An Analytical Description of Longitudinal Phase Space Distortions in Magnetic Bunch Compressors

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Abstract

In this paper we consider a possible mechanism of strong distortions of longitudinal phase space due to collective effects in an electron bunch passing a magnetic bunch compressor. Analytical expressions are derived for the case of a linear compression. The main emphasis is put on analysis of coherent synchrotron radiation (CSR) effects.

1 Introduction

Magnetic bunch compressors are designed to obtain short electron bunches with a high peak current for linac-based short-wavelength FELs [1–5] and future linear colliders [5–7]. The basic principle of compression is very simple. A relativistic electron bunch accumulates energy chirp while passing RF accelerating structures off-crest and then gets longitudinally compressed due to an energy-dependent path length in the magnetic compressor (for instance, in a chicane). Since, however, electron bunches are very short and intensive, collective effects like coherent synchrotron radiation (CSR) [8] can seriously influence beam dynamics in compressors [9].

In the recent experiments with bunch compressors [10–12] the fragmentation of longitudinal phase space has been observed. The self-consistent simulations [13] of beam dynamics in the TESLA Test Facility (TTF) bunch compressor chicane (BCC), taking into account CSR effects, have also shown phase space fragmentation. It has been explained by strong enhancement of CSR effects due to the locally peaked (non-Gaussian) density distribution created during compression process because of RF nonlinearity (see also [14]).

It has been mentioned in [13] that another mechanism can be considered which is also relevant for the ideal linear RF modulation (or, even without modulation). Namely, high-frequency components of the beam current spectrum (higher than typical inverse pulse duration) cause energy modulations at the same frequencies due to wakefields. The energy modulation is converted into an induced density modulation while the beam is passing the bunch compressor. If the wakefields are strong enough, the induced modulation can be much larger than the initial one. In other words, the system can be treated as a high-gain klystron-like amplifier. The general tendency is that higher frequencies (to some extent) are going to get amplified stronger so that they may become much better pronounced in comparison with the case of undisturbed compression. Thus, the charge distribution and, more generally, the longitudinal phase space can be essentially modified.

In this paper we study such a mechanism analytically in linear approximation. Since it is difficult to measure (simulate) small high-frequency perturbations in the initial state of the beam, one cannot exactly predict its final state. Thus, our goal is to calculate (estimate) the gain as a function of frequency. If the gain is large then one may expect significant modifications of longitudinal phase space in the bunch compressor, and vice versa. In section 2 we study the dynamical aspect of the problem assuming linear energy chirp along the beam and the given amplitude of parasitic energy modulation at some frequency. In section 3 we consider the case when these energy perturbations are created due to wakefields upstream of bunch compressor and in section 4 we thoroughly study CSR in the bunch compressor chicane.

2 Compression of the beam with linear energy chirp and superimposed sinusoidal modulation

In this paper we consider 1-D model of the electron beam neglecting all the transverse effects. An undisturbed phase space distribution of the beam with dc current, linear energy chirp along the beam and Gaussian energy spread can be described with the following function:

$$f(s, \delta\gamma) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma} \exp\left[-\frac{(\delta\gamma - h\gamma_0 s)^2}{2\sigma_\gamma^2}\right] , \qquad (1)$$

where s is the coordinate along the beam (particles with positive values of s are placed behind the particle with $s = 0$), $\gamma_0 = \mathcal{E}_0/(mc^2)$ is the nominal energy in units of the rest energy, m is electron's mass, c is the velocity of light, $\delta\gamma = (\mathcal{E} - \mathcal{E}_0)/(mc^2)$ is the energy deviation from the nominal value, $\sigma_\gamma = \sigma_\mathcal{E}/(mc^2)$ is the rms local energy spread, $h = \mathrm{d}(\delta\gamma)/(\gamma_0 \, \mathrm{d}\, s)$ describes linear energy chirp along the beam, I_0 is the beam current. Normalization is chosen in such a way that after integration over $\delta\gamma$ we get the current. We assume γ_0 to be large and consider small energy deviations $\delta\gamma \ll \gamma_0$ although formally we let $\delta\gamma$ extend from $-\infty$ to ∞. The model of dc current allows us to exclude edge effects from consideration and to deal with small sinusoidal modulations.

