On the Existence of Kink-(Soliton-)States

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Abstract

There are several two dimensional quantum field theory models which are equipped with different vacuum states. For example the Sine-Gordon- and the ϕ_2^4-model. It is known that in these models there are also states, called soliton- or kink-states, which interpolate different vacua. We consider the following question: Which are the properties a pair of vacuum sates must have, such that an interpolating kink-state can be constructed? Since we are interested in structural aspects and not in specific details of a given model, we are going to discuss this question in the framework of algebraic quantum field theory which includes, for example, the $P(\phi)_2$-models. We have shown that for a large class of vacuum states, including the vacua of the $P(\phi)_2$-models, there is a natural way to construct an interpolating kink-state.

Introduction

In quantum field theory there are several models in $1+1$ dimensional space-time which are equipped with different vacuum states. For example the Sine-Gordon-model, the ϕ_2^4-theory and the Skyrme-model in $1+1$ dimensions. Further candidates are special types of $P(\phi)_2$-models. It is known that in the Sine-Gordon- and ϕ^4_2-model there are also states which interpolates different vacuum states. These states are called soliton- or kink-states. An construction in the framework of algebraic quantum field theory was done by J. Fröhlich [14] in the 70s where he also discussed the existence of kink-states in general $P(\phi)_2$-models. However, these construction leads only to kink-states which interpolates vacua which are connected by an (special) internal symmetry transformation. Since there are also candidates for models with different vacua which can not be connected by a symmetry transformation, we can ask the following question:

If we consider any quantum field theory in $1+1$ dimensions which is equipped with different vacuum states. Which are the properties a pair of vacuum sates must have, such that an interpolating kink-state can be constructed?

Since we are interested in structural aspects and not in specific details of a given model, we are going to discuss this question in the framework of algebraic quantum field theory which includes, for example, the $P(\phi)_2$-models. For this purpose, let us describe the main aspects of algebraic quantum field theory in $1+1$ space-time dimensions.

A $1+1$ dimensional quantum field theory is given by a prescription which assigns to each region $\mathcal{O} \subset \mathbb{R}^2$ a C*-algebra $\mathfrak{A}(\mathcal{O})$ and the elements in $\mathfrak{A}(\mathcal{O})$ represent physical operations which are localized in $\mathcal{O}$. These prescription has to satisfy a list of axioms which are motivated by physical principles.

(1) A physical operation which is localized in a region $\mathcal{O}$ should also localized in each region which contains $\mathcal{O}$. Therefore, we require that if a region $\mathcal{O}_1$ is contained in a lager region $\mathcal{O}$, then the algebra $\mathfrak{A}(\mathcal{O}_1)$ is a sub-algebra of $\mathfrak{A}(\mathcal{O})$.

(2) Two local operations which take place in space-like separated regions should not influence each other. Hence the *principle of locality* is formulated as follows: If a region $\mathcal{O}_1$ is space-like separated from a region $\mathcal{O}$, then the elements of $\mathfrak{A}(\mathcal{O}_1)$ commute with those of $\mathfrak{A}(\mathcal{O})$.

(3) Each operation which is localized in $\mathcal{O}$ should have an equivalent counterpart which is localized in a translated region $\mathcal{O}+x$. The *principle of translation covariance* is described by the existence of a two-parameter automorphism group $\{\alpha_x; x \in \mathbb{R}^2\}$ which acts

on the C*-algebra $\mathfrak{A}$, generated by all local algebras $\mathfrak{A}(\mathcal{O})$, such that α_x maps $\mathfrak{A}(\mathcal{O})$ onto $\mathfrak{A}(\mathcal{O}+x)$.

A prescription $\mathcal{O} \to \mathfrak{A}(\mathcal{O})$ of this type is called a *translationally covariant Haag-Kastler-net.* To introduce the notion of kink-states, we first should discuss the notion of a (physical) state in our framework. A state is a positive linear functional ω on $\mathfrak{A}$ with $\omega(1) = 1$. Using the *GNS-construction*, we obtain a Hilbert-space $\mathfrak{H}$, a representation π of $\mathfrak{A}$ on $\mathfrak{H}$ and a vector Ω, such that $\omega(a) = \langle\Omega, \pi(a)\Omega\rangle$ for each $a \in \mathfrak{A}$. There may be many states on $\mathfrak{A}$ and we need a criterion to select the states of physical interest. For our purpose, we use the *Borchers-criterion* which requires:

(1) There exists a unitary strongly continuous representation of the translation group $U : x \mapsto U(x)$ on the GNS-Hilbert-space $\mathfrak{H}$ which implements α_x in the GNS-representation π, i.e. $\pi(\alpha_x a) = U(x)\pi(a)U(-x)$, for each $a \in \mathfrak{A}$.

(2) The spectrum (of the generator) of $U(x)$ is contained in the closed forward light cone.

A special class of states which satisfy the Borchers-criterion are the *vacuum-states* which fulfill the additional property of translation invariance, i.e. $\omega \circ \alpha_x = \omega$.

We are now prepared to describe the properties of a kink-state:

Particle-like properties: We require that a kink-state fulfills the Borchers-criterion. These property guarantees that one has the possibility to "move" a kink like a particle. If the lower bound of the spectrum of $U(x)$ is an isolated mass-shell, then a kink-state "behaves" completely like a particle.

The interpolation property: A pair of vacuum states ω_1, ω_2 is interpolated by a kink-state ω if there is a bounded region $\mathcal{O} \subset \mathbb{R}^2$, such that $\omega(a) = \omega_1(a)$, if a is localized in the left space-like complement of $\mathcal{O}$, and $\omega(a) = \omega_2(a)$, if a is localized in the right space-like complement of $\mathcal{O}$. In other words, in one space-like direction ω "looks like" the vacuum ω_1, in the other space-like direction ω "looks like" the vacuum ω_2.

We come now to the question which conditions a pair of vacuum-states has to satisfy, such that a interpolating kink-state can be constructed.

In the first two sections we introduce the notion of *admissible, locally equivalent* vacuum-states. As we will see later, these condition is sufficient for the existence of an interpolating kink-state. As an example of an *admissible* vacuum-state, one may think of the vacuum of a massive free scalar field [7, 1, 23]. But for a direct application of our construction to the $P(\phi)_2$-models, one has to check that these property remains also true for the interacting case.

Therefore, we consider also a lager class of vacuum-states which we call *weakly admissible*. The disadvantage of these condition is that it is much more technical. On the other hand, it can be proven for the vacuum-states of $P(\phi)_2$-models [22].

The main result of this paper is presented in section 3. It states that for each pair of *weakly admissible* vacuum states there exists a kink-state which interpolates them. The following diagram shows the logical structure:

admissible and locally equivalent

$\Downarrow$

weakly admissible and locally equivalent

$\Downarrow$

existence of interpolating kink-state

$\Downarrow$

locally equivalent

To motivate the following analysis, we give here the main steps of the construction of a kink-state. A detailed discussion of our program is carried out in section 4 and 5. Let as briefly summarize the strategy of it.

