

A sonic boom in bubble wall friction

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Abstract

We revisit the computation of bubble wall friction during a cosmological first-order phase transition, using an extended fluid *Ansatz* to solve the linearized Boltzmann equation. A singularity is found in the fluctuations of background species as the wall approaches the speed of sound. Using hydrodynamics, we argue that a discontinuity across the speed of sound is expected on general grounds, which manifests itself as the singularity in the solution of the linearized system. We discuss this result in comparison with alternative approaches proposed recently, which find a regular behaviour of the friction for all velocities.

1 Introduction

The recent detection of gravitational waves, and the forthcoming launch of the space-based interferometer LISA, have recently boosted the interest on the dynamics of phase transitions in the early Universe and their observational consequences. If the phase transition is first order, it will proceed via bubble nucleation, expansion and percolation, producing sound waves and turbulence in the plasma which, together with the kinetic energy of the Higgs field, could source stochastic gravitational waves observable by LISA [1, 2]. Other relics could also be produced during this process, such as a dark matter abundance [3] and a matter-antimatter asymmetry [4, 5, 6], and hence the detection of primordial radiation by LISA could also give us information on the underlying particle physics model. Crucially, the abundance of relics produced by this phase transition depends, among other few parameters, on the expansion velocity of these bubbles. The determination of this velocity depends on the microphysics of the interactions of the bubble wall with particles in the plasma. The passage of the bubble will drive the plasma out of equilibrium, and this will backreact on the bubble, acting as a dissipative friction force which tends to halt its expansion. Thus, an accurate determination of these relics, especially the stochastic gravitational wave spectrum generated at the early Universe, relies on an adequate modelling of this out-of-equilibrium dynamics between the Higgs field and the hot plasma.

The calculation of this plasma friction has been previously tackled in the literature using the so-called *fluid Ansatz* for the non-equilibrium particle distribution function [7, 8]. This approach describes non-equilibrium via three fluctuations — chemical potential, temperature and velocity fluctuations —, each with a specific momentum dependence. This allows for the computation of the collision terms (at least in a leading-log approximation), thus converting the integro-differential Boltzmann equation into a more manageable set of ODE’s for the three parameters appearing in the *Ansatz*. It is then found that the friction becomes singular as the wall velocity approaches the speed of sound in the plasma [7, 8] — an effect akin to a sonic boom. Furthermore, the formalism also predicted that baryogenesis would be utterly impossible at any supersonic wall velocity¹.

In fact, both effects have one and the same origin: a vanishing eigenvalue in the kinetic matrix of the Boltzmann system when the wall velocity equals the sound speed. At this singular point, damping becomes strong, meaning that all modes which tend to be brought back to equilibrium by mutual collisions will do so very close to the wall. Baryogenesis then vanishes because it cannot work in equilibrium. However, energy-momentum conservation requires some of the fluctuations of the light species (such as photons, gluons and light fermions) to actually be undamped, and in this case the vanishing eigenvalue in the kinetic matrix will translate to a discontinuity in the temperature and velocity of the plasma across the wall boundary. The need for such a

¹Some local mechanisms of baryon generation are still active regardless of details of charge transport along the bubble, such as shown in [9, 10]. However, these mechanisms typically predict an insufficient amount of baryon asymmetry, incompatible with observations.

discontinuity is in fact confirmed by macroscopic hydrodynamical analyses [11, 12, 13, 14].

Recently, however, an alternative approach for treating the dynamics of microscopic non-equilibrium has been put forward [15], claiming to solve the Boltzmann equation without specifying any *Ansatz* for the particle abundances². Instead, the authors introduce an implicit truncation scheme in the mode expansion of the system, assuming that all higher moments of the distribution function are proportional to the first order fluctuation in velocities. It is then shown that the formalism predicts a continuous behaviour for observables at all wall velocities, including across the speed of sound boundary. In particular it predicts that electroweak baryogenesis can even work with supersonic walls, even if at a somewhat reduced rate. When applied to a study of the friction and the wall velocity, the formalism again predicted a continuous behaviour [27]³.

One is then led to wonder about the origin of such discrepancies and the reliability of two different methods that yield so contrasting results. For the baryogenesis computation this issue has been recently tackled in [29], where it was shown that supersonic baryogenesis is indeed possible (even if suppressed) if one considers an *extended fluid Ansatz* with more than just three perturbations. Moreover, a continuous behaviour does indeed emerge across all velocities as more and more perturbations are included in the description, thus bringing the predictions of the fluid *Ansatz* in agreement with those of the *Ansatz*-less approach of [15].

However, it still remains to be settled whether the two approaches would lead to similar predictions for the friction calculation as well, i.e. whether the discontinuity predicted in the three-fluid approximation for the background fluctuations could be brought under control by extending the *Ansatz* to include more perturbations. The goal of the present paper is to assess this question. It will be found that, unfortunately, the answer is negative.

Even though the friction in the bubble wall during the phase transition constitutes a quite similar problem to baryogenesis, there are some notable differences. One main difference being that, for the friction calculation, one is interested in CP-even deviations from equilibrium, whereas for baryogenesis they are CP-odd, and this leads to some qualitative differences in the collision terms. Furthermore, energy-momentum conservation plays a prominent role in the friction calculation. At the same time, as mentioned above, the singularity in the friction close to the speed of sound is caused by essentially the same reason that suppresses the baryon asymmetry for supersonic walls: some of the eigenvalues of the linearized fluid system change sign as the wall speed becomes supersonic. For baryogenesis, this leads to the fact that deviations are only non-zero behind that wall, thus suppressing the BAU. For the friction calculation the sign change of the eigenvalue results in a divergence of the friction (and hence the breakdown of the perturbative scheme).

