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Generalized Parton Distributions I

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I give a brief introduction to generalized parton distributions and to the phenomenology of the exclusive processes where they can be measured.

1. INTRODUCTION

Much progress has been made in the theory of generalized parton distributions (GPDs) since their introduction some years ago [1–4]. Unifying the concepts of parton distributions and of hadronic form factors, they contain a wealth of information about how quarks and gluons make up hadrons. Advances in experimental technology raise hope to study the exclusive processes where these functions appear, and the first results from HERA and JLAB are encouraging [5].

A good way to highlight the similarities and differences between usual parton densities and their generalization is to look at the Compton amplitude. Via the optical theorem, the cross section for inclusive deep inelastic scattering can be obtained from the imaginary part of the *forward* amplitude $\gamma^* p \to \gamma^* p$. In the Bjorken region of large photon virtuality Q^2 and collision energy, this amplitude factorizes into a parton distribution and a perturbatively calculable scattering process at the level of quarks and gluons. The simplest diagram for this is shown in Fig. 1a. The amplitude for deeply virtual Compton scattering $\gamma^* p \to \gamma p$, a completely exclusive process, factorizes in an analogous way if in addition to the Bjorken limit we require a small invariant momentum transfer t to the proton. Since the two proton momenta in the diagram of Fig. 1a are now different, the non-perturbative dynamics is not described by ordinary parton distributions, but by quantities which generalize them. In addition, the finite momentum transfer to the proton makes a second space-time structure of the process possible. Whereas in Fig. 1a the partonic subprocess is the scattering of a photon on a quark or antiquark, the virtual photon can also annihilate a quark-antiquark pair with transverse separation of order $1/Q$ in the proton target, as shown in Fig. 1b.

2. SOME PHYSICS ASPECTS

2.1. Parton distributions and wave functions

The key difference between the usual parton distributions and their generalizations is seen by representing them in terms of the quark and gluon light-cone wave functions of the hadron [6,7]. The usual parton distributions are obtained from the squared wave functions for all partonic configurations containing a parton with specified polarization and

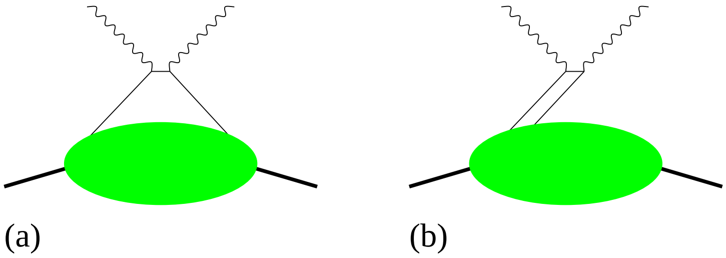

Figure 1. Feynman diagrams for the Compton amplitude in the regime where it factorizes into a parton distribution and a hard partonic subprocess: (a) quark-photon scattering, (b) annihilation of a quark-antiquark pair.

longitudinal momentum fraction x in the fast moving hadron (Fig. 2a). This represents the *probability* for finding such a parton. In contrast, GPDs represent the *interference* of different wave functions, one where a parton has momentum fraction $x + \xi$ and one where this fraction is $x - \xi$ (Fig. 2b). GPDs thus correlate different parton configurations in the hadron at the quantum mechanical level. There is also a kinematical regime where the initial hadron emits a quark-antiquark or gluon pair (Fig. 2c). This has no counterpart in the usual parton distributions and, as we already mentioned, carries information about $q\bar{q}$ or gg-components of small transverse separation in the hadron wave function. It is reminiscent of the $q\bar{q}$ or gg distribution amplitudes for a meson, i.e., of the $q\bar{q}$ or gg light-cone wave functions integrated over the transverse parton momentum.

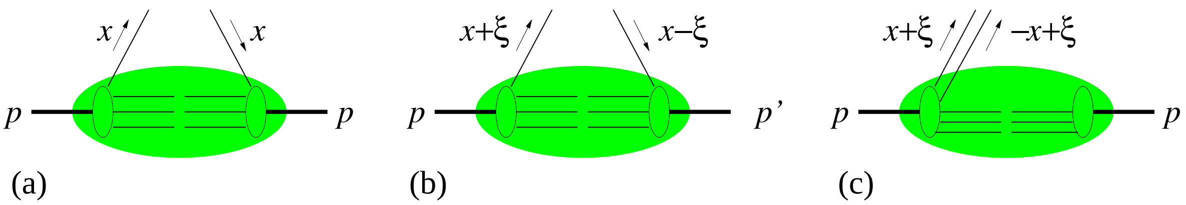

Figure 2. (a) Usual parton distribution, representing the probability to find a parton with momentum fraction x in the nucleon. All configurations of the spectator partons are summed over. (b) GPD in the region where it represents the emission of a parton with momentum fraction $x+\xi$ and its reabsorption with momentum fraction $x-\xi$. (c) GPD in the region where it represents the emission of a parton pair. Here $x+\xi > 0$ and $x-\xi < 0$.

