New physics effects on the W-boson mass from a doublet extension of the SM Higgs sector*

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(Dated: April 12, 2022)

Recently, the CDF collaboration has reported a new precision measurement of the W-boson mass, M_W , showing a large deviation from the value predicted by the Standard Model (SM). In this paper, we analyse possible new physics contributions to M_W from extended Higgs sectors. We focus on the Two-Higgs-Doublet Model (2HDM) as a concrete example. Employing predictions for the electroweak precision observables in the 2HDM at the two-loop level and taking into account further theoretical and experimental constraints, we identify parameter regions of the 2HDM in which the prediction for M_W is close to the new CDF value. We assess the compatibility of these regions with precision measurements of the effective weak mixing angle and the total width of the Z boson.

INTRODUCTION

Electroweak precision observables (EWPOs) are crucial for our understanding of the electroweak interactions. Among the EWPOs, the W-boson mass M_W , the effective weak mixing angle $\sin^2\theta_{\rm eff}^{\rm lep}$, and the total decay width of the Z boson, Γ_Z , are especially important. Because of their precise experimental measurements, where the highest accuracy has been reached for M_W , they have a large sensitivity to deviations from their Standard Model (SM) predictions that can be caused by quantum effects of physics beyond the SM (BSM).

Recently, the CDF collaboration has presented a new measurement of the W boson mass [1] with an unprecedented precision:

$$M_W = 80.4335 \pm 0.0094 \text{ GeV}$$
. (1)

This value deviates by about 7σ from the SM prediction. It is, however, also in tension with some of the previous measurements of M_W and with the average that was obtained in Ref. [2] prior to the announcement of the CDF result. While it will be mandatory to assess the compatibility of the different measurements and to carefully analyse possible sources of systematic effects, the new result from CDF is a strong additional motivation for investigating BSM contributions to the prediction for M_W . In fact, since many years the combined experimental value of M_W has always been consistently above the SM prediction for a Higgs boson mass of about 125 GeV, favouring a non-zero BSM contribution to M_W . The new CDF measurement significantly strengthens the preference for an upward shift compared to the SM prediction arising from BSM effects.

In this paper, we assess the possibility that an extended Higgs sector shifts M_W upwards with respect to

the SM. As a concrete example, we focus on the Two-Higgs-Doublet Model (2HDM). The M_W prediction in the 2HDM has been studied at the one- and two-loop level in Refs. [3–15] finding sizeable BSM corrections. In the present paper, we investigate the 2HDM M_W prediction in the context of the CDF measurement, employing the state-of-the-art two-loop predictions from Refs. [14, 15], as well as other theoretical and experimental constraints. Since loop effects in the 2HDM affecting M_W will manifest themselves also in the predictions for the effective weak mixing angle and the total width of the Z boson, we also assess the compatibility of those (pseudo-)observables with the experimental data.

NON-STANDARD CORRECTIONS TO M_W IN THE 2HDM

We consider a \mathcal{CP} -conserving 2HDM containing two $SU(2)_L$ doublets Φ_1 and Φ_2 of hypercharge 1/2. We impose a \mathbb{Z}_2 symmetry in the Higgs potential under which the two doublets transform as $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$, but that is softly broken by an off-diagonal mass term. This potential reads

$$V_{2\text{HDM}}(\Phi_{1}, \Phi_{2}) =$$

$$= m_{11}^{2} \Phi_{1}^{\dagger} \Phi_{1} + m_{22}^{2} \Phi_{2}^{\dagger} \Phi_{2} - m_{12}^{2} \left(\Phi_{1}^{\dagger} \Phi_{2} + \Phi_{2}^{\dagger} \Phi_{1} \right)$$

$$+ \frac{1}{2} \lambda_{1} (\Phi_{1}^{\dagger} \Phi_{1})^{2} + \frac{1}{2} \lambda_{2} (\Phi_{2}^{\dagger} \Phi_{2})^{2} + \lambda_{3} (\Phi_{1}^{\dagger} \Phi_{1}) (\Phi_{2}^{\dagger} \Phi_{2})$$

