# Calibration of Waveguide Beam Position Monitors

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#### Abstract

To ensure overlap between the photon beam and electron beam at the SASE-FEL at the TESLA Test Facility [1], several position sensitive diagnostics components are installed along the beamline of the FEL. For the undulator part, a new type of waveguide beam position monitors (BPMs) is designed, tested, and installed inside the beam pipe of one undulator module. This paper proposes a method to calibrate these monitors with beam based measurements

#### 1. Introduction

The waveguide beam position monitor (BPM) system for one undulator module consists of ten individual BPM units, each with four coupling channels. The operation principle of the BPM is based on the coupling to the magnetic field copropagating with the electron beam with small slots in the aperture [2]. Fig.1 shows a schematic profile of one BPM unit in the plane perpendicular to the beam direction.

The output data from a BPM shows the electron beam offset relative to the electrical center of the BPM, not to the geometrical. The electrical center may drift due the imbalance among the output signals, because signals must travel through different paths, transducers, cables and signal processing electronics elements, and are then measured by detectors. To find the relation between the geometrical and electrical centers is the object of the calibration. This paper proposes a method to estimate the imbalance from four output signals of one BPM unit using beam signals.

#### 2. Modelling BPM Output Signals

Assuming that the overall effect of transducer, cables, and electronics elements can be described by a gain factor  $g_i$ , which is unique for each BPM unit channel, the amplitude of the beam induced signal  $V_i$  from a bunch charge q at a position (x, y) reads like

$$V_i = q \cdot g_i \cdot F_i(x, y),$$

where the response function  $F_i(x,y)$  describes the response of a particular slot. This response function can be calculated analytically via the beam induced wall current density model [3]. The wall current on a spot  $(b,\alpha)$  on a circular beam pipe with radius b at an angle  $\alpha$  induced by a beam charge q with current  $I_b$  at  $(r,\theta)$  can be expressed with

$$I_w(r,\theta) = \frac{I_b}{2\pi b} \cdot \frac{b^2 + r^2}{b^2 - r^2 - 2br\cos(\alpha - \theta)}$$

in a cylindrical reference frame centred in the BPM. In a circular region  $r \leq b$  the response

function can be given by the wall current density at the location of the coupling slot sensitive center normalised to the beam current. For cartesian coordinates the response function for a beam at (x,y) for the ith slot of a BPM reads

$$F_i(x,y) = \frac{b^2 - (x^2 + y^2)}{b^2 + (x^2 + y^2) - 2(x_s x + y_s y)}.$$
 (1)

with slot position  $(x_s, y_s)$ . This function is normalised to  $F_i(0,0) =$ . Further analysis is based on two assumptions. One is that the response function will not change in time. Since the response function only depends on the geometrical structure of the BPM, and in particular on the location of the sensitive centers of the coupling channels, this is reasonable. The second assumption is that all effects which displace the electrical center relative to the geometrical center can be included into the gains  $g_i$ . This assumption is acceptable when the amplification by the signal processing electronics is stabilised by an internal feedback. When all  $g_i$ 's are known, the offset between electrical and geometrical monitor center is determined. This idea can be applied to the calibration of a BPM unit itself for modelling its response function [4].

## 3. Gain Estimation

Considering  $j = 1 \cdots m$  measurements for the induced voltage into all channels for m different beam positions. Then, the voltage from the ith channel at the jth measurements is

$$V_{ij} = g_i \cdot q_j \cdot F_i(x_j, y_j) \tag{2}$$

where  $F_i(x_j, y_j)$  is the response function for an ideal BPM channel for an electron beam at the position  $(x_j, y_j)$  according to Eq.1. Because only the relative imbalance between all gains is of interest for calibration,  $g_1$  can be set to 1, resulting in 3 unknown parameters for the gains  $(g_2, g_3, g_4)$ . Beside this, for each measurement a set of 3m unknown parameters  $(q_j, x_j, y_j)$  is generated while 4m quantities  $(V_{1j}, V_{2j}, V_{3j}, V_{4j})$  are measured. At m=3 measurement the system of equations has a unique solution, for m>4 the number of known