To describe phase space transformation in the bunch compressor we assume a linear dependence of path length S in the compressor on $\delta\gamma/\gamma_0$ described by compaction factor [1]

$$R_{56} = \frac{\mathrm{d}\,S}{\mathrm{d}(\delta\gamma/\gamma_0)} = \frac{\mathrm{d}\,s}{\mathrm{d}(\delta\gamma/\gamma_0)} \ .$$

Then a particle position in the beam before and after compression, s_{i} and s_{f}, are connected by

$$s_{\mathrm{f}} = s_{\mathrm{i}} + R_{56}\frac{\delta\gamma}{\gamma_0} \ .$$

Therefore, to describe the final state of the beam we should substitute s in (1) by $s - R_{56}\delta\gamma/\gamma_0$. Then the new distribution will have the form of (1) where I, σ_γ, and h are substituted by CI, $C\sigma_\gamma$, and Ch, respectively. Here C is the compression factor:

$$C = \frac{1}{1 + hR_{56}} \ .$$

For compression one should provide $hR_{56} < 0$. For instance, $R_{56} < 0$ for the chicane so that h has to be positive in this case. In addition, in this paper we restrict our consideration by the condition $1 + hR_{56} > 0$, i.e. the beam is undercompressed.

Now let us consider an energy modulation at some frequency ω on top of the linear chirp. In front of the bunch compressor the phase space distribution has the form:

$$f(s,\delta\gamma) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma}\exp\left\{-\frac{[\delta\gamma - h\gamma_0 s + \Delta\gamma\sin(ks)]^2}{2\sigma_\gamma^2}\right\} \ ,$$

where $k = \omega/c$ and $\Delta\gamma$ is the amplitude of energy modulation. As it was done above, we substitute s by $s - R_{56}\delta\gamma/\gamma_0$ to describe the change of distribution function in the bunch compressor. Then we integrate over $\delta\gamma$ in order to get current as a function of s:

$$I(s) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma}\int\limits_{-\infty}^{\infty}\mathrm{d}\,\delta\gamma\exp\left\{-\frac{[\delta\gamma(1 + hR_{56}) - h\gamma_0 s + \Delta\gamma\sin(ks - kR_{56}\delta\gamma/\gamma_0)]^2}{2\sigma_\gamma^2}\right\}$$

[1] Compaction factor is generally defined for momentum deviations $\delta p/p_0$. For considered here ultrarelativistic case $\delta p/p_0 = \delta\gamma/\gamma_0$.

After change of variables $x = [\delta\gamma(1 + hR_{56}) - h\gamma_0 s]$ the integral takes the following form:

$$I(s) = \frac{CI_0}{\sqrt{2\pi}\sigma_\gamma} \int\limits_{-\infty}^{\infty} \mathrm{d}\, x \exp\left\{ -\frac{[x + \Delta\gamma \sin(Cks - CkR_{56}x/\gamma_0)]^2}{2\sigma_\gamma^2} \right\}$$

The integral of such a form is known to describe the process of density bunching starting from initial sinusoidal energy modulation but without linear energy chirp (see, for instance, [15]). Making integration and Fourier expansion, one gets:

$$I(s) = CI_0 \left[1 + 2\sum_{n=1}^{\infty} J_n \left(nCkR_{56}\frac{\Delta\gamma}{\gamma_0} \right) \exp\left(-\frac{1}{2} n^2 C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2} \right) \cos(nCks) \right] . (2)$$

Here J_n is the Bessel function of nth order. Without compression ($h = 0, C = 1$) the expression (2) is reduced to the well-known one [15].

