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# Neutrinos in the early universe

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**Abstract**

The contribution of neutrinos to the energy density of the universe is negligible. Instead, neutrinos influence significantly non-equilibrium processes in the early universe: the formation of structure, nucleosynthesis and baryogenesis. From the cosmic microwave background and from the large scale structure of matter one infers an upper bound of 1 eV on the sum of neutrino masses. The analysis of the abundances of light elements determines the number of neutrino flavours to be $N_\nu = 3$. Finally, decays of heavy Majorana neutrinos at very high temperatures can naturally explain the cosmological baryon asymmetry. Successful baryogenesis, independent of initial conditions, is possible for neutrino masses in the range $10^{-3} \leqslant m_i \lesssim 0.1$. This mass window is consistent with results from neutrino oscillations and will soon be probed by cosmological observations. ***To cite this article: W. Buchmüller, C. R. Physique 6 (2005).***


**Résumé**

**Les neutrinos et l'Univers primordial.** La contribution des neutrinos à la densité d'énergie de l'Univers est négligeable. Pourtant, ils jouent un rôle important dans les processus hors équilibre dans l'Univers primordial : la formation des structures, la nucléosynthèse et la baryogénèse. A partir de l'étude du fond diffus cosmologique et de la structure à grande échelle de la matière, on peut déduire une limite supérieure de 1 eV sur la somme des masses des neutrinos. L'analyse des abondances des éléments légers contraint le nombre de saveurs de neutrinos ($N_\nu = 3$). Enfin, la désintégration de neutrinos de Majorana lourds à très haute température peut naturellement expliquer l'asymétrie baryonique cosmologique. Un scénario de baryogénèse indépendant des conditions initiales est possible pour des masses de neutrinos dans les limites $10^{-3} \leqslant m_i \lesssim 0.1$. Cette fenêtre de masses est en accord avec les résultats des expériences d'oscillation des neutrinos et sera bientôt explorée par les observations cosmologiques. ***Pour citer cet article : W. Buchmüller, C. R. Physique 6 (2005).***


*Keywords:* Neutrino; Early universe; Leptogenesis

*Mots-clés :* Neutrino ; Univers primordial ; Leptogénèse

## 1. Introduction

Neutrinos play a key role in particle physics and in cosmology. Particle physics unravels the structure of matter at short distances. The characteristic energy scales of the different layers of structure are of order 1 eV, the binding energy of atoms, 1 MeV, the binding energy of nuclei, and 100 GeV, the energy scale of weak interactions. In recent years, solar and atmospheric

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neutrino oscillations have provided evidence for very small neutrino masses [1]. In the context of grand unified theories these new results in neutrino physics probe much larger energy scales up to $10^{15}$ GeV [2].

Knowing the laws of nature which govern the interactions of elementary particles at these energies allows us to calculate the properties of a plasma of particles at the corresponding temperatures.[1] Extrapolating the observed Hubble expansion of the universe back to early times one then concludes that such temperatures were indeed realized in the very early universe. As we shall see, neutrinos are of crucial importance for the evolution of the universe at all the energy scales relevant for the structure of matter.

At a temperature $T \sim 1$ eV electrons and nuclei combined to form neutral atoms and the universe became transparent to photons. The discovery and the recent detailed studies [3] of the corresponding *cosmic microwave background* (CMB) are a corner stone of early-universe cosmology. Most remarkably, the CMB data have shown that the expanding universe is spatially flat. It is described by the Robertson–Walker metric [4],

$$ds^2 = dt^2 - a(t)^2\left(dr^2 + r^2 d\theta^2 + r^2\, d\phi^2\right) \tag{1}$$

where $t$, $r$, $\theta$ and $\phi$ are time, radial and angular coordinates, respectively. The time dependence of the cosmic scale factor $a(t)$ is determined by the Friedmann equation,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_R + \rho_M + \rho_\Lambda) \tag{2}$$

where $\rho_R$, $\rho_M$ and $\rho_\Lambda$ are the contributions of radiation, non-relativistic matter and the 'vacuum' to the total energy density $\rho$, and $G$ is Newton's constant. The radiation is dominated by photons and neutrinos, $\rho_R = \rho_\gamma + \rho_\nu$, and the main components of the non-relativistic matter are baryons and cold dark matter, $\rho_M = \rho_B + \rho_{DM}$. The various energy densities are conveniently normalized to the critical density, i.e., $\Omega_i = \rho_i/\rho_c$, with $\rho_c = 3H_0^2/(8\pi G)$. Here $H_0$ is the present value of the Hubble parameter: $H_0 = 100\,h$ km s$^{-1}$ Mpc$^{-1}$, with $h \simeq 0.72$. For a flat universe, one then has

$$\Omega_\gamma + \Omega_\nu + \Omega_B + \Omega_{DM} + \Omega_\Lambda = 1 \tag{3}$$

During the past years a wealth of new data [5] from supernovae, the microwave background and large scale structure have demonstrated that the energy density of the universe is dominated by dark matter and 'dark energy', with $\rho_M + \rho_\Lambda \simeq 1$ (cf. Fig. 1). From a combined analysis of all data one finds [5]: $\Omega_\gamma \simeq 0.05\%$, $\Omega_\nu < 2\%$, $\Omega_B \simeq 4\%$, $\Omega_{DM} \simeq 25\%$ and $\Omega_\Lambda \simeq$