parameters exceeds the number of unknowns. Using a nonlinear multiparameter chi-square method the 3m+3 unknown parameters, including the gains, can be estimated. Rewriting Eq.2 to

$$V_{ij} = g_i \cdot q_j \cdot F_i(x_j, y_j) \equiv V(i, j; \mathbf{a})$$

$$i = 1 \cdots 4 \quad \text{and} \quad j = 1 \cdots m$$

$$\mathbf{a} = (g_2, g_3, g_4, q_1, x_1, y_1, \cdots, q_m, x_m, y_m)$$

where  $\mathbf{a}$  is the array of fitting parameters to be determined. This can be estimated by minimising the chi-square function

$$\chi^{2}(\mathbf{a}) = \sum_{i=1}^{4} \sum_{j=1}^{m} \frac{[V_{ij} - V(i, j; \mathbf{a})]^{2}}{\sigma_{ij}^{2}}$$

where  $\sigma_{ij}^2$  is the error of the *i*th channel at the *j*th measurement and is assumed to be equal for all measurements and thus negligible. The diagonal elements of the covariant matrix build by second derivatives to each fit parameter of the chi-square function are the errors of each fitted quantity.

#### 4. Test with Measured Data

The method described was tested with data from testbench measurements with a prototype BPM. The prototype BPM is mounted on a two dimensional precision stage with a wire strung coaxially through the BPM. A beam simulating TEM-wave is induced into the system, and all four output signals are detected and recorded. With this setup mapping scans have been performed on a square field with 1.2 mm edge length and 20  $\mu$ m step width. The relative error for each voltage measurement is 0.25% and the signal source amplitude jitter is 1% of the peak value.

Several seeds of five, eight, and twelve position were randomly drawn from the data field, simulating five, eight, and twelve different orbit measurements with randomly spread beam positions in the transverse plane of the BPM. Fig.2 shows a typical result for a five point random selection.

There is a systematic offset between set and back calculated wire positions. This offset is caused by an imperfect alignment of the wire with respect to the mechanical center of the BPM. After correcting the set values for this offset, a typical result for an eight point selection is depicted in Fig.3 showing good agreement between set and back calculated wire position.

In total, 30 seeds were tested and convergence was always reached. The average error for the fitted wire position is  $\sigma_{fit} = 8 \ \mu \text{m}$  and  $\sigma_{offsety} = 15 \ \mu \text{m}$  for the average offset error resulting in a total error of  $\sigma_{tot} = 17 \ \mu \text{m}$ . In Tab.1 results for estimated gain factors are summarised.

The average bunch charge parameter was calculated to  $q=88.7\pm0.6$ , where error reflects the jitter of the signal source.

#### 5. Conclusion

Tests values from testbench measurements show that the proposed procedure is useful to determine beam positions in an absolute reference frame centred in the geometrical center of a BPM.

### References

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- [3] J. H. Cuperus, Monitoring of Particle Beams at High Frequencies, NIM 145 (1977) 219-231, 1977.
- [4] K. Satoh, M. Tejima, Recalibration of Position Monitors with Beams, Proc of the PAC95, Dallas.

Table 1 Results for gain factors.

Quantity	Fit Estimate		
$g_2$	1.1920	±	0.0143
$g_3$	0.7684	$\pm$	0.0115
$g_4$	0.9705	$\pm$	0.0059

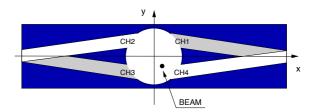


Fig. 1. Waveguide BPM profile.

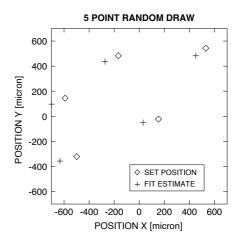


Fig. 2. Set and back calculated wire positions.

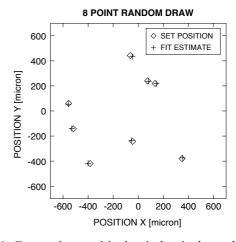


Fig. 3. Corrected set and back calculated wire positions.