PROCEEDINGS OF SCIENCE

Using classical bit-flip correction for error mitigation in quantum computations including 2-qubit correlations

Constantia Alexandrou,[a,b] **Lena Funcke,**[c,d] **Tobias Hartung,**[e] **Karl Jansen,**[f,*] **Stefan Kühn,**[b] **Georgios Polykratis,**[a,b] **Paolo Stornati**[f,g] **and Xiaoyang Wang**[h]

[a] *Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus*

[b] *Computation-based Science and Technology Research Center, The Cyprus Institute, 20 Kavafi Street, 2121 Nicosia, Cyprus*

[c] *Center for Theoretical Physics, Co-Design Center for Quantum Advantage, and NSF AI Institute for Artificial Intelligence and Fundamental Interactions, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA*

[d] *Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada*

[e] *Department of Mathematical Sciences, University of Bath, 4 West, Claverton Down, Bath, BA2 7AY, UK*

[f] *Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, 15738 Zeuthen*

[g] *Institut für Physik, Humboldt-Universität zu Berlin, Zum Großen Windkanal 6, D-12489 Berlin, Germany*

[h] *School of Physics, Peking University, 5 Yiheyuan Rd, Haidian District, Beijing 100871, China*

E-mail: alexand@ucy.ac.cy, lfuncke@mit.edu, tobias.hartung@desy.de, karl.jansen@desy.de, s.kuehn@cyi.ac.cy, g.polykratis@cyi.ac.cy, paolo.stornati@desy.de, zzwxy@pku.edu.cn

We present an error mitigation scheme which corrects readout errors on Noisy Intermediate-Scale Quantum (NISQ) computers [1, 2]. After a short review of applying the method to one qubit, we proceed to discuss the case when correlations between different qubits occur. We demonstrate how the readout error can be mitigated in this case. By performing experiments on IBMQ hardware, we show that such correlations do not have a strong effect on the results, justifying to neglect them.

Preprint number: MIT-CTP/5352

The 38th International Symposium on Lattice Field Theory, LATTICE2021 26th-30th July, 2021 Zoom/Gather@Massachusetts Institute of Technology

*Speaker

PoS(LATTICE2021)327

1. Introduction

In recent years, quantum computing has started to become a very active area of research in lattice field theory. The reason is that quantum computations offer the exciting possibility to solve problems which are either extremely hard or even impossible to address on classical computers. This includes systems with a non-zero chemical potential, topological terms, and real-time evolutions.

This promising avenue is, however, blocked by the fact that current quantum computers, so-called Noisy Intermediate-Scale Quantum (NISQ) computers, only have a small number of qubits which, in addition, are very noisy. This leads to various kind of errors in quantum computations, which often prevent obtaining solutions with a desired accuracy. However, already in current and near-term NISQ devices, these errors can be partly corrected through error mitigation schemes (see, e.g., Refs. [1–15]). These works include various ideas to mitigate the noise, e.g., by altering the quantum circuit, by post-processing the data, or by measuring modified operators. As demonstrated in the above references, in this way the quantum noise can indeed be mitigated leading to more reliable estimates of physical results. Another important improvement is the construction of minimal but maximally expressive quantum circuits, which has been developed by some us in Refs. [16, 17].

2. Readout error mitigation

In Refs. [1, 2], we have developed a general error mitigation scheme for readout errors. Our method is based on a readout error calibration of qubits and a reinterpretation of the measurements as measuring "noisy operators". In particular, we have demonstrated that our error mitigation method scales efficiently, i.e., polynomially. Our method can be applied either as a pre-processing or as a post-processing step. Further experiments and a comparison of our error mitigation scheme on IBMQ and Rigetti hardware is provided in Ref. [18] in this conference. As we will show below and discussed in Ref. [1], the method can also take correlations between qubits into account. We will demonstrate this both theoretically and in practical experiments. In order to explain the main idea of the method, we will start, however, with a description where correlations are neglected.