Analyzing (2) we see that the frequency range (of initial modulation), in which the beam can be effectively bunched, is limited by $k \leq (CR_{56}\sigma_\gamma/\gamma_0)^{-1}$. Within this range the condition $Ck|R_{56}|\Delta\gamma/\gamma_0 \geq 1$ means that the beam is completely bunched and the phase space is fragmented. For $k \simeq (CR_{56}\sigma_\gamma/\gamma_0)^{-1}$ this happens when $\Delta\gamma \geq \sigma_\gamma$. It is worth mentioning that σ_γ always stands for the initial energy spread (before compression).

In this paper we will use linear approximation assuming that $Ck|R_{56}|\Delta\gamma/\gamma_0 \ll 1$. This leaves us with only the first harmonic of the beam current ($J_1(X) \simeq X/2$):

$$I(s) \simeq CI_0 \left[1 + \rho_{\mathrm{ind}}\, \mathrm{sgn}(\mathrm{R_{56}}) \cos(\mathrm{Cks}) \right] \ , \qquad (3)$$

where $\mathrm{sgn}(\mathrm{R_{56}})$ is the sign of R_{56} and ρ_{ind} is the amplitude of the first harmonic in the final state of the beam:

$$\rho_{\mathrm{ind}} = Ck|R_{56}|\frac{\Delta\gamma}{\gamma_0} \exp\left(-\frac{1}{2} C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2} \right) \ . \qquad (4)$$

We have considered here the model of infinitely long beam. The results of this paper can be used for a bunch with finite length σ as soon as the following condition is satisfied:

$$k\sigma \gg 1 \ . \qquad (5)$$

The influence of beam emittance ϵ on longitudinal dynamics is negligible when

$$k\epsilon S_{\mathrm{c}}/\beta \ll 1 \ , \qquad (6)$$

where S_{c} is the length of a path through compressor and β is the beta-function.

3 Wakefields upstream of a bunch compressor

Let us assume that upstream of the bunch compressor there is a small density perturbation ρ_{i} at some frequency:

$$I(s) = I_0 \left[1 + \rho_{\mathrm{i}} \cos(ks)\right] \ . \tag{7}$$

Due to some wakefields upstream of compressor the beam gets modulated in energy at the same frequency with the amplitude $\Delta\gamma$. Describing the action of wakefields by longitudinal impedance $Z(k)$ we can connect the amplitudes of energy and density modulations as follows:

$$\Delta\gamma = \frac{|Z(k)|}{Z_0} \frac{I_0}{I_{\mathrm{A}}} \rho_{\mathrm{i}} \ , \tag{8}$$

where $Z_0 = 377\ \Omega$ is the free-space impedance and $I_{\mathrm{A}} = 17$ kA is the Alfven current. Then, using (4) we calculate the amplitude of the induced density modulation at the end of bunch compressor. In general case, to find final density modulation ρ_{f} one should sum up induced modulation and (transformed to the end of compressor) initial one, taking care of phase relations. But in this paper we use approximation

$$\rho_{\mathrm{i}} \ll \rho_{\mathrm{ind}} \ll 1 \ .$$

In other words, $\rho_{\mathrm{f}} \simeq \rho_{\mathrm{ind}}$ and the gain in density modulation

$$G = \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{i}}} \simeq \frac{\rho_{\mathrm{ind}}}{\rho_{\mathrm{i}}}$$

is assumed to be high, $G \gg 1$ (otherwise the effect, considered in this paper, is not of great importance). Under this approximation the gain depends neither on phase of $Z(k)$ nor on sign of R_{56} and is equal to

$$G = Ck|R_{56}| \frac{I_0}{\gamma_0 I_{\mathrm{A}}} \frac{|Z(k)|}{Z_0} \exp\left(-\frac{1}{2} C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2}\right) \tag{9}$$

For broadband nonresonant wakefields the product $k|Z(k)|$ is usually a growing function of k. For such cases the maximal gain is achieved at

$$k_{\mathrm{opt}}^{-1} \simeq \frac{\sigma_\gamma}{\gamma_0} |R_{56}| C \ . \tag{10}$$

