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Interface tension of the electroweak phase transition

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Abstract

In our nonperturbative lattice investigation we study the interface tension of the finite-temperature electroweak phase transition. In this analysis the Higgs mass has been chosen to be about 35 GeV. At the transition point of a finite volume system, tunnelling between the symmetric and the Higgs phase takes place. This phenomenon leads to a splitting of the ground state, which can be used to determine the interface tension. The result obtained this way agrees with the result of the two-coupling method and with the prediction of the perturbative approach.

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1 Introduction

At high temperatures the electroweak symmetry is restored. In the early universe a cosmological phase transition took place between the high-temperature –"symmetric"– and low-temperature –"Higgs"– phases. Since the baryon violating processes are unsuppressed at high temperatures, the observed baryon asymmetry of the universe has finally been determined at the electroweak phase transition [1].

In recent years quantitative studies of the electroweak phase transition have been carried out by means of resummed perturbation theory [2]–[4] and lattice Monte Carlo simulations [5]–[9]. It has been clarified that in the SU(2)-Higgs model for Higgs masses (m_H) below 50 GeV, the phase transition is of first order. However, its strength rapidly decreases for increasing Higgs boson masses.

One of the most important quantities that characterize the strength of the phase transition is σ, the interface tension between the two phases. The course of the cosmological electroweak phase transition, thus the nucleation rate and temperature, is substantially influenced by σ.

The determination of σ for $m_H \approx 35$ GeV in the SU(2)-Higgs model is the main goal of the present letter. We propose and apply some new methods different from those of [8]. Comparing the results we strengthen the confidence in the correctness of the determination of σ.

The plan of the paper is as follows. In Section 2 we give some details of the simulations and suggest a new method to determine the transition point. Section 3 contains the determination of σ based on the finite volume tunnelling phenomena between the two phases. In Section 4 a brief description of the two-coupling study of the interface tension with a bootstrap error-analysis is presented. Section 5 is devoted to our conclusions.

2 Simulation

The lattice action of the SU(2)-Higgs model is given by

$$\begin{aligned} S[U,\varphi] &= \beta \sum_{pl} \left(1 - \tfrac{1}{2}\mathrm{Tr}\, U_{pl}\right) \\ &\quad + \sum_x \left\{ \tfrac{1}{2}\mathrm{Tr}\,(\varphi_x^+ \varphi_x) + \lambda \left[\tfrac{1}{2}\mathrm{Tr}\,(\varphi_x^+ \varphi_x) - 1\right]^2 - \kappa \sum_{\mu=1}^{4} \mathrm{Tr}\,(\varphi_{x+\hat{\mu}}^+ U_{x,\mu}\, \varphi_x) \right\} , \end{aligned} \tag{1}$$

where $U_{x,\mu}$ denotes the SU(2) gauge link variable and U_{pl} is the path-ordered product of the four $U_{x,\mu}$ around a plaquette. The symbol φ_x stands for the Higgs field, which is also written as $\varphi_x = \rho_x \cdot \alpha_x$, with $\rho_x \in \mathbf{R}^+$ and $\alpha_x \in \mathrm{SU}(2)$. Details of the simulation algorithms can be found in [8, 10].

The order parameters under study are ρ^2, $s_{\log}$, the density of $S_{\log} \equiv S[U,\varphi] - 3\sum_x \log(\rho_x)$, and L_φ, the density of $\sum_{x\mu} L_{\varphi,x\mu} \equiv \sum_{x\mu} \frac{1}{2}\mathrm{Tr}\,(\varphi_{x+\hat{\mu}}^+ U_{x,\mu}\, \varphi_x)$. The latter plays an important rôle

in our reweighting [11], which has been used to obtain data in the vicinity of the simulation point [8].

In our finite-temperature simulations we have used elongated lattices: $V = L_t \times L_{xy}^2 \times L_z$, where $L_t = 2,\ L_{xy} \ll L_z$. Keeping the bare parameters $\beta = 8.0$, $\lambda = 3.0 \cdot 10^{-4}$ and $L_t = 2$ fixed we tuned κ to its critical value. These parameters corresponds to a Higgs boson mass of roughly 35 GeV at $T = 0$.

