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# Theoretical Update of the Semileptonic Branching Ratio of B Mesons

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**Abstract:**

We reconsider the prediction of the semileptonic branching ratio of B mesons, using the recent calculation [1] of the radiative corrections with account for finite quark masses in nonleptonic decays and taking into account $1/m_b^2$ corrections. For the semileptonic branching ratio we obtain $B_{SL} = (11.8 \pm 1.6)\%$ using pole quark masses and $B_{SL} = (11.0 \pm 1.9)\%$ using running $\overline{\rm MS}$ quark masses. The uncertainty is dominated by unknown higher order perturbative corrections. We conclude that the present accuracy of the theoretical analysis does not allow to state a significant disagreement with the experimental results. However, our re-analysis of the decay $b \to ccs$ yields an increase of $(35 \pm 11)\%$ due to next-to-leading order corrections including mass dependent terms, which further emphasizes the problem of the average charm quark content of the final states in B decays.

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**1.** The theoretical description of inclusive weak decays of heavy hadrons has made considerable progress over the recent years, see [2] for a review. It could be shown that in the limit of infinite heavy quarks the decay rate coincides with that of the corresponding free quark decay; corrections to that limit are of nonperturbative origin and suppressed by at least two powers in the heavy quark mass [3]. Today there is increasing confidence that QCD predictions of heavy particle decays rest on a firm theoretical foundation. In view of these apparent advances and with the availability of new and more precise data on the semileptonic branching ratio $B_{SL}$ of the B meson [4], the long felt discrepancy between its measured and its predicted value becomes more and more baffling [5, 6]. Over the last years, the main efforts were concentrated on the determination of the size of nonperturbative power-suppressed corrections to the free quark decay, which, however, turned out to be small, of natural size $\sim 1\,\mathrm{GeV}^2/m_b^2 \sim 5\%$, and cannot explain the experimental value of $B_{SL}$. Thus, it seems timely to place more emphasis on *perturbative* radiative corrections to the free quark decay, which since the well-known analysis by Altarelli and Petrarca [7] have not been receiving proper attention in the literature. In this letter we update the theoretical prediction of the semileptonic branching ratio of B mesons using a recent calculation [1] of the charm quark mass dependence of radiative corrections to nonleptonic decays. In addition we re-analyze the decay rate $\Gamma(b \to ccs)$, taking into account the quark mass dependence of radiative corrections and the contributions of penguins. The semileptonic branching ratio is then evaluated using both pole masses and running quark masses. The latter procedure was advocated in [8] on the evidence of the cancellation of renormalon singularities [9]. Finally, we discuss shortly the problem of fixing a proper renormalization scale in heavy quark decays.

**2.** The semileptonic branching ratio of B mesons is defined by

$$B_{SL} \equiv \frac{\Gamma(B \to Xe\nu)}{\sum_{\ell=e,\,\mu,\,\tau}\Gamma(B \to X\ell\nu_\ell) + \Gamma(B \to X_c) + \Gamma(B \to X_{c\bar{c}}) + \Gamma(\text{rare decays})}. \qquad (1)$$

The heavy quark expansion (HQE) allows to relate the inclusive decay rate of a B meson to that of the underlying b quark decay process, apart from $1/m^2$ corrections:

$$\Gamma(B \to X) = \Gamma(b \to x) + \mathcal{O}(1/m_b^2). \qquad (2)$$

The power-suppressed correction terms to the total inclusive widths of both semi– and nonleptonic decays were calculated in [3, 5].

