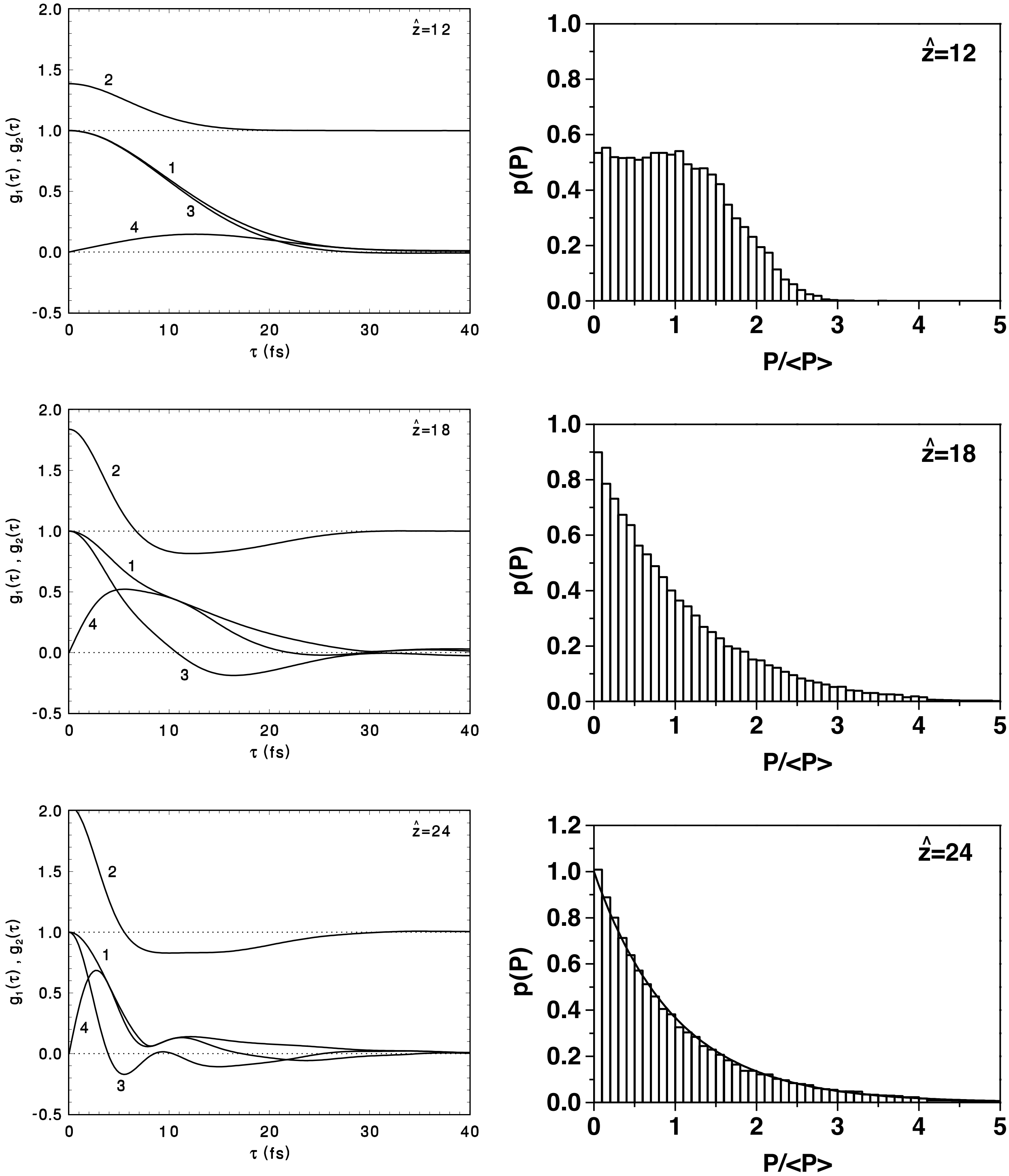

Fig. 2. The first and the second order time correlation functions, $|g_1(\tau)|$ (curve 1), $g_2(\tau)$ (curve 2), $\mathrm{Re}(g_1(\tau))$ (curve 3) and $\mathrm{Im}(g_1(\tau))$ (curve 4) of the radiation pulse (at left) and histograms of the probability density distribution, $p(P)$, of the instantaneous output power at different reduced length of the FEL amplifier (at right). Calculations have been performed with nonlinear simulation code over 10^4 independent statistical events. $\langle P\rangle$ denotes the average power.

