



Reduction of the coupling to external sources and modes of propagation by a nearly confocal resonator

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Abstract

This paper presents a numerical and experimental study of a nearly confocal resonator with spherical mirrors at 12 GHz. The geometry was chosen in order to have a large quality factor for the diffraction losses, and thereby a weak coupling to external parasitic TE and TM modes, that propagate in a pipe on which the resonator may be installed. In turn, this allows a significant improvement of its signal-to-noise ratio, e.g. when used as a beam monitor.

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1 Introduction

Open resonators with spherical mirrors were originally investigated in the early 1960s, when the first laser oscillators in the microwave and optical regimes appeared. The distribution of the electromagnetic field in such cavities were first calculated by Boyd and Gordon in 1961 [1, 2] and checked experimentally in 1970 [3, 4]. One interesting property of the open resonators with spherical mirrors is their potentially high quality factor, especially for the diffraction losses that result from the finite size of the mirrors. These were first calculated by Fox and Li in 1961 [5]. The highest quality factor, and thereby the smallest diffraction losses, are achieved when the distance between the mirrors is close or equal to their curvature radius. This design produces the smallest possible beam diameter at the mirrors for a given cavity length. A very readable tutorial on such confocal resonators is given in [6] and an overview of the formulas can be found in [7]. Reciprocity [8] then suggests that, as a result of their high quality factor for the diffraction losses, open resonators with spherical mirrors only couple weakly to external fields, especially close to confocality.

In turn, this should allow a significant improvement of the signal-to-noise ratio for diagnostic devices based on an open resonator with spherical mirrors. This would, for instance, allow measurement of the dielectric properties of a material located inside or flowing through the open resonator without being disturbed by external fields. Also, beam monitors in high intensity accelerators are often perturbed by microwave fields generated by the beam itself upstream of the detection device and that propagate in the wake of the bunches. A resonator pick-up with spherical mirrors situated transversely to the direction of propagation of the beam, with a high quality factor for the diffraction losses, is likely to be almost insensitive to parasitic TE or TM fields, while keeping a significant coupling to the direct quasi-TEM fields of the beam. A conceptual design of such a beam monitor for the third Compact Linear Collider (CLIC) test facility CTF3 [9] was reported in [10] and [11]. Its purpose is to monitor the evolution of the 12 GHz beam harmonic during the interleaving of 3 GHz bunch trains and the subsequent multiplication of the beam frequency by a factor four in the CTF3 Combiner Ring. A comparison of the strictly confocal and nearly confocal configurations was performed, and the impact of a small coupling iris between one mirror and a signal extraction waveguide was discussed.

In this paper, we perform a complete analytical, numerical and experimental investigation of a nearly confocal resonator pick-up with spherical mirrors, designed for microwaves with a frequency of 12 GHz, with special emphasis on the coupling between external parasitic modes and the field in the nearly confocal resonator. In Section 2, we describe the geometry of this resonator and estimate its quality factors. We then report on numerical and experimental studies of the coupling of external TE and TM modes to such a device in Sections 3 and 4. Finally, conclusions are given in Section 5.

2 Analytical determination of the geometry and of the quality factors

A schematic layout of an open resonator with spherical mirrors is shown in Figure 1, together with the parameters that are relevant for its design.

