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Measuring the Spin of the Higgs Boson*

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Abstract

By studying the threshold dependence of the excitation curve and the angular distribution in Higgs-strahlung at e^+e^-colliders, $e^+e^- \to ZH$, the spin of the Higgs boson in the Standard Model and related extensions can be determined unambiguously in a model-independent way.

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1. Establishing the Higgs mechanism for generating the masses of the fundamental particles, leptons, quarks and gauge bosons, in the Standard Model and related extensions, is one of the principal aims of experiments at prospective e^+e^- linear colliders [1]. After the experimental clarification of tantalizing indications of a light Higgs boson at LEP [2] has been stopped, the particle can be discovered at the Tevatron [3] or later at the LHC [4].

Assuming the positive outcome of these experiments, we address in this letter the question of how the spinless nature and the positive parity of the Higgs boson[1] can be established in a model independent way. Higgs-strahlung,

$$e^+e^- \to ZH, \tag{1}$$

provides the mechanism for the solution of this problem. The rise of the excitation curve near the threshold and the angular distributions render the spin-parity analysis of the Higgs boson unambiguous in this channel. Without loss of generality, we can assume the Higgs boson to be emitted from the Z-boson line, Fig. 1(a). Were it emitted from the lepton line[2], the required Hee coupling would be so large that the state could have been detected as a resonance at LEP, $e^+e^- \to H(\gamma)$, or could be detected at the LHC via resonant $H \to e^+e^-$ decays, dominating over the $H \to ZZ^{(*)} \to 4l$ decay mode which involves two small Z branching ratios.

The cross section for Higgs-strahlung in the Standard Model is given by the expression [7]

$$\sigma[e^+e^- \to ZH] = \frac{G_F^2 M_Z^4}{96\pi s}\,(v_e^2 + a_e^2)\,\beta\,\frac{\beta^2 + 12M_Z^2/s}{(1-M_Z^2/s)^2}, \tag{2}$$

where $v_e = -1 + 4\sin^2\theta_W$ and $a_e = -1$ are the vector and axial-vector Z charges of the electron; M_Z is the Z-boson mass, $\sqrt{s}$ the centre-of-mass energy, and $\beta = 2p/\sqrt{s}$ the Z/H three-momentum in the centre-of-mass frame, in units of the beam energy, *i.e.* $\beta^2 = [1-(M_H+M_Z)^2/s][1-(M_H-M_Z)^2/s]$. The excitation curve rises linearly with β and therefore steeply with the energy above the threshold[3]:

$$\sigma \sim \beta \sim \sqrt{s-(M_H+M_Z)^2}. \tag{3}$$

This rise is characteristic of the production of a scalar particle in conjunction with the Z boson (with only two exceptions, to be discussed later).

[1] The determination of the parity and the parity mixing of a spinless Higgs boson has been extensively investigated in Refs. [5, 6].

[2] We thank H. Murayama and T. Rizzo for alerting us to this potential loophole.

[3] Non-zero width effects can easily be incorporated; see A. Para, note in preparation.

The second characteristic is the angular distribution of the Higgs and Z bosons in the final state [8],

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = \frac{3}{4}\frac{\beta^2\sin^2\theta + 8M_Z^2/s}{\beta^2 + 12M_Z^2/s}. \tag{4}$$

The distribution of the polar angle θ is isotropic near the threshold and it develops into the characteristic $\sin^2\theta$ law at high energies which corresponds to dominant longitudinal Z production, congruent with the equivalence theorem.