In the present work, we will argue that the discontinuity in the friction calculation is

²For other recent developments on bubble wall friction see [16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

³Recent applications of this approach can be found in [26, 28].

actually physical. We will study in detail the origin of the singularity of the linearized system. We give some general arguments why such a ‘sonic boom’ should occur in the plasma for wall velocities close to the speed of sound. Finally, we support our arguments with an explicit calculation using higher moments and a generalized Ansatz in the Boltzmann equation, as done in [29] for the analysis of the baryon asymmetry.

The paper is organized as follows. In section 2 we review the hydrodynamics of the phase transition to show that a discontinuity in temperature and fluid velocity across the bubble wall is expected on the grounds of energy-momentum conservation. Our point is that the macroscopic analysis confirms the discontinuity predicted by the microphysical approach with the fluid *Ansatz*. In section 3 we review the friction calculation with the three-fluid *Ansatz* of [8] and discuss how the discontinuity emerges in this approach. The extended fluid *Ansatz* is then reviewed in section 4, and the friction calculation is performed in section 5, where we also show our main results. Finally, section 6 is left for discussions and conclusions.

2 The origin of the discontinuity

Before we study the friction calculation using an extended Ansatz for the fluid, we will provide some general arguments based on hydrodynamic considerations.

As we will see later, in the linearized system the origin of the singularity/discontinuity in the friction arises from a sign change in the eigenvalues of the Liouville operator in conjunction with energy-momentum-conservation. This system can also be studied on length scales much larger than the longest damping scale in the problem. Since the wall thickness is many orders smaller than the bubble size, this separation of scales is easily achieved. This system is then in local equilibrium and fulfills the hydrodynamic equations.

This hydrodynamic setup is typically studied to analyze the overall energy budget of the phase transition and the expansions mode. In the following we follow the conventions of [14].

Consider a system where the plasma in both phases is described by a radiation component and a bag constant ϵ in one of the two phases. The pressure of the system then reads

$$p_{\pm} = \frac{a_{\pm}}{3} T^4 - \epsilon_{\pm} . \quad (1)$$

Defining the enthalpy $\omega = e + p$, conservation of the energy momentum tensor

$$T_{\mu\nu} = u_{\mu} u_{\nu} \omega - g_{\mu\nu} p \quad (2)$$

then reads (in the wall frame)

$$\Delta T_{0z} = \Delta T_{zz} = 0, \quad (3)$$

or explicitly

$$\gamma_+^2 v_+^2 \omega_+ + p_+ = \gamma_-^2 v_-^2 \omega_- + p_-, \quad \gamma_+^2 v_+ \omega_+ = \gamma_-^2 v_- \omega_- . \quad (4)$$

A change in the bag constant will be accompanied with a change in the plasma temperature and velocity, $T_+ \neq T_-$ and $v_+ \neq v_-$. Using the notation $\alpha = (\epsilon_+ - \epsilon_-)/(a_+ T_+^4)$ and eliminating the temperatures then leads to the relation [14]

$$v_+ = \frac{1}{1+\alpha} \left[\left(\frac{v_-}{2} + \frac{1}{6v_-} \right) \pm \sqrt{\left(\frac{v_-}{2} + \frac{1}{6v_-} \right)^2 + \alpha^2 + \frac{2}{3}\alpha - \frac{1}{3}} \right]. \quad (5)$$

The equation has per se two solutions but only one solution is consistent globally. For wall velocities beyond the Jouguet velocity, the fluid in front of the wall has to be at rest in the plasma frame. This implies $v_+ > v_-$ and is called a detonation. For wall velocities below the speed of sound, the fluid behind the wall has to be at rest in the plasma frame. This implies $v_+ < v_-$ and is called a deflagration. For wall velocities in between, hybrid solutions are the most likely outcome which technically are also deflagrations, $v_+ < v_-$. See [14] for details.

The relation (5) has some interesting properties. One can rewrite it as

$$v_+ = \frac{1}{1+\alpha} \left[X_+ \pm \sqrt{X_-^2 + \alpha^2 + \frac{2}{3}\alpha} \right] \quad (6)$$

using $X_{\pm} = v_{\pm}/2 \pm 1/(6v_{\pm})$. For $\alpha = 0$, the relation reads $v_+ = X_+ \pm X_-$, which yields $v_+ = v_-$ and $v_+ v_- = 1/3$. For $v_- \rightarrow c_s = 1/\sqrt{3}$, one obtains $X_+ \rightarrow c_s$ and $X_- \rightarrow 0$, which translates into

$$v_+ = \frac{1}{1+\alpha} \left[c_s \pm \sqrt{\alpha^2 + \frac{2}{3}\alpha} \right] \quad \text{for } (v_- \rightarrow c_s). \quad (7)$$

However, in the limit of very weak phase transitions and expanding in α one obtains

$$v_+ \simeq \frac{1}{1+\alpha} \left[X_+ \pm X_- \pm \frac{1}{3} \frac{\alpha}{X_-} \right] \quad \text{for } (\alpha \rightarrow 0). \quad (8)$$

Selecting the physical branch then leads in leading order

$$v_- - v_+ = \frac{3v_-(1-v_-^2)}{1-3v_-^2} \alpha, \quad \text{for } (\alpha \rightarrow 0). \quad (9)$$

This is basically the expression one will necessarily obtain once the Boltzmann equations are linearized and expanded around the equilibrium. Unfortunately, this limit shows a divergence for $X_- \rightarrow 0$ or $v_- \rightarrow c_s$. The divergence stems from the fact the square root in (5) is non-analytic in this limit. Figure 1 shows the exact and linearized relations between v_+ and v_- , see (5) and (9). The strength of the phase transition is $\alpha = 0.002$ (cf. section 3 below) and only the physical branch is shown for the linearized relation.