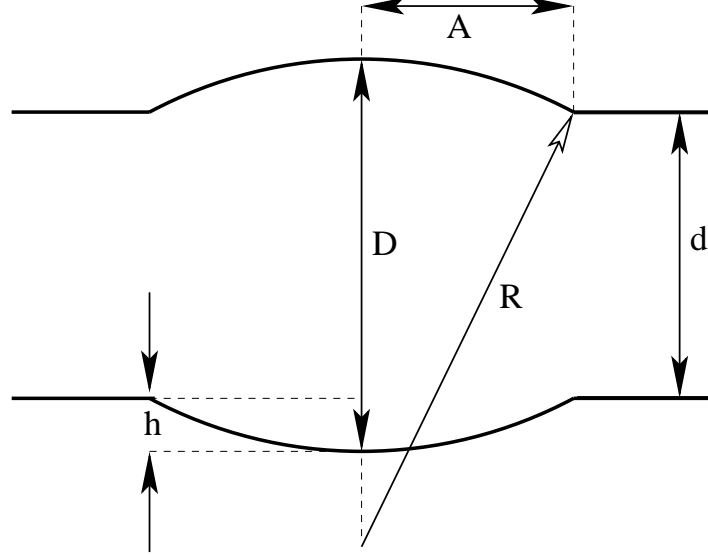


Figure 1: Schematic layout of an open resonator with spherical mirrors, inserted into a (rectangular) pipe.

In cylindrical coordinates, the paraxial solution of the wave equation between the spherical mirrors of an open resonator is described by Gaussian beams modulated with associated Laguerre polynomials L_n^m . When solving the wave equation with vanishing fields on the mirrors, the following resonance condition is obtained:

$$f = \frac{c}{2D} \left[q + 1 + \frac{1}{\pi} (1 + m + 2n) \arccos \left(1 - \frac{D}{R} \right) \right]. \quad (1)$$

Here, q is the number of nodes between the spherical mirrors, and (m, n) are the indices of the associated Laguerre polynomials L_n^m .

The relevant parameter to describe the diffraction losses due to the finite size of the spherical mirrors is the Fresnel number:

$$N_F = \frac{A^2}{D\lambda} \times \sqrt{\frac{2D}{R} - \left(\frac{D}{R} \right)^2}. \quad (2)$$

The fractional power loss per bounce at each mirror due to the diffraction losses can be written as follows:

$$\alpha_d = \frac{2\pi (8\pi N_F)^{1+m+2n} e^{-4\pi N_F}}{(m+n)!n!}. \quad (3)$$

The quality factor Q_d corresponding to the diffraction losses is then given by:

$$Q_d = \frac{2\pi D}{\alpha_d \lambda}. \quad (4)$$

For a given resonator geometry (and thereby a given Fresnel number), the smallest diffraction losses occur for the fundamental mode, which has $m = n = 0$.

The resistive losses, which are due to the finite conductivity at the mirror surfaces, are determined by the surface resistivity of the mirror $R_s = 1/\sigma\delta$ (where σ is the conductivity and δ is the skin depth) and by a geometry factor G , which is the same for all eigen-modes in the resonator:

$$G = Z_0 \times \frac{\pi}{2} \times \frac{D}{\lambda}, \quad (5)$$

where $Z_0 = 377 \Omega$ is the impedance of free space.

The quality factor Q_r associated to the resistive losses is simply given by:

$$Q_r = \frac{G}{R_s}. \quad (6)$$

In a purely confocal resonator ($R = D$), the resonance condition becomes:

$$f = \frac{c}{4D} \times (3 + m + 2n + 2q). \quad (7)$$

Several combinations of the m , n and q indices therefore lead to the same resonant frequency. Since a purely confocal resonator is over-moded and was found to be significantly unstable, a nearly confocal configuration is considered instead. Although it may lead to somewhat larger diffraction losses, it nevertheless allows to solve some problems that one faces with a strictly confocal resonator pick-up [11].

In this study, we use a mirror distance $D = 6.78$ cm and a curvature radius $R = 8.69$ cm. This ensures that the only eigen-mode at 12 GHz has $m = n = 0$ and $q = 4$. The vertical dimension of the pipe is $d = 3.7$ cm. The elevation of the zenith of the mirror domes above their edges and the mirror radius are $h = 1.54$ cm and $A = 4.94$ cm.

For the mode with $m = n = 0$, the quality factor associated to the diffraction losses is $Q_d = 3.6 \times 10^6$. As for the resistive losses at the surface of the Aluminium mirrors, we expect $Q_r = 4.1 \times 10^4$, which is two orders of magnitude smaller than the quality factor of the diffraction losses. In turn, reciprocity suggests that it should ensure a good reduction of the coupling to parasitic modes propagating in the pipe.

