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R. Hempfling

Deutsches Elektronen-Synchrotron DESY, Hamburg

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Yukawa Coupling Unification with Supersymmetric Threshold Corrections

Ralf Hempfling

Deutsches Elektronen-Synchrotron, Notkestraße 85, 22603 Hamburg, Federal Republic of Germany

Abstract. Radiative corrections to the down-type fermion masses at the supersymmetric threshold are enhanced by the ratio of vacuum expectation values, $\tan\beta$. This will have a profound impact on the unification of Yukawa couplings in supersymmetric grand unified theories (SUSY-GUT). We present an example of such a model with a horizontal gauge symmetry that naturally explains the fermion mass hierarchy and the small mixing angles of the Kobayashi-Maskawa (KM) matrix. The unification of the lepton and the down-quark Yukawa couplings is achieved without introducing large Higgs multiplets.

One of the most puzzling features of the quark mass matrices is the large ratio of quark masses (*e.g.*, $m_t/m_u \approx 3\times 10^4$) and the small off-diagonal entries of the KM matrix. There have been many attempts to explain these properties by imposing additional symmetries. The light quark masses in these scenarios are generated radiatively[1] or suppressed by ratios of vacuum expectation values[2-5]. Additional problems arise if we try to embed the $\mathrm{SU(3)_c\times SU(2)_L\times U(1)_Y}$ standard model gauge group in the simple gauge group of a grand unified theory (GUT)[6]. In this letter we consider a GUT model based on SU(5)[7]. In the minimal version of an SU(5) GUT theory the right-handed down-type quarks and the left-handed leptons are different components of the same fields. As a result, the down-type quarks and the leptons have the same Yukawa couplings at $M_{\rm GUT}$. By running the coupling constants from $M_{\rm GUT}$ to m_z one obtains a prediction for the down-type quark masses in terms of the lepton masses.

It has been shown recently[8] that in the minimal supersymmetric model (MSSM)[9;10] the $\mathrm{SU(3)_c\times SU(2)_L\times U(1)_Y}$ gauge couplings unify at a scale $M_{\rm GUT} = \mathcal{O}(10^{16}$ GeV). Additionally, the unification of τ and bottom Yukawa couplings at $M_{\rm GUT}$ within the MSSM is rather successful[5;11;12]. However, the predictions for the first two generations are clearly incompatible with experiment.

One way out is to introduce a large Higgs representation such as the 45 under SU(5). However, such an extension introduces many new interactions and it would be desirable to find alternatives with a smaller particle content. In this letter we will consider a model with a fourth family of fermions and a family of mirror fermions (a more general, and therefore less predictive, class of models has been considered in ref. 13). Such additional fields are naturally present in many extensions of SU(5). According to the "survival" hypothesis[14] the additional fermions and their mirror fermions combine and acquire masses of the order of $M_{\rm GUT}$. Nonetheless, their existence will affect the parameters of the low energy effective Lagrangian. In particular, we will show that the existence of a fourth family of fermions and a family of mirror fermions suffices in order to reconcile the bad GUT predictions for the down-type fermion masses of the first two generations. Furthermore, we will constrain the Yukawa matrices by introducing a horizontal $\mathrm{U(1)_h}$[15] gauge symmetry and a discrete Z_3 symmetry. This will explain naturally the quark mass hierarchy and the Kobayashi Maskawa (KM) matrix. In Table 1 we list the full particle content of the theory and their transformation properties under SU(5), $\mathrm{U(1)_h}$ and Z_3 [$\phi \to \exp(iz\pi/3)\phi$]. This set of fields is manifestly anomaly-free. In addition, we impose R-invariance in order to avoid baryon number violating interactions. Here the four generation of quarks and leptons are denoted by U_i and D_i ($i=1,2,3,4$) and the mirror quarks and mirror leptons are denoted by U' and D'. The adjoint representation responsible for the breaking of SU(5) is denoted by Φ and the Higgs field responsible for the electroweak breaking are denoted by H_1 and H_2. In addition, we need to introduce SU(5) singlets N_i ($i=1,2,3,4$) and N' to break $\mathrm{U(1)_h}$ and Z_3. The superpotential of this theory can be written as