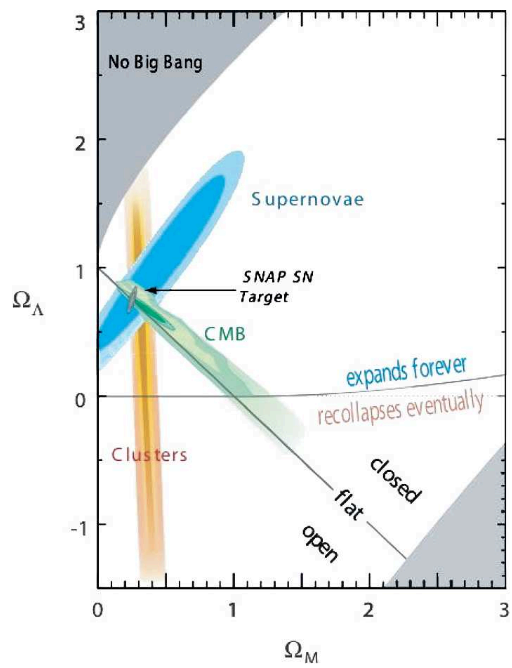


Fig. 1. Evidence for a cosmological vacuum energy density from supernovae data, cluster measurements and CMB data. Also shown is the expected confidence region from the SNAP supernovae program. From [6].

[1] We use units where the Boltzmann constant $k_B = 0.86 \times 10^{-4}$ eV/K is set equal to one.

70%. A further, very important result is the precise determination of the baryon number density relative to the photon number density, $\eta_B = n_B/n_\gamma$, from CMB data [7] combined with measurements of large scale structure [8]:

$$\eta_B^{\rm CMB} = \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10} \tag{4}$$

Since no significant amount of antimatter is observed in the universe, the baryon density coincides with the cosmological baryon asymmetry, $\eta_B = (n_B - n_{\bar{B}})/n_\gamma$.

The cosmological energy density of neutrinos with standard weak interactions is determined by the sum of the neutrino masses [9],

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{92.5\ {\rm eV}} \tag{5}$$

From neutrino oscillations we know $\sum m_\nu > 0.05$ eV [1], whereas cosmological observations imply $\sum m_\nu < 1$ eV [9], which yields a small contribution to the energy density, $\Omega_\nu < 1\%$. Hence neutrinos are irrelevant for the expansion of the universe. However, as we shall see in the following sections, they play a key role in non-equilibrium processes.

In Section 2 we briefly discuss the effect of neutrinos on the formation of large scale structure, which leads to an upper bound on neutrino masses. During the epoch of nucleosynthesis neutrinos strongly influence the helium abundance. The resulting bound on the number of neutrino flavours is discussed in Section 3. At temperatures above the critical temperature of the electroweak phase transition, $T > T_{\rm EW}$, baryon and lepton number violating processes are in thermal equilibrium. Majorana neutrinos then affect the cosmological baryon asymmetry. In Section 4 we describe in some detail, how the observed baryon density can be explained by decays of heavy Majorana neutrinos.

## 2. Neutrino masses and structure formation

Although the universe appears homogeneous and isotropic when averaged over distances $\mathcal{O}(100\ {\rm Mpc})$,[2] it shows a very interesting structure on smaller scales, which started to form at a temperature $T \sim 1$ eV, when the radiation dominated phase turned into a matter dominated phase [4]. This *large scale structure* (LSS) of the universe can be analysed statistically by studying the density perturbation field

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \langle\rho\rangle}{\langle\rho\rangle} \tag{6}$$

Here $\rho(\vec{x})$ is the local mass density, and $\langle\rho\rangle$ is the mass density averaged over the total volume.

A characteristic feature of the density perturbations is the correlation function

$$\xi(\vec{r}) = \big\langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\big\rangle \tag{7}$$

which is conveniently analysed in terms of its Fourier transform,

$$\xi(r) = \frac{V}{(2\pi)^3}\int {\rm d}^3k\, |\delta_k|^2(k)\, {\rm e}^{-{\rm i}\vec{k}\cdot\vec{r}} \tag{8}$$

Here we have used the isotropy of the universe on very large scales, which implies that $\xi$ and $|\delta_{\vec{k}}|^2$ depend only on $r = |\vec{r}|$ and $k = |\vec{k}|$, respectively. The Fourier transform of the correlation function is the so-called power spectrum,

$$P(k) = |\delta_k|^2(k) \tag{9}$$

The leading theory of structure formation starts from an approximately scale invariant primordial spectrum of density perturbations,

$$P_0(k) \propto k^n \tag{10}$$

with $n \simeq 1$. Structure formation takes place during the matter dominated phase of the early universe, when the constituents of cold dark matter and baryons are non-relativistic, i.e., at temperature below $\sim 1$ eV. Small density inhomogeneities grow and form the large inhomogeneities observed today via the gravitational, so-called Jeans instability. During this epoch photons and, depending on their mass, also neutrinos are relativistic. This modifies the density perturbations in a way which has to be

[2] 1 Mpc $= 10^6$ pc $= 3.086 \times 10^{24}$ cm.

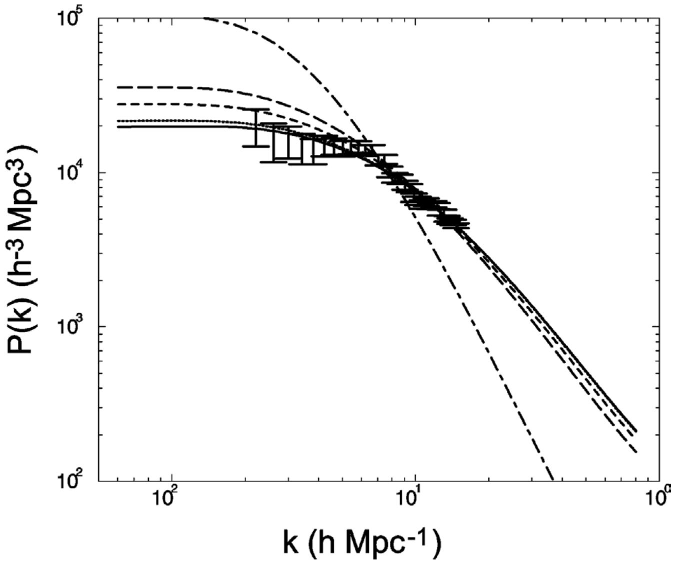


Fig. 2. Comparison of the 2dF power spectrum with theoretical predictions for different neutrino masses: $m_\nu = 0$ (full line), $m_\nu = 0.1$ eV (dotted line), $m_\nu = 0.3$ eV (small-dashed line), $m_\nu = 0.5$ eV (long-dashed line), $m_\nu = 3$ eV (dashed-dotted line). The other parameters are $\Omega_M = 0.3$, $\Omega_B = 0.04$ and $h = 0.7$. From [11].