As will be shown in Section 3, the calculation of σ by use of the two-coupling method presupposes a precise determination of the critical hopping parameter κ_c on large lattices. We suggest here a new method, which in practice gives the same and very precise κ_c as the multicanonical method [13]. However, using this method the notoriously large autocorrelation time of the multicanonical simulation can be reduced dramatically.

The method is based on the observation that, for sufficiently large elongated lattices, a flat regime appears between the peaks in the probability distribution of an order parameter [8]. The peaks correspond to pure phase configurations. Between the peaks the system is dominated by configurations with two interfaces perpendicular to the long direction z, and two bulk phases. At the transition point the free energy densities of these bulk phases are the same, the ratio of the occupied volumes is arbitrary, thus the distribution is flat. By use of constrained simulation [12] one can enforce that an order parameter of the system stays in a given short interval between the peaks. Close to the transition point one can use a reweighting procedure to determine the value of the hopping parameter at which the distribution is flat.

In practice we have chosen $s_{\log}$ as the order parameter of the constrained simulation. Since $s_{\log}$ remains unchanged by the overrelaxation algorithms [10], only the heatbath algorithms have to be modified. Every proposal that leads to expectation values of $s_{\log}$ outside the selected interval, is rejected. The observed acceptance rate of the algorithm is larger than 98%. The lattice size under study was $2 \times 24^2 \times 256$. In a series of two short runs without any bounds on $s_{\log}$, the position of the two peaks, belonging to the pure phases, was estimated. As a consequence the interval $s_{\log} \in [4.90, 4.95]$ has been chosen to determine κ_c. We have performed 18000 measuring sweeps at $\kappa = 0.12865$, and by using reweighting we have obtained results in the vicinity of this point. The integrated autocorrelation time turned out to be $\tau_{\rm int} = 21(2)$ sweeps for $s_{\log}$ and $L_{\varphi,x\mu}$. This value is much smaller than a typical autocorrelation time for a multicanonical simulation. The reason is the following. In the case of a multicanonical simulation the system must completely go from one phase to the other in order to obtain an independent configuration. In our method only the ratio of the bulk phases changes slightly. However, this is enough to tell whether the free energy densities are the same or not.

The transition point is defined by the implicit relation for κ_c

$$\langle s_{\log} \rangle_{\kappa_c} \equiv \tfrac{1}{2} \left(s_{\log,-} + s_{\log,+} \right) , \tag{2}$$

with $s_{\log,-}$ and $s_{\log,+}$ denoting the bounds on $s_{\log}$. In our case this definition is equivalent to the condition that at κ_c the distribution is flat in the given interval.

For the determination of the statistical errors on $\langle s_{\log} \rangle_\kappa$ we have used the results of a bootstrap procedure with 3000 iterations and 30 independent subsamples (see Section 4). The final result of this analysis is $\kappa_c = 0.1286565(7)$. Within the error bars, it agrees with the result from the

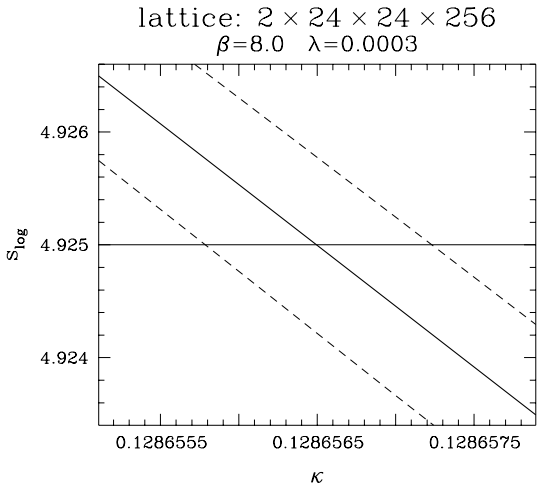

Figure 1: *The order parameter $s_{\log}$ as a function of the hopping parameter. The dashed lines give the range of the statistical error. From $4.90 < s_{\log} < 4.95$ the result $\kappa_c = 0.1286565(7)$ is obtained.*

$2 \times 16^2 \times 256$ lattice (see Section 3). Thus, the finite-size effects of κ_c beyond $L_{xy} = 24$ are expected to be small. The values of $s_{\log}$ as a function of κ are plotted in figure 1.

The above procedure gives the same κ_c for the upper half and for the lower half of the interval. This fact can be considered as a check that the system was in the flat regime. We frequently tested that the configurations contained two bulk phases and two interfaces in between.

