Transverse self-fields within an electron bunch moving in an arc of a circle

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Self-interaction within an electron bunch moving under the action of external forces may spoil the high brightness required for a SASE-FEL operating in the x-ray regime. Here we present part of the results achieved in [1], which deals with transverse self-interactions. We address the problem of a 1D line bunch moving in an arc of a circle analytically and from a fully electrodynamical viewpoint comparing our results with the code TRAFIC[4].

1. TWO PARTICLES IN AN ARC OF A CIRCLE

We follow [1] (see the latter for detailed treatment and references), starting from the expressions of the Lorentz force in the transverse (in the orbital plane, orthogonally with respect to the test particle velocity) direction:

$$\vec{F}_{\perp C[orR]} = e\vec{E}_{\perp C[R]}(\vec{r_{\mathrm{T}}}, t) + ec[\vec{\beta}_{\mathrm{T}} \times \vec{B}_{C[R]}(\vec{r_{\mathrm{T}}}, t)]_{\perp}. \quad (1)$$

Here the subscript T stands for test particle, C and R stand for Coulomb and Radiative field. $\vec{B}$ and $\vec{E}$ are understood to be the Lienard-Wiechert formulae for the electromagnetic fields. Using explicit expressions for the quantities in Eq. (1) together with the proper retardation conditions we found analytic equations for the transverse force exerted by a source electron on a test particle in all the cases in Fig. 1, both with the test particle in front (the cases actually depicted in the figure) or behind the source. These expressions are in perfect agreement with simulations by the code TRAFIC[4] (see [1]). As an example we show, in Fig. 2, the normalized transverse force ($\hat{F} = F_{\perp}/[e^2/(4\pi\varepsilon_0\Delta s)]$) for a two-particle system entering a hard-edge bending magnet (situation in Fig. 1a) as a function of the position after injection. The solid lines show analytical results; the circles describe the outcome from TRAFIC[4]. We plotted several outcomes from different values of the normalized distance between the two particles.

2. LINE BUNCH AND TEST PARTICLE

By integration of the latter results over a given bunch density distribution function one can calculate analytical expressions for the transverse force acting on a test particle behind or in front of the bunch (Tail-Head interaction). Fig. 3 shows the normalized transverse force $F_{\perp}/f_2$ ($f_2 = e^2\lambda_0/(4\pi\varepsilon_0 R)\ln(\Delta\hat{s}_{\mathrm{max}}), \Delta\hat{s}_{\mathrm{max}}$ being the maximal source-test distance) acting on a test particle in front of a bunch with rectangular density distribution which enters a hard-edge bending magnet as a function of the position of the test particle inside the magnet. The solid lines show analytical results; the circles describe the outcome from TRAFIC[4]. We choose $\Delta s_{\mathrm{max}} = 100\mu$m, $\gamma = 100$, $R = 1$ m; graphs are plotted for several values of the parameter $\Delta\hat{s}_{\mathrm{min}}$. Similar studies can be performed with the test particle behind the bunch (giving completely different results, due to the change of the retardation condition, [1]). In the case of tail-head interaction, by composition of rectangular bunches we found an expression for

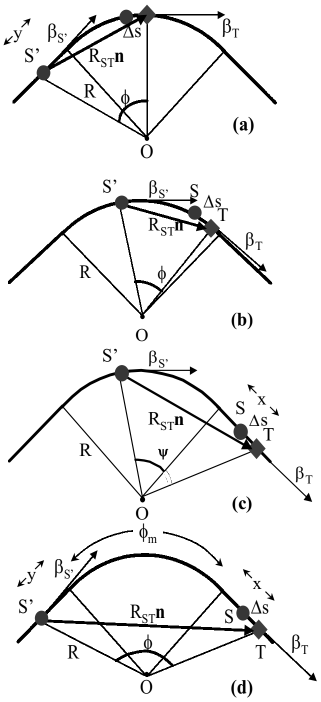

Figure 1. Possible configurations source-test particle.

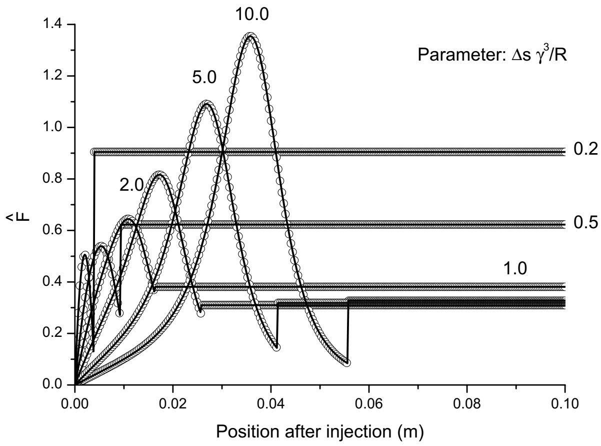

Figure 2. Example of transverse self-force for a two-particle system. The source particle is behind the test electron (Tail-Head interaction).

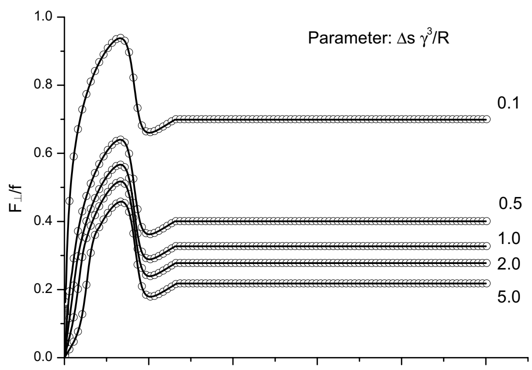

Figure 3. Example of transverse self-force acting on a self particle in front of a rectangular bunch (Tail-Head interaction).

the transverse force exerted by a bunch with arbitrary density distribution. Such expression is regularized to a formula independent of the distance between test particle and bunch by subtraction of the steady-state transverse self-interaction:

$$\widetilde{F}_{\perp}^{\rm tot} \simeq \frac{e^2}{2\pi\varepsilon_0 R}\left[\lambda(s - R\phi^3/6) - \right.$$

$$\left. -\frac{1}{6}\int_{-\infty}^{s-R\phi^3/24} \ln\left(\frac{24(s-s')}{R\phi^3}\right)\frac{d\lambda(s')}{ds'}ds'\right] , \qquad (2)$$

λ being the particle density distribution, s being the test particle position in the bunch, and ϕ the retarded angle (see [1]).

REFERENCES

1. G. Geloni et al., DESY 02-048, May 2002. Also, to be submitted to Phys. Rev. E