Analytical Treatment of the Radiative Interaction of Electrons in a Bunch Passing a Bending Magnet

E.L. Saldin[a], E.A. Schneidmiller[a], M.V. Yurkov[b]

[a] *Automatic Systems Corporation, 443050 Samara, Russia*

[b] *Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia*

Abstract

The paper presents some analytical results of the theory of coherent synchrotron radiation (CSR) describing the case of finite curved track length.

1. Introduction

Analysis of project parameters of linear colliders [1, 2] and short-wavelength FELs [1, 3, 4] shows that the effects of coherent synchrotron radiation of short electron bunches passing bending magnets influence significantly on the beam dynamics (see, e.g., [5, 6]). The first investigations in the theory of coherent synchrotron have been performed about fifty years ago [7, 8, 9]. In these papers the main emphasis was put on the calculations in far zone of CSR produced by a bunch of relativistic electrons moving on a circular orbit. Another part of the problem, namely that of the radiative interaction of the electrons inside a bunch has been studied for the first time in refs. [10, 11] and later in refs. [5, 12] where the energy loss along the bunch has been calculated. The results of the above mentioned CSR theories are valid for a model situation of the motion of an electron bunch on a circular orbit and do not describe the case of an isolated bending magnet. The first analytical results describing this case have been presented in ref. [13]. In particular, analytical expressions have been obtained for the radiative interaction force, for the energy loss distribution along the bunch and for the total energy loss of the bunch. The criterium for the applicability region of the previous theories to the case of a finite magnet length has been derived. In this report some analytical results of ref. [13] are presented.

2. Results for a rectangular bunch

Let us consider a rectangular bunch of the length l_b passing a magnet with the bending angle ϕ_m and the bending radius R. We use the model of ultrarelativistic electron bunch with a linear distribution of the charge (zero transverse dimensions) and assume the bending angle to be small, $\phi_m \ll 1$. We neglect the interaction of the bunch with the chamber walls assuming the electrons to move in free space. The total number of particles in the bunch is equal to N and the linear density is equal to $\lambda = N/l_b$.

When the electron bunch passes the magnet, the electromagnetic field slips over the electrons due to the curvature and the difference between the electron's velocity and velocity of light c. The slippage length L_{sl} is given by the expression:

$$L_{sl} \simeq \frac{R\phi_m}{2\gamma^2} + \frac{R\phi_m^3}{24} , \qquad (1)$$

where γ is relativistic factor. When applying the results of steady-state CSR theory (periodical circular motion) to the case of isolated magnet it is assumed usually that the bunch length is much shorter than the slippage length. To obtain more correct criterium for the applicability region one has to develop more general theory including transient effects when the bunch enters and leaves the magnet. Such an investigation has been performed in ref. [13]. In particular, it has been stressed that the radiation formation length of the order of $l_b\gamma^2$ before and after magnet plays an important role in CSR effects. In practically

TESLA FEL-Report 1997-06

important case when the conditions $\gamma\phi_m \gg 1$ and $R/\gamma^3 \ll l_b < L_{sl}$ are satisfied, the expression for the total energy loss of the bunch can be written in the following form [13]:

$$\Delta\mathcal{E}_{tot} = -\left(\frac{3^{2/3}e^2N^2}{l_b^{4/3}R^{2/3}}\right)(R\phi_m) \times\left\{1+\frac{3^{1/3}4}{9}\frac{l_b^{1/3}}{R^{1/3}\phi_m}\left[\ln\left(\frac{l_b\gamma^3}{R}\right)+C\right]\right\} \quad (2)$$

where e is the charge of the particle and

$$C = 2\ln 2 - \frac{1}{2}\ln 3 - \frac{11}{2} \simeq -4 \ .$$

The first term in eq. (2) is the solution obtained in the framework of steady-state approach (see, e.g., refs. [9, 11, 5]). Therefore, with logarithmical accuracy we can set the applicability region of the results of the steady-state theory for the case of a finite curved track length:

$$\frac{l_b^{1/3}}{R^{1/3}\phi_m}\ln\left(\frac{l_b\gamma^3}{R}\right) \ll 1 \ . \quad (3)$$

In particular, the steady-state theory provides completely incorrect results for the case of the electron bunch much longer than the slippage length, $l_b \gg L_{sl}$. In this case the energy losses of the particles in the bunch due to CSR are proportional to the local linear density and take place mainly after the magnet [13]. For a "short" magnet, $\gamma\phi_m \ll 1$, the total energy loss of rectangular bunch is equal to

$$\Delta\mathcal{E}_{tot} = -\frac{2}{3}\frac{e^2N^2}{l_b}\ \gamma^2\phi_m^2 \ . \quad (4)$$

The energy loss of the rectangular bunch passing a "long" magnet, $\gamma\phi_m \gg 1$, is equal to:

$$\Delta\mathcal{E}_{tot} = -\frac{N^2e^2}{l_b}\left[4\ln(\gamma\phi_m) - 2\right] \ . \quad (5)$$