Apart from the momentum fraction variables x and ξ GPDs depend on the invariant momentum transfer t. This is an independent variable because the momenta p and p' may differ not only in their longitudinal components but also in their transverse ones, p_T and p'_T. GPDs thus contain information on how the transverse distribution of partons in a fast moving hadron is correlated with their longitudinal momentum fraction. The former is connected with t and the latter with ξ, which is related to the energy of the overall process, so that at small ξ this can also be viewed as a particular instance of the shrinkage phenomenon in diffractive processes. A perhaps more intuitive picture of the transverse structure can be obtained by Fourier transforming from $p_T - p'_T$ to impact parameter space [8].

2.2. Matrix elements

Generalized parton distributions are defined as Fourier transforms of matrix elements between different hadron states, such as

$$\int d\lambda\, e^{i\lambda x(p+p')\cdot n} \langle p(p',s')|\, \bar{\psi}(-\lambda n)\, \gamma\cdot n\, \psi(\lambda n)\, |p(p,s)\rangle. \tag{1}$$

The quark and antiquark operators are separated by a light-like distance λn (i.e., $n^2 = 0$). These matrix elements, represented by a blob in fig. 1, describe the transition between hadronic and partonic degrees of freedom. In the limit where the two states $|p(p,s)\rangle$ and $|p(p',s')\rangle$ become equal, one recovers the usual parton densities, which thus provide boundary values of GPDs.

On the other hand, taking moments of these distributions in the momentum fraction x gives the matrix elements of *local* currents, for instance of the vector current in (1). The moments of GPDs are thus given by elastic form factors, just as the moments of ordinary parton distributions give expectation values of operators, i.e., form factors at zero momentum transfer.

As in the case of usual parton distributions, the operators appearing in the definition of GPDs need to be renormalized. This induces a dependence on a factorization scale, which in a physical process provides the "borderline" between the hard subprocess explicitly calculated in perturbation theory and the long-distance physics described by the parton distribution. This dependence, which is perturbatively calculable, is described by evolution equations, which are of interest in themselves. It is in fact this aspect on which the earliest studies of GPDs have focused on [1]. In the regime of fig. 2b, the evolution is similar to the standard DGLAP evolution of parton densities, with a kernel that depends on the parameter ξ describing the longitudinal momentum transfer to the partons. In the regime of fig. 2c, evolution acts in a similar way as the ERBL evolution of meson distribution amplitudes [9]. The evolution equations for ordinary parton distributions and for distribution amplitudes, although acting in quite different ways, are thus intimately related, and the evolution of GPDs interpolates between these two extremes.

2.3. Spin and orbital angular momentum

GPDs have a rich structure in the polarization of both the hadrons and the partons. For quarks four different combinations contribute to the Compton amplitude. The functions H_q and E_q are summed over the quark helicity, and $\tilde{H}_q$ and $\tilde{E}_q$ involve the difference between right and left handed quarks. H_q and $\tilde{H}_q$ conserve the helicity of the proton, whereas E_q and $\tilde{E}_q$ allow for the possibility that the proton helicity is flipped. In that case the overall helicity is not conserved: the proton changes helicity but the quarks do not, so that angular momentum conservation has to be ensured by a transfer of *orbital* angular momentum. This is only possible for nonzero transverse momentum transfer, and therefore cannot be observed with ordinary parton distributions, where the momenta p and p' are equal. That GPDs deeply involve the orbital angular momentum of the partons is reflected in Ji's sum rule [2], which states that the second moment $\int dx\, x\, [H_q(x,\xi,t) + E_q(x,\xi,t)]$ is a form factor whose value at $t=0$ gives the *total* angular momentum carried by quarks, both its spin and orbital part.

3. MODELING GPDS

The wealth of information contained in GPDs comes with the challenge to handle functions of three kinematical variables x, ξ, and t. A good understanding of what this functional dependence may look like is important to formulate realistic models of these functions. It is also needed in order to devise physically motivated parameterizations for them, which will probably be the starting point in trying to extract information from data, just as is the case for the ordinary parton densities.

Efforts to model GPDs are proceeding along different lines. Many current parameterizations are based on the concept of double distributions [1,3], which may be seen as "generating functions" for the GPDs. They incorporate the nontrivial requirements on the x and ξ dependence imposed by Lorentz invariance, and present a convenient way to use the known parton densities as an input to the model. One can however not expect to obtain the physics of the $q\bar{q}$ or gg region by "extrapolating" information from the usual parton densities, so that such approaches are typically supplemented with terms that only have support in the region depicted in Fig. 2c. Examples are the Polyakov-Weiss D-term and contributions describing pion exchange in the t-channel [10].

Other approaches include models based on the representation of generalized distributions in terms of light-cone wave functions [11]. The possible structure of GPDs can also be explored in field theories such as QED [6] or two-dimensional QCD [12], which are simpler than full QCD. Finally, there have been attempts to calculate GPDs in approximations of QCD such as the MIT bag [13] and the chiral quark soliton model [14]. Let us remark that several of these approaches do *not* support a simple factorizing ansatz of the type $f(x,\xi,t) = g(x,\xi)F(t)$ for the GPDs. In the light of our discussion above, this reflects that the correlation between transverse and longitudinal structure in the proton in indeed nontrivial.

