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Operator product expansion for the non-local gluon condensate

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Abstract: We consider the short-distance expansion of the product of two gluon field strength tensors connected by a straight-line-ordered Wilson line. The vacuum expectation value of this nonlocal operator is a common object in studies of the QCD vacuum structure, whereas its nucleon expectation value is known as the gluon quasi-parton distribution and is receiving a lot of attention as a tool to extract gluon distribution functions from lattice calculations. Extending our previous study [1], we calculate the three-loop coefficient functions of the scalar operators in the operator product expansion up to dimension four. As a by-product, the three-loop anomalous dimension of the nonlocal two-gluon operator is obtained as well.

Contents

1 Introduction

In this work, we construct the operator product expansion (OPE) of the non-local two-gluon operator

$$\mathbb{G}_{\mu\nu\alpha\beta}(z) = g^2 F_{\mu\nu}(zv)\,[zv,0]\,F_{\alpha\beta}(0)\,, \tag{1.1}$$

where g is the gauge coupling, $F_{\mu\nu}(x)$ is the gluon field strength tensor, v^μ is an auxiliary four-vector, with $v^2 \neq 0$, and z is a real number. In addition, $[zv,0]$ is a straight-line-ordered Wilson line connecting the two field strength tensors,

$$[zv,0] = \mathcal{P}\exp\left[ig\int_0^z dz'\,v^\mu A_\mu(z'v)\right]\,, \tag{1.2}$$

with $A_\mu(x)$ being the gluon field in the adjoint representation of the color gauge group.

The motivation for this study is twofold. On the one hand, the vacuum expectation value (VEV) of the non-local operator in Eq. (1.1) (non-local gluon condensate) describes the correlation of gluon fields in QCD vacuum as a function of their distance and is the basic quantity, e.g., in the stochastic model of the QCD vacuum [2, 3]. It also governs the effect of gluon condensation on the mass spectra of heavy quarkonia and the short-distance expansion of the heavy-quark potential [4–7]. Lattice QCD studies of the non-local gluon condensate exist aiming at extracting the gluon correlation length [8, 9], and also its behavior at the deconfinement phase transition at high temperatures (see, e.g., Ref. [10]). A similar two-gluon correlator, albeit with a different Wilson line contour, appears in the definition of the rapidity anomalous dimension (AD), alias Collins–Soper kernel [11].

On the other hand, the nucleon matrix elements of the same non-local operator, usually referred to as gluon quasi parton distribution functions (qPDFs), are attracting increasing interest (see Ref. [12] for a review). They can be calculated on the lattice for spacelike separations [13] and matched to the usual collinear gluon parton distribution functions (PDFs) using continuum perturbation theory [14–16]. This technique is attractive, as it allows one to probe the gluon PDF more directly than with other approaches, but it is also challenging. In particular, the renormalization of the non-local gluon operator involves subtleties [17, 18], and also lattice calculations are very challenging due to high statistical noise and the necessity to inject a very large momentum in the nucleon, which requires the use of very fine lattices. Using the ratio of the nucleon to vacuum matrix elements in such calculations can be advantageous [19], as in this way all linear ultraviolet (UV) divergences related to the Wilson line renormalization [20] get canceled.

In this paper, we consider the OPE of the non-local operator in Eq. (1.1) to three-loop accuracy, taking into account all scalar operators up to dimension four. As a by-product of this calculation, the three-loop AD (matrix) of the non-local gluon operator is obtained. From the technical point of view, this calculation is an extension of our work in Ref. [1], where the perturbative contribution to the OPE was calculated to three-loop accuracy and the two-loop AD was derived. We will mostly adopt the conventions and the notation of Ref. [1], a short summary of which is given in Sect. 2. The calculation is described in Sect. 3. The results for the relevant ADs and coefficient functions (CFs) are presented in Sect. 4. The renormalization group (RG) evolution equations for the computed CFs as well as the RG improvement of the purely perturbative contributions are considered in Sects. 5 and 6. Section 7 is reserved for a summary and conclusions.