2.1 Neglecting correlations

Let $p_{q,1}$ be the probability of erroneously reading out a qubit q such that an incorrect outcome 1 is measured instead of a correct outcome 0, and $p_{q,0}$ the probability of erroneously reading out 0 instead of 1. These probabilities can be obtained by preparing each qubit in the computational basis states and measuring the outcomes (see Fig. 1). For example, to obtain $p_{q,1}$ we prepare the qubit q multiple times in the state $|0\rangle$ and record the number of outcomes 1. Then, if we want to measure an operator, e.g., the $Z_2 \otimes Z_1$ operator on 2 qubits (where Z represents the third Pauli matrix σ_Z), the correct expectation value for $Z_2 \otimes Z_1$ can be determined by measuring the operator $Z_2 \otimes Z_1$ itself, the operator $Z_2 \otimes \mathbb{1}_1$, and the operator $\mathbb{1}_2 \otimes Z_1$. From the measurement of the probabilities $p_{q,0/1}$ in the calibration process, we can obtain the coefficients of these operators, and we find

$$\begin{aligned} Z_2 \otimes Z_1 = & \frac{1}{\gamma(Z_2)\gamma(Z_1)} \mathbb{E}\left(\tilde{Z}_2 \otimes \tilde{Z}_1\right) - \frac{\gamma(\mathbb{1}_1)}{\gamma(Z_2)\gamma(Z_1)} \mathbb{E}\left(\tilde{Z}_2\right) \otimes \mathbb{1}_1 \\ & - \frac{\gamma(\mathbb{1}_2)}{\gamma(Z_2)\gamma(Z_1)} \mathbb{1}_2 \otimes \mathbb{E}\left(\tilde{Z}_1\right) + \frac{\gamma(\mathbb{1}_2)\gamma(\mathbb{1}_1)}{\gamma(Z_2)\gamma(Z_1)} \mathbb{1}_2 \otimes \mathbb{1}_1 \,, \end{aligned} \tag{1}$$

where the tilde denotes a noisy operator measured on noisy quantum hardware, $\mathbb{E}$ denotes the expectation value of the noisy operator subject to bit flips, which should not be confused with the quantum mechanical expectation value, and we defined

$$\gamma\left(O_q\right) := \left\{ \begin{array}{ll} 1 - p_{q,0} - p_{q,1} & \text{for } O_q = Z_q \\ p_{q,1} - p_{q,0} & \text{for } O_q = \mathbb{1}_q . \end{array} \right\} . \tag{2}$$

Thus, the correct expectation value of a two-qubit operator can be obtained by measuring noise-afflicted expectation values of Z_1, Z_2, and $Z_1 \otimes Z_2$ on the quantum device and by combining them with coefficients that only depend on the known bit-flip probabilities. A key element is that $\mathbb{E}\left(\tilde{Z}_Q \cdots \tilde{Z}_1\right)$ can be factorized into single-qubit expectation values $\mathbb{E}\left(\tilde{Z}_Q \cdots \tilde{Z}_1\right) = \mathbb{E}\tilde{Z}_Q \cdots \mathbb{E}\tilde{Z}_1$. The iterative proof of this equation is given in Ref. [1] and eventually leads to the fact that the readout error mitigation method scales polynomially.

2.2 Taking correlations into account

2.2.1 Theoretical framework

The construction of the error-mitigated $Z_2 \otimes Z_1$ operator for 2 qubits in Eq. (1) is based on a probabilistic description at the operator level. For example, if a single Z_q operator is measured on qubit q, then a bit-flip of $1 \to 0$ and $0 \to 0$ occurs with a probability of $p_{q,1}(1 - p_{q,0})$. In this case, the measurement outcome is always 0, such that measuring Z_q is equivalent to measuring the identity operator $\mathbb{1}$ with probability $p_{q,1}(1 - p_{q,0})$. Considering all four bit-flip cases yields a probability distribution of random operators $\tilde{Z}_q$. The expectation of $\tilde{Z}_q$ with respect to this probability distribution is

$$\mathbb{E}(\tilde{Z}_q) = (1 - p_{q,0} - p_{q,1})Z_q + (p_{q,1} - p_{q,0})\mathbb{1}_q . \tag{3}$$

Rearranging Eq. (3) yields an expression for the operator Z_q that we wish to measure, in terms of constants depending on $p_{q,0/1}$ and of the expectation of $\tilde{Z}_q$ subject to the bit-flip distribution measured on the quantum device. If no correlations between qubits exist, then we can build the two-qubit result of Eq. (1) by tensoring the expressions for Z_q obtained from Eq. (3) on both qubits.