Independent information on the helicity of the Z state is encoded in the final-state fermion distributions in the decay $Z \to f\bar{f}$. Denoting the fermion polar angle[4] in the Z rest frame with respect to the Z flight direction in the laboratory frame by θ_*, the double differential distribution in θ and θ_* is predicted by the Standard Model to be

$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta\, d\cos\theta_*} &= \frac{9M_Z^2\gamma^2/4s}{\beta^2 + 12M_Z^2/s}\Big\{\sin^2\theta\sin^2\theta_* + \frac{1}{2\gamma^2}[1+\cos^2\theta][1+\cos^2\theta_*] \\ &\quad + \frac{1}{2\gamma^2}\frac{2\,v_e a_e}{(v_e^2+a_e^2)}\frac{2\,v_f a_f}{(v_f^2+a_f^2)}4\cos\theta\cos\theta_*\Big\}, \end{aligned} \tag{5}$$

with $\gamma^2 = E_Z^2/M_Z^2 = 1 + \beta^2 s/4M_Z^2$. Again, for high energies, the longitudinal Z polarization is reflected in the asymptotic behaviour $\propto \sin^2\theta_*$.

2. The helicity formalism is the most convenient theoretical tool for defining observables which uniquely prove the scalar nature of the Standard-Model Higgs boson. Denoting the basic helicity amplitude [9] for arbitrary H spin-$\mathcal{J}$, with the azimuthal angle set to zero, by

$$\langle Z(\lambda_Z)H(\lambda_H)|Z^*(m)\rangle = \frac{g_W M_Z}{\cos\theta_W}\, d^1_{m,\lambda_Z-\lambda_H}(\theta)\,\Gamma_{\lambda_Z\lambda_H}, \tag{6}$$

the reduced vertex $\Gamma_{\lambda_Z\lambda_H}$ is dependent only on the helicities of the Z and Higgs bosons, λ_Z and λ_H respectively, and is independent of the Z^* spin component m along the beam-axis by rotational invariance. The standard coupling is split off explicitly.

The normality of the Higgs state,

$$n_H = (-1)^{\mathcal{J}}\,\mathcal{P}, \tag{7}$$

which is the product of the spin signature $(-1)^{\mathcal{J}}$ and the parity $\mathcal{P}$, plays an important rôle in classifying these helicity amplitudes. The normality determines the relation between

[4]Azimuthal distributions provide supplementary information, see Ref. [8]; to match the definitions used in the formulae, the azimuthal angle shown in Fig. 9(a) of Ref. [8] should be denoted $(\pi - \phi_*)$.

helicity amplitudes under parity transformations. If the interactions which determine the vertex (6) are $\mathcal{P}$ invariant, equivalent to $\mathcal{CP}$ invariance in this specific case, the reduced vertices are related by

$$\Gamma_{\lambda_Z \lambda_H} = n_H \, \Gamma_{-\lambda_Z \, -\lambda_H}. \tag{8}$$

The total cross section for a $\mathcal{CP}$ invariant theory is in this formalism then given by,

$$\sigma = \frac{G_F^2 M_Z^6 \,(v_e^2 + a_e^2)}{24\pi s^2 \,(1 - M_Z^2/s)^2} \,\beta \left[|\Gamma_{00}|^2 + 2\,|\Gamma_{11}|^2 + 2\,|\Gamma_{01}|^2 + 2\,|\Gamma_{10}|^2 + 2\,|\Gamma_{12}|^2 \right], \tag{9}$$

Correspondingly, the polar angular distributions introduced above can be written,

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\cos\theta} = \frac{3}{4\,\Gamma^2} \left\{ \sin^2\theta \left[|\Gamma_{00}|^2 + 2\,|\Gamma_{11}|^2 \right] + [1 + \cos^2\theta] \left[|\Gamma_{01}|^2 + |\Gamma_{10}|^2 + |\Gamma_{12}|^2 \right] \right\}, \tag{10}$$

and

$$\begin{aligned} \frac{1}{\sigma}\,\frac{d\sigma}{d\cos\theta\, d\cos\theta_*} &= \frac{9}{16\,\Gamma^2} \Bigg\{ \sin^2\theta \sin^2\theta_* \,|\Gamma_{00}|^2 + \frac{1}{2}[1+\cos^2\theta][1+\cos^2\theta_*] \left[|\Gamma_{10}|^2 + |\Gamma_{12}|^2 \right] \\ &\quad + \sin^2\theta\,[1+\cos^2\theta_*]|\Gamma_{11}|^2 + [1+\cos^2\theta]\,\sin^2\theta_* |\Gamma_{01}|^2 \\ &\quad + \frac{2\,v_e a_e}{(v_e^2 + a_e^2)}\,\frac{2\,v_f a_f}{(v_f^2 + a_f^2)}\, 2\cos\theta\cos\theta_* \left[|\Gamma_{10}|^2 - |\Gamma_{12}|^2 \right] \Bigg\}, \end{aligned} \tag{11}$$

where Γ^2 corresponds to the square bracket of Eq. (9).