$$+ \lambda_{4} (\Phi_{1}^{\dagger} \Phi_{2}) (\Phi_{2}^{\dagger} \Phi_{1}) + \frac{1}{2} \lambda_{5} \left((\Phi_{1}^{\dagger} \Phi_{2})^{2} + (\Phi_{2}^{\dagger} \Phi_{1})^{2} \right).$$

$$(2)$$

All parameters can be assumed to be real, because we focus on the \mathcal{CP} -conserving case. After minimization of the Higgs potential, the Higgs doublets are decomposed

as $\Phi_i^T = (\phi_i^+, (v_i + \phi_i + i\chi_i)/\sqrt{2})$ with $v_1^2 + v_2^2 \equiv v^2 \simeq 246 \text{ GeV}$ and $v_2/v_1 \equiv \tan \beta$.

After rotating to the mass eigenstate basis, the Higgs boson spectrum consists of the \mathcal{CP} -even Higgs bosons h and H (obtained by rotating the $\phi_{1,2}$ states by the angle α), the \mathcal{CP} -odd A boson and the neutral Goldstone boson G (obtained by rotating the $\chi_{1,2}$ states by the angle β), as well as the charged Higgs boson H^{\pm} and the charged Goldstone boson G^{\pm} (obtained by rotating the $\phi_{1,2}^{\pm}$ states by the angle β). We identify the \mathcal{CP} -even mass eigenstate h with the observed SM-like Higgs boson and work in the so-called alignment limit by enforcing $\alpha = \beta - \pi/2$ [16]. The remaining input parameters for our numerical analysis are m_H , m_A , $m_{H^{\pm}}$, $\tan \beta$, and $M^2 \equiv m_{12}^2/(\sin \beta \cos \beta)$. Relations between these parameters and the parameters of Eq. (2) are listed e.g. in Ref. [17].

The leading 2HDM corrections to the EWPOs are induced via corrections to the ρ parameter, which is defined as the ratio of the neutral and charged current four-fermion interactions. In the 2HDM, ρ is equal to one at the tree-level. This tree-level value is, however, affected by loop corrections, which are associated with a breakdown of the custodial symmetry. As discussed in detail in Refs. [14, 18–23], the custodial symmetry is restored in the scalar sector at the one-loop level if either $m_H = m_{H^{\pm}}$, a restoration of the custodial symmetry in the scalar sector at two loops happens only if the additional constraint of either $\tan \beta = 1$ or $m_H^2 = M^2$ is fulfilled.

The non-SM one-loop corrections to the ρ parameter (assuming massless external fermions), $\Delta \rho_{\text{non-SM}}^{(1)}$, in the \mathcal{CP} -conserving 2HDM (and for $\alpha = \beta - \pi/2$) are given by [14]

$$\begin{split} \Delta \rho_{\text{non-SM}}^{(1)} &= \frac{\alpha}{16\pi^2 s_W^2 M_W^2} \left\{ \frac{m_A^2 m_H^2}{m_A^2 - m_H^2} \ln \frac{m_A^2}{m_H^2} \right. \\ &\left. - \frac{m_A^2 m_{H^\pm}^2}{m_A^2 - m_{H^\pm}^2} \ln \frac{m_A^2}{m_{H^\pm}^2} \right. \\ &\left. - \frac{m_H^2 m_{H^\pm}^2}{m_H^2 - m_{H^\pm}^2} \ln \frac{m_H^2}{m_{H^\pm}^2} + m_{H^\pm}^2 \right\}, \end{split} \tag{3}$$

where s_W and c_W are the sine and cosine of the weak mixing angle, respectively, $\alpha \equiv e^2/(4\pi)$, and e is the electric charge. The quantity $\Delta \rho$ enters the prediction for the W boson mass approximately via

$$\Delta M_W \simeq \frac{1}{2} M_W \frac{c_W^2}{c_W^2 - s_W^2} \Delta \rho. \tag{4}$$