calculated by solving the Boltzmann equations for this plasma. The power spectrum is then modified by a transfer function $T$ which depends on wave number and redshift:

$$P(k,z) = P_0(k)T^2(k,z) \tag{11}$$

The qualitative effect of neutrinos on $T(k,z)$ can be easily understood. As long as neutrinos are relativistic ($t < t_{nr}$), they can freely stream out of overdense regions into underdense regions, where the time $t_{nr}$ depends on the neutrino mass. This damps inhomogeneities on scales smaller than $\lambda_{\mathrm{FS}}(t_{nr})$. For larger scales, inhomogeneities amplify and contribute to structure formation. In a flat universe, dominated by dark matter and dark energy, one finds for the corresponding wave number [4]:

$$k_{\mathrm{FS}} = \frac{2\pi}{\lambda_{\mathrm{FS}}} \sim 0.03\left(\frac{m_\nu}{1\ \mathrm{eV}}\right)\mathrm{Mpc}^{-1} \tag{12}$$

For $\Omega_\nu \ll \Omega_M$, the suppression of the power spectrum on small scales due to the free streaming of neutrinos is given by ($k \gg k_{\mathrm{FS}}$) [10]:

$$\frac{\Delta P(k,z)}{P(k,z)} \simeq -8\frac{\Omega_\nu}{\Omega_M} \tag{13}$$

In Fig. 2 the 2dF (2 degree Field Galaxy Redshift Survey) data for the power spectrum are compared with the theoretical predictions for different neutrino masses.[3] Neutrino masses larger than 0.5 eV are clearly in conflict with data. For the sum of neutrino masses $\sum m_\nu$ combined analyses of CMB and LSS yield currently upper bounds in the range 0.6–1.8 eV [9,11]. These results will be considerably improved in the coming years. Weak lensing of the CMB [12] and of galaxies [13] are expected to probe $\sum m_\nu$ down to 0.10 eV [9,11,14].

## 3. Neutrino flavours and nucleosynthesis

At a temperature $T \sim 1$ MeV the weak interactions between neutrinos, electrons and positrons freeze out. At the corresponding decoupling temperature $T_D$ the rate of the weak interactions is equal to the Hubble parameter, $\Gamma_{\mathrm{weak}} = H|_{T_D}$. Here the Hubble parameter depends on $g_*$, the effective number of degrees of freedom in the plasma, $H = 1.66 g_* T^2/M_p$, with $M_p = 1.2 \times 10^{19}$ GeV and $g_* = 43/4$ in the standard model [4]. Since the electroweak couplings of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ to $e^-$ and $e^+$ are not identical, one finds slightly different decoupling temperatures for the different neutrino flavours, $T_D(\nu_e) = 2.4$ MeV and $T_D(\nu_\mu, \nu_\tau) = 3.7$ MeV [9].

[3] The analysis assumes no running of the scalar spectral index, no tensor perturbations and $N_\nu = 3$ degenerate neutrinos.

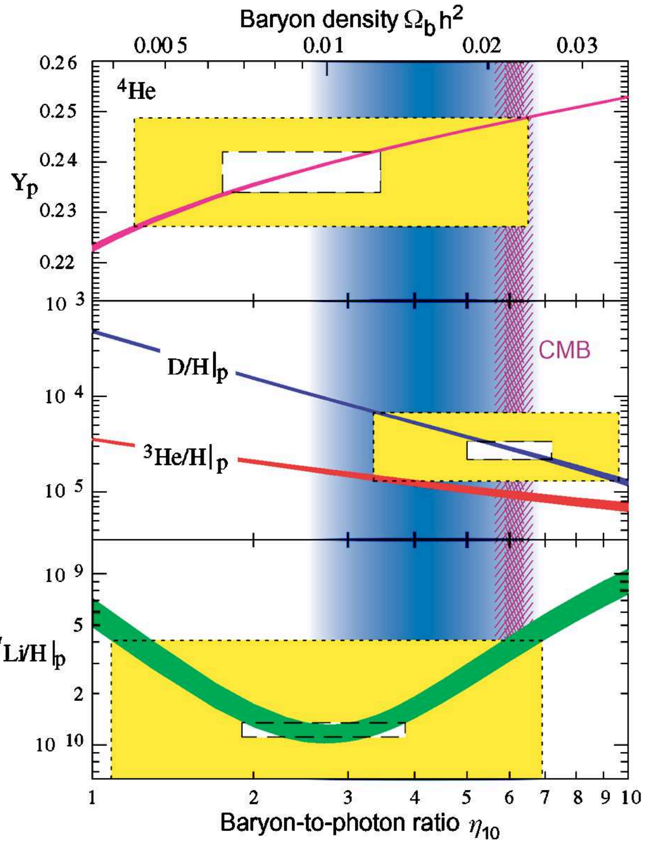


Fig. 3. The primordial abundances (subscript 'p') of $^4$He, D, $^3$He and $^7$Li as predicted by the standard model of nucleosynthesis. Boxes indicate the observed light element abundances (smaller boxes: $2\sigma$ statistical errors; larger boxes: $2\sigma$ statistical and systematic errors added in quadrature). The narrow vertical band indicates the CMB measurement of the comic baryon density. $\eta_{10}$ is the baryon to photon ratio $\eta_B$ in units $10^{-10}$. From [5].