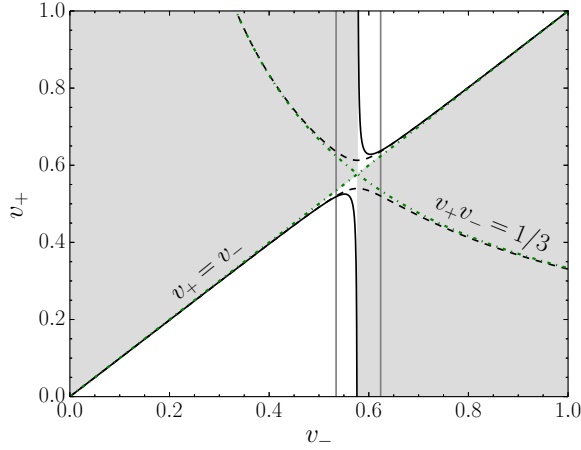


Figure 1: The exact (black dashed) and linearized (black solid) curves for the relation between v_+ and v_- . The gray shaded region are unphysical. For the linearized relation only the physical branch is shown, and the discontinuity across the speed of sound is clear. The strength of the phase transition is $\alpha = 0.002$. The region in between the vertical lines in the middle satisfies $\alpha/X_-^2 > 1$, where we expect a breakdown of the linearization procedure. Indeed, it is clear that, outside this region, the solid and dashed curves agree.

In the Boltzmann equations we will solve, the difference in the equation of state between the two phases is actually not a bag constant but a mean field term and a bag constant, so

$$p_+ = \frac{a}{3}T^4 - \epsilon, \quad p_- = \frac{a}{3}T^4 - b m^2 T^2, \quad (10)$$

with a , b and ϵ given by the SM particle content and Higgs potential. The phase transition will happen close to the critical temperature, so

$$b m^2 T^2 \simeq \epsilon, \quad (11)$$

since it is weak. This will be enforced by the Higgs equation in the Boltzmann setup. Defining the strength of the phase transition in terms of the trace of the energy-momentum tensor [33],

$$\alpha = \frac{\Delta\theta}{3w_+} = \frac{2\epsilon - b m^2 T_-^2}{2a T_+^4} \simeq \frac{b m^2}{2a T_+^2}, \quad (12)$$

then leads again to the result (9) that was obtained with only a bag constant. We will see later that one can recover this relationship in the linearized Boltzmann equations.

In summary, the physical solution shows a discontinuity close to the speed of sound (or more precisely the Jouguet velocity) because the globally consistent solutions are changing the branch of solutions in (5). Linearizing the equation in the bag constant and the mean field term (which encapsulate the energy injection) then leads to a singularity

in the solution for a wall velocity close to the speed of sound, which stems from the non-analyticity of the square root in (7).

The solution to the Boltzmann equation will be valid in the regime $\alpha \ll X_-^2$ and beyond this point the actual solution would saturate to the value in (7) rather than to blow up. One should also explicitly check that the solution to the Boltzmann equation selects the globally consistent branch, meaning $v_+ \leq v_-$ for $v_- \leq c_s$. This is automatically fulfilled for the linearized system if it abides to (9).

3 Linearized Boltzmann setup

The results from the hydrodynamic analysis discussed in section 2 can also be obtained from a microphysical perspective, studying the non-equilibrium dynamics of the plasma during the phase transition. For not-too-strong phase transitions, with $\alpha \ll 1$, the Higgs bubble wall is typically thicker than the de Broglie wavelength of plasma particles, and the non-equilibrium distribution function of particle species i can be found by solving the semi-classical Boltzmann equation

$$p^\mu \partial_\mu f_i(x^\mu, p^\mu) + m F^\mu \partial_{p^\mu} f_i(x^\mu, p^\mu) + \mathcal{C}[f_j] = 0, \quad (13)$$

where p^μ is the four-momentum of the particles evaluated on-shell ($p^0 = E = \sqrt{\vec{p}^2 + m^2}$), F^μ is the semi-classical force from the wall driving the particles out of equilibrium, and \mathcal{C} is the collision term which tends to bring the system back to equilibrium.

This is a partial integro-differential equation, and in order to make progress we need to make an *Ansatz* for the f_i 's, such that we can actually solve the integrals in the collision term and reduce the problem to a set of ordinary differential equations for the fluctuations characterizing the displacement from equilibrium. We will postpone a detailed discussion on this issue until the next section, focusing here instead on the general argument which will allow us to recover the results of section 2.

First, note that, in principle, there is one such set of equations for each particle species in the plasma. However, only the heavier species, such as the W^\pm 's, Z^0 's and top quarks, feel the passage of the bubble and have a significant non-vanishing source term. The other species, namely photons, gluons, light quarks and leptons, remain very close to mutual equilibrium among themselves, and characterize the so-called *background*. They are collectively driven out of equilibrium due to collisions with the heavy particles, which communicate the passage of the bubble to the background. We will parametrize the fluctuations of the heavy species relative to the background fluctuations.

