Application of the Optical Theorem for Quasi-Elastic Scattering
to the Photoproduction of Vector Mesons

by

G. Knies

Deutsches Elektronen-Synchrotron DESY, Hamburg

Abstract

We investigate some consequences of the unitarity of the S-matrix to inelastic diffraction scattering processes. The application to the photoproduction of vector mesons leads to model independent lower limits on the total cross section for the production of hadrons by photons.

I. The Optical Theorem for Quasi-Elastic Scattering

Let a_1, a_2, b_1 and b_2 be arbitrary particles. With these we consider the following scattering processes in the forward direction ($\theta=0$):

$$a_1 + a_2 \rightarrow a_1 + a_2 \quad a \rightarrow a \quad (1.1)$$

$$b_1 + b_2 \rightarrow b_1 + b_2 \quad b \rightarrow b \quad (1.2)$$

$$a_1 + a_2 \rightarrow b_1 + b_2 \quad a \rightarrow b \quad (1.3)$$

For the unitarity relation of the S-matrix ($S = 1 + iT$)

$$-i(T - T^\dagger) = T^\dagger T = T^\dagger |n\rangle \langle n| T \quad (2)$$

the matrix elements of the above reactions read

$$-i\langle a|T - T^\dagger|a\rangle = \langle Ta|Ta\rangle = \langle a|T^\dagger|n\rangle \langle n|T|a\rangle \quad (3.1)$$

$$-i\langle b|T - T^\dagger|b\rangle = \langle Tb|Tb\rangle = \langle b|T^\dagger|n\rangle \langle n|T|b\rangle \quad (3.2)$$

$$-i\langle a|T - T^\dagger|b\rangle = \langle Ta|Tb\rangle = \langle a|T^\dagger|n\rangle \langle n|T|b\rangle, \quad (3.3)$$

where $|a\rangle$ and $|b\rangle$ are composed of plane waves for the particles a_1, a_2 and b_1, b_2 , respectively.

Between the three equations (3) we have the Schwarz inequality

$$|\langle Ta|Tb\rangle| \leq (\langle Ta|Ta\rangle \langle Tb|Tb\rangle)^{1/2} \\ = \kappa (\langle Ta|Ta\rangle \langle Tb|Tb\rangle)^{1/2}, \quad 0 \leq \kappa \leq 1. \quad (4)$$

κ assumes extremal values

$$\kappa = 1, \quad \text{if } |Ta\rangle = z|Tb\rangle; \quad z \neq 0, \quad \text{complex}$$

$$\kappa = 0, \quad \text{if } |Ta\rangle \perp |Tb\rangle.$$

The quantity κ^2 measures the probability for identical final states being produced by an interaction from the initial states $|a_1 + a_2\rangle$ and $|b_1 + b_2\rangle$.

In order to express the contents of (3) and (4) in terms of measurable quantities let σ' denote that part of the cross section in Reactions (1) which comes from the imaginary part of the scattering amplitude. Eqs.(3.1) and (3.2) may then be expressed as the optical theorem.

$$d\sigma'/dt (a \rightarrow a, \theta = 0) = (1/16\pi) \cdot \sigma_{\text{tot}}(a) \sigma_{\text{tot}}(a) \quad (5.1)$$

$$d\sigma'/dt (b \rightarrow b, \theta = 0) = (1/16\pi) \cdot \sigma_{\text{tot}}(b) \sigma_{\text{tot}}(b) \quad (5.2)$$

If the state $|a\rangle$ contains particles with spin, it is the meaning of Eqs.(5.1) and (3.1) that $|a\rangle$ in all cases is the same superposition of helicity states. The same applies for b . For the measurement of the experimental quantities in (5), this implies a precise prescription as to which helicity states are to be included into the observation. This prescription becomes redundant if either the scattering is independent of spin or if there is only one (independent) helicity amplitude different from zero in the forward direction. The imaginary part of the strong interaction amplitude, or $d\sigma'/dt$, may be determined from interference with electromagnetic processes.

