Statistical Properties of Radiation from SASE FEL Driven by Short Electron Bunches

E.L. Saldin[a], E.A. Schneidmiller[a], M.V. Yurkov[b]

[a]Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, D-22607 Hamburg, Germany

[b]Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia

We analyze statistical properties of the radiation from a SASE FEL driven by short electron bunches. In the linear regime the radiation from a SASE FEL is a Gaussian random process. When approaching saturation point, statistical properties of the radiation change drastically on a scale of one field gain length. Particular attention is devoted to the analysis of fluctuations of total energy in the radiation pulse and after a narrow-band monochromator. It was found that fluctuations at saturation are significantly suppressed when electron pulse length becomes comparable with cooperation length.

1. Introduction

Fluctuations of the electron beam current density can serve as the input signal in the FEL amplifier. These fluctuations always exist in the electron beam due to the effect of shot noise. An FEL amplifier which starts up from shot noise is frequently known as Self-Amplified Spontaneous Emission (SASE) FEL. It is accepted that one-dimensional theory describes in the clearest way the effects connected with longitudinal coherence. The results of the one-dimensional theory can be applied to the high-gain FEL amplifier when the transverse coherence of the radiation is settled. Some averaged output characteristics of SASE FEL in the framework of one-dimensional model have been obtained in [1,2]. An approach for 1-D time-dependent numerical simulations of SASE FEL have been developed in [3,4]. Realization of this approach allowed to obtain some statistical properties of radiation from a SASE FEL operating in linear and nonlinear regime [5,6].

At present there is significant progress in the description of the SASE FEL operation in terms of statistical optics (time and spectral correlation functions, time of coherence, interval of spectral coherence, probability density functions of the instantaneous radiation power and of the finite-time integrals of the instantaneous power, probability density function of the radiation energy after the monochromator installed at the exit of SASE FEL) [7]. Mostly these studies refer to the case of a long bunch, and up to now there is rather limited knowledge of short-pulse effects. Actually, there is only one study [4] where an asymptotic solution for average energy in the radiation pulse was found. Study of nonlinear mode of SASE FEL operation given in that paper is rather incomplete. In this paper we perform comprehensive study of SASE FEL driven by short electron bunches. Main emphasis is put on nonlinear mode of operation. It was found that slippage effects result in a set of novel features of the radiation. In particular, for very short pulses we found the effect of stabilization of fluctuations of energy in the radiation pulse and fluctuations of the energy after narrow-band monochromator. Suppression factor scales as a square root of pulse length.

2. Basic notations

The one-dimensional model describes the amplification of the plane electromagnetic wave by the electron beam in the undulator. When space charge and energy spread effect can be neglected, operation of FEL amplifier is described in terms of the gain parameter $\Gamma = \left[\pi j_0 K^2/(I_A \lambda_{\mathrm{w}} \gamma^3)\right]^{1/3}$, efficiency parameter $\rho = \lambda_{\mathrm{w}}\Gamma/(4\pi)$, and detuning parameter $\hat{C} = [2\pi/\lambda_{\mathrm{w}} - \omega(1+K^2)/(2c\gamma^2)]/\Gamma$ (see, e.g. [9,10]). Here λ_{w} is undulator period, $K = e\lambda_{\mathrm{w}} H_{\mathrm{w}}/2\pi mc^2$ is undulator parameter, γ is relativistic factor, H_{w} is undulator field, j_0 is the beam current density, $(-e)$ and m are charge and mass of electron, $I_A = mc^3/e \simeq$ 17 kA, and ω is frequency of electromagnetic wave. When describing start-up from shot noise, one more parameters of the theory appears – number of particles in coherence volume, $N_\mathrm{c} = I/(\rho\omega e)$, where I is beam current.

To be specific, we consider an electron beam with a Gaussian axial profile of the current density: $S(\hat{s}) = j(\hat{s})/j_\mathrm{max} = \exp\left[-\hat{s}^2/(2\hat{\sigma}_\mathrm{b}^2)\right]$, where $\hat{\sigma}_\mathrm{b} = \rho\omega_0\sigma_\mathrm{b}/c$ and σ_b is the rms bunch length. Here and below, the normalization is performed with respect to the maximal current density, j_max. The rms bunch length is assumed to be large, $\omega_0\sigma_\mathrm{b}/c \gg 1$ or, in normalized form: $\hat{\sigma}_\mathrm{b} \gg \rho$. Under this assumption we can neglect the contribution of the coherent seed to the input signal of the FEL amplifier starting from the shot noise. Since ρ is always much less than unity, we can investigate short-pulse effects, when the bunch is comparable to (or even much shorter than) the typical slippage distance $c/(\rho\omega_0)$.