Following [7, 8] we can model the non-equilibrium functions f_i as

$$f_i(x, p) = \frac{1}{e^{\beta(p^\mu u_\mu - \delta_p - \delta_{p, \text{bg}})} \pm 1}, \quad (14)$$

with $\beta^{-1} = T$ the temperature of the plasma and u_μ its four-velocity relative to the observer. Expanding the fluctuations in powers of momenta and keeping only first order

terms one obtains

$$\delta_p^{\text{perfect fluid}} = \delta\mu + p^\mu (\delta u_\mu - u_\mu \delta T/T), \quad (15)$$

and analogously for $\delta_{p,\text{bg}}$. The plasma four-vector is in the wall frame given as $u^\mu = \gamma_w(1, v_w)$, while we parametrize $\delta u_\mu = \delta v \bar{u}_\mu = \delta v \gamma_w(v_w, 1)$.

Thus $\delta\mu$ encodes information on the chemical potential of the fluid composed of species i , $\delta T/T$ parametrizes its temperature deviation relative to the plasma, and δv are fluctuations of the fluid velocity. Photon and gluon numbers quickly equilibrate which implies $\delta\mu_{\text{bg}} = 0$ [7], so the background is described only by temperature and velocity fluctuations.

To solve for the three fluctuations of gauge bosons and tops we take three moments of the Boltzmann equation, multiplying it by 1, $p^\mu u_\mu$ and $p^\mu \bar{u}_\mu$ and integrating over momenta. The resulting equations correspond to conservation equations for particle number and energy-momentum [8]. The *Ansatz* allows for solving the collision terms and, after linearizing in the fluctuations, and assuming a planar wall (such that the problem becomes essentially one-dimensional in the coordinate z) we are left with a system, as seen in the rest frame of the wall, of the form [8]

$$\begin{aligned} A_W \cdot (q_W + q_{bg})' + \Gamma_W \cdot q_W &= S_W, \\ A_t \cdot (q_t + q_{bg})' + \Gamma_t \cdot q_t &= S_t, \\ A_{\text{bg}} \cdot q'_{bg} + \Gamma_{\text{bg},W} \cdot q_W + \Gamma_{\text{bg},t} \cdot q_t &= 0, \end{aligned} \quad (16)$$

where $q = (\delta\mu, -\delta T/T, \delta v)^T$ and prime denotes derivative with respect to the coordinate z . The matrices A_W , A_t and A_{bg} take the form

$$A_i = \frac{\gamma_w}{2\pi^2} \begin{pmatrix} v_w c_2 & v_w c_3 & c_3/3 \\ v_w c_3 & v_w c_4 & c_4/3 \\ c_3/3 & c_4/3 & v_w c_4/3 \end{pmatrix}, \quad A_{\text{bg}} = \gamma_w \frac{c_4^{\text{bg}}}{2\pi^2} \begin{pmatrix} v_w & 1/3 \\ 1/3 & v_w/3 \end{pmatrix}, \quad (17)$$

with coefficients c_n differing for bosons and fermions (see section 4 below), and $\gamma_w = (1-v_w^2)^{-1/2}$ the relativistic factor associated to the wall velocity. The background matrix corresponds to the bottom right 2×2 block of the A_i , summed over all fermionic and bosonic degrees of freedom in the background⁴, so $c_4^{\text{bg}} = 78c_4^f + 19c_4^b$. The collision matrices Γ_W , Γ_t are then 3×3 while $\Gamma_{bg,X}$ are 2×3 . Notice that the matrices A_i and A_{bg} have a vanishing eigenvalue at $v_w = 1/\sqrt{3} = c_s$.

Finally, the source terms are given by moments over the forces in the Boltzmann equation

$$S = \gamma_w v_w \frac{(m^2)'}{4\pi^2 T^2} \begin{pmatrix} c_1 \\ c_2 \\ 0 \end{pmatrix}, \quad (18)$$

and is present for all species that significantly change mass during the phase transition.

⁴The background contains 78 fermionic degrees of freedom (5 light quarks \times 12 d.o.f each, 3 charged leptons \times 4 d.o.f and 3 neutrinos \times 2 d.o.f each) and 19 bosonic (16 gluons, 2 photons and 1 Higgs).

The singularity in the friction arises then from an interplay of energy-momentum conservation, the peculiar form of the collision terms, and the zero eigenvalue in the kinetic matrix. First, notice that the equation of motion of the two background degrees of freedom can be uniquely fixed using energy-momentum conservation. In other words, there are two vectors χ such that $\chi \cdot \Gamma = 0$ which relates the matrices $\Gamma_{\text{bg},X}$ to the corresponding 2×3 parts of the matrices Γ_X . The collision terms drop out in the corresponding equations

$$\chi \cdot A \cdot \Delta q = \chi \cdot \int S dz. \quad (19)$$

These two equations represent the linearized form of the energy-momentum conservation (3). The source terms parametrize the difference in the equation of state between the two phases due to the mean field term, $p_+ - p_- \propto m^2 T^2 \propto \int S dz$. At the same time, almost all fluctuations Δq are damped by the collision terms and tend to zero away from the source. The sole exceptions are the two fluctuations that parametrize a collective shift in the local equilibrium, parameterized by a temperature shift δT and a shift in the fluid motion δv . Hence, away from the source, the constraint (19) becomes essentially a 2×2 system involving only δT_{bg} and δv_{bg} . This will produce a singularity in the fluctuations once the wall velocity approaches the speed of sound, in analogy to (9).