Firstly, we build the tensor-product of two copies of our QFT, i.e. the net $\mathcal{O} \mapsto \mathfrak{A}_2(\mathcal{O}) := \mathfrak{A}(\mathcal{O}) \otimes \mathfrak{A}(\mathcal{O})$ which we call in the sequel the *squared theory*. We consider the map α_F, called the *flip automorphism*, which is given by interchanging the tensor-factors, i.e.:

$$\alpha_F : a_1 \otimes a_2 \mapsto a_2 \otimes a_1$$

The requirement that the pair of vacuum-states ω_1, ω_2 satisfies our technical assumptions guarantees the existence of an automorphism α_θ which has the following properties:

(1) The automorphism α_θ is an involution, i.e. $\alpha_\theta^2 = \mathsf{id}$.

(2) There exists a bounded region $\mathcal{O}$, such that for each observable a which is localized in the left space-like complement of $\mathcal{O}$ we

have $\alpha_\theta(a) = a$, and for each observable a which is localized in the right space-like complement of $\mathcal{O}$ we have $\alpha_\theta(a) = \alpha_F(a)$.

In the final step we introduce the algebra homomorphism

$$\Delta_\theta : a \in \mathfrak{A} \mapsto \alpha_\theta(a \otimes \mathbf{1}) \in \mathfrak{A} \otimes \mathfrak{A}$$

and show that the state

$$\omega := \omega_1 \otimes \omega_2 \circ \Delta_\theta$$

is a kink-state which interpolates ω_1 and ω_2. The interpolating property of ω follows directly from its definition. The hard part is to prove that ω satisfies the Borchers-criterion. We can also consider the kink-state

$$\bar{\omega} := \omega_2 \otimes \omega_1 \circ \Delta_\theta$$

which is the anti-kink-state with respect to ω.

Let $x \mapsto U(x) = e^{iPx}$ be the representation of the translation group which corresponds to the kink-state ω and $x \mapsto U_j(x) = e^{iP_jx}$ the representation of the translation group which corresponds to the vacuum-state ω_j, $j = 1, 2$. If the vacuum states ω_1 and ω_2 are different, then ω is not translationally invariant which implies that 0 is not contained in the spectrum of the generator P. We show that the square of the mass-operator $M^2 = P_\mu P^\mu$ in the kink-representation is bounded from below by $1/2 \cdot \min(\inf(U_1), \inf(U_2))$, with $\inf(U_j) := \inf(\mathrm{sp}(P_{j,\mu}P_j^\mu)\backslash\{0\})$. If there is a mass-gap in the theory, then we have $\inf(U_j) = \mu_j > 0$ and obtain a lower bound estimate for the *kink-mass* m:

$$m \geq 1/2 \min(\mu_1, \mu_2)$$

These estimate is discussed in section 6.

We conclude this paper with section 7 "Conclusion and Outlook".

1 Preliminaries

Let us consider a quantum field theory in two dimensions which is described by a translationally covariant Haag-Kastler-net. We repeat the axioms of such a net briefly.

A Haag-Kastler-net with translation covariance is a prescription that assigns to each open double cone $\mathcal{O}$ a C*-algebra $\mathfrak{A}(\mathcal{O})$ such that this prescription is isotonous and respects locality, i.e. $\mathcal{O}_1 \subset$

$\mathcal{O}_2$ implies $\mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$ and $\mathcal{O}_1 \subset \mathcal{O}_2'$ implies $[\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = \{0\}$. Here $\mathcal{O}'$ denotes the space like complement of a double cone $\mathcal{O}$. Furthermore, let $\mathfrak{A}$ be the C*-inductive limit of the net $\mathcal{O} \mapsto \mathfrak{A}(\mathcal{O})$ we assume the existence of a group-homomorphism $\alpha : \mathbb{R}^2 \to \mathrm{Aut}\mathfrak{A}$ from the translation group into the automorphism group of $\mathfrak{A}$, such that α_x maps $\mathfrak{A}(\mathcal{O})$ onto $\mathfrak{A}(\mathcal{O}+x)$.

We consider now a class of states of $\mathfrak{A}$ which are of interest for our sequel analysis. Let us select the set S_A (S_A^w) of all *admissible (weakly admissible) vacuum states* which consists of *pure vacuum states* ω such that

(a) wedge duality holds as well as Haag duality in the GNS-representation, i.e.

$$\mathfrak{A}_\omega(W_\pm + x) = \mathfrak{A}_\omega(W_\mp + x)'$$

and for $\mathcal{O} = W_+ + x \cap W_- + y$ we have

$$\mathfrak{A}_\omega(\mathcal{O}) = \mathfrak{A}_\omega(W_+ + x) \cap \mathfrak{A}_\omega(W_- + y) \quad .$$

(b) The GNS-representation π of ω is faithful. [1]

(c) Admissible: for $W_\pm + x \subset W_\pm + y$ the inclusion

$$\mathfrak{A}_\omega(W_\pm + x) \subset \mathfrak{A}_\omega(W_\pm + y)$$

is a split inclusion or

(d) Weakly admissible: there exists an automorphism β of the C*-algebra

$$\overline{\bigcup_{\mathcal{O}} \mathfrak{A}_\omega(\mathcal{O}) \otimes \mathfrak{A}_\omega(\mathcal{O})}^{\|\cdot\|}$$

such that there is a space-like vector $r \in W_\pm$ and for each $a_1, a_2 \in \mathfrak{A}_\omega(\mathcal{O})$ holds the relation

$$\beta(a_1 \otimes a_2) = \begin{cases} a_2 \otimes a_1 & \text{if } \mathcal{O} \subset W_\pm + r \\ a_1 \otimes a_2 & \text{if } \mathcal{O} \subset W_\mp - r \end{cases}$$

and the automorphism

$$\alpha_x \circ \beta \circ \alpha_{-x} \circ \beta$$

is inner.

[1]Condition *(c)* is only a proper assumption if $\mathfrak{A}$ is not simple.

We now give a few comments on the notation used above. For a region $G \subset \mathbb{R}^2$, $\mathfrak{A}(G)$ denotes the C*-algebra which is generated by all $\mathfrak{A}(\mathcal{O})$'s with $\mathcal{O} \subset G$. For a state ω we write $\mathfrak{A}_\omega(G)$ for the v.Neumann-algebra $\pi(\mathfrak{A}(G))''$, where $(\mathfrak{H}, \pi, \Omega)$ is the GNS-triple of ω and the double-prime denotes the weak closure in $\mathfrak{B}(\mathfrak{H})$. Moreover, $W_\pm$ denotes the wedge-region $\{x| \, |x^0| < \pm x^1\}$. It is clear that in two dimensions each double-cone $\mathcal{O}$ is an intersection of two unique wedge-regions, i.e. $\mathcal{O} = W_+ + x \cap W_- + y$.

Remark: The condition *weakly admissible* looks rather technical and one might worry that it can never be fulfilled. However, these condition can be proven for a large class of vacuum-states, namely the vacuum-states of the $P(\phi)_2$-models, whereas *admissibility* can only be proven for the massive free scalar field [23, 22]. Furthermore, we are going to see that each vacuum state which is admissible is also weakly admissible.

