DESY 94-020 hep-ph/9402314 ISSN 0418-9833
RAL/94-021
February 1994

QUANTUM EFFECTS ON HIGGS-BOSON PRODUCTION AND DECAY DUE TO MAJORANA NEUTRINOS

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ABSTRACT

We analyze the phenomenological implications for new electroweak physics in the Higgs sector in the framework of $SU(2)_L \otimes U(1)_Y$ theories that naturally predict heavy Majorana neutrinos. We calculate the one-loop Majorana-neutrino contributions to the decay rates of the Higgs boson into pairs of quarks and intermediate bosons and to its production cross section via bremsstrahlung in e^+e^- collisions. It turns out that these are extremely small in three-generation models. On the other hand, the sizeable quantum corrections generated by a conventional fourth generation with a Dirac neutrino may be screened considerably in the presence of a Majorana degree of freedom.

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1 Introduction

The well-established "see-saw" mechanism, as suggested by Yanagida, Gell-Mann, Ramond, and Slansky [1] in grand unified theories [2], could serve as a natural solution to the problem of the smallness in mass of the three known neutrinos, ν_e, ν_μ, and ν_τ, provided that these are Majorana particles. Such a solution entails the existence of very heavy neutral leptons, which also have to be of Majorana type. If heavy Majorana neutrinos are assumed to be realized in nature at the mass scale of 1–10 TeV, they may manifest themselves in lepton-number violating processes involving the Z and Higgs bosons [3,4] or through non-universality effects in diagonal leptonic Z-boson decays [5]. Their presence may also influence [6,7] the electroweak oblique parameters, S, T, U (or ε_1, ε_2, ε_3) [8], as well as the parameters X, Y, Z [9] introduced recently. The masses of such heavy Majorana neutrinos and their couplings to ordinary matter should satisfy a large set of stringent constraints coming from a global analysis of charged-current universality, neutral-current effects, and other low-energy data [10], which set severe limits on the prospects of observing these particles in high-energy experiments.

Making use of the full power of existing data to constrain new electroweak physics, we find that, in a large class of extensions of the minimal Standard Model (SM) by Majorana neutrinos, the Higgs sector is feebly confined. We demonstrate this by elaborating minimal scenarios which extend the field content of the SM by introducing right-handed neutrinos [1,11]. In addition, new electroweak physics may arise from the possible existence of fourth-generation Majorana neutrinos [12], which can be added in a natural way without conflicting with the data from the CERN Large Electron Positron Collider (LEP) and the SLAC Linear Collider (SLC) [12,13]. Such scenarios can account also for the mass hierarchy problem of the light neutrinos, since the light neutrinos acquire their masses radiatively at the two-loop level [14]. By the same token, this resolves the solar-neutrino deficit problem [15], through the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [16]. However, it has recently been argued that the Hill-Paschos model [12] containing Majoron fields (J) violates astrophysical constraints by predicting too big Jee, Juu, and Jdd couplings [17]. Subsequently, it has been shown that, even in Majoron models with *three* generations, astrophysical constraints limit considerably the testability of such models by terrestrial experiments [18]. In order to avoid that our analysis depends on whether Majoron scalars are present in the model or not, we shall consider Majoronless models by assuming that the Majorana mass terms in the Yukawa sector are bare. Similar Majoronless scenarios can be realized if the SM gauge group is extended by an additional hypercharge group, $U(1)_{Y'}$ [19]. In such theories, Majoron fields are completely absent. For simplicity, we shall also assume the absence of Majoron-triplet scalars [20], as they seem to be ruled out by the present LEP data.

This work is organized as follows: In Sect. 2, we shall give a short description of the basic low-energy structure of the SM with right-handed neutrinos. In Sect. 3, we shall compute the quantum corrections to the $q\bar{q}H$, WWH, ZZH, and ZAH vertices induced by heavy Majorana neutrinos along with additional charged leptons and quarks. We shall also briefly outline our renormalization scheme. In Sect. 4, we shall discuss our numerical results and assess the possibility of discovering in the Higgs sector new electroweak physics beyond the minimal SM. Section 5 contains our conclusions.

2 $SU(2)_L \otimes U(1)_Y$ theories with right-handed neutrinos

Heavy Majorana neutrinos can naturally be predicted in extensions of the SM where $\Delta L = 2$ operators have been introduced in the Yukawa sector by the inclusion of right-handed neutrinos. The quark sector of such extensions is similar to that of the minimal SM. For the leptonic sector, we shall adopt the conventions of Ref. [11]. Specifically, the interactions of the Majorana neutrinos, n_i, and charged leptons, l_i, with the gauge bosons, $W^\pm$ and Z, and the Higgs boson, H, are described by the following Lagrangians [11]:

$$\mathcal{L}^W_{int} = -\frac{g_W}{2\sqrt{2}} W^{-\mu} \sum_{i=1}^{n_G} \sum_{j=1}^{2n_G} \bar{l}_i \; B_{l_i j} \gamma_\mu (1-\gamma_5) \; n_j \; + \; \text{h.c.} \,, \tag{2.1}$$

$$\mathcal{L}^Z_{int} = -\frac{g_W}{4\cos\theta_W} Z^\mu \sum_{i,j=1}^{2n_G} \bar{n}_i \gamma_\mu \Big[i\mathrm{Im}C_{ij} \; - \; \gamma_5 \mathrm{Re}C_{ij} \Big] n_j \,, \tag{2.2}$$

$$\mathcal{L}^H_{int} = -\frac{g_W}{4M_W} \; H \sum_{i,j=1}^{2n_G} \bar{n}_i \Bigg[(m_i + m_j)\mathrm{Re}C_{ij} + \; i\gamma_5 (m_j - m_i)\mathrm{Im}C_{ij} \Bigg] n_j \,, \tag{2.3}$$

where m_i are the masses of n_i and B and C are $n_G \times 2n_G$ and $2n_G \times 2n_G$ dimensional mixing matrices, respectively, with n_G being the number of generations. These matrices are defined as