For the free quark decay rates we introduce the following notations:

$$\Gamma(b \to c\ell\nu) = \Gamma_0\,\mathrm{PH}(x_c, x_\ell, 0)\,I(x_c, x_\ell, 0), \qquad (3)$$

$$\Gamma(b \to cud + cus) = 3\Gamma_0\,\mathrm{PH}(x_c, 0, 0)\,\eta(\mu)\,J(x_c, \mu), \qquad (4)$$

$$\Gamma(b \to ccs + ccd) = 3\Gamma_0\,\mathrm{PH}(x_c, x_c, x_s)\,\kappa(x_c, x_s, \mu)\,K(x_c, x_s, \mu). \qquad (5)$$

Here $\Gamma_0$ is defined by $\Gamma_0 = G_F^2|V_{cb}|^2 m_b^5/(192\pi^3)$. $\mathrm{PH}(x_1, x_2, x_3)$ is the tree level phase space factor of the decay $b \to q_1 + W \to q_1 + \bar{q}_2 + q_3$; for arbitrary masses $x_i = m_i/m_b$ it is given by:

$$\mathrm{PH}(x_1,x_2,x_3)=12\int\limits_{(x_2+x_3)^2}^{(1-x_1)^2}\frac{ds}{s}\,(s-x_2^2-x_3^2)\,(1+x_1^2-s)\,w(s,x_2^2,x_3^2)\,w(s,x_1^2,1) \qquad (6)$$

with

$$w(a,b,c)=(a^2+b^2+c^2-2ab-2ac-2bc)^{1/2}. \qquad (7)$$

The functions $\eta$ and $\kappa$ contain the leading-order QCD corrections to the nonleptonic rates $b\to cuq$ and $b\to ccq$, respectively. In particular, $\eta$ is given by [10]

$$\eta(\mu)=\frac{1}{3}\left\{2\left(\frac{\alpha_s(m_W)}{\alpha_s(\mu)}\right)^{4/\beta_0}+\left(\frac{\alpha_s(m_W)}{\alpha_s(\mu)}\right)^{-8/\beta_0}\right\} \qquad (8)$$

with $\beta_0=11-2n_f/3$ for $n_f$ running flavours, $n_f=5$ in our case. The expression for $\kappa(\mu)$ is given below. Finally, $I$, $J$ and $K$ contain the next-to-leading QCD corrections to the decay rates. The function $I$ can be written as

$$I(x_1,x_2,x_3)=1+\frac{2}{3}\frac{\alpha_s}{\pi}\,g(x_1,x_2,x_3), \qquad (9)$$

where $g$ has been calculated in Ref. [11] for arbitrary arguments in terms of an one-dimensional integral. Analytic expressions are available for the special cases $g(x_1,0,0)$ [12] and $g(0,x_2,0)$ [1]. The complete calculation of $J(x_c,\mu)$ was first done in [1], while $J(0,\mu)$ is also available from [13]; some of the terms for arbitrary $x_c$ have been also calculated in [11]. The function $K$ is not yet known completely; we will discuss it below.

Summarizing existing calculations of the radiative corrections, we give the numerical values of $g(x_c,0,0)$, $g(x_c,x_\tau,0)$ and $J(x_c,m_b)$ in Table 1. The numbers are evaluated for $\alpha_s(m_Z)=0.117$, i.e. $\Lambda^{(4)}_{\overline{\rm MS}}=312\,\mathrm{MeV}$, and at the renormalization scale $\mu=m_b=4.8\,\mathrm{GeV}$. With these parameters, we find $\eta(m_b)=1.10$.

**3**. As explained above, all the decay rates entering the semileptonic branching ratio (1) are known to next-to-leading order in the strong interaction including final state particle mass effects, except for $\Gamma(b\to ccs+ccd)$ and the rare decays. Whereas the latter can safely be neglected, the channel $b\to ccs$ deserves a closer consideration. In addition to the contributions studied in Ref. [1], where a 30% increase of the decay rate $b\to ccs$ by radiative corrections was obtained[1], we take into account the dependence of these corrections on the s quark mass and discuss the contributions of penguins.

