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Scaling and Diffraction in Deep Inelastic Scattering

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Abstract

We pursue the hypothesis that the events with a large rapidity gap, observed at HERA, reflect the scattering of electrons off lumps of wee partons inside the proton. A simple scaling behaviour is predicted for the diffractive structure functions, which are related to the inclusive structure function $F_2(x, Q^2)$ at small values of the scaling variable x . The results are compared with recent measurements of the diffractive structure function $F_2^D(x, Q^2, M^2)$.

In the so-called “rapidity gap” events, observed and studied at HERA [1]-[3], a system of hadrons is observed with small invariant mass and with a gap in rapidity in the hadronic energy flow adjacent to the proton beam direction. This suggests, that in the scattering process a colour neutral part of the proton with small momentum fraction is stripped off, which then fragments into the hadrons visible in the detector. The proton remnant, carrying most of the energy, escapes undetected close to the proton direction. The rapidity gap reflects the absence of a colour flow between proton and current fragments. In analogy to hadronic processes of similar kind the “rapidity gap” events are also referred to as “diffractive” events.

It is a remarkable feature of this new class of events, that the cross section at large momentum transfer Q^2 is not suppressed relative to the total inclusive cross section. Naively, one might expect that the rate for extracting more than one parton from the proton should rapidly decrease with increasing Q^2 . This, however, is not the case. It is a theoretical challenge to derive the observed “leading twist” behaviour of the diffractive cross section from QCD, the theory of strong interactions.

In a recent paper [4], we have proposed to describe the multi-parton processes underlying the diffractive events by means of an effective lagrangian which specifies the coupling between the virtual photon, the colour singlet wee parton cluster inside the proton and the hadronic final state. Together with further information on the cluster density and the mass spectrum of the final states, one then obtains a prediction for the diffractive differential cross section. Recently, the H1 collaboration at HERA has published a first measurement of the diffractive structure function F_2^D for a large range of the kinematic variables [3]. In this letter, we therefore extend our previous work [4] and compare the results with the recent measurements as well as predictions of other theoretical approaches.

We consider the inelastic scattering process

$$e(k) + p(P) \rightarrow e(k') + \tilde{p}(P') + X(P_X) , \quad (1)$$

where \tilde{p} and X denote the proton remnant and the detected hadronic system, respectively. From the four momenta k , P , $P' = (1 - \xi)P$, $q = k - k'$ and $P_X = q + \xi P$ one obtains the Lorentz invariant kinematic variables

$$s = (k + P)^2 , \quad Q^2 = -q^2 , \quad x = \frac{Q^2}{2q \cdot P} , \quad M^2 = (q + \xi P)^2 . \quad (2)$$

In addition to the first three variables, which characterize ordinary deep inelastic scattering, the invariant mass M of the detected hadronic final state occurs as fourth variable.

Diffractive events have been observed for small values of Bjorken's scaling variable x . In this case one has,

$$\xi = \frac{M^2 + Q^2}{W^2 + Q^2} \simeq x \frac{M^2 + Q^2}{Q^2} , \quad W^2 = (q + P)^2 \simeq \frac{Q^2}{x} , \quad (3)$$

where W is the invariant mass of the total hadronic final state including the proton remnant.

The effective lagrangian used in [4] is based on the hypothesis that the wee parton clusters inside the proton can be described by a scalar field σ , carrying vacuum quantum numbers. The hadronic system in the final state is represented by a spectrum of massive vector states, analogous to generalized vector meson dominance [5]. The coupling between virtual photon, scalar and vector fields is then given the unique dimension five operator,

$$\mathcal{L}_I = -\frac{\infty}{\Delta_*} \sigma(\xi) \mathcal{F}_{\mu\nu}(\xi) \mathcal{V}^{\mu\nu}(\xi) , \quad (4)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ are the field strength tensors of the photon field and the hadronic vector field, respectively. The physical picture, that the virtual photon acts like a disk of radius $1/Q$ [6], determines the length in eq. (4) as $1/\Lambda = e\kappa/Q$, where e is the electric charge and κ is an unknown constant.

The total diffractive cross section can now be expressed as

$$\sigma_D(ep \rightarrow e\tilde{p}X) = \int dx dQ^2 dM^2 d\xi \frac{d\hat{\sigma}(e\sigma \rightarrow eV(M))}{d\xi dQ^2} \rho(Q^2, M^2) f_\sigma(\xi, Q^2) , \quad (5)$$

where $d\hat{\sigma}$ is the quasi-elastic cross section for the production of a vector state of mass M , calculated according to eq. (4), $\rho(Q^2, M^2)$ is the spectral density of vector states, and $f_\sigma(\xi, Q^2)$ is the probability density of finding a wee parton cluster with momentum fraction ξ inside the proton. Here we have assumed that the spectral density only depends on the transverse size and the mass of the produced final state.