2 Definition of Kink-States

In this section, we give a mathematical definition of kink-states and consider some immediate implications.

Definiton 2.1 : A state ω of $\mathfrak{A}$ is called a *kink-state* interpolating vacuum states ω_1, ω_2 if

(a) ω satisfies the Borchers criterion.

(b) Let $(\mathfrak{H}, \pi, \Omega), (\mathfrak{H}_j, \pi_j, \Omega_j)$ be the GNS-triples of $\omega, \omega_j; j = 1, 2$.

$$\pi|_{\mathfrak{A}(W_-)} \cong \pi_1|_{\mathfrak{A}(W_-)}$$

$$\pi|_{\mathfrak{A}(W_+)} \cong \pi_2|_{\mathfrak{A}(W_+)}$$

Here $\cong$ means *unitarily-equivalent.*

The set of all kink-states which interpolate ω_1 and ω_2 is denoted by $\mathsf{S}_{kink}(\omega_1, \omega_2)$.

Remark: In the sequel we write $(\mathfrak{H}, \pi, \Omega), (\mathfrak{H}_j, \pi_j, \Omega_j)$ for the GNS-triples of $\omega, \omega_j; j = 1, 2$, unless we state something different.

Of course, since ω is translationally covariant, we conclude for each $x \in \mathbb{R}^2$:

$$\pi|_{\mathfrak{A}(W_- + x)} \cong \pi_1|_{\mathfrak{A}(W_- + x)} \tag{1}$$

$$\pi|_{\mathfrak{A}(W_+ + x)} \cong \pi_2|_{\mathfrak{A}(W_+ + x)} \tag{2}$$

We show now that if there exists a kink-state $\omega \in \mathsf{S}_{kink}(\omega_1, \omega_2)$, then the GNS-representations π_1 and π_2 are unitarily equivalent on each algebra $\mathfrak{A}(\mathcal{O})$. We call two states which satisfies these relation *local unitarily equivalent* and write: $\omega_1 \cong_{loc} \omega_2$.

Lemma 2.1 : *If there exists a kink-state $\omega \in \mathsf{S}_{kink}(\omega_1, \omega_2)$ for $\omega_1, \omega_2 \in \mathsf{S}_A$, then $\omega, \omega_1, \omega_2$ are local unitarily equivalent, i.e.:*

$$\mathsf{S}_{kink}(\omega_1, \omega_2) \ni \omega \quad \Rightarrow \quad \omega \cong_{loc} \omega_1 \cong_{loc} \omega_2$$

Proof. As mentioned above, each double-cone can be written as $\mathcal{O} = W_+ + x \cap W_- + y$. Now, let $\omega \in \mathsf{S}_{kink}(\omega_1, \omega_2)$ be a kink state. We obtain from equ. (1) and (2):

$$\pi|_{\mathfrak{A}(W_- + y)} \cong \pi_1|_{\mathfrak{A}(W_- + y)} \tag{3}$$

$$\pi|_{\mathfrak{A}(W_+ + x)} \cong \pi_2|_{\mathfrak{A}(W_+ + x)} \tag{4}$$

Using Haag duality we obtain:

$$\pi|_{\mathfrak{A}(\mathcal{O})} \cong \pi_1|_{\mathfrak{A}(\mathcal{O})} \cong \pi_2|_{\mathfrak{A}(\mathcal{O})}$$

Since the double-cone $\mathcal{O}$ can be chosen arbitrarily, the result follows.
□

Remark: Lemma 2.1 states that local unitary equivalence of two vacuum states ω_1, ω_2 is an essential condition for the existence of an interpolating kink-state.

3 The Existence of Kink-States

In this section we formulate the main result of our paper which states that for two weakly admissible vacuum states ω_1, ω_2 local unitary equivalence is not only essential but also sufficient for the existence of kink-states.

Theorem 3.1 : *For each pair of admissible or weakly admissible vacuum states $\omega_1, \omega_2 \in \mathsf{S}_A$ which are local unitarily equivalent, i.e. $\omega_1 \cong_{loc} \omega_2$, there exists a kink-state $\omega \in \mathsf{S}_{kink}(\omega_1, \omega_2)$ which interpolates ω_1 and ω_2.*

We do not give a proof of the theorem in this section because we need some further results for preparation. The complete proof is given in section 5. To motivate the sequel steps, we describe the construction of kink-states briefly.

(1) The first step is to tensor two copies of our QFT, i.e. we consider the net $\mathcal{O} \mapsto \mathfrak{A}_2(\mathcal{O}) := \mathfrak{A}(\mathcal{O}) \otimes \mathfrak{A}(\mathcal{O})$. We introduce the map α_F which is given by prescription

$$\alpha_F : a_1 \otimes a_2 \mapsto a_2 \otimes a_1$$

and extends to an automorphism of $\mathfrak{A}_2$ which is called the *flip automorphism*.

(2) Using the split property (in case of admissible vacuum-states), we can choose an unitary operator $\theta \in \mathfrak{A}_{11}(W_+ + r)$ which implements the flip-automorphism α_F on $\mathfrak{A}_2(W_+)$ $(W_+ \subset W_+ + r)$. Here and in the sequel we write $\mathfrak{A}_{ij}(G) := [\ \pi_i \otimes \pi_j(\mathfrak{A}_2(G))\ ]''$ for a region $G \in \mathbb{R}^2$ and $i, j = 1, 2$.

(3) We define the representation $\alpha_\theta := \pi_{11}^{-1} \circ \mathsf{Ad}(\theta) \circ \pi_{11}$ and show that α_θ is an automorphism of an extension $\mathfrak{F}_2 \supset \mathfrak{A}_2$ of $\mathfrak{A}_2$ and satisfies condition *(d)* of section 1. In case of weakly admissible vacuum-states the existence of such an automorphism is required.

(4) In the final step we define the representation $\rho : \mathfrak{A} \to \mathfrak{B}(\mathfrak{H}_1 \otimes \mathfrak{H}_2)$ by

$$\rho(a) := \pi_1 \otimes \pi_2(\Delta_\theta(a))$$

with $\Delta_\theta := \alpha_\theta|_{\mathfrak{A}\otimes\mathbf{1}}$. We prove that ρ is well defined and show that the state

$$\omega := \omega_1 \otimes \omega_2 \circ \Delta_\theta$$

is a kink-state which interpolates ω_1 and ω_2.

4 Kink-States in the Squared Theory

We consider now the tensor-product $\mathcal{O} \mapsto \mathfrak{A}_2(\mathcal{O}) := \mathfrak{A}(\mathcal{O}) \otimes \mathfrak{A}(\mathcal{O})$ of the theory with itself which is also called the squared theory. We show that for two local unitarily equivalent admissible vacuum states ω_1, ω_2 of $\mathfrak{A}$ there is a canonical construction for a kink-state on the squared theory which interpolates the vacuum states $\omega_1 \otimes \omega_2$ and $\omega_2 \otimes \omega_1$.