The leading order decay rate can be written as

$$\Gamma(b\to c\bar{c}s)\Big|_{\rm LO}=3\Gamma_0|V_{cs}|^2\,\mathrm{PH}(x_c,x_c,x_s)\left\{\sum_{i=1}^{6}c_i^2(\mu)+2\left[\frac{1}{3}\,c_1(\mu)c_2(\mu)+c_1(\mu)c_3(\mu)\right.\right.$$

[1]See also [14].

$$+\frac{1}{3}\,c_1(\mu)c_4(\mu)+\frac{1}{3}\,c_2(\mu)c_3(\mu)+c_2(\mu)c_4(\mu)+\frac{1}{3}\,c_3(\mu)c_4(\mu)+\frac{1}{3}\,c_5(\mu)c_6(\mu)\Big]$$

$$-\,2f(x_c,x_c,x_s)\Big[c_1(\mu)c_5(\mu)+\frac{1}{3}\,c_1(\mu)c_6(\mu)+\frac{1}{3}\,c_2(\mu)c_5(\mu)+c_2(\mu)c_6(\mu)$$

$$+c_3(\mu)c_5(\mu)+\frac{1}{3}\,c_3(\mu)c_6(\mu)+\frac{1}{3}\,c_4(\mu)c_5(\mu)+c_4(\mu)c_6(\mu)\Big]\Big\} \qquad (10)$$

$$\equiv 3\Gamma_0|V_{cs}|^2\,\mathrm{PH}(x_c,x_c,x_s)\,\kappa(x_c,x_s,\mu). \qquad (11)$$

The coefficients $c_i(\mu)$, $1\leq i\leq 6$, are the leading order Wilson-coefficients multiplying the operators $Q_i$ in the effective Lagrangian and can be found in tabulated form in [15]. Unlike the expression in Ref. [7], our Eq. (10) also takes into account the interference of four-quark operators having the usual $(V-A)\otimes(V-A)$ structure with penguin operators of the structure $(V-A)\otimes(V+A)$. These interference terms are explicitly of order $x_c^2$ and enter the decay rate with a weight-function $f$, given by

$$f(x_c,x_c,x_s)=\frac{1}{\mathrm{PH}(x_c,x_c,x_s)}\int\limits_{(x_c+x_s)^2}^{(1-x_c)^2} ds\,\frac{6x_c^2}{s^2}\,w(s,x_c^2,x_s^2)\,w(1,s,x_c^2)\,(s+x_s^2-x_c^2)\,(1+s-x_c^2). \qquad (12)$$

For reasonable quark masses $x_c=0.3$ and $x_s=0.04$ we find $f=0.24$ and $\kappa(\mu=m_b=4.8\,\mathrm{GeV})=1.07$. Neglecting the penguin-contributions, i.e. for $c_i(\mu)\equiv 0$ for $i\geq 3$, $\kappa(m_b)$ coincides with $\eta(m_b)=1.10$, so that the penguins interfere destructively and reduce the decay rate by $\sim 3\%$ similarly to what was observed in [7].

In next-to-leading order, the decay rate can be written as in Eq. (5), where $K$ is defined by

$$\kappa(x_c,x_s,\mu)K(x_c,x_s,\mu)\equiv\sum_{i,j=1}^{6} f_{ij}(x_c,x_s)c_i(\mu)c_j(\mu)d_{ij}(x_c,x_s,\mu), \qquad (13)$$

the weight-factors $f_{ij}$ being given in (10), whereas the $d_{ij}$ have the structure

$$d_{ij}=1+k_{ij}\,\frac{\alpha_s(\mu)}{\pi}+r_{ij}\,\frac{\alpha_s(m_W)-\alpha_s(\mu)}{\pi}+\mathcal{O}(\alpha_s^2). \qquad (14)$$