$$B_{l_i j} = \sum_{k=1}^{n_G} V^l_{l_i k} U^{\nu *}_{kj} \,, \tag{2.4}$$

$$C_{ij} = \sum_{k=1}^{n_G} U^\nu_{ki} U^{\nu *}_{kj} \,, \tag{2.5}$$

where V^l and U^ν are the leptonic Cabbibo-Kobayashi-Maskawa (CKM) matrix and the unitary matrix that diagonalizes the $2n_G \times 2n_G$ "see-saw" neutrino mass matrix, respectively. B and C satisfy a number of identities, which guarantee the renormalizabilty of our minimally extended model, namely [4,11]

$$\sum_{k=1}^{2n_G} B_{l_1 k} B^*_{l_2 k} = \delta_{l_1 l_2}, \qquad \sum_{k=1}^{2n_G} C_{ik} C^*_{jk} = C_{ij}, \qquad \sum_{k=1}^{2n_G} B_{lk} C_{ki} = B_{li}, \qquad \sum_{k=1}^{n_G} B^*_{l_k i} B_{l_k j} = C_{ij}, \tag{2.6}$$

$$\sum_{k=1}^{2n_G} m_k C_{ik} C_{jk} = 0, \qquad \sum_{k=1}^{2n_G} m_k B_{lk} C^*_{ki} = 0, \qquad \sum_{k=1}^{2n_G} m_k B_{l_1 k} B_{l_2 k} = 0. \tag{2.7}$$

Equations (2.6) and (2.7) allow us to verify the cancellations of ultraviolet divergences in the one-loop renormalizations of the $q\bar{q}H$, WWH, ZZH, and ZAH vertices. The only information used to obtain all these identities is the gauge structure of the SM. Therefore, our theoretical analysis does not depend on the explicit form of the Majorana-neutrino mass matrix. In fact, a vast number of possible mass ansätze have been proposed in the literature during the last few years [21]. However, all these mass models possess the very same low-energy gauge structure discussed here and thus obey the sum rules of Eqs. (2.6) and (2.7). In other words, the specific form of any such mass matrix produces only supplementary relations between m_i, B_{li}, and C_{ij} on top of the identities of Eqs (2.6) and (2.7). Similarly, the quark mass matrices given, *e.g.*, by the Fritzsch texture lead to additional relations between the quark masses and CKM mixings [22].

Another interesting feature of the SM with right-handed neutrinos is that out-of-equilibrium lepton-number-violating decays of heavy Majorana neutrinos can generate a non-zero lepton number (L) [23] in the universe through the L-violating interactions of Eqs. (2.1)–(2.3). This excess in L then gives rise to a baryon-number (B) asymmetry of the universe via the $(B + L)$-violating sphaleron interactions, which are in thermal equilibrium above the critical temperature of the electroweak phase transition [24]. This mechanism has received much attention recently, and many studies have been devoted to constrain the $(B-L)$-violating mass scale and so to derive a lower mass bound for the heavy Majorana neutrinos [25–29], exploiting the dramatic effect of out-of-equilibrium conditions for the $\Delta L = 2$ operators. Yet, it was conceivable that certain scenarios predicting heavy Majorana neutrinos with m_N = 1–10 TeV could naturally account for the existing B asymmetry in the universe (BAU) [25]. Furthermore, on the basis of a two-generation scenario of right-handed neutrinos, it was argued [26] that the m_N lower bound of ~ 1 TeV had been underestimated by many orders of magnitude. As a consequence, the opportunity of probing Majorana-neutrino physics at LEP energies would be extinguished practically. Fortunately, very recently, careful inspection of chemical potentials in the framework of three generations with lepton-flavour mixings have revealed that all these stringent bounds can be weakened dramatically and are quite model dependent [28,29]. In particular, it is sufficient that one individual lepton number, L_e say, is conserved in order to protect any primordial BAU generated at the GUT scale from being erased by sphalerons, even if operators with $\Delta L_{l_i} \neq \Delta L_e$ were in thermal equilibrium [28]. The reason is that sphalerons generally conserve the quantum numbers $B/3 - L_{l_i}$ [27,28] and thus protect any pre-existing BAU from being washed out. Similar conclusions have been drawn in Ref. [29], where it was suggested that the BAU may be preserved by an asymmetry in the number of right-handed electrons. These new observations support our assumptions concerning viable scenarios of heavy Majorana neutrinos with masses in the TeV range, which couple very feebly to a separate leptonic quantum number, so that, for instance, $\Delta L_e = 0$.

3 Heavy Majorana neutrinos and Higgs phenomenology

In this section, we shall analyze quantitatively the implications of Majorana neutrinos for the Higgs sector at the quantum level. Specifically, we shall consider the Higgs-boson decays $H \to q\bar{q}$, $H \to WW$, and $H \to ZZ$ as well as the production mechanism $e^+e^- \to ZH$. Since all these processes have been studied at one loop in the SM already [30], we shall focus attention on the quantum corrections induced by the extended fermion sector described in Sect. 2. Each generation contains two Majorana neutrinos, one charged lepton, an up-type quark, and a down-type quark, so that the anomalies cancel. Since we wish to estimate the size of new physics both in three- and four-generation models, we shall keep the number of generations arbitrary.

Loop calculations in electroweak physics are frequently carried out in the on-mass-shell scheme, which uses the fine-structure constant, α, and the physical particle masses as basic parameters [31]. One drawback of this scheme is the occurrence of large logarithms connected with light charged fermions, which drive the renormalization scale of α from m_e to M_Z and artificially enhance the corrections. These logarithms may be removed from the corrections and resummed by expressing the Born result in terms of the Fermi constant, $G_F = \left[\pi\alpha/\sqrt{2}s_w^2 M_W^2(1-\Delta r)\right]$, where Δr embodies the non-photonic corrections to the muon decay rate [32]. As a consequence, a multiple of Δr is added to the correction in such a way that the large logarithms are exactly cancelled. This procedure is sometimes called modified on-mass-shell (MOMS) scheme.