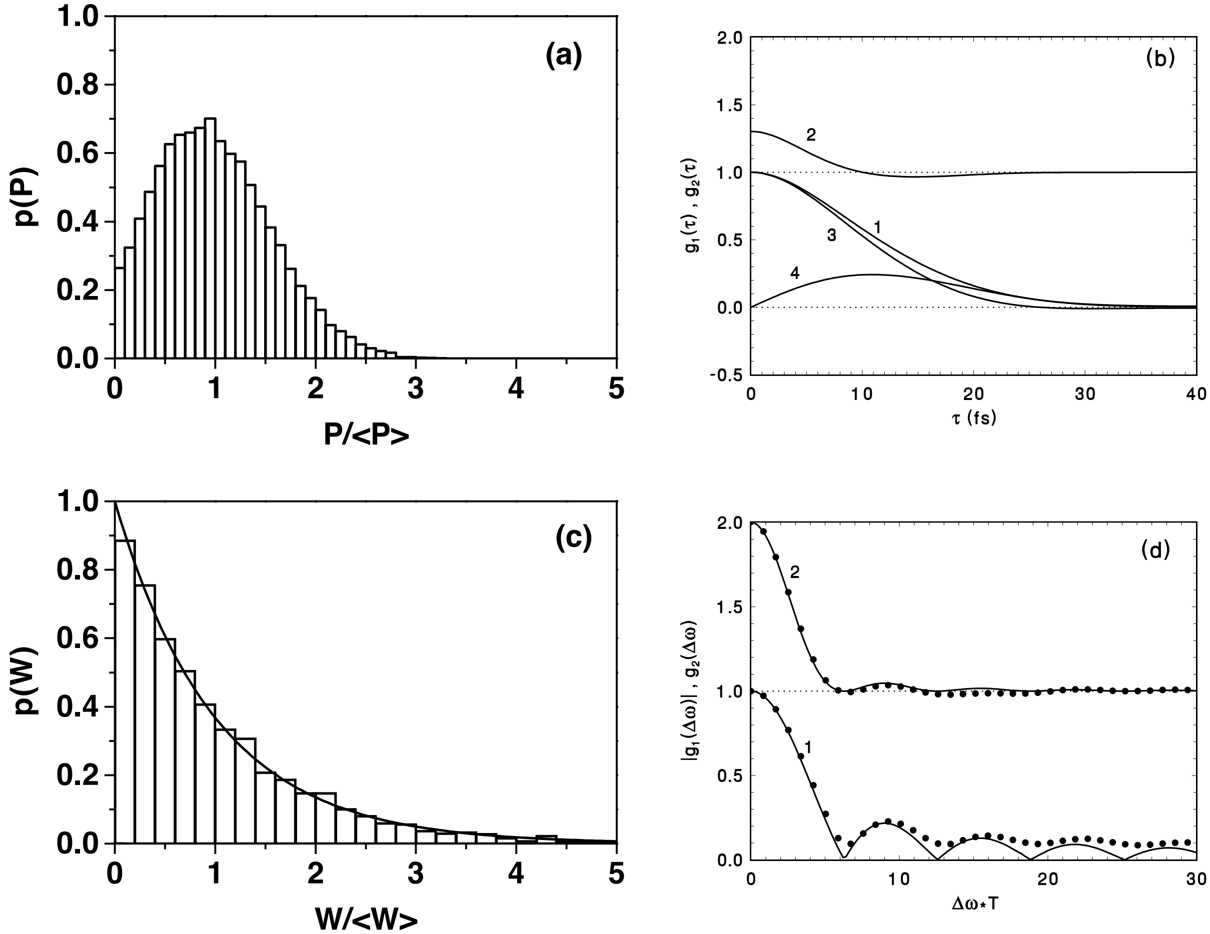

Fig. 3. Characteristics of the output radiation from SASE FEL operating at saturation: probability density function of the instantaneous radiation power (plot (a)), the first and the second order time correlation functions, $|g_1(\tau)|$ (curve 1), $g_2(\tau)$ (curve 2), $\mathrm{Re}(g_1(\tau))$ (curve 3) and $\mathrm{Im}(g_1(\tau))$ (curve 4) (plot (b)), a histogram of the probability density distribution, $p(W)$, of the radiation energy after narrow bandwidth monochromator (plot (c)) and the first and the second order spectral correlation functions, $|g_1(\Delta\omega)|$ (curve 1)and $g_2(\Delta\omega)$ (curve 2) (plot (d)).