As a non-trivial cross-check of the method we should mention that, for $\lambda = 0.0001$ ($m_H \approx 18$ GeV) the results of [8] obtained by the multicanonical method have been reproduced with this new procedure.

3 Tunnelling and energy splitting

The electroweak symmetry is spontaneously broken; however, this phenomenon does not appear in lattice simulations, since at finite volume the symmetry is never broken. In this case a small energy splitting (E_0) appears between the ground state and the first excited state, which vanishes in the infinite volume limit. The appearance of this E_0 at finite volume is due to the tunnelling effect between the symmetric and the Higgs phase. In the z-direction different domains can be observed, with domain walls between them. The associated interface energy determines the tunnelling mass (E_0) via

$$E_0 = C \exp(-L_t L_{xy}^2 \sigma). \tag{3}$$

This tunnelling effect has been used to determine, for instance, the interface tension of the four-dimensional Ising model in the broken phase [14, 15], to calculate the interface tension in three-dimensional systems [16], to determine the interface tension of the finite-temperature

Lattice	sweeps	κ_c
$2\times4\times4\times128$	243200	0.12882(1)
$2\times8\times8\times128$	156000	0.12869(1)
$2\times12\times12\times128$	512000	0.128663(2)
$2\times12\times12\times256$	512000	0.128665(2)
$2\times16\times16\times128$	448200	0.128659(1)
$2\times16\times16\times256$	419200	0.128658(1)

Table 1: *Critical hopping parameter κ_c for various lattice sizes.*

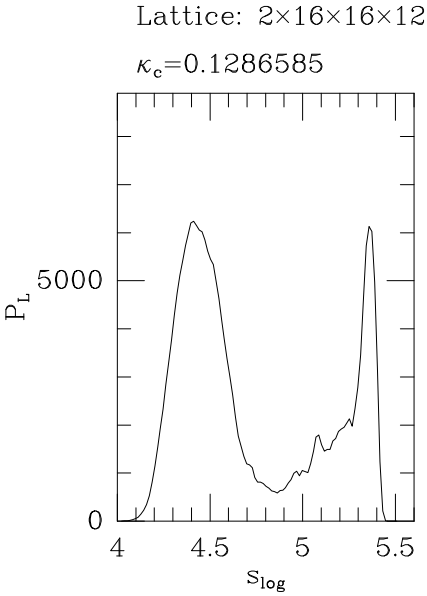

Figure 2: *Distribution of the order parameter $s_{\log}$ at the transition point.*

confinement–deconfinement phase transition of the pure SU(3) gauge theory [17], and to determine the interface tension of the electroweak phase transition [18]. In general, the prefactor C of eq. (3) can depend on L_{xy}; but in our case the leading contribution is L_{xy}-independent.

According to eq. (3) we have measured the energy splitting on different lattice sizes. The number of sweeps and the critical hopping parameters are listed in table 1. The observed exponential autocorrelation times are $\mathcal{O}(100)$ in the case of smaller volumes ($L_{xy} = 4, 8$) and $\mathcal{O}(1000)$ in the case of larger volumes ($L_{xy} = 12, 16$). For larger volumes, the clear two-peak structure with the equal-height criterion for the $s_{\log}$-distribution has been used to determine the transition point (cf. figure 2). In the case of smaller lattices, κ_c has been provided by the peak in the susceptibility.

For the determination of the smallest mass in the Higgs channel the operators $L_{\varphi,x\mu}$ and ρ_x^2 have been used at the transition point. The correlation functions have been determined in the z-direction with a relative error of $\mathcal{O}(10^{-2})$. In the case of smaller lattices ($L_{xy} = 4, 8$), mass fits with one exponential can be used, and the determination of the splitting mass does not cause

L_{xy}	4	8	12	16
E_0 from L_φ	0.147(3)	0.066(4)	0.034(1)	0.0040(6)
E_0 from ρ_x^2	0.149(4)	0.076(4)	0.033(6)	0.0069(1)