In the presence of inter-qubit correlations, a similar method can be employed. For example, if we wish to measure the operator $Z_2 \otimes Z_1$, then we can calibrate the probabilities $p(b|b')$ of finding the bitstring b given that the underlying bitstring was b', i.e., $p(10|00)$ is the probability of flipping the second qubit from 0 to 1 and keeping the first qubit in 0. We then compute the expected operators $\mathbb{E}(\widetilde{Z_2 \otimes Z_1})$, $\mathbb{E}(\widetilde{Z_2 \otimes \mathbb{1}_1})$, and $\mathbb{E}(\widetilde{\mathbb{1}_2 \otimes Z_1})$ by considering the induced probability distribution, e.g.,

$$\langle b'|\mathbb{E}(\widetilde{Z_2 \otimes Z_1})|b'\rangle = \sum_b \langle b|Z_2 \otimes Z_1|b\rangle p(b|b'). \tag{4}$$

This yields diagonal operators for $\mathbb{E}(\widetilde{Z_2 \otimes Z_1})$, $\mathbb{E}(\widetilde{Z_2 \otimes \mathbb{1}_1})$, and $\mathbb{E}(\widetilde{\mathbb{1}_2 \otimes Z_1})$, which can be expressed in terms of the noise-free operators $Z_2 \otimes Z_1$, $Z_2 \otimes \mathbb{1}_1$, $\mathbb{1}_2 \otimes Z_1$, and $\mathbb{1}$. Using the trivial equation $\mathbb{E}(\widetilde{\mathbb{1}_2 \otimes \mathbb{1}_1}) = \mathbb{1}_2 \otimes \mathbb{1}_1$, we obtain as a final result for the noisy expectations

$$\begin{pmatrix} \mathbb{E}(\widetilde{Z_2 \otimes Z_1}) \\ \mathbb{E}(\widetilde{Z_2 \otimes \mathbb{1}_1}) \\ \mathbb{E}(\widetilde{\mathbb{1}_2 \otimes Z_1}) \\ \mathbb{E}(\widetilde{\mathbb{1}_2 \otimes \mathbb{1}_1}) \end{pmatrix} = \Omega \begin{pmatrix} Z_2 \otimes Z_1 \\ Z_2 \otimes \mathbb{1}_1 \\ \mathbb{1}_2 \otimes Z_1 \\ \mathbb{1}_2 \otimes \mathbb{1}_1 \end{pmatrix} \quad \text{with} \quad \Omega_{j,k} = \sum_{b,b'} \langle b|\, O_j \,|b\rangle \, \langle b'|\, O_k \,|b'\rangle \, p(b|b'), \tag{5}$$

PoS(LATTICE2021)327

where $\sum_{b,b'}$ ranges over $b, b' \in \{00, 01, 10, 11\}$, and the operators O_j and O_k are given by $O_1 = Z_2 \otimes Z_1$, $O_2 = Z_2 \otimes \mathbb{1}_1$, $O_3 = \mathbb{1}_2 \otimes Z_1$, and $O_4 = \mathbb{1}_2 \otimes \mathbb{1}_1$.

If the bit-flip error rate $\varepsilon = 1 - \min_b p(b|b)$ is below 0.05, the matrix Ω is strictly diagonally dominant, so Eq. (5) can be inverted to obtain the equivalent of Eq. (1) for two correlated qubits.