At a temperature about one order of magnitude below the neutrino decoupling, $T \sim m_e/3 \sim 0.2$ eV, $e^+e^-$ pairs annihilate into photons, which increases the temperature of the photon gas. The number $g_*$ of relativistic degrees of freedom of the electromagnetic plasma before and after annihilation is $11/2$ and 2, respectively. Since entropy is conserved, this implies for the ratio of the temperatures of neutrinos and photons: $T_\nu/T_\gamma = (4/11)^{1/3} = 0.71$—a prediction whose experimental verification remains a great challenge! The theoretical predictions for $T_\nu$ and for the decoupling temperatures $T_D$ depend only on electroweak processes and have therefore negligible errors. This implies that, within the standard model, the energy density of the universe is rather precisely known during the subsequent process of nucleosynthesis.

The formation of the light elements D, $^3$He, $^4$He and $^7$Li started at a temperature $T \sim 0.2$ MeV. A crucial quantity for these processes is the neutron to proton ratio

$$\frac{n_n}{n_p} \simeq \mathrm{e}^{-\frac{m_n - m_p}{T}} \tag{14}$$

at the freeze-out temperature $T_D(n)$ for the weak interactions which keep protons and neutrons in equilibrium. One finds $T_D(n) \simeq 0.5 g_*^{1/6}$ MeV [4], and therefore $n_n/n_p \simeq 1/6$. At $T \sim 0.2$ MeV deuterium began to form and almost all free neutrons were bound into $^4$He. This yields the corresponding mass fraction:

$$Y \simeq \frac{4n_{4\mathrm{He}}}{n_n + n_p} \simeq \frac{2n_n/n_p}{1 + n_n/n_p} \simeq \frac{1}{4} \tag{15}$$

in good agreement with observation (cf. Fig. 3).

Detailed calculations have been performed for the abundances of the light elements, D, $^3$He, $^4$He and $^7$Li. The results depend via $g_*$ on the energy density of relativistic particles at the time of nucleosynthesis and the relative baryon to photon number density $\eta_B = n_B/n_\gamma$ (cf. Fig. 3). With only photons and neutrinos as relativistic particles, as predicted by the standard model, one finds for the number of neutrino flavours:

$$N_\nu = 2.5^{+1.1}_{-0.9} \quad \text{and} \quad N_\nu = 3.08^{+0.74}_{-0.65} \tag{16}$$

in the analyses [15] and [16], respectively. The relative baryon to photon number density is given by [5]

$$\eta_B^{\mathrm{BBN}} = \frac{n_B}{n_\gamma} = (3.4\text{–}6.9) \times 10^{-10} \tag{17}$$

It is very remarkable that this determination of the baryon asymmetry at a temperature $T \sim 1$ MeV is consistent with the baryon asymmetry $\eta_B^{\mathrm{CMB}}$ measured at a temperature $T \sim 1$ eV. This gives us confidence that our current picture of the early universe is correct at least up to temperatures of a few MeV.

## 4. Neutrino masses and baryon asymmetry

### *4.1. Conditions for baryogenesis*

Based on the validity of the standard model up to energies of order 100 GeV and the indications for physics at energies of $10^{10}$–$10^{15}$ GeV from neutrino oscillations, it is tempting to consider also processes in the early universe at the corresponding very high temperatures, where baryogenesis may have taken place.

A matter-antimatter asymmetry can be dynamically generated in an expanding universe if the particle interactions and the cosmological evolution satisfy Sakharov's conditions [17]:

- baryon number violation,
- $C$ and $CP$ violation,
- deviation from thermal equilibrium.

Here $C$ and $CP$ are charge conjugation and the combined charge conjugation and parity transformation, respectively. At present there exist a number of viable scenarios for baryogenesis. In grand unified theories baryon number ($B$) and lepton number ($L$) are broken by the interactions of gauge bosons and leptoquarks. This is the basis of classical GUT baryogenesis [4]. In a similar way, the lepton number violating decays of heavy Majorana neutrinos lead to leptogenesis [18,19]. In the simplest version of leptogenesis the initial abundance of the heavy neutrinos is generated by thermal processes. Alternatively, heavy neutrinos may be produced in inflaton decays [20] or in the reheating process after inflation [21]. For classical GUT baryogenesis and for thermal leptogenesis the departure from thermal equilibrium is due to the deviation of the number density of the decaying heavy particles from the equilibrium number density. How strong this departure from equilibrium is, depends on the lifetime of the decaying heavy particles and the rate of the cosmological expansion.