While Eqs. (3) and (4) allow for a qualitative understanding of the 2HDM effects on the M_W prediction, a precise higher-order calculation is essential for a comparison with the experimental results. In order to predict M_W (as well as $\sin^2 \theta_{\rm eff}^{\rm lep}$ and Γ_Z) we use the code THDM_EWPOS which

is based on Refs. [14, 15]. It incorporates the full one-loop non-SM corrections as well as the leading non-SM two-loop corrections. To be more specific, the two-loop non-SM corrections are calculated in the limit of vanishing electroweak gauge couplings (keeping the ratio of M_W and M_Z constant). Moreover, all quarks and leptons except for the top quark are treated as massless for the non-SM two-loop corrections. For the calculation of the two-loop corrections, all Higgs boson masses are renormalized in the on-shell scheme. The SM corrections are included via the parameterization given in Ref. [24]. They contain the complete one-loop [25, 26] and the complete two-loop results [27–42], as well as partial higher-order corrections up to four-loop order [43–52].

The remaining theoretical uncertainties of the predictions for M_W , $\sin^2\theta_{\rm eff}^{\rm lep}$, and Γ_Z arise on the one hand from unknown higher-order contributions. On the other hand, a parametric uncertainty is induced by the experimental errors of the input parameters, e.g. the top-quark mass. Since the discrepancy between the CDF value for M_W and the SM prediction is much larger than those theoretical uncertainties we will not give a detailed account of those uncertainties in the following.

NUMERICAL RESULTS

While we expect similar results for all 2HDM types,² we concentrate here for our numerical study on the 2HDM of type I. Regarding our predictions for M_W , we apply various other constraints of both experimental and theoretical nature on the considered parameter space:

- vacuum stability [58] and boundedness-frombelow [59] of the Higgs potential,
- NLO perturbative unitarity [60, 61],
- compatibility of the SM-like scalar with the experimentally discovered Higgs boson using HiggsSignals [62, 63],
- limits from direct searches for BSM scalars using HiggsBounds [64–68],
- b physics [69].³

 1 See also Refs. [53–56] for further higher-order contributions involving fermion loops and Ref. [57] for a prediction of M_W employing the $\overline{\rm MS}$ renormalisation scheme.

³ In practice, the fit results of Ref. [69] are used to obtain 2σ constraints in the $(m_{H^{\pm}}, \, \tan\beta)$ plane of the 2HDM parameter space.

² The difference between the 2HDM types appears only in the down-type and lepton Yukawa couplings. Since the two-loop non-SM correction implemented in THDM_EWPOS uses the approximation of massless down-type quarks and leptons, the choice of the 2HDM type does not affect the EWPO calculation.

We use ScannerS [70] to evaluate all of these constraints apart from the NLO perturbative unitarity constraint, which is evaluated separately. If applicable, we demand that the constraints be fulfilled at the 95% C.L. Taking into account these constraints on the parameter space, we obtain for each parameter point the one- and two-loop predictions for M_W , $\sin^2\theta_{\rm eff}^{\rm lep}$, and Γ_Z . We note that as ScannerS does not define a renormalisation scheme for the 2HDM mass parameters, we choose to interpret these as on-shell renormalised inputs when used in the two-loop calculations of the EWPOs.

We perform a random scan of the 2HDM parameter space. While we fix $m_h = 125.09$ GeV and $\alpha = \beta - \pi/2$, we scan over values of m_H and m_A in the range between 30 and 1500 GeV, m_{H^\pm} between 150 and 1500 GeV, $\tan \beta$ between 0.8 and 50, and m_{12}^2 between 0 and $4\cdot 10^6$ GeV². All points shown in the Figures pass the theoretical and experimental constraints outlined above.