2.2.2 Numerical results

In order to test the mitigation scheme and to assess the effect of correlations, we implement our method for the case of two qubits on different quantum devices. In a first step, we calibrate the bit-flip probabilities $p(b|b')$ by repeatedly preparing each of the four possible computational basis states $|b'\rangle$ and by recording the measurement outcomes b (see Fig. 1 for details). After calibrating the bit-flip probabilities, we run the parametric quantum circuit shown in Fig. 2 to prepare a state $|\psi\rangle$, where our choice of circuit is inspired by the typical layered structure employed in many hybrid quantum-classical algorithms, such as the variational quantum eigensolver [19]. Subsequently, we measure the (noisy) expectation values of the four operators appearing on the left-hand side of Eq. (5). In order to obtain the (noisy) estimate of the expectation values $\mathbb{E}\langle\psi|\widetilde{\mathbb{1}_2 \otimes Z_1}|\psi\rangle$, $\mathbb{E}\langle\psi|\widetilde{Z_2 \otimes \mathbb{1}_1}|\psi\rangle$, and $\mathbb{E}\langle\psi|\widetilde{Z_2 \otimes Z_1}|\psi\rangle$ for a fixed set of parameters $\theta_0, \ldots, \theta_3$, we have to run the circuit multiple times and collect statistics of the measurement outcomes. We refer to these number of repetitions as the number of shots s. Following that, we invert Eq. (5) to mitigate the effects of noise and to retrieve the true expectation values of the observables. In addition, to probe for the effect of correlated bit flips, we also use the mitigation scheme neglecting the correlations, as described in Eq. (1). Comparing these results to the ones obtained with the mitigation scheme taking into account correlated bit flips allows us to assess the influence of such correlations.

To acquire statistics, we repeat the experiment described above for 1000 randomly chosen states $|\psi\rangle$, where we draw the angles $\theta_0, \ldots, \theta_3$ uniformly from $[0, 2\pi)$, and we monitor the mean of the

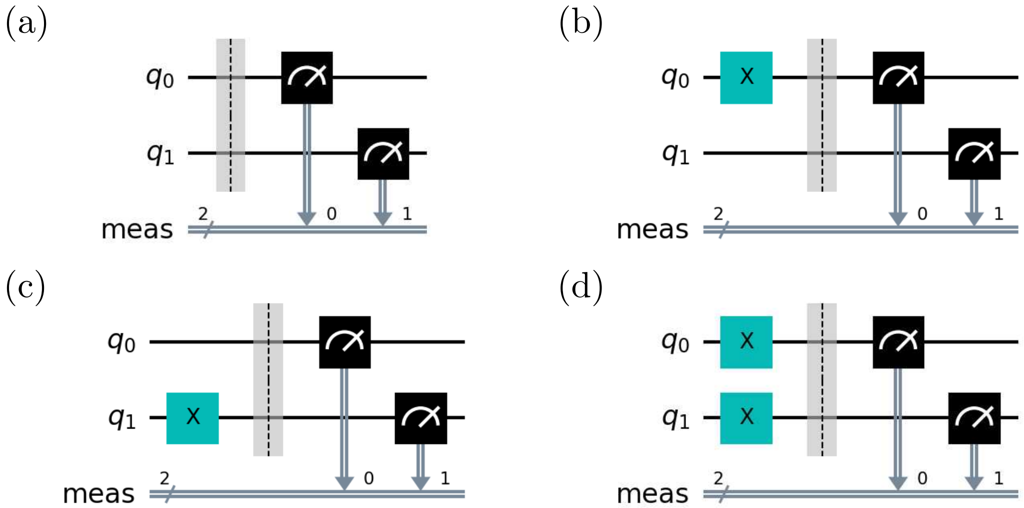

Figure 1: Illustration of the four different quantum circuits required to calibrate the bit-flip probabilities (a) $p(b|00)$, (b) $p(b|01)$, (c) $p(b|10)$, and (d) $p(b|11)$. The green boxes denote X gates, the black boxes are the final measurements, and the vertical dashed lines separate different layers of the quantum circuit.