$$W = W_Q + W_Y + W_H + W_N + W_\Phi\,, \tag{1}$$

where

$$\begin{aligned}
W_Q &= \sum[\lambda_{Q1}N_1Q_1 + \lambda_{Q2}N_2Q_2 \\
&\quad + (\lambda_{Q3}N_3 + \lambda_{Q4}N_4 + \lambda_Q\Phi)Q_3 + m_QQ_4]Q'\,,\\
W_Y &= y_DH_1U_3D_4 + y'_DH_1U_4D_3 + y_UH_2U_3U_4\,,\\
W_H &= H_1(h_3N_3 + h_4N_4 + h_\Phi\Phi)H_2\,,\\
W_N &= \tfrac{1}{3!}\kappa_{ijk}N_iN_jN_k + m_3N_3N' + m_4N_4N' + \tfrac{1}{3!}\kappa'N'^3\,,\\
W_\Phi &= \tfrac{1}{2!}(\lambda_3N_3 + \lambda_4N_4)\Phi^2 + \tfrac{1}{3!}\lambda_\Phi\Phi^3\,,
\end{aligned} \tag{2}$$

where the sum is over $Q = U, D$. All the mass parameters are assumed to be of the order of $M_{\rm GUT}$. The entries

Table 1. The particle spectrum

		SU(5)	U(1)$_h$	Z$_3$
	U_i	10	$1,-1,0,0$	$-1,-1,-1,0$
matter	U'	$\overline{10}$	0	0
$(i=1,2,3,4)$	D_i	$\overline{5}$	$1,-1,0,0$	$-1,-1,-1,0$
	D'	5	0	0
Higgs	H_1, H_2, Φ	5, 5, 24	0	1
singlet	N_i	1	$-1,1,0,0$	1
$(i=1,2,3,4)$	N'	1	0	-1

of κ_{ijk} are constrained by U(1)$_h$ and Z_3. In addition, the fields N_3 and N_4 have the same quantum numbers and can be rotated such that $\kappa_{123}=0$. The potential $\mathcal{V}$ is minimized if the D-terms and the F-terms vanish,

$$D_h = \frac{g_h^2}{2}(N_2^* N_2 - N_1^* N_1) = 0\,, \quad F_\phi^* = -\frac{\mathrm{d}W}{\mathrm{d}\phi} = 0\,, \qquad (3)$$

where $\phi = N_i, N', \Phi$ and g_h is the horizontal gauge coupling. In this basis the potential has a minimum at

$$\begin{aligned}&\langle N_i\rangle = n_i \neq 0\,, \qquad (i=1,2,3)\,,\\ &\langle N'\rangle = n' \neq 0\,,\\ &\langle \Phi\rangle = a\,\mathrm{diag}\left(-\tfrac{2}{3},-\tfrac{2}{3},-\tfrac{2}{3},1,1\right) \neq 0\,,\end{aligned} \qquad (4)$$

and all other fields equal to zero*. Non-zero values of a break SU(5) to SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$ under which the representations decompose as

$$\begin{aligned}&U(10) \to q(3,2,\tfrac{1}{3}) + u^c(\bar{3},1,-\tfrac{4}{3}) + e^c(1,1,2)\,,\\ &D(\bar{5}) \to d^c(\bar{3},1,\tfrac{2}{3}) + \ell(1,2,-1)\,.\end{aligned} \qquad (5)$$

The low energy effective Lagrangian is now obtained by diagonalizing the fermion mass matrices $m_i^f = M_f v_i^f$ and the sfermion mass matrices $m_{ij}^{f2} = m_i^f m_j^f$ [here, $M_f^2 \equiv (\lambda_{f1}n_1)^2 + (\lambda_{f2}n_2)^2 + (\lambda_{f3}n_3 + \lambda_f a)^2 + m_f^2$, $v_1^f \equiv \lambda_{f1}n_1/M_f$, $v_2^f \equiv \lambda_{f2}n_2/M_f$, $v_3^f \equiv (\lambda_{f3}n_3 + Y_f\lambda_f a)/M_f$ and $v_4^f \equiv m_f/M_f$; $Y_f = \frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}, 1, 2$ for $f=q,d,u,\ell,e$] and removing all the fields that acquire masses at $M_{\rm GUT}$. The unitary matrices $\mathcal{U}^f$, defined by

$$v_i^f \to v_j^f \mathcal{U}_{ji}^f = (0,0,0,1)\,, \qquad (6)$$

are given by

$$\mathcal{U}^f = \begin{pmatrix} c_f'' & s_f''c_f' & s_f''s_f'c_f & s_f''s_f's_f \\ -s_f'' & c_f''c_f' & c_f''s_f'c_f & c_f''s_f's_f \\ 0 & 0 & -s_f & c_f \\ 0 & -s_f' & c_f'c_f & c_f's_f \end{pmatrix}. \qquad (7)$$