The helicity amplitudes of Higgs-strahlung in the Standard Model are given by

$$\begin{aligned} \Gamma_{00} &= -E_Z/M_Z, \\ \Gamma_{10} &= -1, \\ \Gamma_{01} &= \Gamma_{11} = \Gamma_{12} = 0, \end{aligned} \tag{12}$$

and the Higgs boson carries even normality:

$$n_H = +1. \tag{13}$$

These amplitudes determine uniquely the spin-parity quantum numbers of the Higgs boson; this will be demonstrated for a $\mathcal{CP}$ invariant theory, for even and odd normality Higgs bosons in **3a** and **3b** respectively. The analysis will be extended to mixed parity assignments in $\mathcal{CP}$ noninvariant theories thereafter.

3a. States of even normality $\mathcal{J}^{\mathcal{P}} = 1^-, 2^+, 3^- \ldots$ can be excluded by measuring the threshold behaviour of the excitation curve and the angular correlations[5].

[5] It is well known that the observation of $H \to \gamma\gamma$ decays or the formation of Higgs bosons, $\gamma\gamma \to H$, in photon collisions rules out the spin-1 assignment as a result of the Landau-Yang theorem.

The most general current describing the Z^*ZH vertex in Fig. 1(a) is given by the expression

$$\mathcal{J}_\mu = \frac{g_W M_Z}{\cos\theta_W} T_{\mu\alpha\beta_1\ldots\beta_S}\, \varepsilon^*(Z)^\alpha\, \varepsilon^*(H)^{\beta_1\cdots\beta_\mathcal{J}}. \tag{14}$$

While ε^α is the usual spin-1 polarization vector, the spin-$\mathcal{J}$ polarization tensor $\varepsilon^{\beta_1\cdots\beta_\mathcal{J}}$ of the state H has the notable properties of being symmetric, traceless and orthogonal to the 4-momentum of the Higgs boson $p_H^{\beta_i}$, and can be constructed from products of suitably chosen polarization vectors. Moreover $T_{\mu\alpha\beta_1\ldots\beta_\mathcal{J}}$, normalized such that $T_{\mu\alpha} = g_{\perp\,\mu\,\alpha}$ in the Standard Model, is transverse due to the conservation of the lepton current. These properties strongly constrain the form of the tensor. The most general tensor for spins ≤ 2 can be seen in Tab.1(top) together with the resulting helicity amplitudes. (The coefficients a_i, b_i and c_i in Tab.1 are independent of the momenta near the threshold.) The leading β dependence of the helicity amplitudes can be predicted from the form of the Z^*ZH coupling. Each momentum contracted with the Z-boson polarization vector or the H polarization tensor will necessarily give zero or one power of β:

$$p_i \cdot \varepsilon_j(\lambda_j) = \begin{cases} \beta s/2M_j & \text{for} \quad i \neq j \quad \text{and} \quad \lambda_j = 0 \\ 0 & \text{for} \quad i = j = Z/H \quad \text{or} \quad \lambda_j = \pm. \end{cases} \tag{15}$$

Furthermore, any momentum contracted with the lepton current will also give rise to one power of β due to the transversality of the current. Then, one need only count the number of momenta in each term of $T^{\mu\alpha\beta_1\ldots\beta_\mathcal{J}}$ to understand the threshold behaviour of the corresponding helicity amplitudes. The β dependence of the excitation curve can be derived from the squared β dependence of the helicity amplitude multiplied by a single factor β from the phase space.