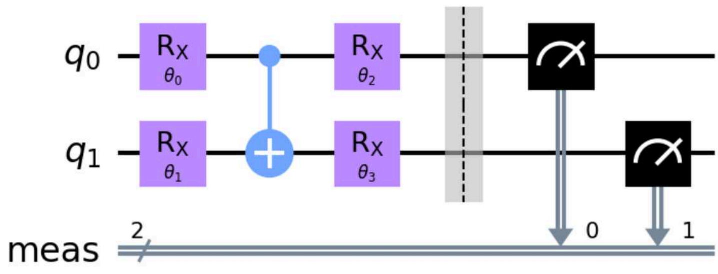

Figure 2: Parametric quantum circuit used in the experiments. The purple boxes denote single-qubit R_X rotation gates with parameters $\theta_0, \ldots, \theta_3$, the blue two-qubit connection is an entangling CNOT gate, the black boxes are the final measurements, and the vertical dashed lines separate different layers of the circuit.

absolute error

$$\left| \langle\psi| \widetilde{Z_2 \otimes Z_1} |\psi\rangle_{\text{measured}} - \langle\psi| Z_2 \otimes Z_1 |\psi\rangle_{\text{exact}} \right| . \tag{6}$$

In this equation, the expression $\langle\psi| \widetilde{Z_2 \otimes Z_1} |\psi\rangle_{\text{measured}}$ refers to either the unmitigated measurement results for the operator $Z_2 \otimes Z_1$ from the quantum device or the results obtained after applying the mitigation scheme, whereas $\langle\psi| Z_2 \otimes Z_1 |\psi\rangle_{\text{exact}}$ refers to the exact solution obtained on a noise-free device with an infinite number of shots.

In Fig. 3, we show our results for the average error as a function of the number of shots obtained on the different quantum devices *ibmq_lima*, *ibmq_quito*, *ibmq_casablanca*, and *ibmq_belem*, which are all IBM Quantum Falcon Processors. In addition to the hardware data, we also provide results from a classical simulation mimicking a quantum device with readout errors only for reference. The bit-flip probabilities for the classical simulation are set to the values obtained from the calibration on quantum hardware. Focusing on the classical simulation first, we observe that the absolute average error for the unmitigated data initially decreases slightly with increasing number of shots s, before quickly reaching a plateau around 7×10^{-2}. In particular, the plateau occurs at a number of shots much smaller than the one that can be executed on real hardware, thus demonstrating that readout errors severely limit the accuracy that can be obtained. Applying the mitigation procedure that neglects the correlations (see Eq. (1)) significantly improves the results, as the orange squares in Fig. 3(a)-(d) reveal. However, around a value of $s = 10^4$ we observe that the absolute value of the average error again tends to stagnate, hence indicating that we have reached the level of accuracy at which the correlated bit flips between qubits become significant and thus the results do not further improve. In contrast, when using the mitigation scheme from Eq. (5) that takes into account the correlations, there is no trend towards a plateau of the average absolute error, and our data show a polynomial decay $\propto 1/\sqrt{s}$ throughout the entire range of shots we study. This scaling of the average error with the number of shots is expected for the ideal noise free case [16], thus showing that the mitigation scheme allows for recovering the noise-free case.

Turning to the data obtained on *ibmq_lima* and *ibmq_quito* without error mitigation (see Figs. 3(a) and 3(b)), we observe good agreement with the simulation taking into account readout errors only, hence indicating that readout errors have a quite significant contribution to the overall