Shot noise in the electron beam is Gaussian random process [7]. FEL amplifier, operating in the linear regime, can be considered as a linear filter which does not change statistics. As a result, radiation is also Gaussian random process. In this case the probability distribution of the instantaneous radiation power should be the negative exponential distribution (the notion of instantaneous power refers to a certain moment of time and a certain $\hat{z}$ coordinate, and that the analysis must be performed over an ensemble of pulses). Also, the finite-time integrals of the instantaneous power and the integrated spectral density (measured after the monochromator) should fluctuate in accordance with the gamma distribution.

Nevertheless, a reasonable question arises what are the features of the radiation from SASE FEL operating in the nonlinear mode and, in particular, at saturation. This question can be answered only on the base of the results obtained with nonlinear simulation codes. Present study of amplification process in SASE FEL is performed with one-dimensional version of code FAST [7,8,10]. In this code the radiation pulse is simulated by discrete representation of the radiation field $E(z,t)$ with the step in longitudinal coordinate equal to radiation wavelength λ. Each simulation run starts from original statistical set of initial data and output results change from run to run. To obtain information about general properties of the output radiation one should use statistical methods [7,10].

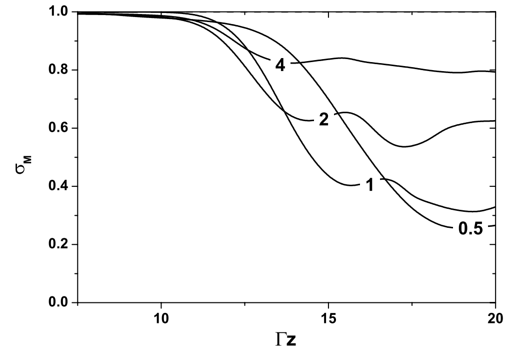

Figure 2. *Normalized rms deviation of energy after narrow-band monochromator versus undulator length. Length of the electron bunch changes in the limits* $\hat{\sigma}_\mathrm{b} = 0.5 - 4$. *Here* $N_\mathrm{c} = 10^8$

3. Average energy in the radiation pulse

Figure 1 shows evolution of the averaged efficiency $\langle\hat{\eta}\rangle = \langle E_\mathrm{rad}\rangle/(\rho\gamma m_\mathrm{e}c^2 N)$ along the undulator length. Here E_rad is energy in the radiation pulse, and N is number of electrons in the bunch. Dashed line in Fig. 1 represents averaged efficiency for the case of long electron bunch with rectangular profile. Amplification process passes three stages: start-up from shot noise, stage of exponential gain, and nonlinear stage. Let us define saturation point as the first maximum of the gain curve. At $\hat{\sigma}_\mathrm{b} \lesssim 2$ saturation length grows as $\hat{z}_\mathrm{sat} \propto 1/\sqrt{\hat{\sigma}_\mathrm{b}}$, and averaged saturation efficiency drops as $\langle\hat{\eta}_\mathrm{sat}\rangle \propto \sqrt{\hat{\sigma}_\mathrm{b}}$. At $\hat{\sigma}_\mathrm{b} \gtrsim 2$ saturation efficiency quickly approaches asymptotical value. Comparison with the case of a long electron pulse with rectangular profile shows that only 60% of electrons produce radiation. This is consequence of gradient profile of electron bunch.

4. Fluctuations of energy in the radiation pulse

Right plot in Fig. 1 shows normalized rms deviation of energy in the radiation pulse. Behaviour of the fluctuations in the linear regime has a straightforward explanation: number of longitudinal modes decreases with the pulse length and undulator length. Radia-

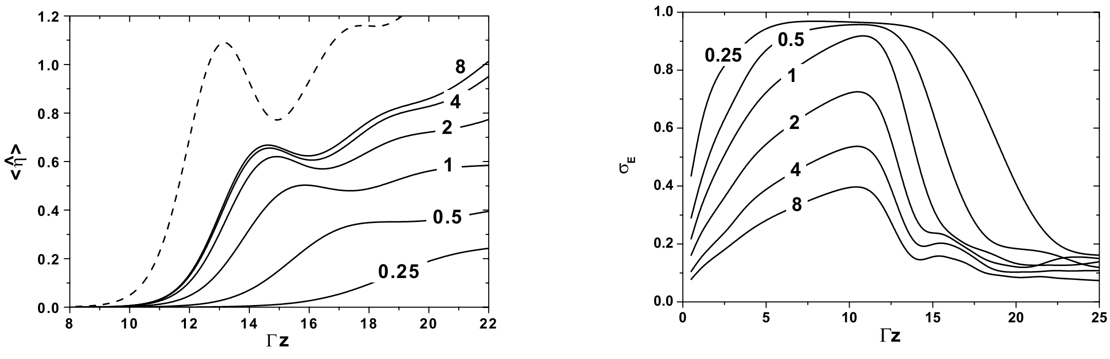

Figure 1. *Averaged efficiency of SASE FEL versus undulator length for different length of electron bunch* $\hat{\sigma}_{\rm b} = 0.25 - 8$ *(left plot). Right plot shows normalized rms deviation of energy in the radiation pulse. Here* $N_{\rm c} = 10^8$*. Dashed line represents the case of long electron pulse with rectangular profile*