We now investigate the question whether (3.3) may also be expressed as a relation between measurable quantities. If T is invariant under time reversal and spatial rotations, then the left-hand side (lhs) of (3.3) is real. Hence the matrix element itself is imaginary and is equal to two times the imaginary part of the (forward) amplitude for the inelastic reaction $a \rightarrow b$, just as in the case of elastic scattering.* In contrast to elastic scattering, however, the rhs of (3.3) may not be interpreted as a cross section. But by virtue of (4) it is related to the total cross sections $\sigma_{\text{tot}}(a)$ and $\sigma_{\text{tot}}(b)$. Thus we have as an optical theorem for quasi-elastic processes (1.3)

$$f(a,b) \cdot d\sigma'/dt (a \rightarrow b, \theta = 0^\circ) \leq (1/16\pi) \sigma_{\text{tot}}(a) \sigma_{\text{tot}}(b)$$

or, with κ from (4)

$$f(a,b) \cdot d\sigma'/dt (a \rightarrow b, \theta = 0) = (\kappa^2/16\pi) \sigma_{\text{tot}}(a) \sigma_{\text{tot}}(b) \quad (5.3a)$$

$$f(b,a) \cdot d\sigma'/dt (b \rightarrow a, \theta = 0) = (\kappa^2/16\pi) \sigma_{\text{tot}}(a) \sigma_{\text{tot}}(b) \quad (5.3b)$$

The factor $f(a,b)$, which is equal to 1 in the case $a=b$, is to account for differences in the cross sections on the lhs of (5.3a) and (5.3b). $f(a,b)$ follows from the reciprocity relation¹

$$\left(\frac{S_a}{S_b}\right)^{1/2} \cdot \frac{p_a}{p_b} \cdot \frac{d\sigma}{dt} (a \rightarrow b) = \left(\frac{S_b}{S_a}\right)^{1/2} \cdot \frac{p_b}{p_a} \cdot \frac{d\sigma}{dt} (b \rightarrow a)$$

with $p_a = |\vec{p}_{a1}|_{\text{CMS}}$ and $p_b = |\vec{p}_{b1}|_{\text{CMS}}$.

$S_a(S_b)$ depends on the spins of the particles in a (b). For example, a state $|a\rangle$ describing a system of two unpolarized particles a_1 and a_2 , with spins S_{a1} and S_{a2} , leads to

$$S_a = (2 S_{a1} + 1) (2 S_{a2} + 1)$$

or $S_a = (2 S_{a2} + 1) 2$ if a_2 is a massless particle with spin. The same applies for S_b .

Eq.(5.3) is to be taken for the case that $|a\rangle$ and $|b\rangle$ in (3.3) are the same superposition of helicity states as in (3.1) and (3.2), respectively. This implies a precise prescription for the measurement of $d\sigma'/dt (a \rightarrow b)$. $\sigma_{\text{tot}}(a)$ and $\sigma_{\text{tot}}(b)$ are identical to those in (5.1) and (5.2).

II. Application to Experiments

Formula (5.3) may be used in two ways. Either, if all three cross sections involved have been measured it may be used to determine κ^2 and to classify the related two-particle states a and b according to the value of κ^2 which measures the similarity of a and b. Up to now, however, no such cases are known.

Or, if one of the cross sections is unknown, with $\kappa^2 \leq 1$ it may be used to establish a limit for it. From reactions like $pp \rightarrow N^*p$, e.g. $N^*(1470)$,² and $\sigma_{\text{tot}}(pp)$ ³ one finds $\sigma_{\text{tot}}(N^*p) > \frac{1}{10} \sigma_{\text{tot}}(pp)$. However, this is not a very interesting lower limit.** From the reaction $\gamma p \rightarrow \rho p$ ⁴⁻⁶ and $\sigma_{\text{tot}}(\rho p)$ ^{7,8} we find, as we shall see below, $\sigma_{\text{tot}}(\gamma p) \geq 99 \mu\text{b}$ at $E_\gamma \approx 4$ GeV, which is a nontrivial lower limit. Let us therefore consider the photoproduction of vector mesons in more detail.

In order to avoid complications from spin, let us first consider the photoproduction on scalar particles such as carbon. Here, the photo-produced ρ mesons in the forward direction can only have the helicities $h = \pm 1$. Because of invariance under space inversion, we have only one independent helicity amplitude and need not worry about the spin states of γ and ρ . With $\sigma_{\text{tot}}(\rho C) = 260 \pm 26 \text{ mb}$ ⁹ and $d\sigma'/dt (\gamma C \rightarrow \rho C, \theta = 0^\circ) = 8.8 \pm 1.2$ ^{6,9,***} $\text{mb}/(\text{GeV}^2, \text{ nucleus})$ at $E_\gamma \approx 4$ GeV, the lower limit relation

$$\sigma_{\text{tot}}(\gamma C) \geq \frac{16\pi}{\sigma_{\text{tot}}(\rho C)} \cdot \frac{d\sigma'}{dt} (\gamma C \rightarrow \rho C, \theta = 0) \cdot f(\gamma, \rho) \quad (6)$$

gives $\sigma_{\text{tot}}(\gamma C) \geq 0.75 \pm 0.12 \text{ mb}$. The lower limit relation (6), which applies to all spinless nuclei, does not depend on any assumption.