The terms $r_{ij}$ contain matching-coefficients and two-loop anomalous dimensions of the operators $Q_i$ and can be obtained from [15]. The terms $k_{11}$ and $k_{22}$ can be obtained from [1, 11], including all dependence on $x_c$ and $x_s$, likewise $k_{12}$ for $x_c=x_s=0$ and, partly, also in dependence on $x_c$ and $x_s$. The other terms are not known. Nevertheless, the knowledge of these three coefficients allows a rather accurate determination of the decay rate to next-to-leading accuracy: for the unknown $d_{ij}$, we most conservatively assume $0<d_{ij}<2$, which corresponds to $|k_{ij}|<15$ for $\alpha_s=0.2$. For $d_{12}$, we replace the uncalculated term, $H_e(x_c,x_s)$ in the notation of [1], by its corresponding value for only one massive c quark, $G_e(x_c)$. We estimate the error introduced by this procedure by $\Delta H_e\approx 2|G_e(x_c)-G_e(0)|$. The values of the relevant $k_{ij}$ are given in Table 2, together with the functions $\kappa$ and

$K$, the latter one yielding the increase of $\Gamma(b \to ccs)$ due to next-to-leading order QCD corrections.

At this point, it is worthwhile to emphasize that the dependence of the decay rate on the s quark mass is rather weak. While the phase space factor is considerably reduced by including the strange mass [7], this effect turns out to be to a large extent compensated by the increase of radiative corrections. For $x_c = 0.3$ we find that a strange quark mass $m_s = 200$ MeV, $x_s = 0.04$, reduces the decay rate by 1.5% only, which is smaller than the effect of the penguins.

Taking everything together, we find that for $x_c = 0.3$ and $x_s = 0.04$, next-to-leading order radiative corrections increase $\Gamma(b \to ccs)$ by $(35 \pm 7^{+8}_{-7})\%$, where the first error is a very conservative estimate of the unknown parts of the next-to-leading corrections and the second error comes from a variation of the renormalization scale $\mu$ within $m_b/2 < \mu < 2m_b$.

**4**. We have now all ingredients at hand to evaluate $B_{SL}$. Before doing so, however, let us make some remarks about the nonperturbative corrections entering the decay rates. They can be expressed in terms of two hadronic matrix elements, $\lambda_1$ and $\lambda_2$. Whereas $\lambda_2$ is directly related to the observable spectrum of beautiful mesons,

$$\lambda_2 \approx \frac{1}{4}\,(m_{B^*}^2 - m_B^2) = 0.12\,\mathrm{GeV}^2, \tag{15}$$

the quantity $-\lambda_1/(2m_b)$, which can be interpreted as the average kinetic energy of the b quark inside the B meson, is only difficult to measure. At present, only a QCD sum rule calculation is available, according to which $\lambda_1 = -(0.5 \pm 0.1)\,\mathrm{GeV}^2$ [16]. For a summary of the discussion about $\lambda_1$ we refer to [17]. The formulas for the decay rates including power-suppressed corrections are given in [5].

We next have to fix the quark masses that enter the decay rates. For the strange quark mass we use $m_s = 0.2\,\mathrm{GeV}$; as emphasized in the last section, the decay rates are not very sensitive to this parameter. As for the heavy quark masses, we make use of the fact that in the framework of HQE the difference between $m_b$ and $m_c$ is fixed:

$$m_b - m_c = m_B - m_D + \frac{\lambda_1 + 3\lambda_2}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right) + \mathcal{O}\left(\frac{1}{m_Q^2}\right). \tag{16}$$

For $m_b$ we use $m_b = (4.8 \pm 0.2)\,\mathrm{GeV}$. Varying the renormalization scale $\mu$ within the range $m_b/2 < \mu < 2m_b$, we find

$$B_{SL} = (11.8 \pm 0.8 \pm 0.5 \pm 0.2 \pm 0.2^{+0.9}_{-1.3})\%\,, \tag{17}$$

which is our main result. Here the first error comes from the uncertainty in $m_b$, the second one from the one in $\alpha_s(m_Z) = 0.117 \pm 0.007$, the third one from the uncertainty in $\lambda_1$ and the fourth from the uncertainty in $\Gamma(b \to ccs)$. The last error comes from the variation of the renormalization scale[2].

[2]Larger values of $m_b$ and (or) of the normalization scale generally yield a larger $B_{SL}$, while the increase of $\alpha_s$ tends to lower the branching ratio.