Table 2: *The measured energy splitting for different lattice extensions.*

any problem. However, for $L_{xy} = 12, 16$ the analysis is more complicated. By use of a one-mass fit we have determined the masses for different z-slice intervals with length of 10 lattice points. Let m_i denote the mass, fitted on the z-slice interval: $i \leq z \leq i + 10$. Since the rôle of the larger masses becomes less and less important for larger distances one obtains the following condition:

$$\text{if} \quad i > j \quad \Longrightarrow \quad m_i < m_j. \tag{4}$$

For the real data, m_i decreases for a while; then, due to the noise, it starts to increase. We have chosen our fit interval in such a way, that monotony holds. Typical results are 5–25 and 5–55 for $L_z = 128$ and $L_z = 256$, respectively. For these intervals we have performed two-mass fits. Since there are several local minima in the χ^2-fit, we take those results, which appear for both z-extensions $L_z = 128$ and $L_z = 256$. The statistical errors are estimated by a jackknife analysis with 256 independent subsamples. The results are listed in table 2. The origin of the difference between the E_0 values for $L_{xy} = 16$ comes from the following fact. In the studied region the decay of the L_φ correlation function is dominated by the smallest mass, however, in case of ρ^2 the decay is still dominated by the next excitation ($E_1 \approx 0.1$). Therefore, it is better to consider E_0 from ρ^2 as an upper bound. Figure 3 contains $-\log(E_0)/L_t$ as a function of L_{xy}^2. The slope of the fitted line gives the interface tension

$$\frac{\hat{\sigma}}{T_c^3} = 0.053(5). \tag{5}$$

The error contains the statistical error and an estimate of the systematics. The latter has been obtained from the difference between the results of the two different operators $L_{\varphi,x\mu}$ and ρ_x^2. The simulations have been performed on the Quadrics-APE machine of DESY, and as a consistency check, some of the calculations have been repeated with 64-bit floating point arithmetics.

The prediction of the perturbative approach [3] is

$$\left.\frac{\sigma}{T_c^3}\right|_{perturbative} = 0.060(6). \tag{6}$$

The error displayed here comes from the uncertainties in the lattice determination of the renormalized masses and coupling [4].

4 Two-coupling method and bootstrap analysis

This method is described in ref. [8], therefore we just sketch the basic ideas.

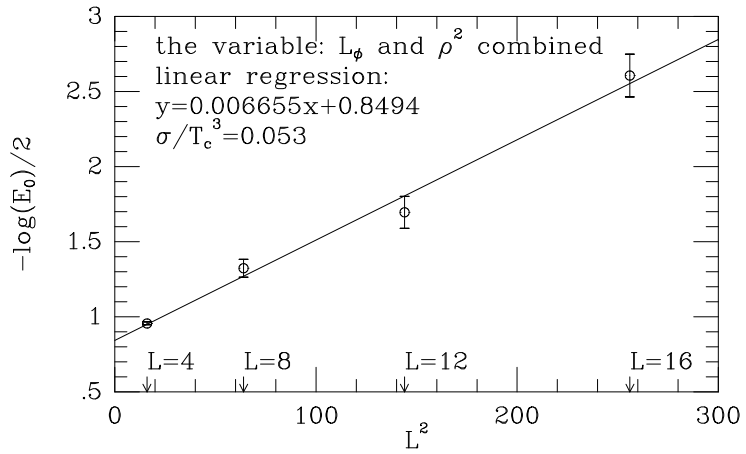

Figure 3: $-\log(E_0)/L_t$ *as a function of* L_{xy}^2. *The slope gives the interface tension.*

The lattice with z-extension L_z much larger than the others, is divided into two halves [19]. In the part with lower z-coordinates we choose the hopping parameter to be $\kappa_1 < \kappa_c$ and in the other one to be $\kappa_2 > \kappa_c$. With a suitable initialization a mixed state with two interfaces is obtained. Let $L_\varphi^{(i)}(\kappa_1, \kappa_2)$ denote the expectation value of L_φ in the part with hopping parameter κ_i. One can estimate the interface tension by

$$\sigma \simeq \frac{1}{2} \lim_{\kappa_2 \searrow \kappa_c} \lim_{\kappa_1 \nearrow \kappa_c} \left\{ (\kappa_2 - \kappa_1) \lim_{L_z \to \infty} L_z \left[L_\varphi^{(1)}(\kappa_1, \kappa_2) - L_\varphi^{(2)}(\kappa_1, \kappa_2) \right] \right\} . \tag{7}$$

With the ansatz

$$L_\varphi^{(i)}(\kappa_1, \kappa_2) = -\frac{c_i}{\kappa_i - \kappa_c} + b_i + a_i \cdot (\kappa_i - \kappa_c) + \mathcal{O}(\kappa_i - \kappa_c)^2 \tag{8}$$

a finite volume estimator of the interface tension is obtained from

$$\hat{\sigma} = L_z \cdot (c_1 + c_2) . \tag{9}$$

As eq. (7) illustrates, σ depends only on the difference $\Delta L_\varphi \equiv [L_\varphi^{(2)} - L_\varphi^{(1)}]$.