* Supersymmetric theories often have multiple degenerate minima with $\mathcal{V}=0$. Here we simply pick the minimum that is phenomenologically viable.

Here we have defined

$$\begin{aligned}&c_f \equiv \cos\theta_f = v_3^f\,,\\ &c_f' \equiv \cos\theta_f' = \frac{v_4^f}{s_f}\,, \qquad c_f'' \equiv \cos\theta_f'' = \frac{v_2^f}{s_f' s_f}\,,\end{aligned} \qquad (8)$$

and $s_f \equiv \sin\theta_f$, *etc.* After decoupling the superheavy mass eigenstates [with masses $\mathcal{O}(M_{\rm GUT})$] we obtain a constrained version of the MSSM. The Higgs mass parameter of the superpotential is $\mu = h_3 n_3 - h_\Phi a$† and the quark and lepton Yukawa couplings are given by

$$\begin{aligned}&y_{ij}^u = y_U \mathcal{U}_{i3}^q \mathcal{U}_{j4}^u + y_U \mathcal{U}_{i4}^q \mathcal{U}_{j3}^u\,,\\ &y_{ij}^d = y_D \mathcal{U}_{i3}^q \mathcal{U}_{j4}^d + y_D' \mathcal{U}_{i4}^q \mathcal{U}_{j3}^d\,, \qquad i,j = 1,2,3\,.\\ &y_{ij}^e = y_D \mathcal{U}_{i3}^l \mathcal{U}_{j4}^e + y_D' \mathcal{U}_{i4}^l \mathcal{U}_{j3}^e\,,\end{aligned} \qquad (9)$$

This equation yields the following texture for the down Yukawa matrix

$$y^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_D' s_d s_q' \\ 0 & y_D s_q s_d' & -y_D s_q c_d' c_d - y_D' s_d c_q' c_q \end{pmatrix}, \qquad (10)$$

and analogous expressions for y^u and y^e. From here the correct m_t/m_c and m_b/m_s ratios are obtained by requiring that $s_q' = \mathcal{O}(\sqrt{m_c/m_t})$ and $s_d' = \mathcal{O}(m_s/(s_q' m_b))$. Note that $c_d' = c_l'$, $c_d'' = c_l''$, $c_q' = c_u' = c_e'$ and $c_q'' = c_u'' = c_e''$. However, we find that $c_d \neq c_l$ ($c_q \neq c_u \neq c_e$) for non-zero values of $\lambda_D a$ ($\lambda_U a$). As a result, the eigenvalues of y^d and y^e are independent. It is, however, clear that this mechanism will affect the unification of the third generation quark and lepton masses less than of the second generation since they are give by a product of the two off-diagonal divided by the diagonal matrix elements in eq. (10).

In this model, the KM-matrix V is very close to unity (*i.e.*, the only non-zero off-diagonal matrix elements are $|V_{cb}|, |V_{ts}| \lesssim \sqrt{m_c/m_t}$). Thus, at tree-level this model is in good agreement with the masses and mixing angles of the second and third generation. However, we still need masses and mixing angles for the first generation.

In any realistic low energy model supersymmetry (SUSY) must be broken. This breaking is assumed to occur in a "hidden" sector. Gravitational coupling then induces explicit soft SUSY breaking terms at a scale

† The fine-tuning required in SUSY-GUT in order to obtain $\mu \lesssim 1$ TeV is a well-known problem [16] and we shall not attempt to solve it within the framework of the model considered here.