A crucial ingredient of baryogenesis is the connection between baryon number and lepton number at temperatures above the critical temperature of the electroweak phase transition, $T > T_{\mathrm{EW}}$, where the symmetry between electromagnetic and weak interactions is restored. Due to the chiral nature of the weak interactions $B$ and $L$ are not conserved [22]. At zero temperature this has no observable effect due to the smallness of the weak coupling. However, for temperatures above the critical temperature $T_{\mathrm{EW}}$, $B$ and $L$ violating processes come into thermal equilibrium [23]. In the standard model one then has an effective interaction between all left-handed fermions [22], which allows processes such as

$$u^c + d^c + c^c \to d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau \tag{18}$$

where $u$, $d$, $c$, $s$, $b$ and $t$ are up, down, charm, strange, bottom and top quarks, respectively, and the superscript $c$ denotes antiparticles. Clearly, such processes violate baryon and lepton number by three units,

$$\Delta B = \Delta L = 3 \tag{19}$$

The transition rate for these so-called *sphaleron processes* in the symmetric high-temperature phase has been evaluated by combining an analytical resummation with numerical lattice techniques [24]. The result is that $B$ and $L$ violating processes are in thermal equilibrium for temperatures in the range

$$T_{\mathrm{EW}} \sim 100 \text{ GeV} < T < T_{\mathrm{SPH}} \sim 10^{12} \text{ GeV} \tag{20}$$

Sphaleron processes have a profound effect on the generation of the cosmological baryon asymmetry. An analysis of the chemical potentials of all particle species in the high-temperature phase yields the following relation between the baryon asymmetry and the corresponding $L$ and $B$–$L$ asymmetries,

$$\langle B\rangle_T = c_S \langle B\text{–}L\rangle_T = \frac{c_S}{c_S - 1}\langle L\rangle_T \tag{21}$$

Here $c_S$ is a number $\mathcal{O}(1)$. In the standard model with three generations and one Higgs doublet one has $c_s = 28/79$.

The relation (21) between baryon and lepton number suggests that $B$–$L$ violation is needed to explain the cosmological baryon asymmetry if baryogenesis took place before the electroweak transition, i.e., at temperatures $T > T_{\text{EW}} \sim 100$ GeV. In the standard model, as well as its supersymmetric extension, $B$–$L$ is a conserved quantity. Hence, no baryon asymmetry can be generated dynamically in these models and one has to consider extensions with $B$–$L$ violation, and therefore lepton number violation. On the other hand, lepton number violation can only be weak, since otherwise any baryon asymmetry would be washed out. The interplay of these conflicting conditions leads to important constraints on neutrino properties and also on possible extensions of the standard model in general.

### 4.2. *Grand unification and leptogenesis*

Lepton number is naturally violated in grand unified theories (GUTs) [25]. The unification of gauge couplings at high energies suggests that the standard model gauge group is part of a larger simple group,

$$G_{\text{SM}} = U(1) \times SU(2) \times SU(3) \subset SU(5) \subset SO(10) \cdots \tag{22}$$

The simplest GUT is based on the gauge group $SU(5)$. Here quarks and leptons are grouped into the multiplets,

$$\mathbf{10} = \left(q, u^c, e^c\right), \qquad \mathbf{5}^* = \left(d^c, l\right), \qquad \mathbf{1} = N \tag{23}$$

where $q = (u, d)$ and $l = (\nu, e)$. Unlike gauge fields, quarks and leptons are not unified in a single multiplet. In particular, the singlet neutrinos $N$ are not needed in $SU(5)$ models. Since the $N$'s have no $SU(5)$ gauge interactions, they can have large Majorana masses $M$ which are not controlled by the Higgs mechanism.

The spontaneous breaking of the electroweak symmetry $U(1) \times SU(2)$ leads to quark and lepton mass matrices. For neutrinos a Dirac mass matrix is generated, $m_D = hv$. Here $v = \langle H \rangle \sim 100$ GeV is the vacuum expectation value of the Higgs field $H$, and $h_{ij}$, $i, j = 1, \ldots, 3$, are the Yukawa couplings which connect the neutrinos $\nu_j$ with the heavy singlets $N_i$ and the Higgs field. The theory predicts six Majorana neutrinos as physical states, three heavy ($N$) and three light ($\nu$), with masses

$$m_N \simeq M, \qquad m_\nu = -m_D \frac{1}{M} m_D^T \tag{24}$$

The explanation of the smallness of the light neutrino masses in terms of the largeness of the heavy neutrino masses is the so-called seesaw mechanism [2].

All quarks and leptons of one generation are unified in a single multiplet in the GUT group $SO(10)$ [25],

$$\mathbf{16} = \mathbf{10} + \mathbf{5}^* + \mathbf{1} \tag{25}$$

In the simplest pattern of symmetry breaking, $B$–$L$, a subgroup of $SO(10)$, is broken at the unification scale $\Lambda_{\text{GUT}}$. If Yukawa couplings of the third generation are $\mathcal{O}(1)$, as it is the case for the top-quark, one finds for the corresponding heavy and light neutrino masses:

$$M_3 \sim \Lambda_{\text{GUT}} \sim 10^{15} \text{ GeV}, \qquad m_3 \sim \frac{v^2}{M_3} \sim 0.01 \text{ eV} \tag{26}$$

It is very remarkable that the light neutrino mass $m_3$ is of the same order as the mass differences $(\Delta m^2_{\text{sol}})^{1/2}$ and $(\Delta m^2_{\text{atm}})^{1/2}$ inferred from neutrino oscillations. This suggests that, via the seesaw mechanism, neutrino masses probe the grand unification scale! Like for quarks and charged leptons, one expects in GUTs a mass hierarchy also for the right-handed neutrinos. For instance, if their masses scale like the up-quark masses one has $M_1 \sim 10^{-5} M_3 \sim 10^{10}$ GeV.

The lightest of the heavy Majorana neutrinos, $N_1$, is ideally suited to generate the cosmological baryon asymmetry [18]. It can decay into final states with lepton or anti-lepton,

$$N_1 \to lH, \qquad N_1 \to l^c H^c \tag{27}$$

thereby violating lepton number conservation. $N_1$ decays to lepton-Higgs pairs then yield a lepton asymmetry $\langle L \rangle_T \neq 0$, which is partially converted to a baryon asymmetry $\langle B \rangle_T \neq 0$ by the sphaleron processes. The generated asymmetry is proportional to the $CP$ asymmetry [26] in $N_1$ decays:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to lH) - \Gamma(N_1 \to l^c H^c)}{\Gamma(N_1 \to lH) + \Gamma(N_1 \to l^c H^c)} \simeq \frac{3}{16\pi} \frac{M_1}{(h^\dagger h)_{11} v^2} \text{Im}(h^\dagger m_\nu h^*)_{11} \tag{28}$$

where the seesaw mass relation (24) has been used.