So let us formulate the result of this section:

Proposition 4.1 : *For each pair of admissible vacuum states $\omega_1, \omega_2 \in \mathsf{S}_A$ which are local unitarily equivalent, i.e. $\omega_1 \cong_{loc} \omega_2$, there exists a kink-state $\omega \in \mathsf{S}_{kink}(\omega_{12}, \omega_{21})$ which interpolates $\omega_{12} := \omega_1 \otimes \omega_2$ and $\omega_{21} := \omega_2 \otimes \omega_1$.*

We start now with the preparation of the proof. Since the inclusion

$$\mathfrak{A}_{11}(W_\pm + x) \subset \mathfrak{N}_{11} \subset \mathfrak{A}_{11}(W_\pm + y)$$

is split with an intermediate type I factor $\mathfrak{N}_{11}$, the flip-automorphism α_F is implemented on $\mathfrak{A}_2(W_+)$ in the representation $\pi_{11} := \pi_1 \otimes \pi_1$ by a unitary involution θ ($\theta^2 = \mathbf{1}$) which is contained in $\mathfrak{A}_{11}(W_+ + r)$, i.e.

$$\pi_{11} \circ \alpha_F|_{\mathfrak{A}_2(W_+)} = \mathsf{Ad}(\theta) \circ \pi_{11}|_{\mathfrak{A}_2(W_+)} \quad . \tag{5}$$

For technical reasons, we introduce an extension of the net of local C*-algebras. For a vacuum state $\omega \in \mathsf{S}$, we define a family of semi norms on $\mathfrak{A}(\mathcal{O})$

$$||a||_T^\omega := |\mathsf{tr}(T\pi(a))| \tag{6}$$

and denote by $\mathfrak{M}_\omega(\mathcal{O})$ the closure of $\mathfrak{A}(\mathcal{O})$ in the topology which is induced by these family. It is clear that the algebra $\mathfrak{M}_\omega(\mathcal{O})$ is a W*-algebra, canonical isomorphic to the v.Neumann algebra $\pi(\mathfrak{A}(\mathcal{O}))''$. The C*-inductive limit, generated by all $\mathfrak{M}_\omega(\mathcal{O})$ is denoted by $\mathcal{A}_\omega$.

Consider now local unitarily equivalent vacuum states ω_1, ω_2 then we obtain

$$\mathfrak{M}(\mathcal{O}) := \mathfrak{M}_{\omega_1}(\mathcal{O}) = \mathfrak{M}_{\omega_2}(\mathcal{O})$$

and hence $\mathcal{A} = \mathcal{A}_{\omega_1} = \mathcal{A}_{\omega_2}$. There are unique extensions of the GNS-representations of ω_1 and ω_2 which are denoted by π_1 and π_2.

Let us consider now the net $\mathcal{O} \mapsto \mathfrak{F}_2(\mathcal{O}) := \mathfrak{M}(\mathcal{O}) \otimes \mathfrak{M}(\mathcal{O})$. We denote the C*-inductive limit which is generated by all algebras $\mathfrak{F}_2(\mathcal{O})$ by $\mathcal{A}_2$ and define the following representation of $\mathcal{A}_2$:

$$\alpha_\theta^1(a) := \theta\pi_{11}(a)\theta \tag{7}$$

Lemma 4.1 : *For each double cone $\mathcal{O}$ which contains $\mathcal{O}_r := W_+ + r \cap W_-$ one has*

$$\alpha_\theta^1(\mathfrak{F}_2(\mathcal{O})) \subset \mathfrak{A}_{11}(\mathcal{O})$$

i.e. α_θ^1 maps local algebras into local algebras.

Proof. We choose an operator $b \in \mathfrak{A}_2(W_+ + y)$ such that for $W_+ \supset W_+ + y$. Then we have:

$$\begin{aligned}
&\theta\pi_{11}(a)\theta\pi_{11}(b) = \theta\pi_{11}(a)\pi_{11}(\alpha_F b)\theta \\
&= \theta\pi_{11}(a\alpha_F b)\theta = \theta\pi_{11}(\alpha_F b\ a)\theta \\
&= \pi_{11}(b)\theta\pi_{11}(a)\theta
\end{aligned}$$

This implies that $\theta\pi_{11}(a)\theta$ is contained in $\mathfrak{A}_{11}(W_+ + y)' = \mathfrak{A}_{11}(W_- + y)$. On the other hand it is clear that for $W_+ + x \subset W_+$ the operator $\theta\pi_{11}(a)\theta$ is also contained in $\mathfrak{A}_{11}(W_+ + x)$. Since each double cone $\mathcal{O}$ which contains $\mathcal{O}_r$ can be written as $\mathcal{O} = W_+ + y \cap W_- + x$, we conclude for each $a \in \mathfrak{A}_2(\mathcal{O})$:

$$\theta\pi_{11}(a)\theta \in \mathfrak{A}_{11}(W_- + y) \cap \mathfrak{A}_{11}(W_+ + x) = \mathfrak{A}_{11}(\mathcal{O}) \tag{8}$$

which completes the proof. □

Since π_{11} is a faithful representation of $\mathcal{A}$, it follows from Lemma 3.1. that the prescription $\alpha_\theta : a \mapsto \pi_{11}^{-1}(\alpha_\theta^1(a))$ is a well defined endomorphism of $\mathcal{A}$.

An immediate consequence of Lemma 3.1 is the following corollary:

Corollary 4.1 : *The automorphism $\alpha_\theta \in \mathrm{Aut}(\mathcal{A})$ has the following properties:*

(1): $\alpha_\theta^2 = \mathsf{id}$

(2): $\alpha_\theta|_{\mathfrak{F}_2(W_+)} = \alpha_F|_{\mathfrak{F}_2(W_+)}$ *and* $\alpha_\theta|_{\mathfrak{F}_2(W_- + r)} = \mathsf{id}_{\mathfrak{F}_2(W_- + r)}$

We define now the following representation of $\mathfrak{A}_2$:

$$\rho := (\pi_1 \otimes \pi_2) \circ \alpha_\theta \tag{9}$$

Since α_θ is an automorphism, we obtain a further corollary:

Corollary 4.2 : *The representation ρ is irreducible.*

We prove now that ρ is a translationally covariant representation.

Lemma 4.2 : *For each $x \in \mathbb{R}^2$ is the automorphism*

$$\alpha_{-x} \circ \alpha_\theta \circ \alpha_x \circ \alpha_\theta$$

inner.