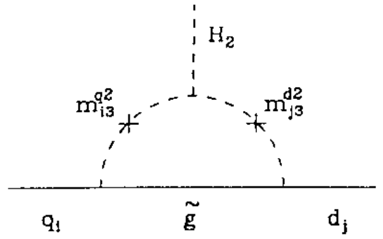

Fig. 1. the dominant contributions to the radiatively generated fermion masses

$m \gtrsim m_s$ into the "visible" sector. In minimal $N = 1$ supergravity models those terms are[17]

$$\mathcal{V}_{\rm soft} = (AmW_3 + BmW_2 + \text{h.c.}) + m^2\phi^\dagger\phi\,, \tag{11}$$

where W_2 (W_3) is the quadratic (trilinear) part of the superpotential [eq. (1)]. A and B are dimensionless constants of order one and m is the universal SUSY-breaking mass parameter for all the scalars ϕ. If we minimize the potential including $\mathcal{V}_{\rm soft}$ we find in general, that the D-term with $D_h = \mathcal{O}(m) \neq 0$ gives rise to additional squark mass terms. Other squark mass terms are derived from the A and B terms

$$\mathcal{V}_{\rm soft} = Am\sum_{f,i} M_f v_{fi}\tilde{f}_i\tilde{f}' + Bm\sum_f M_f v_{f4}\tilde{f}_4\tilde{f}' + \text{h.c.}\,, \tag{12}$$

where the summation is over $f = q, u, d, \ell, e$ and $i = 1, 2, 3$. If we decouple the superheavy states, we find the sfermion mass matrices below $M_{\rm GUT}$

$$m^{f2}_{ij} = m^2\delta_{ij} + m_h^2\left(U^f_{1i}U^f_{1j} - U^f_{2i}U^f_{2j}\right) + m_A^2 U^f_{4i}U^f_{4j}\,, \tag{13}$$

where $m_h^2 \equiv g_h^2 D_h/2$ and $m_A^2 \equiv -(A-B)^2m^2$. Clearly, the mass matrices in eq. (13) are not diagonal in flavor space. The low energy mass parameters are obtained by renormalization group evolution from $M_{\rm GUT}$ to m. While this running of the mass parameters will cause a rather significant splitting of the squark and slepton masses, it will only have a very moderate effect on the ratio of off-diagonal to diagonal matrix elements within the same sfermion mass matrix. In the following discussion, we will assume that these ratios at the scale m are of the same order as the ratios at $M_{\rm GUT}$. If we assume that all the mass parameters in eq. (13) are of the same order we find

$$\begin{aligned} &\frac{m^{q2}_{32}}{m^2}, \frac{m^{q2}_{31}}{m^2}, \frac{m^{d2}_{31}}{m^2} = \mathcal{O}\left(\sqrt{\frac{m_c}{m_t}}\right), \\ &\frac{m^{d2}_{32}}{m^2} = \mathcal{O}\left(\frac{m_s}{m_b}\sqrt{\frac{m_t}{m_c}}\right). \end{aligned} \tag{14}$$

With these off-diagonal matrix elements we can generate the masses for the fermions of the first generation at the one-loop level[13,18]. We will now consider the case of large $\tan\beta$. Here the dominant contributions to the down-type masses arise come from the radiatively generated Yukawa couplings to H_2 [see fig. 1]‡

$$m^d_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{s0} & 0 \\ 0 & 0 & m_{b0} \end{pmatrix} + M_{\rm RC}\begin{pmatrix} \dfrac{m^{q2}_{3i}m^{d2}_{3j}}{6m^4} & \dfrac{m^{q2}_{3i}}{3m^2} \\ \dfrac{m^{d2}_{3j}}{3m^2} & 1 \end{pmatrix}, \tag{15}$$

where $i, j, = 1, 2$. Here we have defined

$$M_{\rm RC} \equiv m_b f_{RG}\tan\beta\frac{\alpha_s}{3\pi}\frac{\mu m_{\tilde{g}}}{m^2} = \mathcal{O}(1 \sim 2\ \text{GeV})\,, \tag{16}$$