The spectral densities ρ_T and ρ_L , assumed in [4] for transversely and longitudinally polarized vector states, were obtained from a fit to the inclusive structure function F_2 at small values of Q^2 and M^2 [7]. In the kinematic range probed at HERA this choice appears no longer appropriate. In the following, we shall instead use the ansatz,

$$\rho_T = \rho_L \equiv \rho = \frac{C}{Q^2 + M^2} . \quad (6)$$

This is a simple interpolation between $\rho \propto 1/Q^2$ for $M^2 \ll Q^2$, and $\rho \propto 1/M^2$ for $Q^2 \ll M^2$, which one may guess based on dimensional analysis. Note, that at large M^2

ρ has to fall off at least as $1/M^2$ in order to satisfy the unitarity bound $\sigma_D \leq \sigma_{max} \propto \ln(W^2/Q^2)/Q^2$. The most general spectral density is obtained by multiplying the ansatz (6) with an arbitrary function of β , where

$$\beta = \frac{Q^2}{Q^2 + M^2} . \quad (7)$$

Some examples and their interpretation will be discussed below.

The diffractive cross section is now easily evaluated. For kinematical reasons, one has

$$\frac{d\hat{\sigma}}{d\xi dQ^2} \propto \delta\left(\xi - x \frac{Q^2 + M^2}{Q^2}\right) . \quad (8)$$

From eqs. (4) and (5) one obtains for the differential cross section in x , Q^2 and M^2 [4],

$$\frac{d\sigma^D}{dx dQ^2 dM^2} = \frac{\pi\alpha^2\kappa^2}{4xQ^4} \left(1 - y + \frac{y^2}{2} - 2\frac{Q^2 M^2}{(Q^2 + M^2)^2} y^2\right) \rho(Q^2, M^2) \xi f_\sigma(\xi, Q^2) . \quad (9)$$

ξ is now the function of x , Q^2 and M^2 , for which the argument of the δ -function in eq. (8) vanishes. It corresponds to the momentum fraction of the parton cluster needed to produce the invariant mass M (cf. (3)).

Defining transverse and longitudinal diffractive structure functions in the usual way,

$$\frac{d\sigma_D}{dx dQ^2 d\xi} = \frac{4\pi\alpha^2}{xQ^4} \left(\left(1 - y + \frac{y^2}{2}\right) F_2^D(x, Q^2, M^2) - \frac{y^2}{2} F_L^D(x, Q^2, M^2) \right) , \quad (10)$$

one obtains ($d\xi = x/Q^2 dM^2$),

$$F_2^D(x, Q^2, M^2) = \frac{\kappa^2 C}{16} f_\sigma(\xi, Q^2) , \quad (11)$$

$$F_L^D(x, Q^2, M^2) = 4\beta(1 - \beta) F_2^D(x, Q^2, M^2) . \quad (12)$$

This result is very simple. Up to an unknown constant, F_2^D is identical with the probability density for finding a wee parton cluster with momentum fraction ξ inside the proton. It is independent of β , which is a consequence of our ansatz for the spectral density. The relation between F_2^D and F_L^D follows from the Lorentz structure of the effective lagrangian (4) and the assumption $\rho_T = \rho_L$. The obtained diffractive cross section scales with Q^2 , i.e., $\sigma_D \propto \int d\xi F_2^D/Q^2 \propto 1/Q^2$. Note, that our calculation does not require the proton remnant \tilde{p} to be a proton. Based on ideas of generalized vector meson dominance, the cross section $\sigma_D(ep \rightarrow epX)$ has been predicted to fall off faster than $1/Q^2$ [8].

In order to proceed further, we have to determine the probability density $f_\sigma(\xi, Q^2)$. Within the parton model, it appears natural to build up f_σ from products of the gluon

density $g(x, Q^2)$ and the sea-quark density $S(x, Q^2)$, which reflect the possible colour singlet states,

$$f_\sigma(\xi, Q^2) = \int_\delta^{\xi-\delta} d\xi' \left(f_g g(\xi', Q^2) g(\xi - \xi', Q^2) + f_S S(\xi', Q^2) S(\xi - \xi', Q^2) + \dots \right) . \quad (13)$$

Here f_g , f_S and $\delta \ll 1$ are constants, and products with three and more parton densities are represented by the dots. The parton densities extracted from deep inelastic scattering are singular at small values of x , such that the integral (13) diverges as the infrared cutoff δ approaches zero. Since the integral is dominated by the contributions from the two regions $\xi' \simeq 0$ and $\xi' \simeq \xi$, one obtains approximately,

$$f_\sigma(\xi, Q^2) \simeq \bar{f}_g g(\xi, Q^2) + \bar{f}_S S(\xi, Q^2) + \dots , \quad (14)$$

where the constants \bar{f}_g and \bar{f}_S depend on the cutoff δ^1 .