To be specific, the relation (19) can be solved for the change of the fluid velocity across the wall,

$$\delta v_{\text{bg}} = \frac{\sum_i N_i m_i^2 c_2^i}{2T^2 \sum c_4} \frac{3v_w}{1 - 3v_w^2}, \quad (20)$$

where the sum over i runs over the heavy particles (tops, W^\pm and Z^0) whereas the sum over c_4 runs over heavy particles as well as over background species. This exactly reproduces the result of the hydrodynamic relation (9), due to

$$\alpha = \frac{b m^2}{2a T^2} = \frac{\sum N_i m_i^2 c_2^i}{2T^2 \sum c_4}, \quad (21)$$

which relates the thermodynamic potentials to the coefficients in the Boltzmann equations, and the relation $(v_- - v_+) \gamma^2 = \delta v_{\text{bg}}$ that stems from $du^\mu/dv_w = \gamma^2 \bar{u}^\mu$ and the definition of δv in (15). For a fiducial phase transition strength $\phi_0/T = 1$ this yields $\alpha \approx 0.002$.

Notice that this conclusion seems unavoidable as long as the linearized Boltzmann equations are solved, even if a generalized Ansatz is used and many more moments are taken. Still, the blow up of these perturbations point to a breakdown of the linearization procedure. This argument only relies on the facts that: a) two moments of the Boltzmann equation represent total energy-momentum conservation, and b) the Ansatz for the distribution functions contains fluctuations that represent collective changes in the local equilibrium (i.e. for the collective temperature and fluid velocity).

4 Generalized fluid *Ansatz*

Let us now consider a generalized fluid *Ansatz*, where the distribution functions still have the form in (14) but now the fluctuations read

$$\delta_p = w^{(0)} + p^\mu w_\mu^{(1)} + p^\mu p^\nu w_{\mu\nu}^{(2)} + \dots \quad (22)$$

and analogously for $\delta_{p,\text{bg}}$. In other words, we expand the non-equilibrium fluctuations in powers of momenta. Keeping only terms up to linear order in momenta, as in section 3, consists in a description of a perfect fluid. Our generalized *Ansatz* allows for more general entropy-producing phenomena [34, 7].

Before proceeding with the analysis, a few comments are in order. First, note that (22) is not the most general *Ansatz* possible, and may not be appropriate in certain circumstances. In particular, additional tensor structures would be best suited for fully resolving the angular dependence of the distribution function in high energy interactions, such as $p_i/|p_i|$, $p_i p_j/|p_i|^2$, and so on. Since we will always compute the collision terms at leading-log, implying small momentum exchange, the parametrization adopted in (22) should be adequate. We will confirm *a posteriori* that the friction converges using our set of fluctuations. Still, even if not the most general, using some *Ansatz* for the f_i 's is absolutely indispensable, since otherwise the momentum dependence of the collision terms cannot be resolved. Even in approaches that claim *Ansatz*-independence, a choice has to be made for the collision terms, which is ultimately equivalent to choosing an *Ansatz* for the distribution function. In such approaches one actually has to be careful that the implicit *Ansatz* chosen for the collision terms is consistently applied in the other terms of the Boltzmann equation as well. For instance, in [15] the collision terms were computed using the perfect fluid *Ansatz* discussed in section 3, whereas the kinetic term was computed differently. This consistency is difficult to accomplish without knowing the *Ansatz* explicitly. Here it is guaranteed by construction.

Collision terms and background equations

With the *Ansatz* (22) the linearized collision term has the form

$$\begin{aligned} \mathcal{C}[f] = & \frac{1}{2T} \sum_{\text{processes}} \int \frac{d^3k d^3p' d^3k'}{(2\pi)^9 2E_k 2E_{p'} 2E_{k'}} |\mathcal{M}_{pk \rightarrow p'k'}|^2 (2\pi)^4 \delta^4(p + k - p' - k') \times \\ & \times f_{0p} f_{0k} (1 \pm f_{0p'}) (1 \pm f_{0k'}) \left[(\delta_p + \delta_k - \delta_{p'} - \delta_{k'}) + (\delta_{p,\text{bg}} + \delta_{k,\text{bg}} - \delta_{p',\text{bg}} - \delta_{k',\text{bg}}) \right]. \end{aligned} \quad (23)$$

Now, an interesting cancellation takes place for some of the background perturbations, which is at the core of the singularity across the speed of sound for the background fluctuations. Recalling that we can set $\delta\mu_{\text{bg}} = w_{\text{bg}}^{(0)} = 0$ since photon and gluon chemical

potentials equilibrate quickly [7], we have

$$\begin{aligned} \delta_{p,\text{bg}} + \delta_{k,\text{bg}} - \delta_{p',\text{bg}} - \delta_{k',\text{bg}} &= w_{\mu,\text{bg}}^{(1)} (p^\mu + k^\mu - p'^\mu - k'^\mu) \\ &+ w_{\mu\nu,\text{bg}}^{(2)} (p^\mu p^\nu + k^\mu k^\nu - p'^\mu p'^\nu - k'^\mu k'^\nu) + \dots, \end{aligned} \quad (24)$$

and we see that the term proportional to $w_\mu^{(1)}$ vanishes due to energy-momentum conservation. This means that the leading-order fluctuations in the background equations are undamped. This cancellation does not happen for higher order fluctuations, which involve more complicated momentum dependences. In fact, we expect them to be severely damped, because of the large number of processes involving background species. In other words, these fluctuations equilibrate quickly, and we can set them to zero from the start. Thus, even if we adopt an extended *Ansatz* for the heavy species, the background can be modelled by two perturbations only, corresponding to the undamped $w_\mu^{(1)}$ — the background behaves as a perfect fluid.