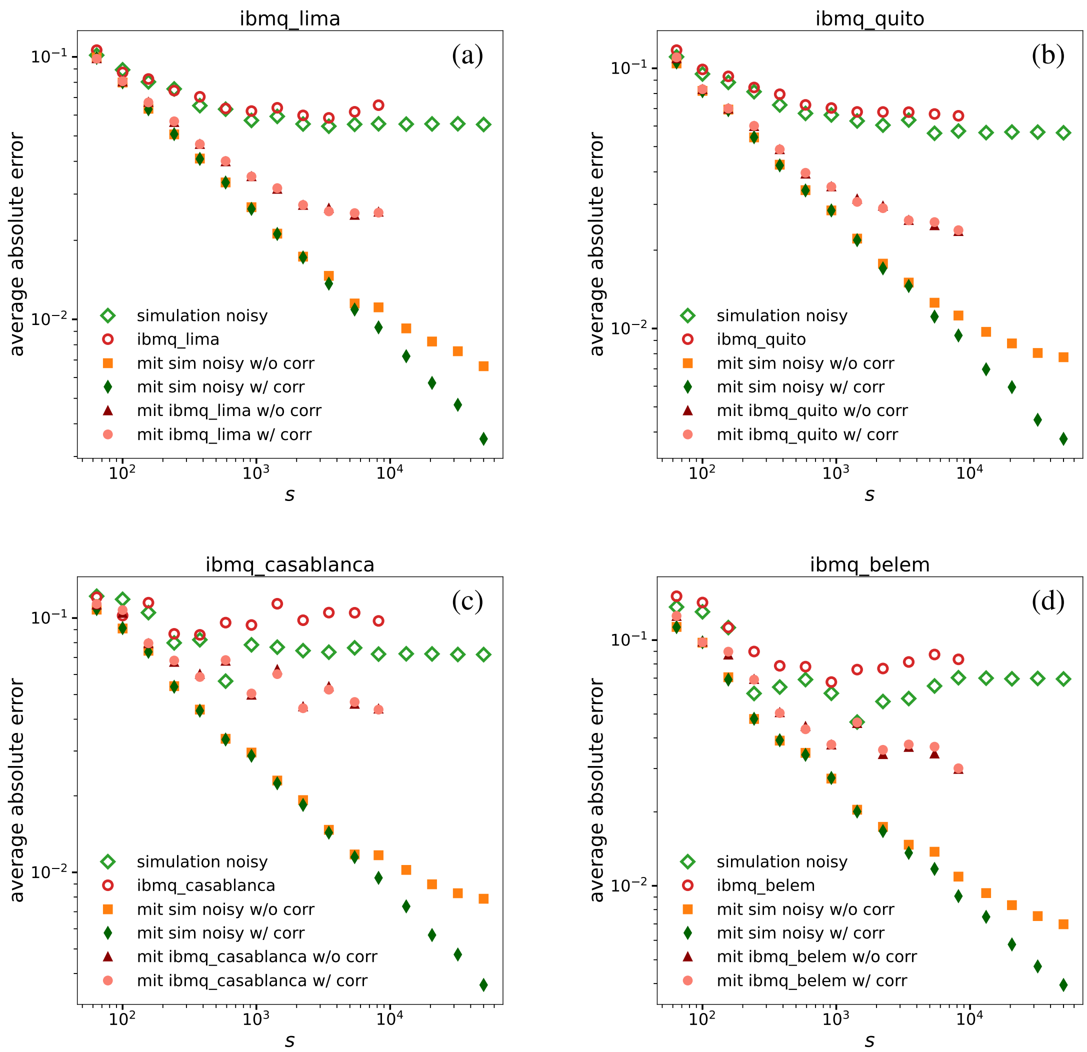

Figure 3: Average absolute errors according to Eq. (6) as a function of the number of shots s for the quantum devices (a) *ibmq_lima*, (b) *ibmq_quito*, (c) *ibmq_casablanca*, and (d) *ibmq_belem*. The open symbols correspond to the unmitigated data, where the diamonds (circles) denote the data from the classical simulation (quantum hardware). The filled symbols correspond to the data obtained after applying the mitigation procedure (abbreviated "mit" in the legend). The orange squares (dark green diamonds) represent the mitigated noisy simulation data without (with) taking two-qubit correlations into account. The dark red triangles (rose circles) correspond to the mitigated hardware data without (with) taking two-qubit correlations into account. The maximum number of shots on hardware devices is limited to 8192; thus, only simulation data is available for values of s exceeding this number.