Proof. Since θ implements the flip-automorphism on $\mathfrak{A}_2(W_+)$ the operator $\theta(x) := U_{11}(x)\theta U_{11}(-x)$ implements α_F on $\mathfrak{A}_2(W_+ + x)$. From this we obtain for $W_+ \supset W_+ + x$ and $a \in \mathfrak{A}_2(W_+ + x)$:

$$\pi_{11}(\alpha_F a) = \theta\pi_{11}(a)\theta = \theta(x)\pi_{11}(a)\theta(x) \tag{10}$$

Hence $\theta(x)\theta$ is contained in $\mathfrak{A}_{11}(W_- + x)$. On the other hand $\theta(x)\theta$ is contained in $\mathfrak{A}_{11}(W_+ + r)$ with $W_+ + r \supset W_+ \supset W_+ + x$ and we obtain:

$$\theta(x)\theta \in \mathfrak{A}_{11}(W_- + x) \cap \mathfrak{A}_{11}(W_+ + r)$$

For the case $W_+ \subset W_+ + x$ we obtain analogously:

$$\theta(x)\theta \in \mathfrak{A}_{11}(W_-) \cap \mathfrak{A}_{11}(W_+ + r + x)$$

By using Haag duality, we conclude that for each x the operator $\theta(x)\theta$ is contained in $\mathfrak{A}_{11}(\mathcal{O}_x)$, where $\mathcal{O}_x$ is a sufficiently large double cone. For each x we define the unitary operator

$$\gamma(x) := \pi_{11}^{-1}(\theta(x)\theta) \in \mathfrak{F}_2(\mathcal{O}_x) \tag{11}$$

This implies the following relation

$$\begin{aligned} &\alpha_x \circ \alpha_\theta \circ \alpha_{-x} \circ \alpha_\theta \\ &= \pi_{11}^{-1} \circ \mathsf{Ad}(\theta(x)\theta) \circ \pi_{11} \\ &= \mathsf{Ad}(\gamma(x)) \end{aligned} \tag{12}$$

which completes the proof. □

Corollary 4.3 : *Each admissible vacuum-state is weakly admissible, i.e.*

$$\mathsf{S}_A \subset \mathsf{S}_A^w \quad .$$

Proof. By Corollary 4.1 and Lemma 4.2, the automorphism $\beta = \alpha_\theta$ satisfies condition *(d)* of section 1. □

In the sequel analysis, it is sufficient to consider vacuum-states which are *weakly admissible.*

Corollary 4.4 : *The representation ρ is translationally covariant, i.e. there exists a strongly continuous representation $x \mapsto U_\rho(x)$ of the translation group which implements α_x:*

$$\rho \circ \alpha_x = \mathsf{Ad}(U_\rho(x)) \circ \rho$$

Proof. In the sequel, we use the following abbreviations:

$$U_{ij}(x) := U_i(x) \otimes U_j(x)$$

where U_i implements the translation group in the vacuum representation π_i; $i, j = 1, 2$. By construction, $\gamma(x)$ satisfies the following *co-cycle condition*:

$$\gamma(x + y) = \alpha_x(\gamma(y))\gamma(x) \tag{13}$$

Moreover, the following relation holds

$$\alpha_\theta \circ \alpha_x = \alpha_x \circ \alpha_{\theta(-x)} \tag{14}$$

and we obtain by using Lemma 4.2

$$\begin{aligned} \rho \circ \alpha_x &= \pi_{12} \circ \alpha_\theta \circ \alpha_x \\ &= \pi_{12} \circ \alpha_x \circ \alpha_{\theta(-x)} \\ &= \mathsf{Ad}(U_{12}(x)) \circ \pi_{12} \circ \alpha_{\theta(-x)} \\ &= \mathsf{Ad}(U_{12}(x)) \circ \pi_{12} \circ \mathsf{Ad}(\gamma(-x)) \circ \alpha_\theta \\ &= \mathsf{Ad}(U_{12}(x)\Gamma_\rho(-x)) \circ \rho \end{aligned} \tag{15}$$

where we have set $\Gamma_\rho(x) := \pi_{12}(\gamma(x))$. Hence for each $x \in \mathbb{R}^2$ we obtain the charge transporter

$$\Gamma_\rho(x) := \pi_{12}(\gamma(x)) \quad . \tag{16}$$

We define

$$U_\rho : x \mapsto U_\rho(x) := U_{12}(x)\Gamma_\rho(-x)$$

which is a strongly continuous representation of the translation group implementing the translations in the representation ρ. $\square$

To study the properties of the representation ρ in more detail we consider a further representation $\bar{\rho} : \mathfrak{A}_2 \to \mathfrak{B}(\mathfrak{H}_2 \otimes \mathfrak{H}_1)$ which is given by

$$\bar{\rho} := \pi_{21} \circ \alpha_\theta \quad . \tag{17}$$

We will see that $\bar{\rho}$ plays the role of an *anti-kink*. The representation $\bar{\rho} : \mathfrak{A}_2 \to \mathfrak{B}(\mathfrak{H}_2 \otimes \mathfrak{H}_1)$ is translationally covariant which can be proven by using the arguments in the proofs of Lemma 4.1 and Lemma 4.2 for $\bar{\rho}$.

To establish the statement of Proposition 4.1, it remains to be proven, that the spectrum of U_ρ is contained in the closed forward light cone. For this purpose, we prove now the additivity of energy momentum spectrum.

Let us consider two representations $\rho_1 = \pi_{12} \circ \alpha_{\theta_1}$ and $\rho_2 = \pi_{21} \circ \alpha_{\theta_2}$, where θ_1 resp. θ_2 are unitary operators which implement α_F on $\mathfrak{F}_2(W_+ + x_1)$ resp. $\mathfrak{F}_2(W_+ + x_2)$, or in case of weakly admissible vacuum-states α_{θ_1} and α_{θ_2} are automorphisms which satisfy condition *(d)* of section 1. Then we define the composition of ρ_1 and ρ_2 as follows:

$$\rho_1\rho_2 := \rho_1 \circ \pi_{21}^{-1} \circ \rho_2 = \pi_{12} \circ \alpha_{\theta_1}\alpha_{\theta_2} \tag{18}$$

The representations can also be composed in the other direction:

$$\rho_2\rho_1 := \pi_{21} \circ \alpha_{\theta_2}\alpha_{\theta_1} \tag{19}$$

Moreover, we write in the sequel $S(\rho)$ for the spectrum of U_ρ.

Remark: The composition described above can be interpreted as the composition of soliton homomorphisms in the sense of [21]. Remember that $\rho \circ \pi_{21}^{-1}$ maps $\mathfrak{A}_{21}$ into $\mathfrak{A}_{12}$.

Lemma 4.3 : *Let ρ_1, ρ_2 and $\rho_1\rho_2$ be defined as described above, then the additivity of the energy-momentum spectrum holds, i.e.*

$$S(\rho_1) + S(\rho_2) \subset S(\rho_1\rho_2)$$

Proof. The proof is standard and uses the same method as in the DHR-framework [9, 10]. The only difference which appears is due to the fact that the representations ρ_j are localized in wedge regions and not in double cones. But for the proof it is sufficient that ρ_j maps local algebras into local algebras.