At small values of x , several sets of parton densities [9, 10] show the universal behaviour,

$$S(x, Q^2) \propto g(x, Q^2) . \quad (15)$$

For the GRV parton densities, for instance, the ratio g/S is constant within 10% for values of x between 10^{-4} and 10^{-3} [11]. In this case one obtains a simple relation between the diffractive structure function F_2^D and the inclusive structure function $F_2(x, Q^2)$, which is proportional to $xS(x, Q^2)$. From eqs. (11), (14) and (15) one then obtains the simple scaling relation,

$$F_2^D(x, Q^2, M^2) \simeq \frac{D}{\xi} F_2(\xi, Q^2) , \quad (16)$$

with an unknown constant D . Although the diffractive structure function is defined as a function of three variables, it only depends on two kinematic variables, the momentum fraction ξ of the wee parton cluster and Q^2 . The simple connection with the inclusive structure function reflects the fact that the momentum of the parton cluster is essentially carried by a single parton (cf. (14)). The remaining ones only screen the colour. The additional factor $1/\xi$ occurs for kinematical reasons.

The scaling relation (16) can be directly compared with recent measurements of the two structure functions by the H1 collaboration. The experimental data for the diffractive structure function F_2^D are consistent with

$$F_2^D(x, Q^2, M^2) \propto \ln(Q^2) \xi^{-n} , \quad (17)$$

¹In [4], the integral (13) was approximated in a different way which, however, ignored the fact that the dominant contributions come from the region close to the end points of integration.

where $n = 1.19 \pm 0.06 \pm 0.07$ [3]. On the other hand, at small values of x , the data for the inclusive structure function can be parametrized as $F_2(x, Q^2) \propto \ln(Q^2)x^{-d}$, with $d = 0.19$ [12]. These measurements of F_2 and F_2^D are in agreement with the scaling relation (16).

The model described above is similar in spirit to “aligned jet” models (AJM) [13]-[15], where the current fragment is produced by a quark-antiquark pair of transverse size $1/Q$ penetrating the proton. The predictions of the two approaches may be compared in terms of the diffractive structure function or, equivalently, in terms of the diffractive cross section, conventionally defined as ($x \ll 1$),

$$\frac{d\sigma_D}{dM^2} = \frac{4\pi^2\alpha}{Q^2} F_2^D(x, Q^2, M^2) \frac{d\xi}{dM^2} . \quad (18)$$

From eqs. (11), (14), and using $g(\xi, Q^2) \sim S(\xi, Q^2) \sim \xi^{-1-\lambda}$, one obtains

$$\frac{d\sigma_D}{dM^2} \propto \frac{1}{Q^2(Q^2 + M^2)} \xi^{-\lambda} . \quad (19)$$

In its simplest form, neglecting perturbative QCD corrections [14], the AJM predicts the diffractive cross section [13],

$$\frac{d\sigma_D^{AJM}}{dM^2} \propto \frac{1}{(Q^2 + M^2)^2} . \quad (20)$$

Integrated over M^2 , the model yields a total diffractive cross section $\sigma_D \propto 1/Q^2$. Note, that this prediction of “hard diffraction” as a “leading twist” effect was made already before the development of QCD! The structure function corresponding to the AJM cross section is $F_2^D \propto \beta/\xi$. In the cluster model described above, the AJM cross section could be obtained with “flat” parton densities ($\lambda = 0$) and with the choice of the spectral density $\rho^{AJM} \propto Q^2/(Q^2 + M^2)^2$ in eq. (6). Another choice, $\rho^{NZ} \propto Q^2 M^2/(Q^2 + M^2)^3$, would lead to the model of Nikolaev and Zakharov, which predicts for the diffractive cross section [15],

$$\frac{d\sigma_D^{NZ}}{dM^2} \propto \frac{M^2}{(Q^2 + M^2)^3} , \quad (21)$$

corresponding to the structure function $F_2^D \propto \beta(1 - \beta)/\xi$.

The occurrence of jets with large transverse momentum in diffractive events was first predicted based on the idea of a “pomeron structure function” [16], which can also account for the HERA results on the diffractive structure function in deep inelastic scattering [16]-[20]. Here, a pomeron structure function $\tilde{F}_2(\beta, Q^2)$ at some fixed value Q_0^2 is needed as non-perturbative input, where β is now interpreted as momentum fraction of a parton inside the pomeron. The predictions of the model are then the Q^2 -dependence

of this pomeron structure function as well as the “pomeron flux factor”, which plays the role of the cluster density in the model described above. More precise measurements of the ξ -dependence and the β -dependence should be able to distinguish between the various models.

Starting from the hypothesis, that the diffractive events in deep inelastic scattering represent the scattering of electrons off lumps of wee partons, we have obtained a diffractive structure function which is consistent with recent measurements. In particular, a simple scaling relation has been derived between the diffractive and the inclusive structure functions, which appears to be in agreement with experimental data. This suggests, that the momentum of the wee parton cluster is essentially carried by a single parton. Hence, like the inclusive cross section in deep inelastic electron-proton scattering, also the diffractive cross section may be essentially due to incoherent electron-parton scattering. In the case of diffractive processes, however, this interpretation requires some non-perturbative mechanism of colour screening.

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