error on these devices. Applying the error mitigation again yields a substantial improvement for the results. After an initial decrease with the number of shots, the average errors of the mitigated data show a trend towards a plateau around 3×10^{-2}. Since the inherent statistical fluctuations of the projective measurements should decrease with an increasing value of s, and our mitigation scheme allows for dealing with readout errors, this hints towards other noise sources becoming dominant at this stage, eventually limiting the accuracy that can be reached on the quantum device. Interestingly, the results obtained from the readout error mitigation scheme that takes correlated bit flips into account are essentially identical to those from the method that neglects the correlations. Thus, our results suggest that for the level of accuracy that can be reached on real quantum devices, correlations between the readout errors of qubits do not play a significant role.

Looking at the hardware results in Figs. 3(c) and 3(d) from our simulations on *ibmq_casablanca* and *ibmq_belem*, the picture is qualitatively similar. However, for these devices we observe larger deviations between the classical simulation that takes only readout errors into account and the actual hardware data. This is giving an indication that other sources of noise have a more significant contribution on these devices compared to *ibmq_lima* and *ibmq_quito*. Using the readout error mitigation again improves the results, albeit the effect for these devices is less drastic, due to noise other than readout errors. Moreover, also for this case we observe that both mitigation schemes yield essentially identical results, as a comparison between the dark red triangles and the rose dots shows. Consequently, correlated readout errors do not play a significant role and can be neglected.

3. Conclusion

In Refs. [1, 2], we have introduced a readout error mitigation scheme which is efficient, scales only polynomially in the number of qubits, and can be practically implemented. In the cited papers, we have also performed experiments on IBMQ hardware and demonstrated the feasibility of our method. Further experiments, including the variances of the error and a comparison between quantum hardware of Rigetti and IBMQ have been performed in Ref. [18] of this conference.

Although the general case that correlations between qubits in the readout process can occur has been discussed in Ref. [1], no numerical experiments had been performed there. In these proceedings, we have filled this gap and have successfully applied our readout error mitigation scheme taking correlations into account. The numerical experiments have been performed on several IBMQ hardware devices for two qubits. When comparing results with and without adding correlations, we obtained similar outcomes, suggesting that correlations can be neglected. This justifies the assumption in Refs. [1, 2] to consider only cases without correlations.

Acknowledgments

We thank Giovanni Ianelli and Tom Weber for many useful discussions. Research at Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Industry Canada and by the Province of Ontario through the Ministry of Colleges and Universities. L.F. is partially supported by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers, Co-design Center for Quantum Advantage (C^2QA) under contract number DE-SC0012704, by the DOE QuantiSED Consortium under subcontract number 675352, by the National Science Foundation under Cooperative Agreement PHY-2019786 (The NSF AI Institute for Artificial Intelligence and Fundamental Interactions, http://iaifi.org/), and by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under grant contract numbers DE-SC0011090 and DE-SC0021006. S.K. acknowledges financial support from the Cyprus Research and Innovation Foundation under project "Future-proofing Scientific Applications for the Supercomputers of Tomorrow (FAST)", contract no. COMPLEMENTARY/0916/0048. G.P. is supported by project NextQCD, co-funded by the European Regional Development Fund and the Republic of Cyprus through the Research and Innovation Foundation (EXCELLENCE/0918/0129) and POST-DOC/0718/0100. Partial support is provided by the Marie Skłodowska-Curie European Joint Doctorate program STIMULATE of the European Commission under grant agreement No 765048. We acknowledge the use of IBM Quantum services for this work. The views expressed are those of the authors, and do not reflect the official policy or position of IBM or the IBM Quantum team.

References

[1] L. Funcke, T. Hartung, K. Jansen, S. Kühn, P. Stornati and X. Wang, *Measurement error mitigation in quantum computers through classical bit-flip correction*, *arXiv:2007.03663* (2020) [2007.03663v1].