We will consider the collision processes shown in figure 2, involving interactions of the heavy species with background particles. In these diagrams the heavy species are incoming particles in the interaction, so the corresponding collision term will enter the equation for this species. Taking moments of the Boltzmann equation will introduce a factor $p^\mu p^\nu \dots$, with p the momentum of this species. Importantly, for the two equations corresponding to energy-momentum conservation, there will be just a single power of p^μ . If we now consider the equation for the other species in the diagram, this external momentum becomes $p \rightarrow k, p'$ or k' , and we then get the corresponding collision term appearing in the background equations. So when summing over the equations for all species, a factor $p + k - p' - k'$ arises in the energy-momentum equations, and the result vanishes. This argument does not rely on the specific *Ansatz* for the distribution functions, and in general the moments of the Boltzmann equation that represent energy-momentum conservation do not contain collision terms. So, overall, we do not need to compute additional collision terms other than Γ_W and Γ_t appearing in the equations for the heavy species, because the corresponding matrices for the background equations will be

$$\Gamma_{\text{bg},W} = -N_W \Gamma_W, \quad \Gamma_{\text{bg},t} = -N_t \Gamma_t, \quad (25)$$

for the two moments that correspond to energy-momentum conservation. Since we deal with only two fluctuations for the background, these are all the collision terms we need to solve the system⁵.

Truncating the expansion (22) to keep terms up to order ℓ in powers of momenta (the usual perfect fluid *Ansatz* corresponding to $\ell = 1$) results in a description with $N = (\ell + 1)(\ell + 2)/2$ fluctuations. We then need to take N independent moments of the Boltzmann equation, which is obtained by multiplying (13) by factors $(p^\mu u_\mu)^m (p^\mu \bar{u}_\mu)^n$, with $0 \leq m + n \leq N$, and integrating over momenta. The resulting matrices appearing in the system (16) to an arbitrary order ℓ are obtained from integrals given in the

⁵This is the justification of the statement in section 3, that there are two vectors χ corresponding to energy-momentum conservation such that $\chi \cdot \Gamma = 0$ for the overall collision matrix Γ .

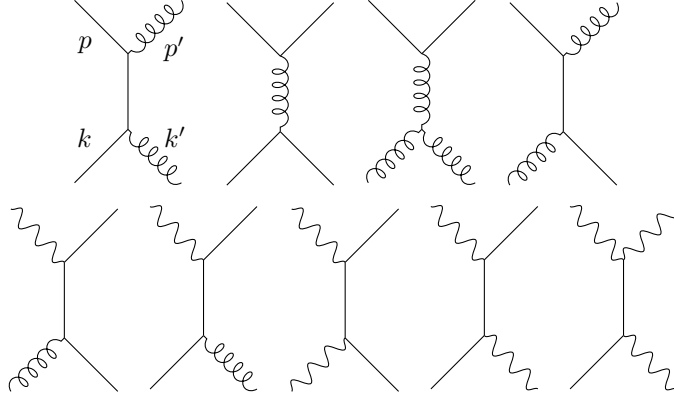


Figure 2: Collision processes taken into account in this study. We compute only the leading-log contributions, which means focusing only on t and u -channel diagrams. The four diagrams in the first row are processes changing the perturbations of the top quark, namely double annihilation into gluons, scattering by light quarks, scattering by gluons, and absorption and reemission of a gluon. The second row are processes changing the distribution of W 's, namely single annihilation (by gluons into quarks and by quarks into gluons), double annihilation into quarks, absorption and reemission, and scattering by quarks.

appendix of reference [29]. At $\ell = 2$, with $N = 6$ fluctuations arranged in a vector as $q = (w^{(0)}, w_0^{(1)}, w_z^{(1)}, w_{00}^{(2)}, w_{0z}^{(2)}, w_{zz}^{(2)})^T$, the Boltzmann equation becomes a matrix system as in (16), with kinetic terms

$$A_i = \frac{\gamma_w}{2\pi^2} \begin{pmatrix} v_w c_2 & v_w c_3 & c_3/3 & v_w c_4 & c_4/3 & v_w c_4/3 \\ v_w c_3 & v_w c_4 & c_4/3 & v_w c_5 & c_5/3 & v_w c_5/3 \\ c_3/3 & c_4/3 & v_w c_4/3 & c_5/3 & v_w c_5/3 & c_5/5 \\ v_w c_4 & v_w c_5 & c_5/3 & v_w c_6 & c_6/3 & v_w c_6/3 \\ c_4/3 & c_5/3 & v_w c_5/3 & c_6/3 & v_w c_6/3 & c_6/5 \\ v_w c_4/3 & v_w c_5/3 & c_5/5 & v_w c_6/3 & c_6/5 & v_w c_6/5 \end{pmatrix}, \quad (26)$$

where, for $n \geq 2$,

$$c_n^b \equiv \frac{1}{T^{n+1}} \int dp p^n f_p^{\text{BE}} (1 + f_p^{\text{BE}}) = n! \zeta_n, \quad (27)$$

$$c_n^f \equiv \frac{1}{T^{n+1}} \int dp p^n f_p^{\text{FD}} (1 - f_p^{\text{FD}}) = \left(1 - \frac{1}{2^{n-1}}\right) n! \zeta_n, \quad (28)$$

and $f^{\text{BE,FD}}$ the Bose-Einstein and Fermi-Dirac distributions. For $n = 1$ one has $c_1^f = \log 2$ and $c_1^b = \log(2T/m)$.