3.1 The decay $H \to q\bar{q}$

The one-loop electroweak corrections to the $H \to q\bar{q}$ decay width are well known within the minimal SM [33]. The contribution due to the fermion sector modified by the presence of Majorana neutrinos as described in Sect. 2, relative to the Born decay width,

$$\Gamma_0(H \to q\bar{q}) \;=\; \frac{N_c G_F M_H m_q^2}{4\pi\sqrt{2}} \left(1 \;-\; \frac{4m_q^2}{M_H^2}\right)^{3/2}, \tag{3.1}$$

with $N_c = 3$, may be calculated in the MOMS scheme from

$$\delta_q \;=\; -\,\mathrm{Re}\Pi'_{HH}(M_H^2) \;-\; \frac{\Pi_{WW}(0)}{M_W^2} \;-\; \frac{2}{s_w c_w}\,\frac{\Pi_{ZA}(0)}{M_Z^2} \;-\; \Delta r_{direct}, \tag{3.2}$$

where Π_{WW}, Π_{ZA}, and Π_{HH} denote unrenormalized vacuum-polarization functions and Δr_{direct} comprises the non-photonic vertex and box contributions to Δr [32]. As a consequence of electromagnetic gauge invariance, $\Pi_{ZA}(0) = 0$ for fermionic contributions. Throughout this work, we shall assume that the novel heavy Majorana neutrinos couple so weakly to the electron and muon that their contribution to Δr_{direct} may be neglected, which agrees with observations by other authors [10,19].

Note that Majorana neutrinos do not contribute through triangle diagrams to $\Gamma(H \to q\bar{q})$ at one loop. This is quite different for lepton pair production. However, the decays into the known lepton flavours are suppressed by the smallness of the Yukawa couplings, and this is not expected to be changed by virtual Majorana-neutrino effects. We shall leave the study of the leptonic decays for future work. Furthermore, it is interesting to observe that the Z and Higgs bosons can mix via loops involving Majorana neutrinos. Such amplitudes, which do not exist in the SM, render the branching ratios of the $t_L\bar{t}_L$ and $t_R\bar{t}_R$ channels, where L and R label the helicity states, different, which is a signal for CP violation [34]. However, these contributions cancel when the helicities are summed over.

The fermionic contributions to the vacuum polarizations may be cast in the general forms,

$$\begin{aligned}
\Pi_{WW}(q^2) &= \frac{G_F M_W^2}{\sqrt{2}} \Big[|B_{li}|^2 \Big(\Pi_V(q^2, m_i, m_l) + \Pi_V(q^2, m_i, -m_l) \Big) \\
&\quad + N_c |V_{ud}|^2 \Big(\Pi_V(q^2, m_u, m_d) + \Pi_V(q^2, m_u, -m_d) \Big) \Big], \qquad (3.3) \\
\Pi_{ZZ}(q^2) &= \frac{G_F M_Z^2}{2\sqrt{2}} \Big[|C_{ij}|^2 \Big(\Pi_V(q^2, m_i, m_j) + \Pi_V(q^2, m_i, -m_j) \Big) \\
&\quad - \mathrm{Re} C_{ij}^2 \Big(\Pi_V(q^2, m_i, m_j) - \Pi_V(q^2, m_i, -m_j) \Big) \\
&\quad + v_l^2 \Pi_V(q^2, m_l, m_l) + \Pi_V(q^2, m_l, -m_l) \\
&\quad + N_c \Big(v_q^2 \Pi_V(q^2, m_q, m_q) + \Pi_V(q^2, m_q, -m_q) \Big) \Big], \qquad (3.4) \\
\Pi_{ZA}(q^2) &= -\sqrt{2} G_F M_Z^2 s_w c_w \Big[- v_l \Pi_V(q^2, m_l, m_l) \; + \; N_c v_q Q_q \Pi_V(q^2, m_q, m_q) \Big], \qquad (3.5) \\
\Pi_{HH}(q^2) &= \frac{G_F}{2\sqrt{2}} \Big[\Big(|C_{ij}|^2 (m_i^2 + m_j^2) + 2 m_i m_j \mathrm{Re} C_{ij}^2 \Big) \Big(\Pi_S(q^2, m_i, m_j) + \Pi_S(q^2, m_i, -m_j) \Big) \\
&\quad + \Big(2 m_i m_j |C_{ij}|^2 + (m_i^2 + m_j^2) \mathrm{Re} C_{ij}^2 \Big) \Big(\Pi_S(q^2, m_i, m_j) - \Pi_S(q^2, m_i, -m_j) \Big) \\
&\quad + 4 m_l^2 \Pi_S(q^2, m_l, m_l) \; + \; 4 N_c m_q^2 \Pi_S(q^2, m_q, m_q) \Big], \qquad (3.6)
\end{aligned}$$

where $v_f = 2T_f - 4s_w^2 Q_f$ is the $Zf\bar{f}$ vector coupling, T_f is the weak isospin of f, Q_f is its electric charge in units of the positron charge, V_{ud} is the $n_G \times n_G$ CKM matrix of the quark sector, and the scalar and vector functions, Π_S and Π_V, are listed in the Appendix. Here and in the following, it is understood that indices are to be summed over when they appear more than once in an expression. For later use, we have presented also Π_{ZZ} and Π_{ZA}. We postpone the numerical discussion of the new virtual effects to Sect. 4.