3 Numerical simulations

The CST Microwave Studio [12] simulation package was used to perform numerical studies of the electromagnetic properties of the nearly confocal resonator. We consider a structure that consists of two thick plates facing each other and separated by 3.7 cm, from which a spherical cavity, with a curvature radius of 8.69 cm and a depth of 1.54 cm, is carved out. A 1.905 cm \times 0.953 cm waveguide is connected to the upper mirror, through the plate.

3.1 Reflection coefficient at the waveguide

A driven method was used in order to search for the eigen-modes of the nearly confocal resonator, at and around the frequency of interest. Figure 2 shows how the reflection coefficient S_{11} at the waveguide varies with frequency between 11 and 13 GHz (in order to allow a fine meshing of the whole structure, resistive losses are not included and open boundaries are used for the lateral sides, between the plates). When there is no resonance, the microwave signals injected through the waveguide do not oscillate between the spherical mirrors and mostly escape through the open boundaries, so S_{11} remains rather small. On the other hand, when the resonance condition is satisfied, the diffraction losses become very small and the eigen-modes are trapped inside the nearly confocal resonator (provided that m and thereby $m + 2n$ are even numbers in order to avoid a zero-field at the centre of the upper mirror, which would prevent from coupling to the field propagating through the waveguide). As a significant fraction of the power oscillating between the mirrors couples back to the waveguide through the large hole in the upper mirror, S_{11} becomes much closer to 0 dB. This is clearly the case at 12 GHz.

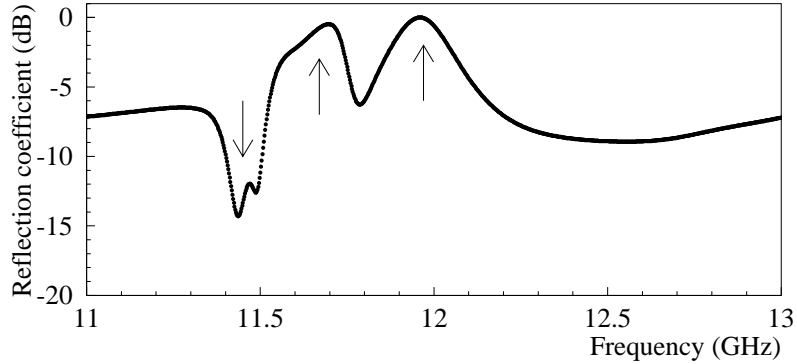


Figure 2: S_{11} coefficient as a function of frequency between 11 and 13 GHz when the input port is the waveguide connected to the upper mirror. The various resonances are identified with arrows.

In the frequency range of Figure 2, one expects another resonance, with both an even value of m (which ensures a non-zero field at the centre of the upper mirror) and small diffraction losses. This occurs at 11.7 GHz, i.e. for $q = 3$ and $m + 2n = 2$, and it is clearly

visible in Figure 2 (for the two associated modes, Q_d is of the order of 10^3). Another resonance condition is satisfied at 11.4 GHz, with $q = 2$ and $m + 2n = 4$, however the three modes have large diffraction losses ($Q_d < 30$), therefore one finds a dip instead of a peak in the S_{11} spectrum.

Figure 3 shows the E-field in the nearly confocal resonator at 12 GHz. The observed pattern agrees very well with the theoretical expectations for the mode with $m = n = 0$ and $q = 4$: there are four nodes between the mirrors and the field has a Gaussian dependence on the radial distance r .