With gauge couplings $\alpha_s \approx 0.12$ and $\alpha_W \approx 1/30$, the collision matrices are

$$\Gamma_t = \begin{pmatrix} 0.00899 & 0.01752 & 0 & 0.05489 & 0 & 0.01830 \\ 0.01752 & 0.05311 & 0 & 0.234580 & 0 & 0.07819 \\ 0 & 0 & 0.01801 & 0 & 0.08331 & \\ 0.05489 & 0.23458 & 0 & 1.38413 & 0 & 0.46138 \\ 0 & 0 & 0.08331 & 0 & 0.51188 & 0 \\ 0.01830 & 0.07819 & 0 & 0.46138 & 0 & 0.34591 \end{pmatrix} T, \quad (29)$$

$$\Gamma_W = \begin{pmatrix} 0.00466 & 0.00903 & 0 & 0.02822 & 0 & 0.00941 \\ 0.00903 & 0.03101 & 0 & 0.13043 & 0 & 0.04348 \\ 0 & 0 & 0.0123 & 0 & 0.0479 & 0 \\ 0.02822 & 0.13043 & 0 & 0.71107 & 0 & 0.23702 \\ 0 & 0 & 0.0479 & 0 & 0.25481 & 0 \\ 0.00941 & 0.04348 & 0 & 0.23702 & 0 & 0.15841 \end{pmatrix} T. \quad (30)$$

These numbers correct some mistakes in reference [7] that are mentioned in [35].

Finally, the source term for fluctuations of a heavy species reads

$$S = \gamma_w v_w \frac{(m^2)'}{4\pi T^2} (c_1 \ c_2 \ 0 \ c_3 \ 0 \ c_3/3)^T. \quad (31)$$

With this setup, we solve the Boltzmann equation for the fluctuations using the shape *Ansatz*

$$\phi(z) = \frac{\phi_0}{2} \left(1 - \tanh \frac{z}{L_w} \right), \quad (32)$$

which gives the functional dependence $m(z)$ and its derivative. We can then plug the fluctuations into the Higgs equation to find the friction.

5 Friction

The passage of the bubble drives the plasma away from equilibrium, which then back-reacts on the Higgs field, acting as a friction against the bubble expansion. Indeed, from energy-momentum conservation one can show that the Higgs equation of motion takes the form [7, 8]

$$-\phi'' + \frac{dV_T}{d\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3 2E_i} \delta f_i(x^\mu, p^\mu) = 0, \quad (33)$$

with V_T the Higgs thermal potential in equilibrium. We see that the non-equilibrium contribution from δf_i corresponds to a dissipative term, a friction which counters the bubble expansion.

There are two independent variables to be solved for, namely v_w and L_w , so we take two moments of (33), multiplying it by ϕ' and by $(2\phi - \phi_0) \times \phi'$ and integrating. This yields

$$\frac{\Delta V_T}{T^4} = f_{\text{fl}} + f_{\text{bg}}, \quad (34)$$

$$-\frac{2}{15(L_w T)^2} \left(\frac{\phi_0}{T}\right)^3 + \frac{1}{T^5} \int_0^{\phi_0} \frac{dV_T}{d\phi} (2\phi - \phi_0) d\phi = g_{\text{fl}} + g_{\text{bg}}, \quad (35)$$

with $\Delta V_T = V_T(\phi_0, T) - V_T(0, T)$ the free energy released during the phase transition. Equation (34) states the balance in total pressure on the wall, equating the inside pressure to that of friction against it. The second equation (35) is a balance of pressure gradient behind and in front of the wall, which also has to vanish overall because otherwise the wall would stretch or compress, which does not happen in the stationary state.

The functions f_{fl} and g_{fl} (respectively f_{bg} and g_{bg}) are the contributions to friction coming from the fluctuations of the heavy species in the fluid (resp. of the background). They are shown in figure 3 as a function of the wall velocity v_w , for various orders in momentum expansion of the fluid *Ansatz* up to $\ell = 5$, which takes into account a total of 21 fluctuations.

First, note that the friction contribution from fluctuations of the heavy species is non-singular at all velocities. This happens because, for these fluctuations, the Boltzmann system takes the general form

$$q' + (A^{-1} \cdot \Gamma) \cdot q = A^{-1} \cdot S, \quad (36)$$

for some matrices A and Γ and a source S , whose solution behaves as

$$q(z) \sim \int^z A^{-1} \cdot S(z') e^{-\lambda z'} dz'. \quad (37)$$

Here, λ is an eigenvalue of $A^{-1} \cdot \Gamma$, which sets the rate at which fluctuations decay. One sees that an eventual singular behaviour stemming from the kinetic matrix appears also in the exponential factor, effectively damping the singularity in $A^{-1} \cdot S$ and leading to the continuous behaviour seen in the plots.

On the other hand, as previously discussed, the two background fluctuations $w_{\text{bg}}^{(1)}$ are not damped at all — the collision terms vanish — and one ends up with a solution similar to (37) but with $\lambda = 0$, cf. equation (19). Here, the vanishing eigenvalue in the kinetic matrix, which for these modes occur at $v_w = c_s$, leads to a singularity in the solutions and, consequently, in the friction functions f_{bg} and g_{bg} .

Note that, as we increase the order in the fluid expansion, the kinetic matrix becomes singular for other values of the wall velocity. For instance, the matrix in (26) is singular at $v_w = c_s$ and $v_w = \sqrt{3/5}$. However, these other singularities are associated with higher order fluctuations which *are* damped by non-vanishing collision terms, cf. equations (23) and (24). So we recover the continuous behaviour across these velocities. The only