In Fig. (1), we present the scan results for the EW-POs. In the panels, the light green band indicates the M_W value (and the associated 1σ uncertainty) measured recently by the CDF collaboration. Points located within the 1σ interval of the CDF measurement are coloured in red. We also show as a light orange band the world average for $\sin^2 \theta_{\rm eff}^{\rm lep}$ (and the associated $1\,\sigma$ uncertainty) [71] that was obtained by averaging over the results of the four LEP collaborations and the SLD collaboration, where the two most precise measurements (based on the forward-backward asymmetry of bottom quarks at LEP and the left-right asymmetry at the SLC) showed a discrepancy of more than 3σ . For comparison, we also display the result for $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ (and its associated 1σ uncertainty) that is based on the measurement of the left-right asymmetry by the SLD collaboration [71] as a light yellow band.⁴ In addition, the light purple band shows the world average value for Γ_Z (and its associated 1σ uncertainty).

In the upper left panel, the results are shown in the $(M_W,\sin^2\theta_{\rm eff}^{\rm lep})$ plane. We observe that an M_W value close to the value measured by the CDF collaboration is very well compatible with the predicted range in the 2HDM (while passing other theoretical and experimental constraints). M_W values within the $1\,\sigma$ interval of the CDF M_W measurement are in mild tension with the world average value for $\sin^2\theta_{\rm eff}^{\rm lep}$ but in good agreement with the value measured by the SLD experiment.

In the upper right panel, we show the results in the (M_W, Γ_Z) plane. We find that the points where M_W is close to the measured CDF value are compatible at the level of 1–1.5 σ with the world average value for Γ_Z . This is confirmed in the bottom panel showing the results in

the (sin² $\theta_{\rm eff}^{\rm lep}$, Γ_Z) plane. The red points, for which M_W is within $1\,\sigma$ of the CDF value, are at most in mild tension with the results of the other precision measurements at the Z peak.

In the left panel of Fig. (2), we investigate for which mass configurations the CDF value for M_W can be reached. In this panel, the scan results are shown in the $(m_H - m_{H^{\pm}}, m_A - m_{H^{\pm}})$ plane. For $m_H - m_{H^{\pm}} > 0$ and $m_A - m_{H^{\pm}} > 0$, as well as for $m_H - m_{H^{\pm}} < 0$ and $m_A - m_{H^{\pm}} < 0$, the one-loop non-SM correction to the ρ parameter (see Eq. (3)) is positive (see also Ref. [72]). Since the CDF value for M_W lies above the SM prediction, it favours this part of the parameter region of the 2HDM giving rise to a sizeable upward shift of M_W , see Eq. (4). In contrast, the one-loop non-SM correction is negative for $m_H - m_{H^\pm} > 0$ and $m_A - m_{H^\pm} < 0$ as well as for $m_H - m_{H^{\pm}} < 0$ and $m_A - m_{H^{\pm}} > 0$ implying that the CDF M_W value cannot be reached in this part of the parameter space. For reference, we also give in Tab. (I) the complete parameter values for two exemplary points in the upper right and lower left part of the left plot of Fig. (2). This Table also shows the M_W values obtained if the non-SM contributions are only evaluated at the one-loop level, finding significantly smaller values. The left plot of Fig. (2) furthermore highlights the fact that the CDF value for M_W cannot be reproduced in scenarios of the 2HDM in which the three BSM scalars are mass-degenerate, since as discussed above the custodial symmetry of the non-SM contribution is restored in the limit where all scalar masses are taken to be equal to each other.

We further assess the size of the two-loop non-SM corrections in the right panel of Fig. (2) showing the scan results in the $(M_W, \Delta M_W^{\rm 2L, non-SM})$ plane. Here, $\Delta M_W^{\rm 2L, non-SM}$ denotes the difference between M_W evaluated employing the one- and two-loop non-SM correction and M_W evaluated employing only the one-loop non-SM corrections. We observe that large values for M_W are often associated with sizeable positive non-SM two-loop corrections. This shows the importance of a precise evaluation of the non-SM contributions to M_W taking into account corrections beyond the one-loop level.