Spin 0: The spin-0 helicity amplitudes presented in Tab.1(top) have no dependence on β near threshold. Consequently the excitation curve rises linearly in β at threshold, with the single power of β coming from the phase space. This is also the case for the Standard Model, as described in **1** and obtained from the spin-0 form factors by setting $a_1 = 1$ and $a_2 = 0$.

Spin 1: It is easily seen that all helicity amplitudes vanish near threshold linearly in β, so the excitation curve rises $\sim \beta^3$, distinct from the Standard Model.

Spin 2: The most general spin-2 tensor contains a term with no momentum dependence ($\propto c_1$), resulting in helicity amplitudes which do not vanish at threshold if $c_1 \neq 0$. However, the helicity amplitudes Γ_{01} and Γ_{11} contain c_1 and are consequently non-zero in this case, leading to non-trivial $[1+\cos^2\theta]\sin^2\theta_*$ and $\sin^2\theta[1+\cos^2\theta_*]$ correlations which are absent in the Standard Model. Therefore, if the excitation curve rises linearly, not observing these correlations in experiment rules out the spin-2 assignment to the H state. However, if $c_1 = 0$ in the spin-2 case, the excitation curve rises $\sim \beta^5$ near threshold.

Spin ≥ 3: Above spin-2 the number of independent helicity amplitudes does not increase any more [9]. Consequently, the most general spin-$\mathcal{J}$ tensor $T_{\mu\alpha\beta_1\ldots\beta_\mathcal{J}}$ is a direct product of a tensor $T_{(2)}^{\mu\alpha\beta_i\beta_j}$ isomorphic with the spin-2 tensor and momentum vectors $q^{\beta_k} = (p_Z + p_H)^{\beta_k}$ as required by the properties of the spin-$\mathcal{J}$ wave-function $\varepsilon^{\beta_1\cdots\beta_\mathcal{J}}$,

$$T_{(\mathcal{J})}^{\mu\alpha\beta_1\beta_2\ldots\beta_\mathcal{J}} = \sum_{i<j} T_{(2)}^{\mu\alpha\beta_i\beta_j} q^{\beta_1}\ldots q^{\beta_{i-1}} q^{\beta_{i+1}}\ldots q^{\beta_{j-1}} q^{\beta_{j+1}}\ldots q^{\beta_\mathcal{J}}. \tag{16}$$

Contracted with the wave-function, the extra $\mathcal{J}-2$ momenta give rise to a leading power $\beta^{\mathcal{J}-2}$ in the helicity amplitudes. The cross section therefore rises near threshold $\sim \beta^{2\mathcal{J}-3}$, *i.e.* with a power ≥ 3, in contrast to the single power of the Standard Model.

3b. It is quite easy to rule out particles of odd normality, $\mathcal{J}^\mathcal{P} = 0^-,\ 1^+,\ 2^-,\ \ldots,$ which may mimic the Standard Model Higgs boson in Higgs-strahlung. Since the helicity amplitude Γ_{00} must vanish by Eq. (8), the observation of a non-zero $\sin^2\theta\,\sin^2\theta_*$ correlation in Eq. (11), as predicted by the Standard Model, eliminates all odd normality states. In particular, the assignment of negative parity to the spin-0 state can be ruled out by observing [10] a polar-angle distribution different from the energy-independent $[1+\cos^2\theta]$ distribution which is characteristic for 0^- particle production [8] in contrast to the Standard Model.

Nevertheless, in anticipation of the mixed normality scenario we present the helicity amplitudes also for Higgs bosons of odd normality, and spin ≤ 2 in Tab.1(bottom). We find a similar picture to the even normality case, where here the excitation curve only presents a linear rise for a particle of spin-1. The generalization to higher spins ≥ 3 follows exactly as before, resulting in an excitation curve $\sim \beta^{2\mathcal{J}-1}$, *i.e.* with a power ≥ 5, at threshold.

The above formalism can be generalized easily to rule out a mixed normality state with spin ≥ 1. For a Higgs boson of mixed normality one may no longer use Eq. (8) to obtain the simple form of the (differential) cross sections seen in Eqs. (9–11). In particular, the polar angle distribution, Eq. (10), is modified to include a linear term proportional to $\cos\theta$, indicative of $\mathcal{CP}$ violation [6]. The analysis, however, proceeds as in the fixed normality case, since the most general tensor vertex will be the sum of the even and odd normality tensors given in Tab.1.