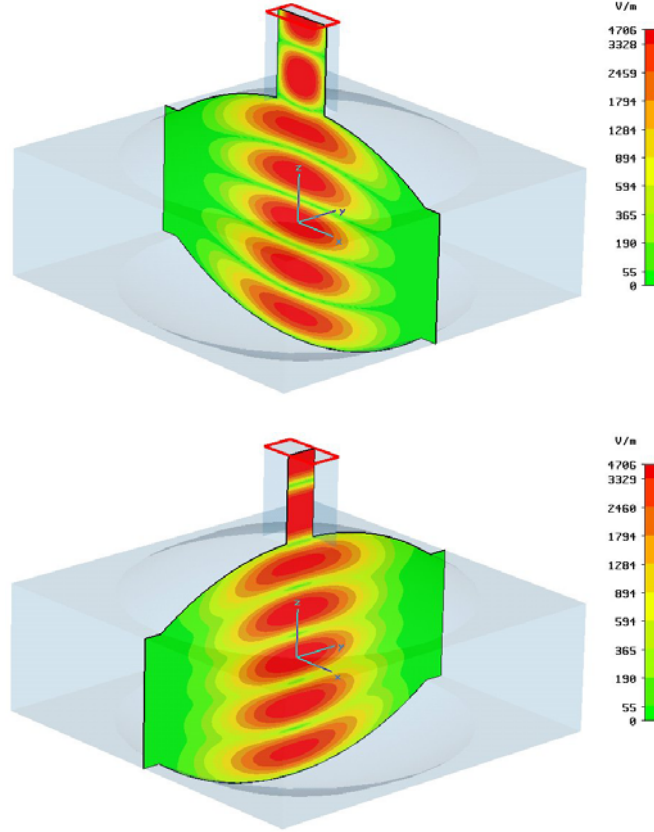


Figure 3: Eigen-mode found by CST Microwave Studio in the nearly confocal resonator at 12 GHz when a TE_{10} mode is injected through the waveguide.

3.2 Transmission coefficient for the incoming modes

Having successfully shown that the eigen-mode oscillating at 12 GHz inside the nearly confocal resonator indeed has a large quality factor for the diffraction losses, the second step of our simulation studies is to demonstrate that the modes propagating inside a pipe do not couple to the nearly confocal resonator at 12 GHz, as suggested by reciprocity.

The nearly confocal resonator is installed on a rectangular pipe with a width of 11.0 cm and a height of 3.7 cm. The number of modes that can propagate through the pipe with a cut-off frequency below 12.1 GHz is 38 (24 TE modes and 14 TM modes). In the waveguide used for signal extraction, only the TE_{10} mode can propagate. As a result, one must compute 38 transmission coefficients around 12 GHz in order to fully characterize the coupling of the external TE and TM modes to the nearly confocal resonator.

Around the frequency of interest, the S_{21} spectrum was first computed with a simple structure consisting of the rectangular pipe with the signal extraction waveguide only (i.e. without the carved spherical mirrors of the nearly confocal resonator). For some of the propagating TE and TM modes, the transmission coefficient S_{21} remains very small (-40 dB or less), because the field structure does not allow significant coupling to the TE_{10} mode of the extraction waveguide. In that case, it is not relevant to compute the S_{21} spectrum in the presence of the nearly confocal resonator. For all TE and TM modes that are likely to have a reasonable coupling to the extraction waveguide, the presence of the two carved spherical mirrors in the upper or lower sides of the pipe leads to a dramatic decrease of S_{21} by more than 40 dB around the resonant frequency of the nearly confocal resonator. This is illustrated in Figure 4 for the TE_{10} mode of the rectangular pipe. A similar behaviour is observed for all other incoming TE and TM modes. We have thus successfully demonstrated the ability of the nearly confocal resonator to reduce the coupling to external modes around the frequency of interest.