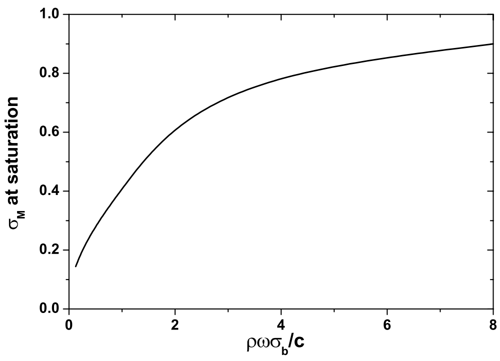

Figure 3. *Normalized rms deviation of energy after narrow-band monochromator versus length of electron bunch. SASE FEL operates at saturation*

tion of the SASE FEL operating in the linear regime is Gaussian random process, so probability distribution of the energy in the radiation pulse is gamma-distribution. Situation changes drastically when amplification process enters nonlinear stage. It is seen that deviation of energy drops quickly on a scale of a field gain length. In the case of a long pulse fluctuations at saturation are suppressed as $1/\sqrt{\hat{\sigma}_{\rm b}}$ which is a consequence of increasing the number of statistically independent spikes in the radiation pulse [4]. Physical picture becomes quite different for the bunch length $\hat{\sigma}_{\rm b} \lesssim 2$: deviation of energy in the radiation pulse at saturation starts to decrease with the bunch length as $\sqrt{\hat{\sigma}_{\rm b}}$. Nature of this phenomenon can be understood by analyzing structure of the radiation pulse. In the end of linear mode of operation SASE FEL driven by short electron bunch produces radiation pulses of nearly the same shape, but with amplitudes fluctuating nearly by negative exponential distribution. When amplification process enters nonlinear stage, radiation power is saturated, and pulses sleep forward. Further growth of the total energy occurs due to the radiation of bunched electron beam. Since maximal bunching of the electron beam is limited to the unity, this additional radiation is well stabilized, leading to the overall stability of the total energy in the radiation pulse.

Simulations show that statistics of the radiation also change drastically near the saturation point on a scale of one field gain length.

5. Fluctuations of energy after narrow-band monochromator

Let us study statistics of SASE FEL radiation filtered through narrow-band monochromator. The plots of normalized rms deviation of energy after narrow-band monochromator versus undulator length are presented in Fig. 2. In the linear stage of SASE FEL operation the value of normalized energy deviation is equal to unity, and energy fluctuates in accordance with negative exponential distribution. This is consequence of the fact that in this case radiation is Gaussian random process. However, in the nonlinear mode of operation we obtain significant decrease of fluctuations when the pulse length goes down (see Fig. 3). This effect has simple physical explanation. When SASE FEL driven by short bunch operates in the linear regime, every time radiation pulses have similar shape, but amplitude fluctuates nearly by negative exponential distribution. When amplification process enters nonlinear stage, amplitudes of different pulses are equalized due saturation effects, while keeping close shape. Spectrum of the radiation pulse is given by Fourier transform of the radiation field, and at saturation we obtain nearly similar spectrum envelope for different pulses. As a result, we can expect that fluctuations of the radiation energy after narrow-band monochromator should follow fluctuations of the total energy in the radiation pulse. Comparison of Figs. 1 and 2 confirms this simple physical consideration. At saturation fluctuations of the energy after narrow-band monochromator drop as $\sigma_{\mathrm{M}} \simeq \sigma_{\mathrm{E}} \propto \sqrt{\hat{\sigma}_{\mathrm{b}}}$.

6. Conclusion

Study of amplification process in the SASE FEL driven by electron bunch of finite pulse duration allows us to make the following conclusions. For the bunch length $\hat{\sigma}_{\mathrm{b}} \gtrsim 4$ asymptotical results [7] for a long rectangular bunch are applicable. At $\hat{\sigma}_{\mathrm{b}} \lesssim 2$ SASE FEL exhibits quite different behaviour caused by strong influence of slippage effects. In addition to reduction of the FEL gain and efficiency [4], short-pulse effects strongly influence statistical properties of the radiation in the nonlinear regime. In particular, for very short pulses we found the effect of stabilization of fluctuations of energy in the radiation pulse and fluctuations of the energy after narrow-band monochromator. When SASE FEL operates at saturation, suppression factor scales as $\sqrt{\hat{\sigma}_{\mathrm{b}}}$ with electron bunch length.

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