We choose test functions f_j with $\mathrm{supp}\tilde{f}_j \subset S(\rho_j)$ and a local operator $a \in \mathfrak{F}_2(\mathcal{O})$. The operators

$$a_j := \int dx\ f_j(x)\gamma(x)\alpha_x a$$

have energy-momentum transfer in $\mathrm{supp}\tilde{f}_j$. Here $\gamma(x)$ is defined as in equ.(11) above. Now $\Psi_1 = \pi_{12}(a_1)\Omega_{12} \in \mathfrak{H}_1 \otimes \mathfrak{H}_2$ has energy-momentum support in $\mathrm{supp}\tilde{f}_1$ and $\Psi_2 = \pi_{21}(a_2)\Omega_{21} \in \mathfrak{H}_2 \otimes \mathfrak{H}_1$ has energy-momentum support in $\mathrm{supp}\tilde{f}_2$. Moreover, the vector

$$\Psi := \rho_1(a_2)\pi_{12}(a_1)\Omega_{12}$$

has energy-momentum support in supp$\tilde{f}_1$+supp$\tilde{f}_2$ which remains also true for

$$\Psi_y := \rho_1(a_2) U_{\rho_1}(y) \pi_{12}(a_1) \Omega_{12} \quad .$$

We compute now:

$$||\Psi_y||^2 = \langle \Omega_{12}, \pi_{12}(a_1^*) \rho_1(\alpha_{-y}(a_2^* a_2)) \pi_{12}(a_1) \Omega_{12} \rangle$$

Since ρ_1 acts as $\pi_{12} \circ \alpha_F$ on $\mathfrak{F}_2(W_+ + y)$ with $W_+ + y \subset W_+ + x_1$, we conclude by using the cluster theorem:

$$\begin{aligned} &\lim_y ||\Psi_y||^2 = ||\Psi_1||^2 \ \langle \Omega_{12}, \pi_{12}(\alpha_F(a_2^* a_2)) \Omega_{12} \rangle \\ &= ||\Psi_1||^2 \ \langle \Omega_{21}, \pi_{21}(a_2^* a_2) \Omega_{21} \rangle \\ &= ||\Psi_1||^2 \ ||\Psi_2||^2 \end{aligned}$$

as y tends to minus space like infinity. Hence for $||\Psi_j|| \neq 0$ we obtain $\Psi_y \neq 0$ for one $y \in \mathbb{R}^2$ and the result follows. $\square$

Let us have a closer look at the anti-kink representation $\bar{\rho}$. We denote by J_{kl} the modular conjugation with respect to the pair $(\mathfrak{A}_{kl}(W_+), \Omega_{kl})$. For technical reasons, we make the following assumption:

Assumption: Let us assume that there exists a PCT-symmetry, i.e. an involutive anti-automorphism $j : \mathfrak{A}_2 \mapsto \mathfrak{A}_2$ with $j(\mathfrak{A}_2(\mathcal{O})) = \mathfrak{A}_2(-\mathcal{O})$ and $j \circ \alpha_x = \alpha_{-x} \circ j$ which is implemented in each vacuum representation π_{kl} by the modular conjugation J_{kl}, i.e.:

$$\pi_{kl}(ja) = J_{kl} \pi_{kl}(a) J_{kl}$$

Now we define the following representation:

$$\rho^J := j_{21} \circ \rho_F \circ j \tag{20}$$

Here we have set $j_{kl} := \mathsf{Ad}(J_{kl})$ and the representation ρ_F is given by

$$\rho_F := \pi_{21} \circ \alpha_F \circ \alpha_\theta \quad . \tag{21}$$

Remark: If an automorphism β of $\mathcal{A}_2$ satisfies the condition *(d)* of section 1, then the automorphism $\alpha_F \circ \beta$ satisfies it also.

Lemma 4.4 : *The representations $\bar{\rho}$ and ρ^J are unitarily equivalent and in addition we obtain that $S(\rho) = S(\bar{\rho})$.*

Proof. Using the composition role described above, we obtain that $\rho\bar{\rho} = \pi_{12}$ and $\bar{\rho}\rho = \pi_{21}$. By Corollary 4.2 and Lemma 4.2 we can use the results of [16, 19, 21] we conclude that the anti-kink sector is unique. Thus we have $\bar{\rho} \cong \rho^J$. In addition to that, the representations ρ and ρ_F are unitarily equivalent (ρ_F is a PT-conjugate for ρ in the sense of [21]). Since $U_{\rho^J}(x) := J_{21}U_{\rho_F}(-x)J_{21}$ implements the translation group in the representation ρ^J (see also [4, 16, 21]), we conclude

$$S(\bar{\rho}) = S(\rho^J) = S(\rho_F) = S(\rho)$$

which completes the proof. □

Proposition 4.2 : *ρ is a positive energy representation.*

Proof. Corollary 4.2 and Lemma 4.2 state that ρ is translationally covariant and irreducible, in particular factorial. Now by Lemma 4.3 we conclude that $S(\rho) + S(\bar{\rho}) \subset S(\pi_{12}) = S(\pi_{21}) = S(\pi_1) + S(\pi_2)$ and with Lemma 4.4 we obtain finally

$$S(\rho) \subset \bar{V}_+ \tag{22}$$

which completes the proof. □

We are now ready to prove Proposition 4.1.

Proof of Proposition 4.1: We show that $\omega_\theta := \omega_1 \otimes \omega_2 \circ \alpha_\theta$ is a kink-state which interpolates the vacuum states $\omega_1 \otimes \omega_2$ and $\omega_2 \otimes \omega_1$. Since ρ is irreducible, the GNS-representation π_θ of ω_θ is unitarily equivalent to ρ. Hence ω_θ satisfies the Borchers criterion by Proposition 4.2. Furthermore, by Corollary 4.1 we conclude that ω_θ interpolates the vacuum states $\omega_1 \otimes \omega_2$, namely we have

$$\rho|_{\mathfrak{A}_2(W_+)} = \pi_{12} \circ \alpha_F \cong \pi_{21}$$
$$\rho|_{\mathfrak{A}_2(W_- + r)} = \pi_{12}$$

and the result follows. □

5 Kink-States in the Original Theory

We have seen in the last section that each QFT which is equipped with two different (admissible) vacuum states there is a method to construct kink-states in the squared theory. We are now interested in the existence of kink-states for the original theory.

Since we are able to construct kink-states for the squared theory it is easy to obtain kink-states for the original one. Let us consider the automorphism $\alpha_\theta \in \mathrm{Aut}(\mathcal{A}_2)$ with was constructed in the last section and define the following algebra-homomorphisms:

$$\Delta_\theta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A} \quad ; \quad a \mapsto \Delta_\theta(a) := \alpha_\theta(a \otimes \mathbf{1})$$
$$\Delta'_\theta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A} \quad ; \quad a \mapsto \Delta'_\theta(a) := \alpha_\theta(\mathbf{1} \otimes a)$$

We obtain now states

$$\omega_\theta := \omega_1 \otimes \omega_2 \circ \Delta_\theta$$
$$\omega'_\theta := \omega_1 \otimes \omega_2 \circ \Delta'_\theta$$

which has the following localization properties:

$$\begin{aligned} \omega_\theta|_{\mathfrak{A}(W_+)} = \omega_2|_{\mathfrak{A}(W_+)} \quad \omega_\theta|_{\mathfrak{A}(W_-+r)} = \omega_1|_{\mathfrak{A}(W_-+r)} \\ \omega'_\theta|_{\mathfrak{A}(W_+)} = \omega_1|_{\mathfrak{A}(W_+)} \quad \omega'_\theta|_{\mathfrak{A}(W_-+r)} = \omega_2|_{\mathfrak{A}(W_-+r)} \end{aligned} \tag{23}$$

We use now the results of the last section to prove that both ω_θ and ω'_θ are kink states which also proves Theorem 3.1.