The error stemming from the uncertainty in $\mu$ is rather big and shows that higher order perturbative corrections are important. We thus feel motivated to evaluate $B_{SL}$ also in a different scheme, using running short–distance masses, e.g. $\overline{\rm MS}$ masses. This procedure has also been advocated in connection with the cancellation of renormalon contributions [9]. In order to keep the formulas scheme-independent at $\mathcal{O}(\alpha_s)$, the phase-space has to be modified according to

$$m_b^5\,\mathrm{PH}(x_c,0,0) \longrightarrow \bar{m}_b^5\,\mathrm{PH}(\bar{x}_c,0,0)\left\{1+\frac{\bar{\alpha}_s}{\pi}\left(\frac{20}{3}-5\ln\frac{\bar{m}_b^2}{\mu^2}-2\bar{x}_c\ln\bar{x}_c\,\frac{d\ln\mathrm{PH}(\bar{x}_c,0,0)}{d\bar{x}_c}\right)\right\}, \tag{18}$$

where $\bar{x}_i$ denotes running quantities evaluated at the scale $\mu$. With this substitution, we obtain

$$\bar{B}_{SL} = (11.0 \pm 0.6 \pm 0.8 \pm 0.2 \pm 0.2^{+1.0}_{-2.2})\% \tag{19}$$

In Table 3 we give a comparison of theoretical predictions for $B_{SL}$ using different approximations. The main result of our analysis is that the prediction of Altarelli and Petrarca is lowered by more than 1%. It is clearly visible that the main effect comes from taking into account the quark mass dependence of radiative corrections calculated in [1], while the nonperturbative $1/m_b^2$ corrections result in a $\sim 0.2\%$ decrease, in agreement with [5].

The results shown above clearly demonstrate that the main theoretical uncertainty in the predictions for B decays comes from the scale and scheme dependence of the results. As always, the only consistent way to reduce both is to make a next-to-next-to-leading (NNLO) calculation of the decay rate including $\mathcal{O}(\alpha_s^2)$ corrections, which is a formidable enterprise. Lacking this calculation, one is bound to make rather crude estimates of the possible higher-order corrections using some particular prescription to fix the scale. Among these, the BLM prescription [18] seems to us to be the only one that is physically motivated. We now discuss in short its possible outcome on the B decay widths.

The idea underlying the BLM approach is that the major part of higher-order radiative corrections originates from the necessity to evaluate Feynman diagrams with the running coupling at the scale of the gluon virtuality and can be traced by a relatively simple calculation of the diagrams with an extra fermion bubble in the gluon line. The corresponding calculations have been done recently [19, 20] and indicate that the natural scale in the radiative corrections in B decays is significantly smaller than $m_b$. In particular, neglecting the c quark mass, Ref. [20] finds $\mu_{BLM} = 0.07m_b$ for the radiative corrections to the semileptonic decay,[3] while for the final state interaction of quarks in the nonleptonic decays $\mu_{BLM} \sim 0.32m_b$ is found, as indicated by the studies of the $\tau$ lepton hadronic decay width. In fact, the particular scale entering the radiative corrections to the semileptonic width turns out to be not very important for the problem of the semileptonic branching ratio, since these corrections cancel to a large extent between numerator and denominator

[3]Note that this result is contrary to expectations in [5], where the choice of a low scale in the radiative corrections was criticized.

of Eq. (1).[4] The main problem in applying the BLM scheme to nonleptonic B decays is that the radiative corrections in NNLO do not factorize into "semileptonic" and "final-state-interaction" parts. On may thus suspect that a large part of the radiative corrections comes from other types of diagrams than those considered in the BLM method (see the discussion in [14]). In addition, it is not clear how to apply this approach consistently to processes where the relevant operators possess a nontrivial anomalous dimension. Still, we believe that the low scales indicated by the BLM prescription are more natural in B decays. Thus the choice $\mu = 2m_b$ adopted above as one extreme case is in fact rather unlikely, while $\mu = m_b/2$ is presumably more relevant. Adopting this scale, our result for the semileptonic branching ratio in (17) becomes $B_{SL} = (10.5 \pm 1.4)\%$, in perfect agreement with the experimental value $B_{SL}^{\exp} = (10.4 \pm 0.4)\%$ [4]. Summarising, we conclude that there is no evidence for any disagreement between the experimental data and the theoretical prediction for the semileptonic branching ratio of B mesons.