3.2 The decay $H \to VV$

The one-loop renormalization of the $H \to WW$ and $H \to ZZ$ decay widths in the minimal SM is described in Refs. [35,36]. Modifications of the fermion sector affect these decay widths through the WWH and ZZH triangle diagrams depicted in Figs. 1(a) and (b), respectively. Assigning incoming four-momenta and Lorentz indices, (p, μ) and (k, ν), to the vector bosons, V, the renormalized vertex function takes the form

$$\begin{aligned}
\mathcal{T}_{VVH}^{\mu\nu} &= 2^{5/4} G_F^{1/2} M_V^2 \Big[D_{VVH}(a,b,c)\, k^\mu p^\nu \; + \; \Big(1 + \hat{E}_{VVH}(a,b,c) \Big) \; \mathrm{g}^{\mu\nu} \\
&\quad + i F_{VVH}(a,b,c)\, \varepsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma \Big], \qquad (3.7)
\end{aligned}$$

where $a = p^2$, $b = k^2$, $c = (p+k)^2$, and we have discarded terms with p^μ or k^ν anticipating that, in our applications, the vector bosons are real or couple to conserved currents. The hatted quantity has been renormalized by including its counterterm,

$$\hat{E}_{VVH}(a,b,c) \;=\; E_{VVH}(a,b,c) \;+\; \delta E_{VVH}. \tag{3.8}$$

In the MOMS scheme, one has

$$\begin{aligned}\delta E_{VVH} \;&=\; \mathrm{Re}\biggl(\frac{\Pi_{VV}(M_V^2)}{M_V^2} \;-\Pi'_{VV}(M_V^2)\biggr) \;-\; \frac{1}{2}\,\biggl(\mathrm{Re}\Pi'_{HH}(M_H^2) \;+\; \frac{\Pi_{WW}(0)}{M_W^2} \\ &\quad+\;\Delta r_{direct}\biggr) \;-\; \frac{1}{s_w c_w}\,\frac{\Pi_{ZA}(0)}{M_Z^2}.\end{aligned} \tag{3.9}$$

In the SM, $F_{WWH}(a,a,c) = F_{ZZH}(a,b,c) = 0$ for a,b,c arbitrary, due to CP conservation [35,36]. In the presence of Majorana neutrinos, $F_{VVH}(a,a,c)$ does not vanish, in general, so that CP-violating interactions are generated. If the vector-boson polarizations can be determined experimentally, it is possible to construct asymmetries that measure the degree of CP-nonconservation [34]. However, the F_{VVH} term drops out when we sum over all vector-boson polarizations, which we shall do to obtain the total $H\to VV$ decay rates.

In the considered class of models, the most general representations for the D_{VVH} form factors read

$$\begin{aligned}D_{WWH} \;&=\; -\,\frac{G_F}{4\sqrt{2}}\biggl[B^*_{li}C^*_{ij}B_{lj}\,\Bigl(m_i\overline{\mathcal{D}}(m_j,m_l,m_i) \;+\; m_j\overline{\mathcal{D}}(m_i,m_l,m_j)\Bigr) \\ &\quad+\;B^*_{li}C_{ij}B_{lj}\,\Bigl(m_i\overline{\mathcal{D}}(m_i,m_l,m_j) \;+\; m_j\overline{\mathcal{D}}(m_j,m_l,m_i)\Bigr) \\ &\quad+\;2m_l|B_{li}|^2\,(\mathcal{D}(m_l,m_i,m_l) \;+\; \mathcal{D}(m_l,-m_i,m_l)) \\ &\quad+\;2N_cm_u|V_{ud}|^2\,(\mathcal{D}(m_u,m_d,m_u) \;+\; \mathcal{D}(m_u,-m_d,m_u)) \\ &\quad+\;2N_cm_d|V_{ud}|^2\,(\mathcal{D}(m_d,m_u,m_d) \;+\; \mathcal{D}(m_d,-m_u,m_d))\biggr],\end{aligned} \tag{3.10}$$

$$\begin{aligned}D_{ZZH} \;&=\; -\frac{G_F}{4\sqrt{2}}\biggl[\mathrm{Re}(C_{ik}C_{kj}C_{ji})\,\Bigl(m_i\overline{\mathcal{D}}(m_k,m_j,m_i) \;+\; m_k\overline{\mathcal{D}}(m_i,m_j,m_k)\Bigr) \\ &\quad+\;\mathrm{Re}(C^*_{ik}C_{kj}C_{ji})\,\Bigl(m_i\overline{\mathcal{D}}(m_i,m_j,m_k) \;+\; m_k\overline{\mathcal{D}}(m_k,m_j,m_i)\Bigr) \\ &\quad-\;\mathrm{Re}(C_{ik}C^*_{kj}C_{ji})\mathcal{D}_-(m_i,m_j,m_k) \;-\; \mathrm{Re}(C_{ik}C_{kj}C^*_{ji})\mathcal{D}_+(m_i,m_j,m_k) \\ &\quad+\;2m_l\,\Bigl(v_l^2\mathcal{D}(m_l,m_l,m_l) \;+\; \mathcal{D}(m_l,-m_l,m_l)\Bigr) \\ &\quad+\;2N_cm_q\,\Bigl(v_q^2\mathcal{D}(m_q,m_q,m_q) \;+\; \mathcal{D}(m_q,-m_q,m_q)\Bigr)\biggr],\end{aligned} \tag{3.11}$$

$$D_{ZAH} \;=\; \sqrt{2}G_Fs_wc_w\Bigl[-m_lv_l\mathcal{D}(m_l,m_l,m_l) + N_cm_qv_qQ_q\mathcal{D}(m_q,m_q,m_q)\Bigr], \tag{3.12}$$

where we have suppressed the labels a,b,c on both sides of the equations. The auxiliary functions $\mathcal{D}$, $\overline{\mathcal{D}}$, and $\mathcal{D}_\pm$ are listed in the Appendix. For later use, we have also presented the D_{ZAH} form factor appropriate to the Z-photon-Higgs vertex shown in Fig. 1(b). Here b is the photon invariant mass squared. The expressions for E_{WWH}, E_{ZZH}, and E_{ZAH} are similar to Eqs. (3.10)–(3.12), and the corresponding functions $\mathcal{E}$, $\overline{\mathcal{E}}$, and $\mathcal{E}_\pm$ are given in the Appendix.