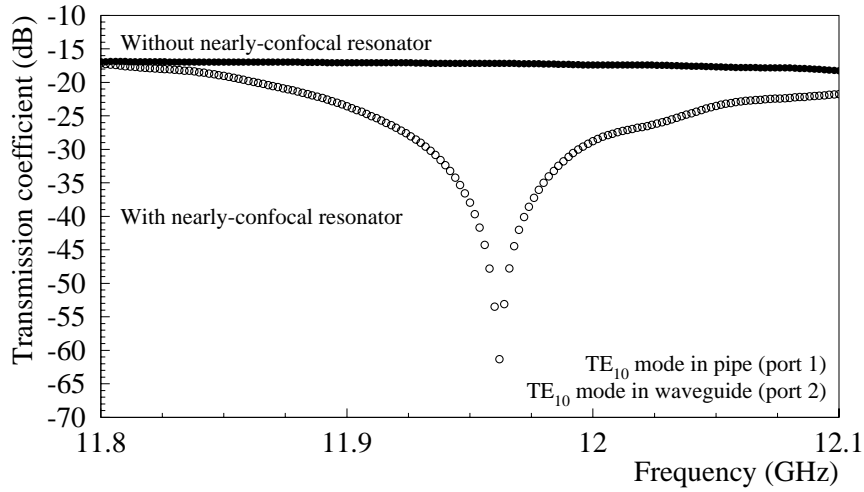


Figure 4: S_{21} coefficient as a function of frequency between 11.8 and 12.1 GHz when a TE_{10} mode is injected at port 1 (one end of the rectangular pipe) and a TE_{10} mode is extracted from port 2 (the waveguide).

4 Experimental results

In order to experimentally test the nearly confocal resonator and to check the validity of our simulations, a simple prototype was built, with the same dimensions as in the previous section. The length of the pipe is 30 cm. A 20 cm long waveguide, with transverse dimensions $1.905\text{ cm} \times 0.953\text{ cm}$, is connected to the upper mirror through the plate, see Figure 5.



Figure 5: Nearly confocal resonator prototype inserted onto a rectangular Aluminium pipe, with its extraction waveguide. The horn antenna used to inject external modes into the nearly confocal resonator through the pipe is shown in the upper picture. Also, one can clearly see the two carved spherical mirrors in the lower picture.

Microwave signals were generated and injected into the waveguide using a network analyzer (Agilent Technologies E8364B PNA series). The S_{11} spectrum of the nearly confocal resonator prototype was measured in the frequency range from 11.5 to 12.5 GHz. The (small) losses in the coaxial transition were also measured and then corrected for off-line, so that port 1 is indeed the waveguide.

Figure 6 compares the measured S_{11} spectrum to the results of our simulations. Despite the absence of resistive losses in the simulations, a good agreement is obtained. Note that the simulated S_{11} spectrum differs slightly from the one shown in Figure 2, because of the presence of the Aluminium pipe, which affects the electromagnetic field that leaks out of the nearly confocal resonator.

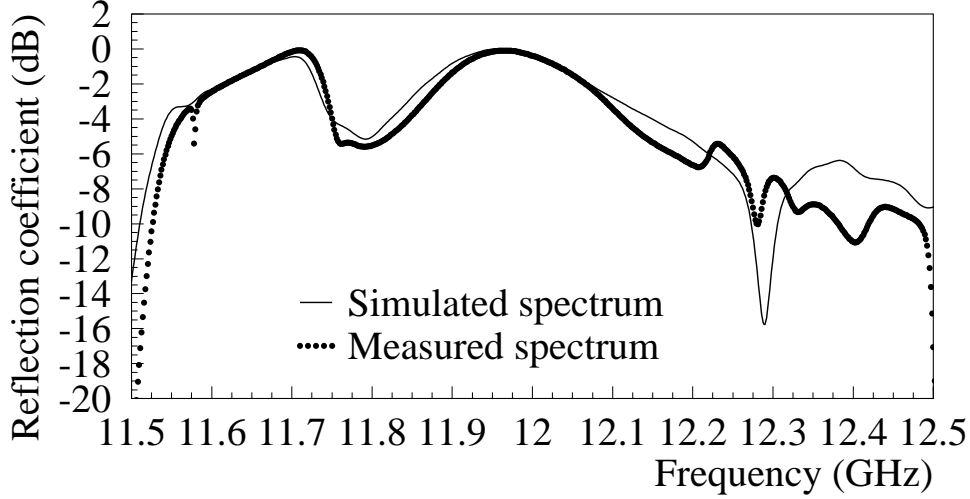


Figure 6: Comparison between the simulated and measured S_{11} spectrum of the nearly confocal resonator (port 1 is the waveguide).