The situation is not so clear, however, with the charm content in the final states. With the 35% increase of the $b \to c\bar{c}s$ rate induced by taking into account the c quark mass in the radiative corrections, this problem is strengthened. From our analysis we get

$$\langle\, n_c \,\rangle = 1.28 \pm 0.08, \tag{20}$$

which is to be confronted with the experimental result $\langle\, n_c \,\rangle^{\exp} = 1.04 \pm 0.07$ [21]. We are not aware of any natural theoretical possibility to lower the value given in (20), unless the c quark mass is much larger than expected, which would conflict, however, with the heavy quark expansion of the meson masses, Eq. (16).

[4]In the case at hand, where the BLM prescription indicates a *very* low scale, we find it more appropriate not to change the scale $\mu = m_b$ to $\mu = 0.07 m_b$, but rather to use the calculation in [20] as an explicit estimate of the $\alpha_s^2(m_b)$ correction.

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## Tables

| $x_c$ | $g(x_c, 0, 0)$ | $g(x_c, x_\tau, 0)$ | $J(x_c, m_b)$ |
|---|---|---|---|
| 0 | $-3.62$ | $-3.37$ | 1.009 |
| 0.1 | $-3.25$ | $-2.89$ | 1.026 |
| 0.2 | $-2.84$ | $-2.42$ | 1.046 |
| 0.3 | $-2.51$ | $-2.08$ | 1.063 |
| 0.4 | $-2.23$ | $-1.81$ | 1.077 |
| 0.5 | $-2.01$ | $-1.61$ | 1.088 |
| 0.6 | $-1.83$ | $-1.45$ | 1.097 |
| 0.7 | $-1.70$ | | 1.105 |
| 0.8 | $-1.59$ | | 1.113 |
| 0.9 | $-1.53$ | | 1.123 |
| 1 | $-1.50$ | | |

Table 1: Next-to-leading order corrections to the semileptonic b quark decay rates and the decay $b \to cud$ as functions of $x_c = m_c/m_b$. Parameters: $\mu = m_b = 4.8\,\text{GeV}$, $\Lambda^{(4)}_{\overline{\text{MS}}} = 312\,\text{MeV}$, corresponding to $\alpha_s(m_Z) = 0.117$; $x_\tau = m_\tau/m_b$. Note that $J(1, \mu)$ diverges like $\sim \ln(1 - x_c)$.

| $x_c$ | $\kappa(x_c, x_s, m_b)$ | $k_{11}$ | $k_{12}(\mu = m_b)$ | $k_{22}$ | $K(x_c, x_s, m_b)$ |
|---|---|---|---|---|---|
| 0 | 1.054 | $-1.33$ | $-7.59 \pm 0.01$ | $-1.26$ | $1.02 \pm 0.05$ |
| 0.1 | 1.056 | $-0.05$ | $-6.65 \pm 0.07$ | $-0.35$ | $1.09 \pm 0.06$ |
| 0.2 | 1.062 | 2.53 | $-4.97 \pm 0.20$ | 1.23 | $1.20 \pm 0.06$ |
| 0.3 | 1.069 | 6.69 | $-2.64 \pm 0.57$ | 3.41 | $1.35 \pm 0.07$ |
| 0.4 | 1.077 | 15.68 | $1.24 \pm 0.96$ | 7.09 | $1.62 \pm 0.09$ |