Defining $r_V = (M_H^2/4M_V^2)$, $n_W = 1$, and $n_Z = 2$, the Born approximation for the $H \to VV$ decay rate reads

$$\Gamma_0(H \to VV) \;=\; \frac{1}{n_V}\,\frac{G_F M_H^3}{8\pi\sqrt{2}}\sqrt{1-\frac{1}{r_V}}\;\left(1 \;-\; \frac{1}{r_V} \;+\; \frac{3}{4r_V^2}\right). \tag{3.13}$$

The fermion sector extended by Majorana neutrinos induces a relative correction to Eq. (3.13), which is given by

$$\delta_V = 2\mathrm{Re}\hat{E}_{VVH}(M_V^2, M_V^2, M_H^2) + \frac{(1-1/r_V)[1-(1/2r_V)]}{1-1/r_V+(3/4r_V^2)} M_H^2 \mathrm{Re} D_{VVH}(M_V^2, M_V^2, M_H^2). \tag{3.14}$$

In Sect. 4, we shall evaluate this expression numerically.

3.3 The reaction $e^+e^- \to ZH$

At LEP200 and future e^+e^- linear colliders with $\sqrt{s} \le 500$ GeV, the bremsstrahlung process, $e^+e^- \to ZH$, will be the most copious source of Higgs bosons in the intermediate mass range [37], and it is important to have the radiative corrections to its cross section well under control. These have been calculated in the SM [38] and in its minimal supersymmetric extension [39]. Here, we shall study the influence of virtual heavy Majorana neutrinos.

To start with, we consider the angular distribution, which, at tree level, is given by

$$\frac{d\sigma_{ZH}^0}{d\cos\theta} \;=\; \frac{G_F^2 M_Z^6 \sqrt{\lambda}}{16\pi s(s-M_Z^2)^2}\,(1+v_e^2)\left(1 \;+\; \frac{\lambda}{8sM_Z^2}\;\sin\theta\right), \tag{3.15}$$

where θ is the angle defined by the electron and Z-boson three-momenta in the centre-of-mass frame and $\lambda = (s - M_Z^2 - M_H^2)^2 - 4M_Z^2 M_H^2$. The corrections due to the fermion sector with Majorana neutrinos described in Sect. 2 may be included by multiplying Eq. (3.15) with $(1 + 2\mathrm{Re}\delta_{ZH})$, where

$$\begin{aligned}
\delta_{ZH} \;=\;& \hat{E}_{ZZH}(M_Z^2, s, M_H^2) + F(\theta) D_{ZZH}(M_Z^2, s, M_H^2) + \frac{s}{s-M_Z^2}\left(\frac{\mathrm{Re}\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{ZZ}(s)}{s}\right) \\
&+ \frac{1}{2}\mathrm{Re}\left(\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \Pi'_{ZZ}(M_Z^2)\right) \;-\; \frac{\Pi_{WW}(0)}{M_W^2} \;-\; \frac{1}{2}\mathrm{Re}\Pi'_{HH}(M_H^2) \\
&+ \frac{4s_w c_w v_e}{1+v_e^2}\left[\frac{s-M_Z^2}{s}\Big(E_{ZAH}(M_Z^2, s, M_H^2) \;+\; F(\theta) D_{ZAH}(M_Z^2, s, M_H^2)\Big) \;-\; \frac{\Pi_{ZA}(s)}{s}\right. \\
&\left. + \frac{c_w}{s_w}\mathrm{Re}\left(\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} \;-\; \frac{\Pi_{WW}(M_W^2)}{M_W^2}\right)\;\right],
\end{aligned} \tag{3.16}$$

Here, all angular dependence is carried by

$$F(\theta) \;=\; \frac{(M_H^2 - M_Z^2 - s)\lambda \sin^2\theta}{2(8sM_Z^2 \;+\; \lambda\sin^2\theta)}. \tag{3.17}$$

As before, we have assumed that the couplings of the electron to the heavy Majorana neutrinos are suppressed [10], so that e^+e^-H triangle and e^+e^-ZH box contributions are shifted from their SM values by insignificant amounts, which may safely be neglected.

As for the integrated cross section, the Born result is

$$\sigma^0_{ZH} \;=\; \frac{G_F^2 M_Z^6 \sqrt{\lambda}}{8\pi s(s-M_Z^2)^2}(1+v_e^2)\left(1 \;+\; \frac{\lambda}{12sM_Z^2}\right), \tag{3.18}$$

and the correction factor is

$$\left(1+2\mathrm{Re}\delta_{ZH}\Big|_{\sin^2\theta=2/3}\right). \tag{3.19}$$

The phenomenological implications of these results will be examined in the next section.

4 Numerical results and discussion

In Sect. 3, we have collected the analytic results for the one-loop corrections to the rates of Higgs-boson production via $e^+e^- \to ZH$ and its decays to $q\bar{q}$, W^+W^-, and ZZ pairs in the context of three- and four-generation models with Majorana neutrinos. We are now in a position to explore the phenomenological consequences of our results.

To start with, we consider extensions of the SM by three right-handed neutrinos. We find that the relative corrections to the Higgs-boson observables under consideration are shifted from their SM values by very small amounts, which are typically of the order of a few tenths of a percent. Similar observations have been made in connection with the oblique parameters S, T, and U [7].

In the following, we shall thus concentrate on models that naturally accommodate a fourth generation with Majorana neutrinos, adopting the scenario proposed by Hill and Paschos [12]. For reasons mentioned in the Introduction, we take the Majorana masses appearing in the Lagrangian to be bare. Specifically, the fourth generation consists of two Majorana neutrinos, N_1 and N_2, one charged lepton, E, one up-type quark, t', and one down-type quark, b'. We assume that E, t', and b' have SM couplings. All these new particles must have masses in excess of $M_Z/2$ so as to escape detection at the LEP/SLC experiments. The Majorana and Dirac masses of the Majorana system are related to the physical masses by $M = m_{N_2} - m_{N_1}$ and $m_D = \sqrt{m_{N_1} m_{N_2}}$, respectively. Conversely, one has $m_{N_{1,2}} = \sqrt{m_D^2 + M^2/4} \mp M/2$.