A mixed normality Higgs boson of spin ≥ 3 may be eliminated by a non-linear rise of the excitation curve at threshold, whereas those of spin-1 and spin-2 may exhibit a linear β dependence, arising from the odd and even tensor contributions respectively. However, these two exceptions can be ruled out by observing neither $[1+\cos^2\theta]\sin^2\theta_*$ nor $\sin^2\theta[1+\cos^2\theta_*]$ angular correlations, since a linear excitation curve in both cases requires that both Γ_{01} and Γ_{11} be non-zero.

4. The analyses described above, can be summarized in a few characteristic observations. The key is the threshold behaviour of the excitation curve which is predicted to be linear in β for the $\mathcal{J}^\mathcal{P} = 0^+$ Higgs boson within the Standard Model. The observation of the linear rise, if supplemented by the angular correlations for two exceptional cases, rules out all other $\mathcal{J}^\mathcal{P}$ assignments:

$\sigma \sim \sqrt{s-(M_Z+M_H)^2}$	(i)	rules out $\mathcal{J}^\mathcal{P} = 0^-, 1^-, 2^-, 3^\pm, \ldots$
threshold:	(ii)	rules out $\mathcal{J}^\mathcal{P} = \quad 1^+, 2^+$
		if no $[1+\cos^2\theta]\sin^2\theta_*$ $\sin^2\theta[1+\cos^2\theta_*]$ correlations

The same rules also eliminate all spin states $\mathcal{J} \geq 1$ for mixed-normality assignments.

The rules can be supplemented by other observables which are specific to two interesting cases. By observing a non-zero $H\gamma\gamma$ coupling, the spin-1 assignment can be ruled out independently. Moreover, the negative-parity assignment in the spin-0 case would give rise to the energy-independent angular distribution $\sim [1+\cos^2\theta]$ in contrast to scalar Higgs production, while mixed $\mathcal{CP}$ noninvariant $0^\pm$ assignments can be probed in a linear $\cos\theta$ dependence of the Higgs-strahlung cross section.

As a result, the measurement of the threshold behaviour of the excitation curve for Higgs-strahlung combined with angular correlations can be used to establish the $\mathcal{J}^\mathcal{P} = 0^+$ character of the Higgs boson in the Standard Model and related extensions unambiguously.