Table 2: The leading and next-to-leading order corrections to the nonleptonic decay $b \to ccs$. The errors rely on a conservative estimate of the unknown parts of the next-to-leading order terms, mostly due to penguin contributions. The last column gives the increase of the decay rate $\Gamma(b \to ccs)$ in next-to-leading order including finite c and s quark effects in the radiative corrections. The input parameters are the same as in Table 1; $x_s = 0.04$.

| $\alpha_s(m_Z)$ | Parton Model [7] pole masses | HQE [5] pole masses | HQE [this work] pole masses | $\overline{\text{MS}}$ masses |
|---|---|---|---|---|
| 0.110 | 0.133 | 0.132 | 0.123 | 0.117 |
| 0.117 | 0.130 | 0.128 | 0.118 | 0.110 |
| 0.124 | 0.125 | 0.123 | 0.113 | 0.102 |

Table 3: Theoretical predictions for the semileptonic branching ratio $B_{SL}$ depending on $\alpha_s(m_Z)$. Input parameters: $m_b = 4.8\,\text{GeV}$, $m_c = 1.33\,\text{GeV}$ (pole masses), corresponding to $\lambda_1 = -0.5\,\text{GeV}^2$, $m_s = 0.2\,\text{GeV}$. Renormalization scale: $\mu = m_b$.

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# Erratum: Theoretical Update of the Semileptonic Branching Ratio of B Mesons [Phys. Lett. B 342 (1995) 362]

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In the third line in Eq. (10) on page 364 there is a sign error: $-2f(x_c, x_c, x_s)$ should read $+2f(x_c, x_c, x_s)$. In addition, we have found an error in the computer program, which affected the average charm content $n_c$ and the scale dependence of the results in the $\overline{\rm MS}$ scheme. We take this opportunity to incorporate the complete results for the quark mass dependence of the radiative corrections to $b \to ccs$ calculated in [22]. The corresponding update of our Table 2 on page 365 is given in Table 2 in [22].

The numerical impact of these corrections is marginal: Eqs. (17) and (19) on page 366 should read:

$$B_{SL} = (12.0 \pm 0.7 \pm 0.5 \pm 0.2^{+0.9}_{-1.2})\% \,, \tag{17}$$

$$\bar{B}_{SL} = (11.3 \pm 0.6 \pm 0.7 \pm 0.2^{+0.9}_{-1.7})\% \,. \tag{19}$$

Table 3 on page 367 has to be replaced by the Table given below.

Since the problem of the average charm content is receiving increasing attention (see, e.g.[23]), we give the corrected result for $n_c$ in a somewhat expanded form. Eq. (20) on page 367 is to be substituted by

$$n_c = 1.24 \pm 0.05 \pm 0.01 \,, \tag{20}$$

which shows the result in the OS scheme. The first error comes from the uncertainty in $m_b = (4.8 \pm 0.2)\,\text{GeV}$, the second one from the uncertainties in the remaining parameters. In the $\overline{\rm MS}$ scheme we get

$$\bar{n}_c = 1.30 \pm 0.03 \pm 0.03 \pm 0.01 \,, \tag{20'}$$

where again the first error comes from the uncertainty in the quark masses, the second one is due to the variation of $\alpha_s$, and the third one comprises the remaining uncertainties.

We have added a figure showing the charm content versus the semileptonic branching ratio, cf. [23], obtained by relaxing the constraint on the quark masses following from the heavy quark expansion in Eq. (16) on page 366 and allowing for a larger range of the ratio $m_c/m_b$. Note that $m_c/m_b$ is scale-independent; both $n_c$ and $B_{SL}$ are functions of $m_c/m_b$, $\mu$ and $\alpha_s(\mu)$.