Since global analyses suggest that the mixings between the new fermions and the established ones are greatly suppressed [10], we neglect these couplings altogether in our analysis. Our new-physics scenario thus effectively reduces to a one-generation model. The interactions between the novel fermions and the weak bosons are characterized by Eqs. (2.1)–(2.3) with $n_G = 1$. The mixing matrices may be determined from the identities of Eqs. (2.6) and (2.7) with the result that

$$C_{N_1N_1} = \frac{m_{N_2}}{m_{N_1}+m_{N_2}}, \quad C_{N_2N_2} = \frac{m_{N_1}}{m_{N_1}+m_{N_2}}, \quad C_{N_1N_2} = -C_{N_2N_1} = i\frac{\sqrt{m_{N_1}m_{N_2}}}{m_{N_1}+m_{N_2}}, \tag{4.1}$$

$$B_{EN_1} = \sqrt{\frac{m_{N_2}}{m_{N_1} + m_{N_2}}}, \quad B_{EN_2} = i\sqrt{\frac{m_{N_1}}{m_{N_1} + m_{N_2}}}. \tag{4.2}$$

Inspired by the fact that the third-generation quarks participate only feebly in the CKM mixing, we ignore the possibility of mixing of t' and b' with quark flavours of the first three generations, *i.e.*, we put $V_{t'b'} = 1$ and $V_{t'd} = V_{ub'} = 0$ otherwise.

Our final results are displayed in Figs. 2–10. Figures 2–4 refer to $H \to t\bar{t}$ [cf. Eq. (3.2)], Figs. 5–7 to $H \to W^+W^-$ [cf. Eq. (3.14) for $V = W$], and Figs. 8–10 to $e^+e^- \to ZH$ [cf. Eq. (3.16) multiplied by two]. Our results for $H \to ZZ$ are very similar to those for $H \to W^+W^-$. In fact, δ_Z differs from δ_W by less than 0.1% in the considered parameter range. This may be understood by observing that, in our analysis, the mass scale of new physics is much larger than the mass difference of the W and Z bosons, so that the custodial symmetry is in effect. In each set of figures, the first two are devoted to a fourth-generation scenario with two mass-degenerate Majorana neutrinos, which are equivalent to one standard Dirac neutrino, *i.e.*, $m_D = m_{N_1} = m_{N_2}$ and $M = 0$, while the third figure deals with the genuine Majorana case, $M > 0$, for which $m_{N_1} < m_D < m_{N_2}$. Since we are mainly interested in the Majorana system, we assume $m_E = m_D$, which has been identified as a natural choice [12], and $m_{t'} = m_{b'} = 400$ GeV in order to reduce the number of parameters to be varied independently. Figures 2 and 5 (3 and 6) examine the dependence of the radiative corrections on M_H (m_D) for selected values of m_D (M_H). In Figs. 4 and 7, the m_{N_1} dependence is analyzed for $m_D = 400$ GeV and several values of M_H. The spikes in Figs. 2–7 arise from threshold effects in the Higgs wave-function renormalization and are an artifact of treating the Higgs boson as an asymptotic state despite its limited lifetime. They occur when $M_H = 2m_i$, where $i = N_1, N_2, E, t', b'$. The corrections remain finite at these points, which may be understood from arguments based on dispersion relations [35]. The $H \to W^+W^-$ triangle diagrams have thresholds at the same points.

In Figs. 2 and 3, we see that a virtual heavy Dirac neutrino, with $m_D \gg M_H/2$, produces a positive correction to $\Gamma(H \to t\bar{t})$, which increases with M_H decreasing and/or m_D increasing. This agrees with previous observations made in connection with additional heavy-fermion doublets [33]. From Fig. 4, we learn that this conventional heavy-flavour effect may be reduced by virtue of a mass splitting between N_1 and N_2, *i.e.*, the possible Majorana nature of the lepton sector. In the mass range considered, the maximum shift in δ_t with respect to the Dirac case is -5% and occurs at $m_{N_1} = M_H/2$. In other words, the influence of new heavy flavours may be screened by the existence of a Majorana degree of freedom. A screening effect of Majorana origin was encountered also in the case of the T parameter, which measures isospin breaking [6,7]. In the present case, however, the heavy flavours generate large corrections even if their Dirac masses are degenerate.

In the case of $H \to W^+W^-$, loop effects due to a Dirac neutrino with $m_D \gg M_H/2$ reduce the decay rate by an amount that increases with M_H and/or m_D; see Figs. 5 and 6. Similar observations have been reported in Refs. [35,36]. Again, the magnitude of this effect may be decreased appreciably by allowing for a nonvanishing Majorana mass, $m_{N_2} - m_{N_1}$; see Fig. 7. In the mass range considered, the maximum shift in δ_W is 7%.

In Fig. 8, the M_H dependence of the shift in $\sigma(e^+e^- \to ZH)$ induced by a conventional fourth generation with $m_{N_1} = m_{N_2} = m_E = m_{t'} = m_{b'} = 400$ GeV is shown for LEP200 energy and three $\sqrt{s}$ values appropriate to future e^+e^- linacs. For $M_H < 700$ GeV, the corrections are negative, decrease in magnitude with $\sqrt{s}$ increasing, and are practically independent of M_H. The spikes at $M_H = 800$ GeV are again due to threshold effects in the Higgs wave-function renormalization. In Figs. 9 and 10, we concentrate on Higgs-boson production at LEP200 and a 500-GeV linac, assuming $M_H = 70$ and 200 GeV, respectively. In Fig. 9, we study how the conventional fourth-generation correction varies with the Dirac-neutrino mass, m_D. In addition to the threshold effects related to the Higgs wave function, there are new possible thresholds at $\sqrt{s} = 2m_i$ $(i = N_1, N_2, E, t', b')$ and $\sqrt{s} = m_{N_1} + m_{N_2}$ originating from the s-channel cut through the ZZH and ZAH triangle diagrams. The one at $\sqrt{s} = 2m_{N_1}$ is visible in both cases considered in Fig. 9. Leaving aside the threshold effects, the correction is negative and its magnitude grows quadratically with m_D. At $m_D = 400$ GeV, it reaches -4.3%. When we now turn on the Majorana mass, we may reduce the effect down to the level of -1% without affecting the invisible Z-boson width; see Fig. 10. Again, the impact of heavy flavours is screened in the presence of genuine Majorana neutrinos.