These results has been obtained in ref. [13] by means of calculation the radiative interaction of the electrons in the bunch. It is interesting to compare the total energy loss of the bunch with the energy of coherent radiation in far zone. The radiation energy in far zone can be calculated as an integral over frequency of the spectral density of the radiation energy:

$$\frac{dW_{coh}}{d\omega} = N^2\eta(\omega)\frac{dW}{d\omega} \ , \quad (6)$$

where $\eta(\omega)$ is the bunch form factor (squared module of the Fourier transform of the linear density distribution). The form factor for the rectangular bunch of the length l_b is given by the expression:

$$\eta(\omega) = \left(\sin\frac{\omega l_b}{2c}\right)^2\left(\frac{\omega l_b}{2c}\right)^{-2} \ . \quad (7)$$

Function $dW/d\omega$ entering eq. (6) is the spectral density of the radiation energy of a single electron. The angular and the spectral characteristics of the radiation of an electron moving in an arc of a circle have been studied in ref. [14][1]. It has been shown that the spectrum of the radiation emitted by an electron moving in an arc of a circle differs significantly from that of conventional synchrotron radiation of an electron executing periodical circular motion. In the latter case the spectral density at low frequencies is proportional to $\omega^{1/3}$ [16]. In the case of a finite curved track length the spectral density is constant at $\omega \to 0$. When the bending angle is small, $\phi_m \ll 1$, the spectral density of the radiation energy emitted by ultrarelativistic electron is function of the only parameter $\gamma\phi_m$ [14]:

$$\frac{dW}{d\omega} = \frac{e^2}{\pi c}f_m \ , \quad (8)$$

where

$$f_m = \left(\mu + \frac{1}{\mu}\right)\ln\frac{1+\mu}{1-\mu} - 2 \ ,$$

and

$$\mu = \frac{\gamma\phi_m/2}{\sqrt{1+(\gamma\phi_m/2)^2}} \ .$$

Formula (8) is valid in the frequency range $\omega \ll c/L_{sl}$. Taking into account formula (7) we can estimate that typical frequencies of the coherent radiation are below the frequency $\omega \sim c/l_b$. It means that we can use the asymptotical expression (8) in the case when $l_b \gg L_{sl}$. Integrating eq. (6) over the frequency, we obtain:

$$W_{coh} = \frac{e^2N^2}{l_b}f_m \ . \quad (9)$$

It is easy to obtain that in the case of a "long" magnet, $\gamma\phi_m \gg 1$, the energy of coherent radiation (9) coincides exactly with the bunch energy loss given by eq. (5) taken with opposite sign. In the limit of a "short" magnet, $\gamma\phi_m \ll 1$, there is also complete agreement between formulae (9) and (4).

3. Bunch with an arbitrary density profile

The solutions obtained in ref. [13] for the rectangular bunch can be generalized for the case of an arbitrary linear charge density. We present here the results of the calculation of the transition process when the bunch enters the magnet. Let the bunch have the density distribution $\lambda(s)$ which satisfies the condition

$$\frac{R}{\gamma^3}\frac{d\lambda(s)}{ds} \ll \lambda(s) \ . \quad (10)$$

[1] The same problem has been considered later in ref.[15], but the results of this paper are incorrect.

Under this condition the rate of the energy change of an electron is given by the expression [13]:

$$
\frac{d\mathcal{E}(s,\phi)}{d(ct)} = -\frac{2e^2}{3^{1/3}R^{2/3}} \left\{ \left(\frac{24}{R\phi^3}\right)^{1/3} \left[\lambda\left(s - \frac{R\phi^3}{24}\right) - \lambda\left(s - \frac{R\phi^3}{6}\right) \right] + \int\limits_{s-R\phi^3/24}^{s} \frac{ds'}{(s-s')^{1/3}} \frac{d\lambda(s')}{ds'} \right\} , \qquad (11)
$$

where s is the position of the electron in the bunch and ϕ is azimuthal angle.

For the Gaussian density distribution:

$$
\lambda(s) = \frac{N}{(2\pi)^{1/2}\sigma}\, exp\left[-\frac{s^2}{2\sigma^2}\right] , \qquad (12)
$$

expression (11) takes the form:

$$
\frac{d\mathcal{E}}{d(ct)} = -\frac{2e^2 N}{3^{1/3}(2\pi)^{1/2}R^{2/3}\sigma^{4/3}} G(\xi,\rho) , \qquad (13)
$$

where function $G(\xi,\rho)$ is given by the expression:

$$
\begin{aligned}
G(\xi,\rho) &= \rho^{-1/3}\left[e^{-(\xi-\rho)^2/2} - e^{-(\xi-4\rho)^2/2}\right] \\
&+ \int\limits_{\xi-\rho}^{\xi} \frac{d\xi'}{(\xi-\xi')^{1/3}} \frac{d}{d\xi'} e^{-(\xi')^2/2} . \qquad (14)
\end{aligned}
$$