The optimal final frequency (when the beam is compressed) roughly does not depend on compression factor C. A crude estimate for the maximal gain is

$$G_{\max} \simeq \frac{I_0}{\sigma_\gamma I_{\mathrm{A}}} \frac{|Z(k_{\mathrm{opt}})|}{Z_0} \ . \tag{11}$$

The term $I_0/(\sigma_\gamma I_\mathrm{A})$ is proportional to a longitudinal brightness (particles density in longitudinal phase space). In practice the phase space distribution can be of complex shape. We note that the local energy spread should be taken for estimations of amplification effect.

4 CSR in the bunch compressor chicane

Wakefields can also exist inside bunch compressors. We consider here coherent synchrotron radiation which is an intrinsic feature of magnetic compressors. CSR effects can be minimized there but not avoided. Recently, CSR-induced beam instability in storage rings has been investigated [16]. That instability develops continuously, in small increments, like most instabilities of relativistic electron beams. We analyze here quite different situation when the longitudinal phase space can be suddenly modified while the electron beam is passing a single element of the beamline. In this sense one can think of such an analogy as a klystron versus a travelling-wave tube.

While the formulae of the previous section are pretty general and do not depend on a type of the bunch compressor, in this section we have to choose a specific model. We consider a simmetric three-dipole chicane [2] where the first and the last dipoles have the length L_d, and the middle one is as long as $2L_\mathrm{d}$. The bending angle in the first dipole θ is small, $\theta = L_\mathrm{d}/R \ll 1$ (R is the bending radius), and the total length L_c of BCC satisfies the condition

$$L_\mathrm{c} \gg 4L_\mathrm{d} \, . \tag{12}$$

The compaction factor can then be expressed in a simple form: $R_{56} = -L_\mathrm{c}\theta^2$. To describe CSR we use the steady-state model neglecting edge effects. The domain of validity of this model can be estimated on the base of results obtained in [17]:

$$L_\mathrm{d} \gg (R^2/k)^{1/3} \ln(\gamma_0^3/kR) \, , \tag{13}$$

assuming that inequality $\gamma_0^3/kR \gg 1$ always holds. We neglect the influence on CSR of transverse beam size and of the screening effect of the vacuum chamber requiring that [9,18]

$$(b^3/R)^{1/2} \ll k \ll (\sigma_\perp^3/R)^{1/2} \, , \tag{14}$$

where b is the transverse size of vacuum chamber and $\sigma_\perp$ is that of electron beam. Under the conditions (13) and (14) the module of CSR impedance in the first and

[2] Under limitations, accepted in this section, all the results are valid for a four-dipole chicane, too.

the last dipoles can be expressed as [18]

$$\frac{|Z(k)|}{Z_0} = \frac{2\Gamma(2/3)}{3^{1/3}} \frac{L_{\mathrm{d}} k^{1/3}}{R^{2/3}} , \tag{15}$$

and in the middle dipole it is two times larger. Here $\Gamma(...)$ is the complete gamma-function.

The accepted model allows us to simplify calculation of the gain. Indeed, under the condition (12) we neglect longitudinal motion inside the dipoles so that the density bunching happens only between dipoles. On the other hand, based on (13) we neglect CSR-induced energy modulation outside the dipoles. Thus, we separate these two processes.

In the framework of this model we consider the two-stage amplification in the bunch compressor. Indeed, initial density perturbation (7) causes energy modulation according to (8), (15). Then we get induced density modulation at the entrance to the middle dipole. We assume it to be much larger than the initial one. Then partially compressed beam (compaction factor is equal to $R_{56}/2$) in the middle dipole gets again modulated in energy at a new frequency:

$$k_{\mathrm{middle}} = \left(1 + \frac{hR_{56}}{2}\right)^{-1} k = \frac{2Ck}{C+1} ,$$

where C is the total compression factor of BCC. As a result, we have an induced density modulation at a final frequency Ck in the end of BCC, which is again assumed to be much larger than the modulation in the middle dipole. So, we assume the gain in each stage to be large and neglect phase relations as we did in the previous section.