Another important quantity regarding the phenomenology of the 2HDM is the trilinear Higgs coupling. As recently emphasised in Ref. [73], also in this case it is crucial to include higher-order corrections in order to evaluate the impact of the experimental results on the parameter space of the model. In Fig. (3), we check whether the parameter region with an M_W value close to the value measured by CDF can be constrained by the experimental limits on the trilinear Higgs coupling. We show the scan results in the (M_W, κ_λ) parameter plane, where $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{\rm SM})^{(0)}$ is evaluated taking into account leading one-loop as well as two-loop corrections [74–76]. We find that the results for κ_λ obtained

⁴ The SLD measurement is the most precise single $\sin^2 \theta_{\rm eff}^{\rm lep}$ measurement and depends only on leptonic couplings.

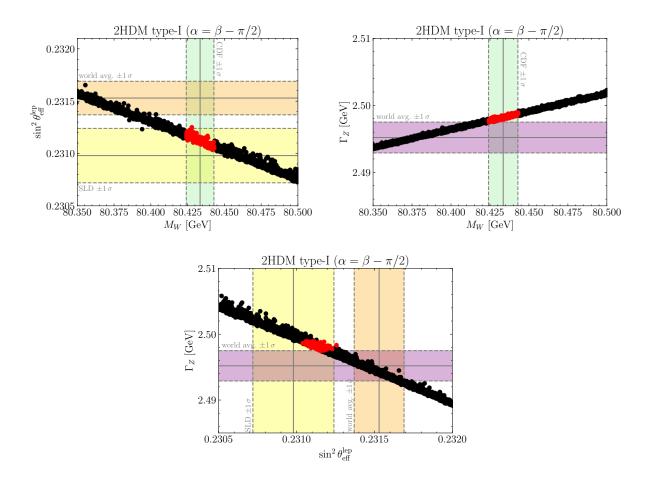


FIG. 1. Upper left: parameter scan of the type-I 2HDM in the $(M_W, \sin^2 \theta_{\rm eff}^{\rm lep})$ plane. The red points are located within the 1 σ interval of the recent M_W measurement by the CDF collaboration. Upper right: same as upper left panel, but the (M_W, Γ_Z) plane is shown. Bottom: same as upper left panel, but the $(\sin^2 \theta_{\rm eff}^{\rm lep}, \Gamma_Z)$ plane is shown.

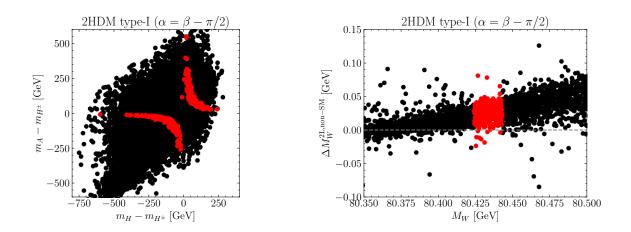


FIG. 2. Parameter scan of the type-I 2HDM, where the red points are located within the $1\,\sigma$ interval of the recent M_W measurement by the CDF collaboration. In the left plot, we show the points from the parameter scan in the $(m_H - m_{H^\pm}, m_A - m_{H^\pm})$ plane. In the right plot, we show for the same points the size of the two-loop non-SM corrections to M_W against the total result for M_W .

m_H	m_A	$m_{H^{\pm}}$	$\tan \beta$	M^2	M_W [GeV]	M_W [GeV]	$\sin^2 \theta_{\mathrm{eff}}^{\mathrm{lep}}$	Γ_Z
[GeV]	[GeV]	[GeV]	_	$[\mathrm{GeV}^2]$	(non-SM@1L)	(non-SM@2L)	_	[GeV]
853.813	928.352	809.047	1.206	444.166×10^3	80.4001	80.4337	0.23113	2.4981
351.962	751.498	762.911	1.255	55.451×10^3	80.3990	80.4339	0.23109	2.4979

TABLE I. Parameter values and results for the electroweak precision observables (calculated including two-loop non-SM corrections if not stated otherwise) for two exemplary points with M_W close to the value measured by the CDF collaboration.