In table 3 the results for $L_\varphi^{(1)}$, $L_\varphi^{(2)}$ and ΔL_φ on a $2 \times 24^2 \times 200$ lattice are listed. We estimated the integrated autocorrelation time from the quotient of true and naive statistical errors. For $L_\varphi^{(1)}$ and $L_\varphi^{(2)}$ the autocorrelation time is always found to be smaller than 50 sweeps. The errors, quoted in the table, are obtained by binning.

The statistical errors of ΔL_φ are always smaller than those of $L_\varphi^{(2)}$. This is due to a kind of shift of the interfaces during the simulation; for instance, in the case of an interface moving from the region with κ_2 into the one with κ_1, both $L_\varphi^{(i)}$ will increase, but the difference between them is not that much affected. Thus, there is a strong correlation between the two $L_\varphi^{(i)}$-values obtained on the same configuration. This has to be taken into account when estimating statistical errors for the interface tension, and is achieved by a bootstrap analysis.

The bootstrap procedure has been discussed in [20] and used e.g. in [21]. We shortly discuss our analysis. Consider a set of N independent measurements for some primary quantities from

κ_1	κ_2	sweeps	$L_\varphi^{(1)}$	$L_\varphi^{(2)}$	ΔL_φ
0.12845	0.12885	10000	1.0607(30)	7.9042(91)	6.8435(83)
0.12849	0.12881	10000	1.0614(32)	7.007(10)	5.9461(92)
0.12853	0.12877	10000	1.0587(41)	6.058(12)	4.999(10)
0.12855	0.12875	12000	1.0684(38)	5.581(12)	4.513(11)
0.12857	0.12873	14000	1.0678(42)	5.035(12)	3.967(11)

Table 3: *Results for* $L_\varphi^{(1)}$, $L_\varphi^{(2)}$ *and* ΔL_φ *on a* $2 \times 24^2 \times 200$ *lattice.*

a single Monte Carlo run. To form a bootstrap sample from the original N measurements we take N objects of them with repetition, thus some may be chosen a couple of times. The number of bootstrap samples is huge, N^N, therefore one randomly generates representatives. Since correlations between the different primary quantities should be considered, these quantities have to emerge from the same Monte Carlo configurations for a given bootstrap sample. One computes in each sample a given secondary quantity a from the sample averages of the primary quantities (e.g. mass fits from correlation functions). An estimate of the statistical error is given by the dispersion of the a-values, thus by the two values left and right to the median containing 68.3% (34.15% in both directions) of the distribution.

In order to eliminate the influence of autocorrelation we have combined 200 measurements to bins for each κ-pair. (E.g. for the first row of table 3 the bin averages provide $N = 50$ independent values for the quantities $L_\varphi^{(1)}$ and $L_\varphi^{(2)}$.) Our data are not obtained in a single Monte Carlo run but in 5 different runs for different κ-pairs. For each κ-pair we generated a bootstrap sample and for the obtained samples we determined $L_\varphi^{(1)}$ and $L_\varphi^{(2)}$ as a function of κ. Using the form of eq. (8) we performed for these functions two χ^2-fits. This procedure has been repeated 10000 times. The statistical error has been determined from the distribution of the $\hat{\sigma}$.

Figure 4 contains the combined data points with the corresponding fits. This yields

$$\frac{\hat{\sigma}}{T_c^3} = 0.065(9+1)\,. \tag{10}$$

The first number in the parenthesis is the statistical error, the second one stands for the uncertainty in the critical hopping parameter. For the individual fits we have $c_1 = 0.00000277(290+5)$ and $c_2 = 0.0000378(61+7)$ with $\chi_1^2 = 1.88$ and $\chi_2^2 = 1.50$, respectively. The error on $(L_z T_c^{-3} \cdot c_2)$ is even larger than that of $\hat{\sigma}/T_c^3$. This is an indication that the correlations have been taken into account. In order to check our error estimate based on the bootstrap procedure, we have also performed a jackknife error analysis with 80 independent jackknife samples. The results of the two different methods are in complete agreement.