5 Conclusions

We have investigated the influence of virtual heavy Majorana neutrinos on some of the most relevant processes involving the Higgs boson, namely, its decays into pairs of quarks and intermediate bosons as well as its production via bremsstrahlung in e^+e^- collisions. We found that the Standard Model predictions are changed insignificantly when the Dirac neutrinos of the established three generations are split into light and heavy Majorana neutrinos. The situation is very different in the fourth-generation scenario proposed by Hill and Paschos [12]. Here, the Majorana nature of the lepton sector manifests itself in a screening of the typical heavy-flavour effects. This feature is familiar from the electroweak parameter T [6,7], which measures the breaking of isospin. In contrast to T, however, the Higgs observables are sensitive to the novel heavy flavours even if they are degenerate in mass.

Acknowledgements. We wish to thank Steven Abel and Wilfried Buchmüller for stimulating discussions concerning cosmological constraints on models with heavy Majorana neutrinos.

A Appendix

In this paper, we evaluate the loop amplitudes using dimensional regularization along with the reduction algorithm of Ref. [40]. In contrast to Ref. [40], we use the Minkowskian metric, $\mathrm{g}^{\mu\nu} = \mathrm{diag}(1, -1, \ldots, -1)$.

The scalar and vector two-point functions occurring in Eqs. (3.3)–(3.6) are defined as

$$\Pi_S(q^2, m_1, m_2) \quad = \quad -\frac{1}{16\pi^2} \int \frac{d^n l}{i\pi^2} \, \mathrm{tr} \left(\frac{1}{\not{l} + \not{q} - m_2} \, \frac{1}{\not{l} - m_1} \right)$$

$$
\begin{aligned}
&= \frac{1}{8\pi^2}\Big[\Big(q^2-(m_1+m_2)^2\Big)\, B_0(q^2,m_1^2,m_2^2) \; - \; m_1^2\Big(1+B_0(0,m_1^2,m_1^2)\Big) \\
&\quad -m_2^2\Big(1+B_0(0,m_2^2,m_2^2)\Big)\Big], \qquad \text{(A.1)}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_V(q^2,m_1,m_2) &= \frac{1}{n-1}\left(\mathrm{g}_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\frac{1}{16\pi^2}\int \frac{d^n l}{i\pi^2}\,\mathrm{tr}\left(\gamma^\nu\,\frac{1}{\not l+\not q-m_2}\,\gamma^\mu\,\frac{1}{\not l-m_1}\right) \\
&= \frac{1}{12\pi^2}\Bigg[\left(q^2-\frac{m_1^2+m_2^2}{2}+3m_1m_2-\frac{(m_1^2-m_2^2)^2}{2q^2}\right)B_0(q^2,m_1^2,m_2^2) \\
&\quad + m_1^2\left(-1+\frac{m_1^2-m_2^2}{2q^2}\right)B_0(0,m_1^2,m_1^2) \; + \; m_2^2\left(-1+\frac{m_2^2-m_1^2}{2q^2}\right) \\
&\quad \times\, B_0(0,m_2^2,m_2^2) \; - \; \frac{q^2}{3} \; + \; \frac{(m_1^2-m_2^2)^2}{2q^2}\Bigg], \qquad \text{(A.2)}
\end{aligned}
$$

where n is the dimensionality of space-time and the standard two-point integral, B_0, is listed, *e.g.*, in Appendix A of Ref. [30]. For the evaluation of the counterterms, we also need Π_V at $q^2=0$,

$$
\begin{aligned}
\Pi_V(0,m_1,m_2) &= \frac{1}{16\pi^2}\Bigg[-2(m_1-m_2)^2\left(\frac{2}{4-n}-\gamma_E-\ln\pi-\frac{1}{2}\ln(m_1^2m_2^2)\right) \\
&\quad -m_1^2-m_2^2+4m_1m_2+\frac{m_1^4+m_2^4-2m_1m_2(m_1^2+m_2^2)}{m_1^2-m_2^2}\ln\frac{m_1^2}{m_2^2}\Bigg], \qquad \text{(A.3)}
\end{aligned}
$$

where γ_E is Euler's constant. In fact, $\Pi_V(0,m,m)=0$ as required by electromagnetic gauge invariance. The pseudo-scalar and axial-vector two-point functions emerge from Eqs. (A.1) and (A.2) by γ_5 reflection, *i.e.*, $\Pi_P(q^2,m_1,m_2)=-\Pi_S(q^2,m_1,-m_2)$ and $\Pi_A(q^2,m_1,m_2)=\Pi_V(q^2,m_1,-m_2)$, respectively. In our calculation, we have used these properties to eliminate the Π_P and Π_A functions.