[2] L. Funcke, T. Hartung, K. Jansen, S. Kühn, M. Schneider, P. Stornati et al., *Towards Quantum Simulations in Particle Physics and Beyond on Noisy Intermediate-Scale Quantum Devices*, *arXiv:2110.03809* (2021) [2110.03809].

[3] J. Cramer, N. Kalb, M.A. Rol, B. Hensen, M.S. Blok, M. Markham et al., *Repeated quantum error correction on a continuously encoded qubit by real-time feedback*, *Nat. Commun.* **7** (2016) .

[4] A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J.M. Chow et al., *Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets*, *Nature* **549** (2017) 242.

[5] Y. Li and S.C. Benjamin, *Efficient variational quantum simulator incorporating active error minimization*, *Phys. Rev. X* **7** (2017) 021050.

[6] K. Temme, S. Bravyi and J.M. Gambetta, *Error mitigation for short-depth quantum circuits*, *Phys. Rev. Lett.* **119** (2017) 180509.

[7] S. Endo, S.C. Benjamin and Y. Li, *Practical quantum error mitigation for near-future applications*, *Phys. Rev. X* **8** (2018) 031027.

[8] S. Endo, Q. Zhao, Y. Li, S. Benjamin and X. Yuan, *Mitigating algorithmic errors in a hamiltonian simulation*, *Phys. Rev. A* **99** (2019) 012334.

[9] A. Kandala, K. Temme, A.D. Córcoles, A. Mezzacapo, J.M. Chow and J.M. Gambetta, *Error mitigation extends the computational reach of a noisy quantum processor*, *Nature* **567** (2019) 491.

[10] S.S. Tannu and M.K. Qureshi, *Mitigating measurement errors in quantum computers by exploiting state-dependent bias*, in *Proceedings of the 52nd Annual IEEE/ACM International Symposium on Microarchitecture*, MICRO '52, (New York, NY, USA), p. 279, Association for Computing Machinery, 2019, DOI.

[11] K. Yeter-Aydeniz, E.F. Dumitrescu, A.J. McCaskey, R.S. Bennink, R.C. Pooser and G. Siopsis, *Scalar quantum field theories as a benchmark for near-term quantum computers*, *Phys. Rev. A* **99** (2019) 032306.

[12] K. Yeter-Aydeniz, R.C. Pooser and G. Siopsis, *Practical quantum computation of chemical and nuclear energy levels using quantum imaginary time evolution and lanczos algorithms*, *npj Quantum Inf.* **6** (2020) .

[13] Z. Chen et al., *Exponential suppression of bit or phase flip errors with repetitive error correction*, *arXiv:2102.06132* (2021) [2102.06132].

[14] E. van den Berg, Z.K. Minev and K. Temme, *Model-free readout-error mitigation for quantum expectation values*, 2012.09738.

[15] M.R. Geller and M. Sun, *Toward efficient correction of multiqubit measurement errors: pair correlation method*, *Quantum Science and Technology* **6** (2021) 025009.

[16] L. Funcke, T. Hartung, K. Jansen, S. Kühn and P. Stornati, *Dimensional expressivity analysis of parametric quantum circuits*, *Quantum* **5** (2021) 422.

[17] L. Funcke, T. Hartung, K. Jansen, S. Kühn, M. Schneider and P. Stornati, *Best-approximation error for parametric quantum circuits*, *arXiv:2107.07378* (2021) [2107.07378].

[18] C. Alexandrou, L. Funcke, T. Hartung, K. Jansen, S. Kühn, G. Polykratis et al., *Investigating the variance increase of readout error mitigation through classical bit-flip correction on IBM and Rigetti quantum computers*, in *38th International Symposium on Lattice Field Theory*, 11, 2021 [2111.05026].

[19] A. Peruzzo, J. McClean, P. Shadbolt, M. Yung, X. Zhou, P.J. Love et al., *A variational eigenvalue solver on a photonic quantum processor*, *Nat. Commun.* **5** (2014) 1.