In order to check whether the measured curvature of $L_\varphi^{(i)}(\kappa_1, \kappa_2)$ as a function of κ_i is really described by eq. (8), we have also tried a 3-parameter quadratic ansatz. The results are $\chi_1^2 = 1.85$ and $\chi_2^2 = 4.15$. The value of χ_2^2 shows that the data cannot be well described by a quadratic fit. Due to the large error bars in the symmetric phase (cf. figure 4) χ_1^2 is not very sensitive to the proposed fit-function. A 4-parameter fit with a quadratic term in addition to eq. (8) results

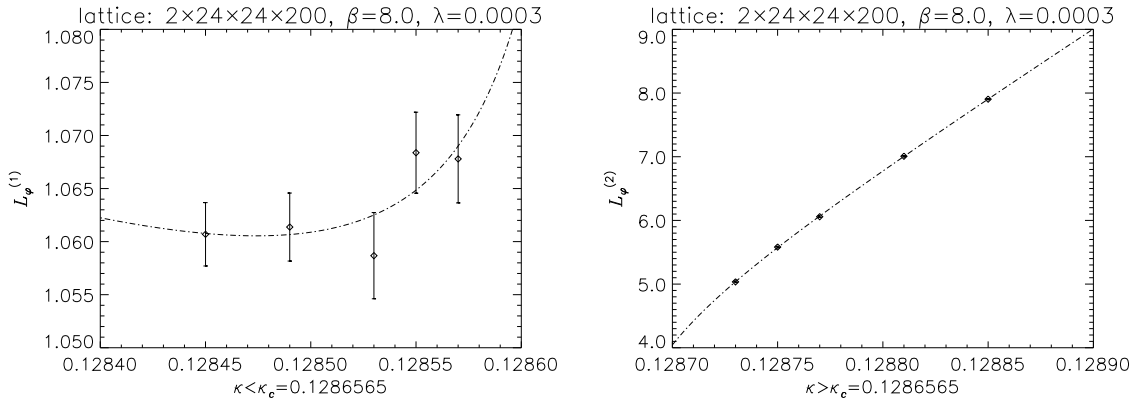

Figure 4: *Least χ^2-fits for the determination of $\hat{\sigma}$ from the two-coupling method. The χ^2-s are $\chi_1^2 = 1.88$ and $\chi_2^2 = 1.50$, respectively.*

in $\chi_1^2 = 1.85$ and $\chi_2^2 = 1.49$. Since the χ^2-s are nearly unchanged, the fourth parameter does not seem to represent a new degree of freedom and the value for $\hat{\sigma}/T_c^3 = 0.063$ does not deviate significantly from that of eq. (10).

It is also possible to determine $\hat{\sigma}$ by using the ΔL_φ-values of table 3 together with the rough estimate $\kappa_c = 0.12865$ for the transition point, which has been obtained by the two-coupling method. One gets $\hat{\sigma}/T_c^3 = 0.075(11)$ with $\chi^2 = 0.77$. In this case the error comes from 1000 normally distributed random data and the uncertainty in κ_c has not been considered. With individual fits for both $L_\varphi^{(i)}$ as previously described, the result is $\hat{\sigma}/T_c^3 = 0.075(11)$ for $\kappa_c = 0.12865$ with $\chi_1^2 = 1.89$ and $\chi_2^2 = 1.54$. From this agreement we conclude that the correlations between the $L_\varphi^{(i)}$-values are treated similarly in both methods. However, the difference between this result and that of eq. (10) emphasizes the importance of a precise κ_c-determination.

5 Conclusions

We have determined the interface tension of the SU(2)-Higgs model with two different methods. Within the error bars the results of eqns. (5) and (10) agree with each other and with the prediction of the perturbation theory. When comparing the errors, it should be kept in mind that the product of the lattice volume and the number of sweeps (thus, CPU-time) is an order of magnitude smaller in the case of the two-coupling method.

Together with the results of ref. [8], in which Binder's method [22] has been used with multicanonical simulation, in total three different approaches for the measurement of the interface tension were studied. No significant difference between the results of these methods has been observed. The two-coupling method turned out to be the most economic way to estimate the interface tension.

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