Proposition 5.1 : *The states ω_θ and ω'_θ are kink-states where ω_θ is contained in $\mathsf{S}_{kink}(\omega_1, \omega_2)$ and ω'_θ is contained in $\mathsf{S}_{kink}(\omega_2, \omega_1)$. Moreover, each state $\hat{\omega}$ which GNS-representation is a sub-representation of the GNS-representation of ω_θ is also contained in $\mathsf{S}_{kink}(\omega_2, \omega_1)$.*

Proof. By construction, ω_θ is the restriction of the state $\omega_1 \otimes \omega_2 \circ \alpha_\theta$ to the first tensor factor, i.e. the algebra $\mathcal{A} \otimes \mathbb{C}\mathbf{1}$. We show now that the GNS-representation σ of ω_θ is unitarily equivalent to $\rho|_{\mathcal{A}\otimes\mathbb{C}\mathbf{1}}$.

The C*-algebra $\mathfrak{A}(W_- + r, W_+)$ which is generated by $\mathfrak{A}(W_- + r)$ and $\mathfrak{A}(W_+)$ is contained in $\mathfrak{A}$. By using the Theorem of Reeh and Schlieder, we obtain that

$$\rho(\mathfrak{A}(W_- + r, W_+)) \otimes \mathbf{1})\Omega_1 \otimes \Omega_2 = \pi_1(\mathfrak{A}(W_- + r))\Omega_1 \otimes \pi_2(\mathfrak{A}(W_+))\Omega_2$$

is dense in $\mathfrak{H}_1 \otimes \mathfrak{H}_2$. Hence the representation $\rho|_{\mathcal{A}\otimes\mathbb{C}\mathbf{1}}$ is cyclic and therefore unitarily equivalent to σ.

Since ρ is a positive energy representation (Proposition 4.2) of $\mathcal{A} \otimes \mathcal{A}$ its restriction $\rho|_{\mathcal{A}\otimes\mathbf{1}} \cong \sigma$ is a positive energy representation of $\mathcal{A}$. By a result of Borchers [5], we can construct a unitary strongly continuous representation $x \mapsto U_\sigma(x)$ of the translation group with the following properties:

1: For each x is the operator $U_\sigma(x)$ contained in $\sigma(\mathcal{A})''$.

2: U_σ implements the translations in the representation σ, i.e. $\sigma(\alpha_x a) = U_\sigma(x)\sigma(a)U_\sigma(-x)$.

3: The spectrum of U_σ is contained in the closed forward light cone.

Now let $(\pi, \mathfrak{H})$ be a sub-representation of σ, i.e. there exists an isometry $v : \mathfrak{H} \to \mathfrak{H}_1 \otimes \mathfrak{H}_2$ such that $\pi = v^*\sigma(\cdot)v$. Since vv^* is a projection which is contained in $\sigma(\mathcal{A})'$ we conclude that $U_\pi(x) := v^*U_\sigma(x)v$ is a unitary strongly continuous representation of the translations which implements the translations in the representation π. In particular the spectrum of U_π is also contained in the closed forward light cone. Thus π satisfies the Borchers criterion.

From equ. (22) we obtain the following relations:

$$\sigma|_{\mathfrak{A}(W_-+r)} \cong \pi_1|_{\mathfrak{A}(W_-+r)} \otimes \mathbf{1} \cong_{quasi} \pi_1|_{\mathfrak{A}(W_-+r)}$$

$$\sigma|_{\mathfrak{A}(W_+)} \cong \mathbf{1} \otimes \pi_2|_{\mathfrak{A}(W_+)} \cong_{quasi} \pi_2|_{\mathfrak{A}(W_+)}$$

Here the symbol $\cong_{quasi}$ means *quasi-equivalent*. Since π is a sub-representation of σ, we conclude:

$$\pi|_{\mathfrak{A}(W_-+r)} \cong_{quasi} \pi_1|_{\mathfrak{A}(W_-+r)}$$

$$\pi|_{\mathfrak{A}(W_+)} \cong_{quasi} \pi_2|_{\mathfrak{A}(W_+)}$$

Using the fact that the v.Neumann-algebras $\pi_1(\mathfrak{A}(W_- + r))''$ and $\pi_2(\mathfrak{A}(W_+))''$ are type III factors, we conclude by using standard-arguments:

$$\pi|_{\mathfrak{A}(W_-+r)} \cong \pi_1|_{\mathfrak{A}(W_-+r)}$$

$$\pi|_{\mathfrak{A}(W_+)} \cong \pi_2|_{\mathfrak{A}(W_+)}$$

Thus the state ω_θ is a kink-state and each state $\hat{\omega}$ which GNS-representation is a sub-representation of the GNS-representation of ω_θ is also contained in $\mathsf{S}_{kink}(\omega_2, \omega_1)$. The proof for ω'_θ works analogously. $\square$

6 Estimates for the Kink- (Soliton-) Mass

To discuss the mass of kink- (soliton-) state, we consider purely massive theories where the admissible vacuum states are *massive vacuum states*.

Let ω be a pure translationally covariant state and $U : x \to U(x)$ the strongly continuous representation of the translations which implements α_x in the GNS-representation of ω. Then ω is called a *massive vacuum state* if the spectrum of $U(x)$ contains $\{0\}$ and a subset of $C_\mu := \{p \in \mathbb{R}^2 : p^2 > \mu\}$ where $\mu > 0$ is a positive real number, called the *mass gap* of ω. We denote the set of all massive weakly admissible vacuum states with mass gap μ by $\mathsf{S}(\mu)$. If the spectrum of $U(x)$ contains the mass shell $H_m := \{p \in \mathbb{R}^2 : p^2 = m^2\}$ and a subset of $C_{\mu+m}$, then we call ω a massive one-particle state with mass $m > 0$.

For a two dimensional QFT it is shown [8, 12, 20], that for each massive one-particle state ω there are massive vacuum states ω_1, ω_2, such that ω interpolates ω_1 and ω_2. The mass m of ω then satisfies the estimate

$$m \geq \frac{1}{2} \min(\mu_1, \mu_2) \tag{24}$$

where μ_1 (resp. μ_2) is the mass gap of ω_1 (resp. ω_2).

Now we consider the situation where two different massive admissible vacuum states $\omega_1 \in \mathsf{S}(\mu_1)$ and $\omega_2 \in \mathsf{S}(\mu_2)$ are given. Then we know by Theorem 3.1 that there exist a kink-state ω which interpolates ω_1 and ω_2.

We denote by $S(\pi)$ the spectrum of $U_\pi(x)$, where U_π is a strongly continuous representation of the translation group which implements α_x in the GNS-representation π of ω.