4. BASIC PHENOMENOLOGY

Apart from deeply virtual Compton scattering (DVCS), a large class of processes where GPDs occur is exclusive electroproduction of a meson M, shown in Fig. 3. In that case, a second non-perturbative ingredient is the meson distribution amplitude. While this makes the theoretical description more involved, meson production processes offer a way to select GPDs with different spin and flavor structure in the target, depending on the quantum numbers of M.

That GPDs occur in the description of the above processes at large Q^2 is the statement of factorization theorems [15]. They also predict the Q^2 behavior of the corresponding $\gamma^* p$ amplitudes at fixed x_B and t. For DVCS the amplitude then becomes independent of Q, whereas it falls like $1/Q$ for meson production, with corrections in $\log Q^2$ due to radiative corrections in both cases. The amplitudes are then proportional to convolutions

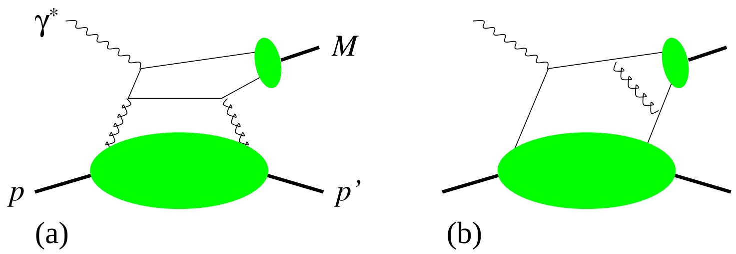

Figure 3. Feynman diagrams for exclusive production of a meson M at large photon virtuality Q^2.

of the GPDs with an expression for the hard scattering. To leading order in α_s this has the form

$$\int dx\, \frac{f(x,\xi,t)}{\xi - x - i\epsilon} \pm \{x \to -x\}, \tag{2}$$

both for DVCS and for meson production, where f stands generically for a GPD. Notice that due to Feynman's $i\epsilon$ prescription, this expression has both a real and an imaginary part. The GPDs themselves are real valued as a consequence of time reversal invariance.

A rather unique opportunity to access this dynamical phase is offered by DVCS. Measured in electroproduction $ep \to ep\gamma$, virtual Compton scattering interferes with the Bethe-Heitler process, where the final state photon is radiated off the lepton, cf. Fig. 4. Making use of this interference term with a completely calculable QED amplitude, one can obtain valuable phase information on the Compton process [16], which would greatly help to unravel the x and ξ dependence of the GPDs from convolutions as in (2).

The large Q^2 limit also selects the helicities of the photon and meson: the leading amplitudes involve a transverse γ^* in DVCS, and longitudinal polarization for both γ^* and M in meson production. In the case of DVCS, the structure and description of

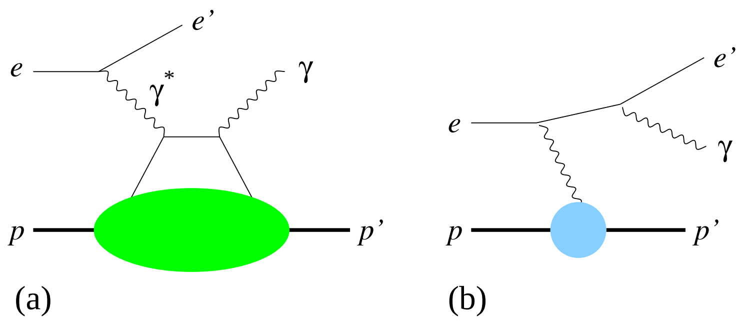

Figure 4. Diagrams for the (a) Compton and (b) the Bethe-Heitler processes, contributing to electroproduction $ep \to ep\gamma$.

the subleading terms going like $1/Q$ is now also known [17]. It turns out that, up to still higher corrections, these terms involve only the longitudinally polarized γ^*. Using angular distributions in the final state of the electroproduction process $ep \to ep\gamma$, one can separate the contributions from the different helicity transitions [16,18]. This means on one hand that in suitable observables the leading terms do not suffer potentially large $1/Q$ corrections, and on the other hand, that at least the first subleading terms in $1/Q$ can be studied for themselves, without a large background from leading contributions. Similar statements hold for meson electroproduction, $ep \to ep\, M$.

5. CONCLUSIONS

The basic properties and the basic physics potential of GPDs are rather well understood by now, and their appearance in the description of exclusive processes rests on solid ground. Progress has been made in understanding the possible shapes of GPDs, but more experience will be necessary in order for theory to keep track with the encouraging experimental developments in the field. The same is true for our understanding of the corrections in α_s and in $1/Q$ to the simple dynamical pictures of Figs. 1 and 3, which are discussed in [17].

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