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$\mathcal{J}^{\mathcal{P}}$	Z^*ZH Coupling	Helicity Amplitudes	Threshold
		Even Normality $n_H = +$	
0^+	$a_1 g_\perp^{\mu\alpha} + a_2 k_\perp^\mu q^\alpha$	$\Gamma_{00} = (-a_1 E_Z - \frac{1}{2} a_2\, s^{3/2}\, \beta^2)/M_Z$	1
		$\Gamma_{10} = -a_1$	1
1^-	$b_1\, g^{\alpha\beta} k_\perp^\mu + b_2\, q^\alpha q^\beta k_\perp^\mu$ $+ b_3\, (q^\alpha g_\perp^{\mu\beta} - q^\beta g_\perp^{\mu\alpha})$ $+ b_4\, (q^\alpha g_\perp^{\mu\beta} + q^\beta g_\perp^{\mu\alpha})$	$\Gamma_{00} = \beta\, [-b_1\, (s - M_Z^2 - M_H^2) - \frac{1}{2}\, b_2\, s^2\, \beta^2 + b_3\, s$ $- b_4\, (M_Z^2 - M_H^2)]\sqrt{s}/(2\, M_Z M_H)$	β
		$\Gamma_{10} = \beta\, (b_3 - b_4)\, s/(2\, M_H)$	β
		$\Gamma_{01} = \beta\, (b_3 + b_4)\, s/(2\, M_Z)$	β
		$\Gamma_{11} = \beta \sqrt{s}\, b_1$	β
2^+	$c_1\, (g^{\alpha\beta_1} g_\perp^{\mu\beta_2} + g^{\alpha\beta_2} g_\perp^{\mu\beta_1})$ $+ c_2\, g_\perp^{\mu\alpha}\, q^{\beta_1} q^{\beta_2}$ $+ c_3\, (g_\perp^{\mu\beta_1} q^{\beta_2} + g_\perp^{\mu\beta_2} q^{\beta_1})\, q^\alpha$ $+ c_4\, (g^{\alpha\beta_1} q^{\beta_2} + g^{\alpha\beta_2} q^{\beta_1}) k_\perp^\mu$ $+ c_5\, k_\perp^\mu q^\alpha q^{\beta_1} q^{\beta_2}$	$\Gamma_{00} = \frac{\sqrt{2/3}}{M_Z M_H^2} \left\{c_1\, E_H(s - M_Z^2 - M_H^2) - \frac{1}{8} c_5\, s^{7/2} \beta^4\right.$ $\left. - \frac{1}{4} s^2 \beta^2 [c_2\, E_Z - 2\, c_3\, E_H + 2\, c_4\, (s - M_Z^2 - M_H^2)/\sqrt{s}]\right\}$	1
		$\Gamma_{10} = \sqrt{2/3}(-c_1 - c_2\, s^2 \beta^2/(4 M_H^2))$	1
		$\Gamma_{01} = (2\, c_1 (s - M_Z^2 - M_H^2) + c_3\, s^2 \beta^2)/(2\sqrt{2} M_Z M_H)$	1
		$\Gamma_{11} = (-c_1\, E_H + \frac{1}{2}\, c_4\, s^{3/2} \beta^2)\sqrt{2}/M_H$	1
		$\Gamma_{12} = -2\, c_1$	1
		Odd Normality $n_H = -$	
0^-	$a_1\, \epsilon^{\mu\alpha\rho\sigma} q_\rho k_\sigma$	$\Gamma_{00} = 0$	
		$\Gamma_{10} = -i\, \beta\, s\, a_1$	β
1^+	$b_1\, \epsilon^{\mu\alpha\beta\rho} q_\rho$ $+ b_2\, \epsilon_\perp^{\mu\alpha\beta\rho} k_\rho$ $+ b_3\, \epsilon^{\alpha\beta\rho\sigma} q_\rho k_\sigma k_\perp^\mu$	$\Gamma_{00} = 0$	
		$\Gamma_{10} = -i\, (b_1\, s\, E_H + b_2\, (E_H\, (M_Z^2 - M_H^2) + \frac{1}{2}\, s^{3/2} \beta^2))/(\sqrt{s}\, M_H)$	1
		$\Gamma_{01} = -i\, (b_1\, s\, E_Z + b_2\, (E_Z\, (M_Z^2 - M_H^2) - \frac{1}{2}\, s^{3/2} \beta^2))/(\sqrt{s}\, M_Z)$	1
		$\Gamma_{11} = -i\, (b_1\, s + b_2\, (M_Z^2 - M_H^2) + b_3\, s^2 \beta^2)/\sqrt{s}$	1
2^-	$c_1\, \epsilon^{\mu\alpha\beta_1\rho} q^{\beta_2} q_\rho$ $+ c_2\, \epsilon_\perp^{\mu\alpha\beta_1\rho} k_\rho q^{\beta_2}$ $+ c_3\, \epsilon^{\alpha\beta_1\rho\sigma} q^{\beta_2} k_\perp^\mu\, q_\rho k_\sigma$ $+ c_4\, \frac{1}{2} \epsilon^{\mu\alpha\rho\sigma} q_\rho k_\sigma q^{\beta_1} q^{\beta_2}$ $+ \beta_1 \leftrightarrow \beta_2$	$\Gamma_{00} = 0$	
		$\Gamma_{10} = -i\, \beta\, (c_1\, s E_H + c_2\, (E_H (M_Z^2 - M_H^2) + \frac{1}{2} s^{3/2} \beta^2)$ $+ \frac{1}{4}\, c_4\, s^{5/2} \beta^2)\sqrt{2s}/(\sqrt{3} M_H^2)$	β
		$\Gamma_{01} = -i\, \beta\, (c_1\, s E_Z + c_2\, (E_Z (M_Z^2 - M_H^2)$ $- \frac{1}{2} s^{3/2} \beta^2))\sqrt{s}/(\sqrt{2}\, M_Z M_H)$	β
		$\Gamma_{11} = -i\, \beta\, (c_1\, s + c_2\, (M_Z^2 - M_H^2) + c_3\, s^2 \beta^2)\sqrt{s}/(\sqrt{2}\, M_H)$	β
		$\Gamma_{12} = 0$	