For a lower limit to the total cross section $\sigma(\gamma p)$ we may use $\sigma(\rho p)$ ^{7,8} and the reaction $\gamma p \rightarrow \rho p$.⁴⁻⁶ In this case, however, complications from spin will arise as we have three independent helicity amplitudes

$\langle \lambda_\rho \lambda_p | | \lambda_\gamma \lambda_p \rangle$ in the forward direction:
 $\langle ++ | T | ++ \rangle$, $\langle +- | T | +- \rangle$ and $\langle +- | T | 0+ \rangle$.

From the experimental result,^{4,5} that the density matrix element $\rho_{00} = 0$, it follows that the last amplitude is zero. Since there remain two independent helicity amplitudes, measurements with polarized beams and targets are necessary. However, if one wants to use the presently known cross sections for unpolarized beams and targets it is sufficient that one of

the following assumptions is fulfilled: (I) $\langle ++|T|++ \rangle = \langle +-|T|+- \rangle$ or (II) for transversal ρ mesons the ρ nucleon cross section does not depend on the nucleon spin.

These two assumptions are made very often. E.g.(I) is used successfully in the strong absorption model for $\gamma p \rightarrow \rho p$,¹¹ and (II) is usually made for high energy scattering. With $d\sigma'/dt (\gamma p \rightarrow \rho p, \theta = 0^\circ) = 134 \pm 9 \mu\text{b}/\text{GeV}^2$ ^{4-6,***} and different determinations of $\sigma(\rho p)$ ^{7,8} we find from relation (6) the corresponding lower limits for $\sigma(\gamma p)$ at $3 < E_\gamma < 5 \text{ GeV}$ as given in Table 1.

$\sigma(\gamma p) (\mu\text{b})$	$\sigma(\rho p) (\text{mb})$
$\geq 92 \pm 9.2$	31.3 ± 2.3^7
$\geq 111 \pm 15$	26 ± 3^8
$\geq 99 \pm 12$	$29 \pm 3 \text{ (mean)}$

Table 1:

Lower limits for $\sigma(\gamma p)$ corresponding to different determinations of $\sigma(\rho p)$

The results for the lower limit of $\sigma(\gamma p)$ are very close to the predictions made for $\sigma(\gamma p)$ in the framework of the vector dominance model^{12,13} and also to a preliminary experimental result on $\sigma(\gamma p)$ ¹⁴ leading to $\kappa^2 \approx 0.9$. In the case of pure ρ dominance, which means $T|\gamma p\rangle = z T|\rho p\rangle$, the lower limits given here would be adopted by $\sigma(\gamma p)$.

Of course, the above reasoning holds for $\gamma \rightarrow \omega$ and $\gamma \rightarrow \phi$, too. But the presently known experimental results on ω and ϕ are not so well established and the corresponding numerical results for the lower limit of $\sigma(\gamma p)$ are much smaller (roughly 10% and 5%, respectively). For combining them with the ρ -value, daring assumptions such as $\langle \omega p|T - T^\dagger|\phi p\rangle = 0$ must be made.

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References and Footnotes

- * At energies high enough the main contributions to the sum in (3.3) come from two and more particle intermediate states. Eq.(3.3) than states what a "shadow" into inelastic channels means and that the diffractive part of a quasi-elastic scattering process is given by the imaginary part of the corresponding amplitude.
- ** Guided by the independent quark model we may assume $\sigma(N^*p) = \sigma(pp)$. Than we find $\kappa^2 \approx 0.1$ for $N^*(1470)$.
- *** That the experimental $d\sigma/dt (\gamma \rightarrow \rho)$ is a good approximation for $d\sigma'/dt$ is established by two experimental results: (a) the phase of the amplitude for $\gamma \rightarrow \rho$ has been measured on carbon¹⁰ and is consistent with being purely imaginary, and (b) $d\sigma/dt (\theta = 0)$ shows the same energy dependence as is expected for diffractive scattering.⁴⁻⁷
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