The calculation of the gain in each stage is similar to that presented in the previous section. The main difference is that now the energy modulation linearly increases inside dipoles. Therefore, a particle with a given energy deviation (at the end of the dipole) gets two times smaller angular kick in comparison with the case when this energy deviation exists in front of the dipole. So, the gain is reduced by a factor of two in each stage. Leaving out the details of calculation, we present here the final result for the BCC gain in density modulation:

$$G\left(\frac{k}{k_{\mathrm{opt}}}\right) = G_{\mathrm{max}} \left(\frac{k}{k_{\mathrm{opt}}}\right)^{8/3} \exp\left\{-\frac{4}{3}\left[\left(\frac{k}{k_{\mathrm{opt}}}\right)^2 - 1\right]\right\} , \tag{16}$$

where the optimal frequency is given by

$$k_{\mathrm{opt}}^{-1} = \sqrt{\frac{3}{8}} |R_{56}| \frac{C(C^2+1)^{1/2}}{C+1} \frac{\sigma_\gamma}{\gamma_0} , \tag{17}$$

and the maximal gain is

$$G_{\max} = a g_0^2 f_1(C) \ . \tag{18}$$

Here a is numerical constant (e is the base of natural logarithm):

$$a = 2^4 \left(\frac{2}{\mathrm{e}^4}\right)^{1/3} \left(\frac{\Gamma(2/3)}{3}\right)^2 \simeq 1.08 \ ,$$

g_0 is the gain parameter:

$$g_0 = \frac{I_0}{\sigma_\gamma I_\mathrm{A}} \left(\frac{\gamma_0}{\sigma_\gamma}\right)^{1/3} \frac{L_\mathrm{d}}{(R^2 |R_{56}|)^{1/3}} \ , \tag{19}$$

and the function of compression factor, $f_1(C)$, has the form:

$$f_1(C) = \frac{2C^{2/3}(C+1)^{1/3}}{(C^2+1)^{4/3}} \ . \tag{20}$$

Since energy modulations are induced inside bunch compressor, one can also calculate the gain in energy modulation. If there is a small perturbation with amplitude $\Delta\gamma$ in front of BCC, it is converted into the density modulation at the entrance to the middle dipole. There CSR induces energy modulation which is assumed to be much larger than the initial one. Then the beam gets further bunched in density while moving to the entrance of the last dipole. In that dipole the energy modulation is induced (much larger than that in the middle dipole). The gain, defined as a ratio between final and initial amplitudes of energy modulation, is then given by (16) with k_opt given by (17). The maximal gain differs from (18) and can be written as

$$G_{\max} = 2a g_0^2 f_2(C) \ , \tag{21}$$

where

$$f_2(C) = \frac{2C^2(C+1)^{1/3}}{(C^2+1)^{4/3}} \ . \tag{22}$$

Let us comment on the behaviour of functions $f_1(C)$ and $f_2(C)$. As we have already mentioned, the final optimal frequency is almost independent of compression factor. The initial optimal frequency is lower when the compression factor is larger, i.e. CSR is weaker in the first dipole. In addition, the bunching process is less effective for lower frequencies. This explains why $f_1(C)$ quickly decreases. The counteracting process is the growth of the beam current during compression. In the last dipole CSR effects are stronger for larger compression factor. This is important for the gain in energy modulation. As a result, function f_2 hardly depends on C in practically interesting region. It is worth mentioning that final density (energy) modulations are defined by the gain and by initial modulations. When C is larger, the gain curve is shifted towards lower frequencies where the components of the beam spectrum are larger, in general. Therefore, one may expect stronger perturbations of the longitudinal phase space in the final state of the beam when compression factor is larger. For the same reason (larger initial modulations) the strongest effect may be observed not at the optimal frequency but at lower frequencies.

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