Transmission measurements were performed as well, by sending microwave signals into the Aluminium pipe with a horn at one of its ends and by measuring the signal in the waveguide. The losses in the waveguide, the horn and the coaxial transitions were corrected for. Figure 7 shows the S_{21} spectrum, where port 1 is the horn output and port 2 is the junction between the upper mirror of the nearly confocal resonator and the waveguide. As expected, a dramatic decrease of S_{21} is observed at the resonant frequency of the nearly confocal resonator. The electromagnetic field coming out from the horn is mostly a TE_{10} mode, but some other modes may be present as well, especially if the horn is tilted. However, even in that case, a clear reduction of the coupling to external fields by the nearly confocal resonator is observed (only the level of the off-resonance coupling is affected by the orientation of the horn with respect to the pipe axis).

Note the presence of a second dip at 11.7 GHz in the S_{21} spectrum, which corresponds to the resonance that has $q = 3$ and $m + 2n = 2$, though with a smaller quality factor for the diffraction losses than at 12 GHz, and thereby a larger coupling to the external modes.

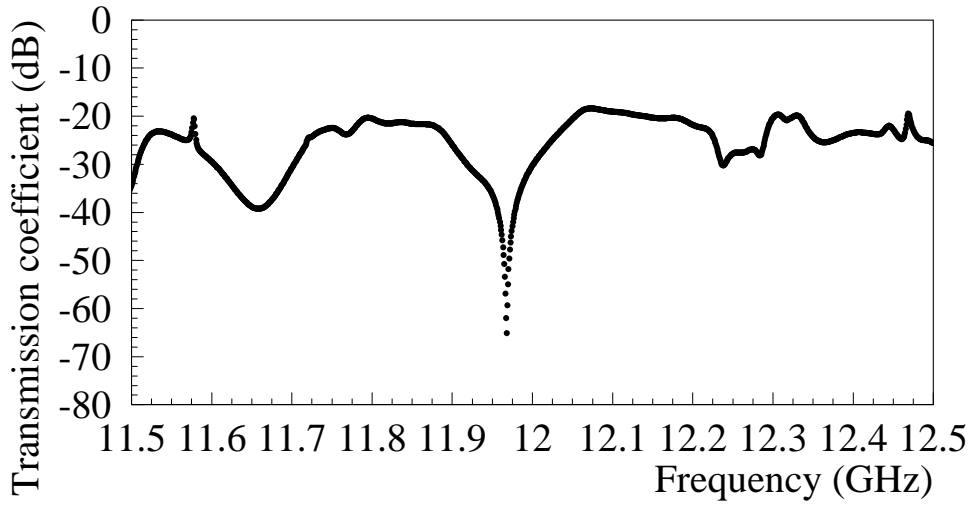


Figure 7: S_{21} coefficient as a function of frequency between 11.5 and 12.5 GHz: port 1 is the horn output at one end of the Aluminium pipe and port 2 is the extraction waveguide.

5 Conclusion

We have studied the electromagnetic properties of a nearly confocal resonator with spherical mirrors inserted onto an Aluminium pipe, by means of both numerical simulations and experimental measurements. As a result of the large quality factor associated to the diffraction losses, a dramatic reduction of the coupling to external sources and modes by the resonator was clearly achieved. In turn, this should allow a significant improvement of the signal-to-noise ratio for diagnostic devices based on such a nearly confocal resonator configuration.

However, in-situ adjustments of the mirror spacing by remotely controlled actuators is likely to be mandatory for a proper operation of the device, as the dip around the resonant frequency is very narrow and as its position in the S_{21} spectrum depends strongly on the mirror distance. Also, damping material in the beam pipe, outside the resonator, can help reduce the influence of some parasitic TE and TM modes, but should meanwhile not affect the eigen-modes with a large Q_d value, since those are confined inside the resonator.

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