Table 1: *The most general tensor couplings of the Z^*ZH vertex and the corresponding helicity amplitudes for Higgs bosons of spin ≤ 2. Here $q = p_Z + p_H$, $k = p_Z - p_H$ and $\perp$ indicates orthogonality of a vector or tensor to q^μ, $t_\perp^{\mu\cdots} = t^{\mu\cdots} - q^\mu/s\, q_\nu t^{\nu\cdots}$. For spin ≥ 3, the helicity amplitudes rise $\sim \beta^{\mathcal{J}-2}$ and $\sim \beta^{\mathcal{J}-1}$ for even and odd normalities respectively.*

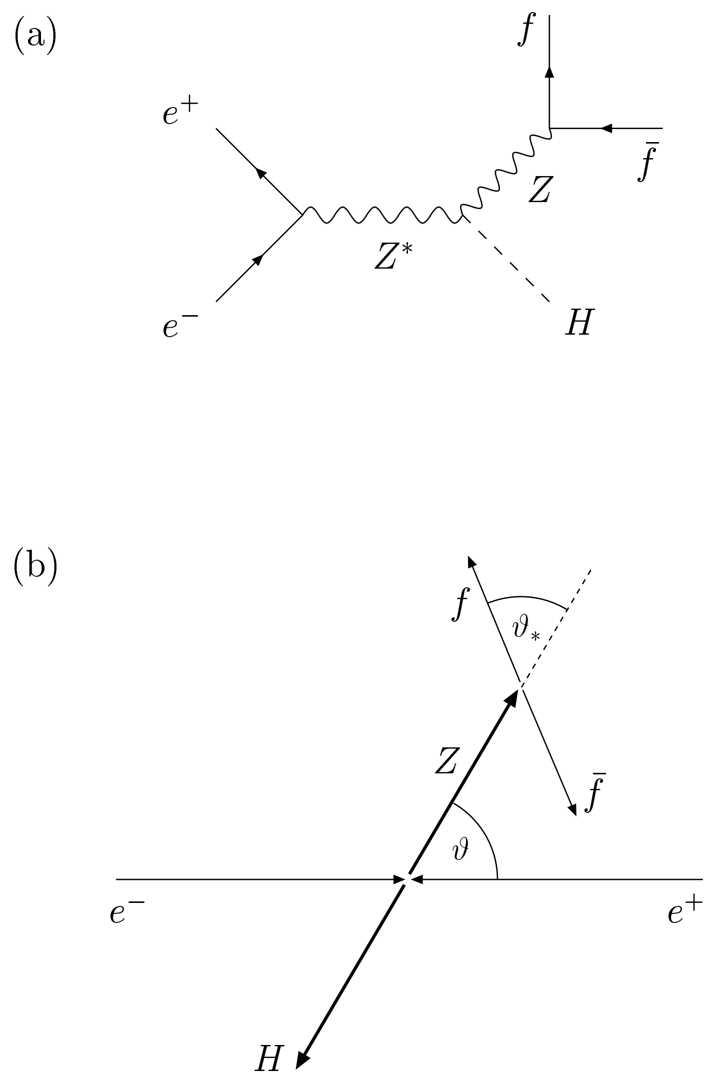

Figure 1: (a) The Higgs-strahlung process, $e^+e^- \to ZH$, followed by the subsequent Z boson decay $Z \to f\bar{f}$, and (b) the definition of the polar angles θ and θ_*, for production and decay respectively.