From the expression (28) one easily obtains a rough estimate for $\varepsilon_1$ in terms of neutrino masses. Assuming dominance of the largest eigenvalue $m_3$ of the mass matrix $m_\nu$, phases $\mathcal{O}(1)$ and an approximate cancellation of Yukawa couplings in numerator and denominator, one finds

$$\varepsilon_1 \sim \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \sim 0.1 \frac{M_1}{M_3} \tag{29}$$

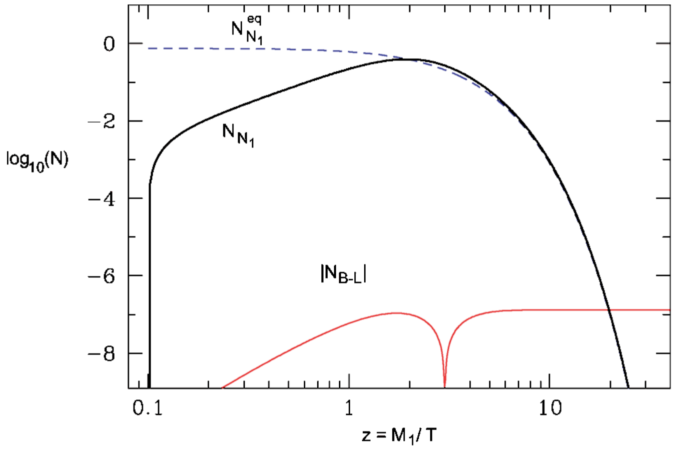


Fig. 4. Evolution of the $N_1$ abundance and the $B–L$ asymmetry for $\varepsilon_1 = -10^{-6}$ and $\overline{m} = 0.05$ eV. The full (dashed) line corresponds to zero (thermal) initial $N_1$ abundance. From [31].

where we have again used the seesaw relation. In this example, the order of magnitude of the $CP$ asymmetry is determined by the mass hierarchy of the heavy Majorana neutrinos. For a mass ratio as for up-type quarks, i.e., $M_1/M_3 \sim 10^{-5}$, one has $\varepsilon_1 \sim 10^{-6}$.

Given the $CP$ asymmetry $\varepsilon_1$, one obtains for the baryon asymmetry,

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -d\varepsilon_1\kappa_f \sim 10^{-10} \tag{30}$$

Here the dilution factor $d \sim 10^{-2}$ accounts for the increase of the number of photons in a comoving volume element between baryogenesis and today, and the efficiency factor $\kappa_f$ represents the effect of washout processes. In the estimate (30) we have assumed a typical value, $\kappa_f \sim 10^{-2}$. The correct value of the baryon asymmetry is then obtained as consequence of a large hierarchy of the heavy neutrino masses, which leads to a small $CP$ asymmetry, and the kinematical factors $d$ and $\kappa_f$ [27]. The baryogenesis temperature

$$T_B \sim M_1 \sim 10^{10}\ \text{GeV} \tag{31}$$

corresponds to the time $t_B \sim 10^{-26}$ s, which characterizes the next relevant epoch before recombination, nucleosynthesis and the electroweak phase transition.

An important question concerns the relation between leptogenesis and neutrino mass matrices which can account for low-energy neutrino data. Many interesting models, some also very different from the example given above, have been discussed in the literature [28]. Of particular interest is the connection with $CP$ violation in other low energy processes [29]. Together with leptogenesis, improved measurements of neutrino parameters will have strong implications for the structure of grand unified theories.

### *4.3. Bounds on neutrino masses*

Leptogenesis is a non-equilibrium process. For a decay width small compared to the Hubble parameter, $\Gamma_1(T) < H(T)$, heavy neutrinos are out of thermal equilibrium, otherwise they are in thermal equilibrium. The borderline between the two regimes is given by $\Gamma_1 = H|_{T=M_1}$ [4]. This is equivalent to the condition that the *effective neutrino mass*,

$$\widetilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} \tag{32}$$

equals the *equilibrium neutrino mass*

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_p} \simeq 10^{-3}\ \text{eV} \tag{33}$$

Here we have used $g_* = 434/4$ as effective number of degrees of freedom. For $\widetilde{m}_1 > m_*$ ($\widetilde{m}_1 < m_*$) the heavy neutrinos $N_1$ are in (out of) thermal equilibrium at $T = M_1$.

It is very remarkable that the equilibrium neutrino mass $m_*$ is close to the neutrino masses suggested by neutrino oscillations, $\sqrt{\Delta m^2_{\rm sol}} \simeq 8 \times 10^{-3}$ eV and $\sqrt{\Delta m^2_{\rm atm}} \simeq 5 \times 10^{-2}$ eV. This suggests that it may be possible to understand the cosmological