In our analysis, all vertex corrections can be reduced to the basic three-point integral,

$$
\begin{aligned}
&\frac{1}{16\pi^2}\int\frac{d^n l}{i\pi^2}\,\mathrm{tr}\left(\frac{1}{\not l+\not p+\not k-m_3}\,\gamma^\nu\,\frac{1}{\not l+\not p-m_2}\,\gamma^\mu\,\frac{1}{\not l-m_1}\right) \\
&\quad = \mathcal{A}p^\mu p^\nu \; + \; \mathcal{B}k^\mu k^\nu \; + \; \mathcal{C}p^\mu k^\nu \; + \; \mathcal{D}k^\mu p^\nu \; + \; \mathcal{E}\mathrm{g}^{\mu\nu}. \qquad \text{(A.4)}
\end{aligned}
$$

As explained in Sect. 3.2, only $\mathcal{D}$ and $\mathcal{E}$ enter our final results. These can be expressed in terms of B_0 and the standard three-point integrals C_0, C_{11}, C_{12}, C_{23}, and C_{24}, viz.

$$
\begin{aligned}
\mathcal{D}(m_1,m_2,m_3) &= \frac{1}{4\pi^2}\Big[-m_1C_0 \; + \; (-m_1+m_2)C_{11} \; - \; (2m_1+m_2+m_3)C_{12} \\
&\quad - \; 2(m_1+m_3)C_{23}\Big], \qquad \text{(A.5)} \\
\mathcal{E}(m_1,m_2,m_3) &= \frac{1}{8\pi^2}\Bigg[(-m_1+m_2)B_0(a,m_1^2,m_2^2) \; + \; (m_2-m_3)B_0(b,m_2^2,m_3^2) \\
&\quad + \; (m_1+m_3)\Big(4C_{24}-B_0(c,m_1^2,m_3^2)\Big) \; + \; \Big[m_1(-b+m_2^2+m_3^2) \\
&\quad + \; m_2(c-m_1^2-m_3^2) \; + \; m_3(-a+m_1^2+m_2^2) \; - \; 2m_1m_2m_3\Big]C_0\Bigg], \qquad \text{(A.6)}
\end{aligned}
$$

where we have suppressed $a = p^2, b = k^2, c = (p+k)^2$ in the argument lists of $\mathcal{D}$ and $\mathcal{E}$ and it is understood that the C functions are evaluated at $(a, b, c, m_1^2, m_2^2, m_3^2)$ in the notation of Ref. [30]. It is convenient to introduce the following short-hand notations:

$$\begin{aligned}
\overline{\mathcal{D}}(m_1, m_2, m_3) &= \mathcal{D}(m_1, m_2, m_3) \; + \; \mathcal{D}(m_1, -m_2, m_3) \\
&\quad + \mathcal{D}(-m_1, m_2, m_3) \; - \; \mathcal{D}(m_1, m_2, -m_3), \qquad \text{(A.7)} \\
\mathcal{D}_\pm(m_1, m_2, m_3) &= (m_1 + m_3)\Big(\mathcal{D}(m_1, m_2, m_3) - \mathcal{D}(m_1, -m_2, m_3)\Big) \\
&\quad \pm (m_1 - m_3)\Big(\mathcal{D}(-m_1, m_2, m_3) + \mathcal{D}(m_1, m_2, -m_3)\Big), \qquad \text{(A.8)} \\
&\qquad \text{(A.9)}
\end{aligned}$$

and similarly for $\overline{\mathcal{E}}$ and $\mathcal{E}_\pm$.

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Figure Captions

Fig. 1: Feynman diagrams pertinent to the fermionic (a) W^+W^-H, (b) ZZH, and $Z\gamma H$ vertex corrections in models with Majorana neutrinos.

Fig. 2: Radiative corrections to the $H \to t\bar{t}$ decay width induced by a fourth generation with Dirac neutrinos ($m_D = m_{N_1} = m_{N_2}$) as a function of M_H for selected values of m_D assuming $m_E = m_D$ and $m_{t'} = m_{b'} = 400$ GeV.

Fig. 3: Radiative corrections to the $H \to t\bar{t}$ decay width induced by a fourth generation with Dirac neutrinos ($m_D = m_{N_1} = m_{N_2}$) as a function of m_D for selected values of M_H assuming $m_E = m_D$ and $m_{t'} = m_{b'} = 400$ GeV.

Fig. 4: Radiative corrections to the $H \to t\bar{t}$ decay width induced by a fourth generation with Majorana neutrinos ($m_{N_1} < m_D < m_{N_2}$) as a function of m_{N_1} for selected values of M_H assuming $m_D = m_E = m_{t'} = m_{b'} = 400$ GeV.

Fig. 5: Radiative corrections to the $H \to W^+W^-$ decay width induced by a fourth generation with Dirac neutrinos ($m_D = m_{N_1} = m_{N_2}$) as a function of M_H for selected values of m_D assuming $m_E = m_D$ and $m_{t'} = m_{b'} = 400$ GeV.

Fig. 6: Radiative corrections to the $H \to W^+W^-$ decay width induced by a fourth generation with Dirac neutrinos ($m_D = m_{N_1} = m_{N_2}$) as a function of m_D for selected values of M_H assuming $m_E = m_D$ and $m_{t'} = m_{b'} = 400$ GeV.

Fig. 7: Radiative corrections to the $H \to W^+W^-$ decay width induced by a fourth generation with Majorana neutrinos ($m_{N_1} < m_D < m_{N_2}$) as a function of m_{N_1} for selected values of M_H assuming $m_D = m_E = m_{t'} = m_{b'} = 400$ GeV.

Fig. 8: Radiative corrections to the total cross section of $e^+e^- \to ZH$ induced by a fourth generation with Dirac neutrinos ($m_D = m_{N_1} = m_{N_2}$) as a function of M_H for selected values of $\sqrt{s}$ assuming $m_D = m_E = m_{t'} = m_{b'} = 400$ GeV.

Fig. 9: Radiative corrections to the total cross section of $e^+e^- \to ZH$ induced by a fourth generation with Dirac neutrinos ($m_D = m_{N_1} = m_{N_2}$) as a function of m_D assuming $m_D = m_E$ and $m_{t'} = m_{b'} = 400$ GeV. The dashed (solid) line refers to low-mass (high-mass) Higgs-boson production at LEP200 (a 500-GeV linac).

Fig. 10: Radiative corrections to the total cross section of $e^+e^- \to ZH$ induced by a fourth generation with Majorana neutrinos ($m_{N_1} < m_D < m_{N_2}$) as a function of m_{N_1} assuming $m_D = m_E$ and $m_{t'} = m_{b'} = 400$ GeV. The dashed (solid) line refers to low-mass (high-mass) Higgs-boson production at LEP200 (a 500-GeV e^+e^- linac).