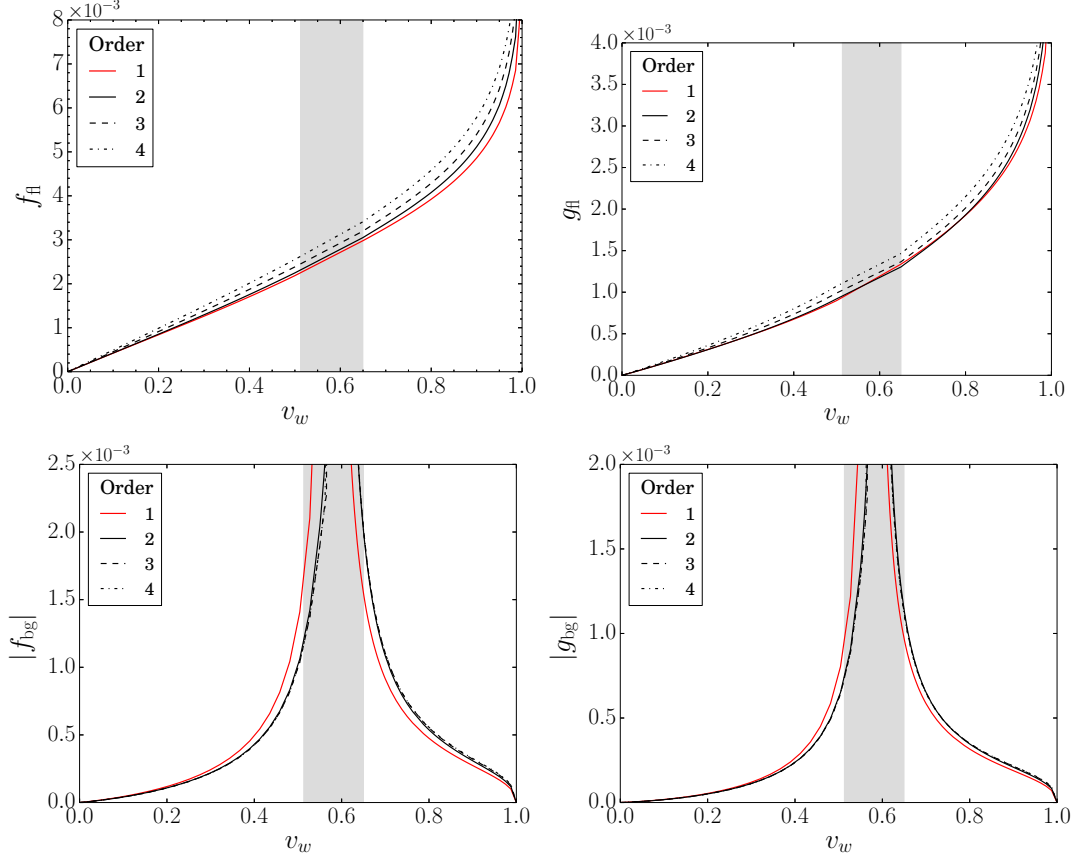


Figure 3: Friction contributions from the fluid and the background in equations (34) and (35) for different orders of the generalized fluid *Ansatz*. Note that only the background contributions have a singularity, and that it appears only at the speed of sound $c_s = 1/\sqrt{3} \approx 0.577$. We chose fiducial values for the wall thickness, $L_w T = 10$, and the phase transition strength, $\phi_0/T = 1$, which yields $\alpha \approx 0.002$. The gray shaded regions satisfy $\alpha/X_-^2 > 1$, where we expect a breakdown of the linearization procedure, see section 2.

remaining singularity happens at $v_w = c_s$, which has a clear physical interpretation, as we argued in sections 2 and 3.

We also remark that the friction functions do converge as we increase the number of fluctuations in the system, confirming that the series expansion in (22) is meaningful and well-behaved. In fact, beyond the second order the friction calculation seems very robust as long as the wall velocity is not too close to the speed of light.

6 Conclusions

We revisited the friction calculation during first-order phase transitions in light of the recent developments in [15] and [27]. In the first part of the paper we argue that a discontinuity in the temperature and fluid background fields is expected on general grounds, just relying on hydrodynamic considerations. This stems from the fact that the global solutions to the fluid profiles transition from deflagrations to detonations when the wall velocity passes the Jouguet velocity.

We also show that this discontinuity turns into a singularity once the hydrodynamic equations are expanded in the (small) change in equation of state α that parametrizes the (weak) phase transition. This unphysical singularity comes from the fact that the hydrodynamic equations are non-analytic in α for wall velocities close to the speed of sound, $v_w \rightarrow c_s$, and $\alpha \rightarrow 0$, see (7).

Based on this analysis, a singularity in the linearized Boltzmann equation is expected. Even if a method could be found to remedy the unphysical singularity for wall velocities close to the speed of sound, this method should still reproduce the discontinuity.

At this point, it is worth comparing our results to the approach presented in [27], which finds a smooth behaviour for friction across the speed of sound. First, it should be noted that this continuity was actually used as an input for constructing the setup in [27], motivating the choices made for the moments and the fluctuations used in their *Ansatz*. In particular, the moments of the Boltzmann equation taken in [27] do not correspond to energy-momentum conservation. But this would imply that expression (25) for the background collision matrix, which is assumed in [27], is no longer valid, and these collision terms would have to be computed separately.

Here we have shown that the interplay of energy-momentum conservation and the inclusion of fluctuations representing collective shifts in temperature and fluid velocity necessarily lead to a singularity close to the speed of sound. Energy-momentum conservation provides two conserved quantities while the collective shifts are not damped by collision terms. These features already imply the occurrence of a singularity, and we reproduce the exact expression for the singularity found in hydrodynamics starting from the Boltzmann equations, see (9) and (20).

Finally, we study the impact of a more general *Ansatz* for the fluctuations. In baryogenesis calculations, adding more fluctuations indeed changes the outcome qualitatively [29], in agreement with [15]. Here we find that generalizing the *Ansatz* only

leads to minor changes in the final outcome for the friction, but the main qualitative feature — the singularity across the speed of sound — still remains.

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