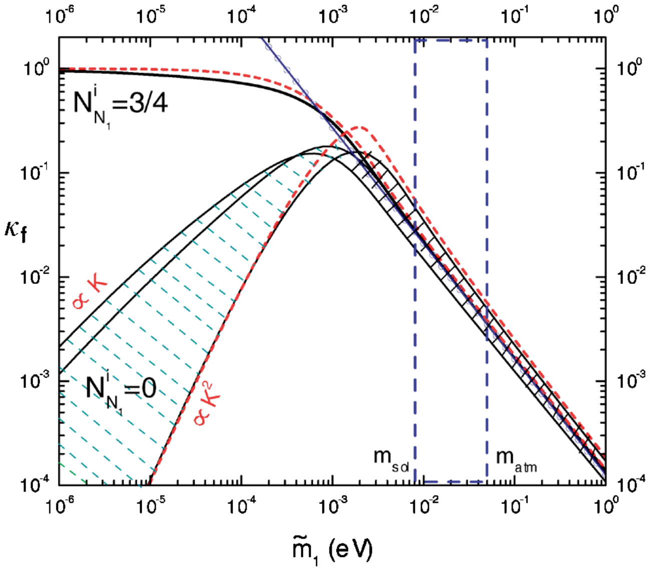


Fig. 5. The efficiency factor $\kappa_f$ for thermal initial abundance ($N^{\rm i}_{N_1} = 3/4$) and zero initial abundance ($N^{\rm i}_{N_1} = 0$); different assumptions about the scattering rate at high temperatures affect $\kappa_f$ only for effective neutrino masses $\widetilde{m}_1 < m_*$. From [34].

baryon asymmetry via leptogenesis as a process close to thermal equilibrium. Ideally, lepton number changing processes would be strong enough at temperatures above $M_1$ to keep the heavy neutrinos in thermal equilibrium and weak enough to allow the generation of an asymmetry at temperatures below $M_1$.

In general, the generated baryon asymmetry is the result of a competition between production processes and washout processes which tend to erase any generated asymmetry. The dominant processes are decays and inverse decays of $N_1$ and lepton number changing scatterings. The Boltzmann equations for leptogenesis can be expressed [30,31]:

$$\frac{{\rm d}N_{N_1}}{{\rm d}z} = -(D+S)\big(N_{N_1} - N^{\rm eq}_{N_1}\big) \tag{34}$$

$$\frac{{\rm d}N_{B-L}}{{\rm d}z} = -\varepsilon_1 D\big(N_{N_1} - N^{\rm eq}_{N_1}\big) - WN_{B-L} \tag{35}$$

Here $N_{N_1}$ and $N_{B-L}$ are number densities, $z = M_1/T$, and $D$, $S$ and $W$ denote decay rate, scattering rate and washout rate, respectively; the variable $z$ plays the role of time. In an expanding universe $z$ increases whereas the temperature $T$ decreases.

In order to understand the dependence of the generated baryon asymmetry on the neutrino parameters, it is crucial to note that the rates $D$ and $S$ depend only on $\widetilde{m}_1$ whereas the washout rate $W$ also depends on $M_1\overline{m}^2$. Here $\overline{m}$ is the absolute neutrino mass scale,

$$\overline{m}^2 = {\rm tr}(m_\nu^\dagger m_\nu) = m_1^2 + m_2^2 + m_3^2 \tag{36}$$

which, for Majorana neutrinos, is a measure of the total amount of lepton number violation. In Fig. 4 the generation of a $B{-}L$ asymmetry is shown for a typical set of parameters, $|\varepsilon_1| = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\widetilde{m}_1 = 10^{-3}$ eV, $\overline{m} = 0.05$ eV, and for two different initial conditions: zero and thermal $N_1$ abundance. The figure demonstrates that the Yukawa interactions are strong enough to bring the heavy neutrinos into thermal equilibrium before leptogenesis takes place. At temperatures below $M_1$ ($z = 2, \ldots, 8$) the number density $N_{N_1}$ exceeds the equilibrium number density $N^{\rm eq}_{N_1}$, and $N_1$ decays generate a $B{-}L$ asymmetry. The resulting asymmetry is consistent with observation, $\eta_B \sim 0.01 \times N_{B-L} \sim 10^{-9}$.

Particularly interesting is the so-called 'strong washout regime', $\widetilde{m}_1 > m_* \simeq 10^{-3}$ eV, where the generated baryon asymmetry is independent of the initial abundance of heavy neutrinos $N_1$. Detailed analyses of the Boltzmann equations (34) have been carried out in [33,34]. It turns out that in the strong washout regime the efficiency factor $\kappa_f$ takes a very simple form which is dominated just by $N_1$ decays and inverse decays [34],

$$\kappa_f = (2 \pm 1) \times 10^{-2} \left(\frac{0.01\ {\rm eV}}{\widetilde{m}_1}\right)^{1.1\pm0.1} \tag{37}$$

Here an estimate of the theoretical uncertainties in the computation of the baryon asymmetry has been included.

The *CP* asymmetry $\varepsilon_1$ satisfies an upper bound [35,36], which is a function of $M_1$, $\widetilde{m}_1$ and $\overline{m}$. Since the rates entering the Boltzmann equations depend on the same quantities, this implies for arbitrary neutrino mass matrices a maximal baryon asymmetry $\eta_B^{\rm max}$ which is a function of only $\widetilde{m}_1$, $M_1$, and $\overline{m}$,

$$\eta_B \leqslant \eta_B^{\max}(\widetilde{m}_1, M_1, \overline{m}) \simeq 0.01\varepsilon_1^{\max}(\widetilde{m}_1, M_1, \overline{m})\kappa_f(\widetilde{m}_1, M_1\overline{m}^2) \tag{38}$$

Requiring the maximal baryon asymmetry to be larger than the observed one,

$$\eta_B^{\max}(\widetilde{m}_1, M_1, \overline{m}) \geqslant \eta_B^{\mathrm{CMB}} \tag{39}$$

yields a constraint on the neutrino mass parameters $\widetilde{m}_1$, $M_1$ and $\overline{m}$.