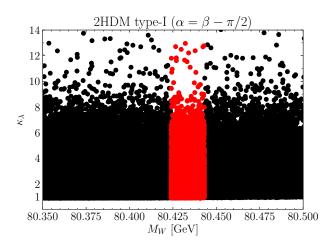


FIG. 3. Parameter scan of the type-I 2HDM, where the red points are located within the $1\,\sigma$ interval of the recent M_W measurement by the CDF collaboration. We show the points from the parameter scan in the (M_W,κ_λ) plane, where both quantities have been computed taking into account corrections up to the two-loop level.

in Ref. [73] are largely independent of the preferred region for M_W . Moreover, only a small fraction of the points within the $1\,\sigma$ interval of the CDF M_W measurement are excluded by the experimental constraint that $-1.0 < \kappa_{\lambda} < 6.6$ [77].

CONCLUSIONS

The mass of the W boson is one of the (pseudo-) observables for which the comparison between its high-precision measurement and accurate theoretical predictions provides the highest sensitivity for discriminating between the SM and possible alternatives or extensions of it. The new measurement released by the CDF collaboration clearly has an important impact in this context. While the establishment of a new world average for M_W will require a careful assessment of the systematic uncertainties of the individual measurements, one can certainly expect that the incorporation of the new result announced by CDF will significantly strengthen the preference for a non-zero BSM contribution to M_W . Indeed, a large deviation in the W-boson mass — if confirmed by other experiments — could be a hallmark of

BSM physics.

In this paper, we have analysed the question of whether a deviation in M_W from the SM prediction as large as the one reported by CDF could actually be accommodated by well-motivated BSM models without spoiling the compatibility with the existing experimental results for other observables from different sectors and with theoretical constraints. Specifically, we have assessed the possibility that such a deviation is due to the effects of an extended Higgs sector. We have focused on the 2HDM as one of the most widely studied extensions of the SM. which can be viewed as a representative case of more complicated Higgs sectors, and we have taken into account other relevant theoretical and experimental constraints. This has led to the remarkable result that BSM quantum corrections can indeed be sufficiently large to obtain a prediction for M_W that is in very good agreement with the CDF measurement while being compatible with other constraints. In particular, we have demonstrated in this context the compatibility with the experimental measurements of the effective weak mixing angle and the total width of the Z boson.

We have found that the mass hierarchies $m_H, m_A <$ $m_{H^{\pm}}$ and $m_{H^{\pm}} < m_H, m_A$ are favoured by the CDF M_W measurement, whereas other mass hierarchies, and especially the case in which $m_H \sim m_A \sim m_{H^{\pm}}$, are disfavoured. Moreover, we have pointed out the importance of a precise evaluation of the non-SM contributions to M_W beyond the one-loop level. Concerning the prediction for the effective weak mixing angle at the Z-boson resonance, it should be noted that a sizeable quantum correction in the 2HDM bringing M_W into agreement with the CDF measurement would be favoured by the SLD measurement of the left-right asymmetry, which is interesting in view of the long-standing discrepancy between the most precise single measurements from LEP and SLC. We have also shown that the region of the 2HDM parameter space that is in agreement with the M_W value of CDF is only slightly affected by the experimental constraints on the trilinear Higgs coupling.

We will present further details of our 2HDM results in an upcoming paper. While we focused on the 2HDM in this paper as a representative case, similar results can also be expected for other extensions of the SM Higgs sector.

ACKNOWLEDGEMENTS

We thank S. Hessenberger for useful discussions and for providing us access to his code THDM_EWPOS. J.B. and G.W. acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2121 "Quantum Universe" – 390833306. H.B. acknowledges support by the Alexander von Humboldt foundation.

- * This paper is dedicated to the memory of Alberto Sirlin, whose work paved the way for precise predictions of the W-boson mass.
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