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# References

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| | Parton Model [7] | HQE [5] | HQE [this work] | |
|---|---|---|---|---|
| $\alpha_s(m_Z)$ | pole masses | pole masses | pole masses | $\overline{\text{MS}}$ masses |
| 0.110 | 0.133 | 0.132 | 0.124 | 0.120 |
| 0.117 | 0.130 | 0.128 | 0.120 | 0.113 |
| 0.124 | 0.125 | 0.123 | 0.114 | 0.105 |

Table 3: Theoretical predictions for the semileptonic branching ratio $B_{SL}$ as a function of $\alpha_s(m_Z)$. Input parameters: $m_b = 4.8\,\text{GeV}$, $m_c = 1.33\,\text{GeV}$ (pole masses), corresponding to $\lambda_1 = -0.5\,\text{GeV}^2$, $m_s = 0.2\,\text{GeV}$. Renormalization scale: $\mu = m_b$.

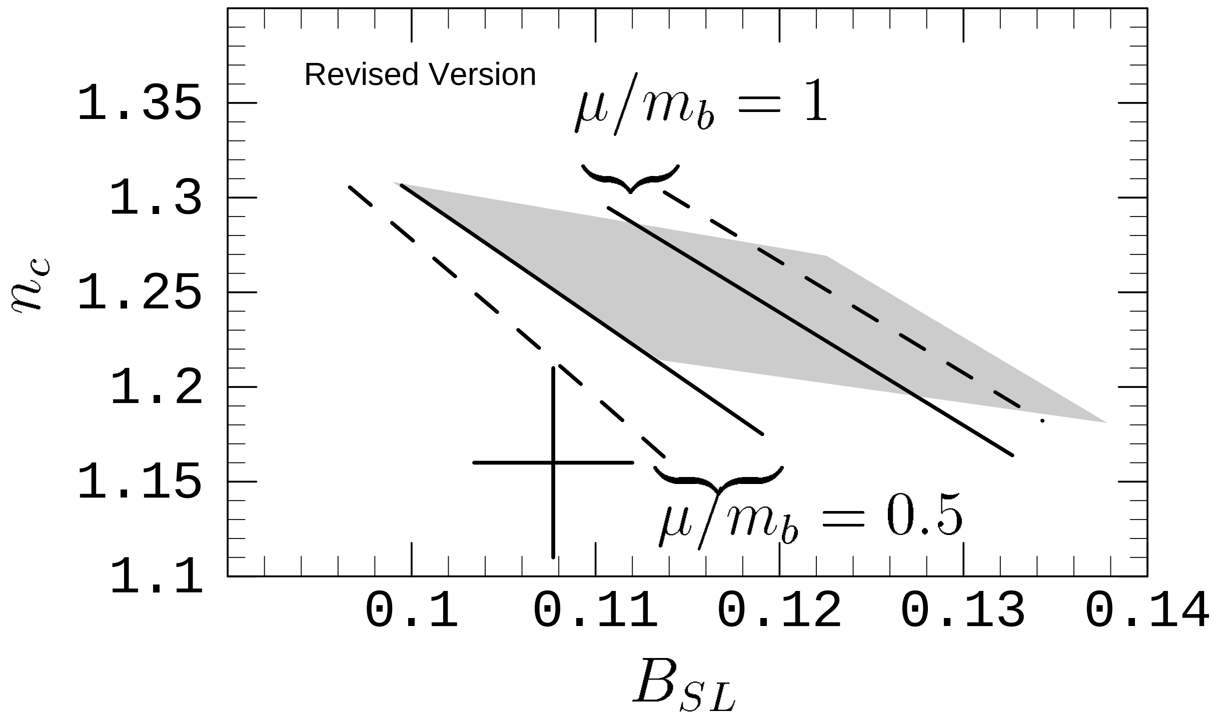


Figure 1: The charm content $n_c$ vs. $B_{SL}$. Solid lines: theoretical predictions in the OS scheme for $0.23 < m_c/m_b < 0.33$, dashed lines: the same in the $\overline{\mathrm{MS}}$ scheme for $0.18 < \bar{m}_c(\bar{m}_c)/\bar{m}_b(\bar{m}_b) < 0.28$. Shaded area: theoretical predictions in the OS scheme with $m_c$ obtained from Eq. (16) and varying $m_b$, $\lambda_1$ and $\mu$ within the range of values given in the text. The experimental data point is taken from [24].