where α_s is the strong coupling constant. The gluino and the Higgs mass parameters are $m_{\tilde{g}}, \mu = \mathcal{O}(m)$ and we have invoked the SO(10) constraint that $\lambda_D = \lambda'_D = \lambda_U$ (*i.e.*, $\tan\beta \equiv \langle H_2\rangle / \langle H_1\rangle \approx m_t/m_b$). The subscript 0 indicates the unrenormalized quark masses. Note that in order to obtain the coupling constants at the scale m we have to run m_b to m and then run the radiatively generated masses from m to the corresponding masses. This is taken into account by the renormalization group factor $f_{RG} = 1 \sim 2$. The strongest phenomenological constraint on the radiatively generated Yukawa matrix element is the requirement that $m^d_{12} \approx |V_{dc}|m_s$. This is in the right order of magnitude of eq. (15) (The correct values is obtained by tuning m_h^2, m_A^2 and m in eq. (13)). The predicted value of the down quark mass[19] is

$$m_d \approx m^d_{11} \approx m^{d2}_{31}/m^{d2}_{32}\,|V_{dc}|\,m_s \approx 10\ \text{MeV} \tag{17}$$

and for the up-quark we find

$$\frac{m_u}{m_d} \approx \frac{m_t}{m_b\tan\beta}\frac{Am}{\mu} \approx \mathcal{O}(1)\,. \tag{18}$$

The off-diagonal squark mass matrix elements are strongly constrained by FCNC processes such as neutral meson mixing[20] and $b \to s\gamma$ decay[21]. The values for the off-diagonal squark mass in eq. (14) impose a lower limit on the squark masses of $m \gtrsim 1$ TeV[9;22]. Note, however, that these are only crude order of magnitude constraints since the squark mass matrices have to be expressed in the basis where the radiatively corrected quark mass matrices [eq. (15)] are diagonal. In addition, the constraints on m^{q2}_{12}/m^2 and m^{d2}_{12}/m^2 coming

‡ In this scenario the second Higgs doublet mediates flavor changing neutral current (FCNC) and thus has to be heavier than $\mathcal{O}$(1 TeV).

from $K - \overline{K}$ mixing is roughly an order of magnitude stronger than the constraints on m_{13}^{q2}/m^2 and m_{13}^{d2}/m^2. While these matrix elements are irrelevant for the radiative generation of the quark masses [eq. (15)] it would require some fine-tuning to suppress them enough such that the experimental bounds are satisfied without a significant increase in m. However, note that even in the absence of the off-diagonal squark mass matrix elements the radiative corrections in eq. (15) will be very significant. Consider again the case of $\tan\beta = m_t/m_b$ and the natural value for $m_{\tilde{g}}\mu/m^2 = -1/4$. Then the value of m_t obtained from the τ and bottom unification[23,5,11,12] can be lowered by $\mathcal{O}(30\text{ GeV})$ and thus is in much better agreement with current experimental data[24].

The radiatively generated electron mass is given by [for a related result see ref. 13]

$$\frac{m_e}{m_d} \approx \frac{\alpha_{\rm em}}{\frac{4}{3}c_W^2\alpha_s}\frac{m_{\tilde{\gamma}}}{m_{\tilde{g}}}\frac{m_\tau/m_{\tilde{\tau}}^2}{m_b/m_{\tilde{b}}^2}\,, \tag{19}$$

where the $\alpha_{\rm em}$ is the fine structure constant, $c_{\rm w} \equiv m_{\rm w}/m_{\rm z}$ and $m_{\tilde{\gamma}}$ the photino mass parameter. This is compatible with the experimental result of $m_e/m_d \approx 1/20$ (assuming that $m_{\tilde{\tau}}/m_{\tilde{b}} \approx 1/3$). While the total lepton number in this model is conserved, there are one-loop induced μ and τ number violating processes such as $\mu \to \gamma e$ and $\tau \to \gamma e$. The resulting lower limits on the slepton masses are well below 1 TeV[25].

We have calculated the dominant $\tan\beta$ radiative corrections to the fermion masses in supersymmetric theories. We find, that they will almost invariably have a profound impact on the unification of τ and bottom Yukawa couplings in SUSY-GUT theories. An example of a theory is presented where the fermion masses of the first generation are generated via these corrections. This explains why $m_e/m_d \ll m_\mu/m_s$ and why $m_u/m_d = \mathcal{O}(1)$. In this model, the tree-level Yukawa matrices [eq. (10)] have a simplified version of the "Fritzsch texture"[2] so that a large ratio of mass eigenvalues for the fermions of the second and third generation is natural. Moreover, the parameters can be tuned in order to yield μ and strange Yukawa coupling unification at $M_{\rm GUT}$ without requiring large Higgs representations.

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