If the vacuum states are ω_1 and ω_2 are inequivalent, then it follows that $0 \notin S(\pi)$. This can be seen as follows: Since ω_1 and ω_2 are inequivalent, there exists an operator $a \in \mathcal{A}$ with $\omega_1(a) \neq \omega_2(a)$. On the other hand, if x tends to space-like infinity we have $\lim_{|x|\to\infty} \omega(\alpha_x a) = \omega_2(a)$ and if x tends to minus space-like infinity we have $\lim_{|x|\to-\infty} \omega(\alpha_x a) = \omega_1(a)$ and ω is not translationally invariant.

From the proofs of Proposition 4.2 and Proposition 5.1 we obtain that $S(\pi)$ is a subset of the closed forward light cone which does not contain the point $k = 0$. Hence we conclude

$$S(\pi) \subset \frac{1}{2}(S(\pi_1) + S(\pi_2)) \tag{25}$$

and obtain for the infimum $\inf(S(\pi))$ of the spectrum $S(\pi)$ the estimate:

$$\inf(S(\pi)) \geq \frac{1}{2} \min(\mu_1, \mu_2) \tag{26}$$

Here the infimum $\inf(S(\pi))$ is defined as the the infimum of the spectrum of the mass operator $M = (P_\mu P^\mu)^{1/2}$, where P is the generator of the translation group U_π.

Let us suppose that ω dominates a massive one particle state with mass $m > 0$, then we obtain from equ. (25) that m satisfies the estimate of equ. (23), namely $m \geq 1/2 \min(\mu_1, \mu_2)$.

We conclude this section by summarizing the discussion above. If we consider a massive one-particle state ω_m with mass $m > 0$, then there are massive vacuum states ω_1 and ω_2 with corresponding mass gaps μ_1, μ_2, such that ω_m interpolates ω_1 and ω_2 and m satisfies the estimate $m \geq 1/2 \min(\mu_1, \mu_2)$. Using the result of section 3 (Theorem 3), we can construct from the vacuum states ω_1, ω_2 a kink-state ω which also interpolates ω_1 and ω_2. If there exists a purification $\omega_{m'}$ of ω which is a massive on particle state with mass m', then m' satisfies the same estimate as the mass m, namely $m' \geq 1/2 \min(\mu_1, \mu_2)$. Here we have assumed that ω_1 and ω_2 are admissible vacuum states.

7 Conclusion and Outlook

We have seen that for each pair of admissible vacuum states which are also locally equivalent there is a natural way to construct an interpolating kink-state. One advantage of this construction is, that we do not need the assumption that the vacua are related by an internal symmetry transformation as in [14]. Furthermore, the construction is purely algebraic and independent of the specific properties of a model.

On the other hand, if we want to apply our result to a concrete model, we have to check that the vacuum states of the model of consideration are admissible or weakly admissible. At the moment, admissibility is only checked for the massive free scalar field [1, 7, 23]. For the vacuum states of the $P(\phi)_2$-models one can prove weak admissibility [22].

If we consider a massive one-particle state ω_m with mass $m > 0$, then there are massive vacuum states ω_1 and ω_2 with corresponding mass gaps μ_1, μ_2, such that ω_m interpolates ω_1 and ω_2. If ω_1 and ω_2 weakly admissible vacuum-states, then we can apply Theorem 3.1 and construct a kink-state ω which also interpolates ω_1 and ω_2. It is not well understood at the moment, what are the relations between ω_m and the reconstructed kink-state ω.

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References

[1] d'Antoni, C. and Fredenhagen, K.: Carges in Spacelike Cones, Commun. Math. Phys. **94**, 537-544 (1984)

[2] d'Antoni, C. and Longo, R.: Interpolation by Type I Factors and the Flip Automorphism, Jour. Func. Anal. **51**, 361-371 (1983)

[3] Araki, H. and Haag, R.: Collision Cross Sections in Terms of Local Observables., Commun. Math. Phys. **4**, 77-91, (1967)

[4] Borchers, H.-J.: CPT-Theorem in the Theory of Local Observables, Commun. Math. Phys. **143**, 315-332, (1992)

[5] Borchers, H.-J.: On the Converse of the Reeh-Schlieder Theorem, Commun. Math. Phys. **10**, 269-273, (1968)

[6] Borchers, H.-J.: Commun. Math. Phys. **4**, 315-323, (1967)

[7] Buchholz, D.: Product States for Local Algebras. Commun. Math. Phys. **36**, 287-304, (1974)

[8] Buchholz, D. and Fredenhagen K.: Locality and the Structure of Particle States. Commun. Math. Phys. **84**, 1-54, (1982)

[9] Doplicher, S., Haag, R. and Roberts, J.E.: Local Observables and Particle Statistics I. Commun. Math. Phys. **23**, 199-230, (1971)

[10] Doplicher, S., Haag, R. and Roberts, J.E.: Local Observables and Particle Statistics II. Commun. Math. Phys. **35**, 49-58, (1971)

[11] Fredenhagen, K.: Generalization of the Theory of Superselection Sectors. Published in Kastler, D.: The Algebraic Theory of Superselection Sectors ... World Scientific 1989

[12] Fredenhagen, K.: Superselection Sectors in Low Dimensional Quantum Field Theory. , DESY-92-133, Sept 1992. Published in the proceedings of 28th Karpacz Winter School of Theoretical Physics: Infinite-Dimensional Geometry in Physics, Karpacz, Poland, 17-29 Feb 1992. J. Geom. Phys. **11** (1993) 337-348

[13] Fredenhagen, K.: On the Existence of Antiparticles. Commun. Math. Phys. **79**, 141-151, (1981)

[14] Fröhlich, J.: New Superselection Sectors (Soliton States) in Two Dimensional Bose Quantum Field Models. Commun. Math. Phys. **47**, 269-310, (1976)

[15] Fröhlich, J.: Quantum Theory of Nonlinear Invariant Wave (Field) Equations. Erice, Sicily, Summer 1977

[16] Guido, D. and Longo, R.: Relativistic Invariance and Charge Conjugation in Quantum Field Theory. Commun. Math. Phys. **148**, 521-551, (1992)

[17] Haag, R.: Local Quantum Physics. Berlin, Heidelberg, New York: Springer 1992

[18] Longo, R.: Index of Subfactors and Statistics I. Commun. Math. Phys. **126**, 217-247, (1989)

[19] Longo, R.: Index of Subfactors and Statistics II. Commun. Math. Phys. **130**, 285-309, (1990)

[20] Schlingemann, D.: Antisolitonen und Mehrsolitonzustände im Rahmen der Algebraischen Quantenfeldtheorie. Diploma thesis, Hamburg 1994

[21] Schlingemann, D.: On the Algebraic Theory of Soliton and Anti-soliton Sectors. DESY-95-012, Feb 1995. 23pp. to appear in Rev. Math. Phys.

[22] Schlingemann, D.: Kink-States in $P(\phi)_2$-Models. (In preparation)

[23] Schlingemann, D.: Remarks on the Split Property for the Massive Free Scalar-Field. (In preparation)