Consider now neutrino masses with normal hierarchy where the dependence on $\overline{m}$ is given by

$$m_3^2 = \frac{1}{3}(\overline{m}^2 + 2\Delta m_{\mathrm{atm}}^2 + \Delta m_{\mathrm{sol}}^2) \tag{40}$$

$$m_2^2 = \frac{1}{3}(\overline{m}^2 - \Delta m_{\mathrm{atm}}^2 + \Delta m_{\mathrm{sol}}^2) \tag{41}$$

$$m_1^2 = \frac{1}{3}(\overline{m}^2 - \Delta m_{\mathrm{atm}}^2 - 2\Delta m_{\mathrm{sol}}^2) \tag{42}$$

In general, one expects $m_1 \leqslant \widetilde{m}_1 \lesssim m_3$. Here the lower bound [37] always holds whereas the upper bound is valid up to strong cancellations between different elements of the neutrino mass matrix. Using the upper bound on the *CP* asymmetry one can calculate the maximal baryon asymmetry. The CMB constraint (39) then yields the upper bound on the neutrino mass scale $\overline{m} \leqslant 0.20$ eV. Using Eqs. (40)–(42) one can easily translate this bound into upper limits on the individual neutrino masses. In a similar way, one finds a lower bound on $M_1$, the mass of the heavy Majorana neutrino $N_1$. The resulting upper and lower bounds are [32]:

$$m_i < 0.1\ \mathrm{eV}, \qquad M_1 > 4 \times 10^8\ \mathrm{GeV} \tag{43}$$

The detailed analyses [33] and [34] have led to the more precise upper bounds 0.15 and 0.12 eV, respectively. These upper bounds also hold in the case of inverted hierarchy. Note that the leptogenesis upper bound on the light neutrino masses is about a factor two more stringent than the recent upper bound obtained from cosmological observations [9,11].

The lower bound on the heavy Majorana neutrinos is particularly important in supersymmetric theories. The corresponding, rather large baryogenesis temperature $T_B \sim M_1 > 4 \times 10^8$ GeV strongly constrains the allowed mass spectrum of superparticles and in particular the nature of dark matter (cf. [19]).

For neutrino masses $m_i > m_*$, the efficiency factor, and therefore the baryon asymmetry $\eta_B$, is independent of the initial $N_1$ abundance. Furthermore, the final baryon asymmetry does not depend on the value of an initial baryon asymmetry generated by some other mechanism [32]. Hence, the present value of $\eta_B$ is entirely determined by neutrino properties. In this way leptogenesis singles out the neutrino mass window

$$10^{-3}\ \mathrm{eV} < m_i < 0.1\ \mathrm{eV} \tag{44}$$

How model dependent is the upper bound on the light neutrino masses from leptogenesis? Measurements in neutrino physics determine the parameters of the neutrino mass matrix,

$$m_\nu = -m_D \frac{1}{M} m_D^T + m_\nu^{\mathrm{triplet}} \tag{45}$$

which in general contains a contribution from $SU(2)$ triplet fields [38] in addition to the seesaw term generated by $SU(2)$ singlet heavy Majorana neutrinos. So far, we have only considered the minimal case, with $m_\nu^{\mathrm{triplet}} = 0$. Clearly, a dominant triplet contribution would destroy the connection between leptogenesis and low energy neutrino physics.

The discovery of quasi-degenerate neutrinos with masses above the bound of 0.1 eV would require significant modifications of minimal leptogenesis and/or the seesaw mechanism. In this case $SU(2)$ triplet contributions to neutrino masses could be a possible way out [39]. One then has no upper bound on the light neutrino masses anymore. Another way to reconcile quasi-degenerate light neutrinos with leptogenesis makes use of the enhancement of the *CP* asymmetry for quasi-degenerate heavy neutrinos. For instance, with a degeneracy comparable to the one of the light neutrinos, i.e., $(M_2 - M_1)/M_1 \sim (m_2 - m_1)/m_1 \sim 5 \times 10^{-4}(\mathrm{eV}/m_2)^2$, $(M_3 - M_2)/M_2 \sim (m_3 - m_2)/m_2 \sim 10^{-3}(\mathrm{eV}/m_2)^2$, the upper bound is relaxed to $m_i < 0.6$ eV [40]. In the extreme case of 'resonant leptogenesis' [41], *CP* asymmetries $\varepsilon = \mathcal{O}(1)$ are reached for degeneracies $\Delta M/M = \mathcal{O}(10^{-10})$. In this case the right-handed neutrino masses can be as small as 1 TeV, which might lead to observable signatures at colliders.

## 5. Conclusions and outlook

Due to their very small masses, the contribution of neutrinos to the energy density of the universe, and therefore to its expansion, is negligible. However, as we have seen in the previous sections, neutrinos play a key role in non-equilibrium processes of the early universe. As relativistic matter they influence the formation of structure, which yields a stringent upper bound on the sum of neutrino masses, $\sum m_\nu < 1$ eV. During the process of nucleosynthesis neutrinos contribute to the relativistic energy density, which implies a lower and an upper bound on the number of neutrino species consistent with $N_\nu = 3$.

Extrapolating the thermal phase of the early universe to temperatures $T \sim 10^{10}$ GeV, one finds that decays of heavy Majorana neutrinos, the seesaw partners of the light neutrinos, can naturally explain the cosmological baryon asymmetry. This leptogenesis mechanism is based on theoretical developments during the last two decades concerning the electroweak phase transition and sphaleron processes, which have firmly established a connection between baryon and lepton number in the high-temperature phase of the standard model.

Thermal leptogenesis is successful for neutrino masses in the range $10^{-3}$ eV $< m_i < 0.1$ eV, which is consistent with results from neutrino oscillations. It is very exciting that in the near future cosmological observations will reach the sensitivity $\sum m_\nu \simeq 0.10$ eV and will therefore probe the neutrino mass window preferred by leptogenesis.

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