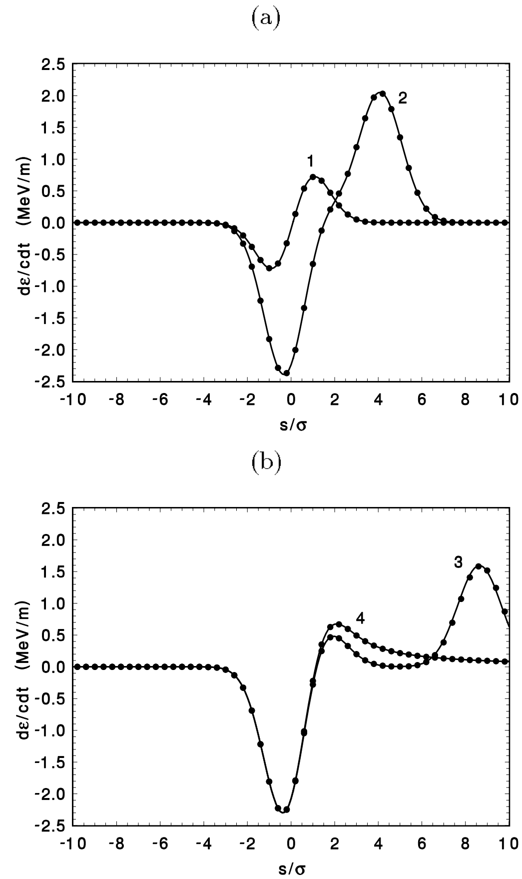
Here $\xi = s/\sigma$ and $\rho = R\phi^3/24\sigma$. Function $G(\xi,\rho)$ reduces to

$$
G(\xi,\rho) \simeq -\frac{9}{2}\xi \exp(-\xi^2/2)\rho^{2/3}
$$

at $\rho \ll 1$. In the opposite case, at $\rho \to \infty$, expression (14) tends to the steady-state solution [11, 5, 12]. In Fig.1 we present the plot of function (13). One can see that there is excellent agreement of analytical results [13] and the results obtained by means of numerical simulation code [6].

Acknowledgement

We wish to thank C.L. Bohn, R. Brinkmann, Ya.S. Derbenev, M. Dohlus, K. Floettmann, D. Jaroszynski, T. Limberg, H. Mais, A. Piwinski, J. Rossbach, V.D. Shiltsev and D. Trines for useful discussions.



The rate of an electron energy change as a function of its position along the Gaussian bunch entering the magnet. Curve (1) in the graph (a): 5 cm after the begin of the magnet. Curve (2) in the graph (a): 14 cm after the begin of the magnet. Curve (3) in the graph (b): 18 cm after the begin of the magnet. Curve (4) in the graph (b): steady state. The curves are the results of calculations with formula (13) and the circles are the results of numerical simulations presented in ref. [6]. The parameters are as follows: $R = 1.5$ m, $\sigma = 50$ μm, $q = 1$ nC.

References

[1] "Conceptual Design of a 500 GeV e+e− Linear Collider with Integrated X-ray Laser Facility" (Editors R.Brinkmann, G. Materlik, J. Rossbach, A. Wagner), DESY 97-048, Hamburg, 1997.

[2] "Zeroth-Order Design Report for the Next Linear Collider", LBNL-PUB-5424,SLAC Report 474, UCRL-ID-124161 (May 1996).

[3] R. Tatchyn et al., Nucl. Instr. and Methods **A375**(1996)274.

[4] J. Rossbach, Nucl. Instr. and Methods **A375**(1996)269.

[5] Ya.S. Derbenev, J. Rossbach, E.L. Saldin, V.D. Shiltsev, "Microbunch Radiative Tail-Head Interaction", DESY Print, TESLA-FEL 95-05, Hamburg(1995).

[6] M. Dohlus, T. Limberg, "Emittance Growth due to Wake Fields on Curved Bunch Trajectories", presented at XVIII International Free Electron Laser Conference (Rome,1996); DESY print, TESLA-FEL 96-13 (1996).

[7] L.I. Schiff, Rev. Sci. Instr. **17**(1946)6.

[8] J. Schwinger, "On radiation by electrons in a betatron", 1945 (unpublished), see references in [9].

[9] J.S. Nodvick and D.S. Saxon, Phys. Rev. **96**(1954)180.

[10] I.E. Tamm, Report of the Institute of Physics, Acad. of Sciences, USSR, 1948 (in Russian).

[11] L.V. Iogansen and M.S. Rabinovich, Sov. Phys. JETP, vol.37(10), 1960, p.83.

[12] J.B. Murphy, S. Krinsky and R.L. Glukstern, "Longitudinal Wakefield for Synchrotron Radiation", Proc. of IEEE PAC 1995, Dallas(1995).

[13] E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, "On the Coherent Radiation of an Electron Bunch Moving in an Arc of a Circle", DESY Print, TESLA-FEL 96-14 (November 1996).

[14] V.G. Bagrov, I.M. Ternov and N.I. Fedosov, Sov. Phys. JETP, vol.55(5), 1982, p.835; Phys. Rev. **D28** (1983)2464.

[15] D.J. Wingham, Phys. Rev. **D35**(1987)2584.

[16] J. Schwinger, Phys. Rev. **75**(1949)1912.