2 Preliminaries

The vacuum expectation value (VEV) of the non-local gluon operator in Eq. (1.1),

$$\Pi_{\mu\nu\alpha\beta}(z) = \langle 0|g^2 F_{\mu\nu}(zv)\,[zv,0]\,F_{\alpha\beta}(0)|0\rangle\,, \tag{2.1}$$

can be written in terms of two invariant functions, $\Pi_{\perp\perp}(z)$ and $\Pi_{\parallel\perp}(z)$, which correspond to contributions with different Lorentz symmetry and do not mix under renormalization [1], as

$$\begin{aligned}\Pi_{\mu\nu\alpha\beta}(z) &= (g^{\perp}_{\mu\alpha}g^{\perp}_{\nu\beta} - g^{\perp}_{\nu\alpha}g^{\perp}_{\mu\beta})\Pi_{\perp\perp}(z) + (g^{\parallel}_{\mu\alpha}g^{\perp}_{\nu\beta} - g^{\parallel}_{\nu\alpha}g^{\perp}_{\mu\beta} - g^{\parallel}_{\mu\beta}g^{\perp}_{\nu\alpha} + g^{\parallel}_{\nu\beta}g^{\perp}_{\mu\alpha})\Pi_{\parallel\perp}(z) \\ &= (g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta})\,\Pi_{\perp\perp}(z) \\ &\quad + \frac{1}{v^2}(v_\mu v_\alpha g_{\nu\beta} - v_\nu v_\alpha g_{\mu\beta} - v_\mu v_\beta g_{\nu\alpha} + v_\nu v_\beta g_{\mu\alpha})\,[\Pi_{\parallel\perp}(z) - \Pi_{\perp\perp}(z)]\,, \end{aligned} \tag{2.2}$$

where

$$g^{\parallel}_{\mu\nu} = \frac{v_\mu v_\nu}{v^2}\,, \qquad g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2}\,. \tag{2.3}$$

The renormalization of $\Pi_{\perp\perp}(z)$ and $\Pi_{\parallel\perp}(z)$ is determined by their respective ADs, $\gamma_{\parallel\perp}$ and $\gamma_{\perp\perp}$, which are currently known to two-loop accuracy [1, 17, 18]. Notice that the renormalization factors are local, i.e., they do not depend on the distance between the fields. They can be interpreted as the renormalization factors of local "heavy-light" operators in an effective field theory (see Refs. [1, 17, 18] for details).

In this work, we consider the OPE in the limit $z \to 0$ of the invariant functions $\Pi_{\perp\perp}(z)$ and $\Pi_{\parallel\perp}(z)$ taking into account contributions of the scalar CP-even operators,

$$O_2 = \sum_i m_i \bar{\psi}_i(0)\psi_i(0)\,, \qquad O_1 = F_{\alpha\beta}(0)F^{\alpha\beta}(0)\,, \tag{2.4}$$

where ψ_i stands for the i-th quark field with mass m_i. We do not consider operators of mass dimension higher than four.* To this accuracy, we have

$$\begin{aligned}\Pi_{\perp\perp}(z) &\underset{z\to 0}{=} \Pi^{m^4}_{\perp\perp}(z)\,\langle\mathbb{1}\rangle + C_2^{\perp\perp}(z)\langle 0|O_2|0\rangle + \frac{g^2}{12}\,C_1^{\perp\perp}(z)\,\langle 0|O_1|0\rangle\,, \\ \Pi_{\parallel\perp}(z) &\underset{z\to 0}{=} \Pi^{m^4}_{\parallel\perp}(z)\,\langle\mathbb{1}\rangle + C_2^{\parallel\perp}(z)\langle 0|O_2|0\rangle + \frac{g^2}{12}\,C_1^{\parallel\perp}(z)\,\langle 0|O_1|0\rangle\,, \end{aligned} \tag{2.5}$$

where $\Pi^{m^4}_{\perp\perp}(z)$ and $\Pi^{m^4}_{\parallel\perp}(z)$ stand for the purely perturbative contributions expanded in the quark masses through order m_q^4. They can be naturally represented as

$$\Pi^{m^4}_{\perp\perp}(z) = \frac{C_0^{\perp\perp}(z)}{z^4} + \frac{C_{m^2}^{\perp\perp}(z)}{z^2}\sum_i m_i^2 + C^{\perp\perp}_{m^4,\mathrm{di}}(z)\sum_i m_i^4 + C^{\perp\perp}_{m^4,\mathrm{nd}}(z)\sum_{i\neq j} m_i^2\, m_j^2\,,$$

*We also do not consider contributions of tensor operators, e.g., $G_{\alpha\xi}(0)\,G_\beta^{\ \xi}(0)$ (symmetrized over the open indices and with the traces subtracted), which do not contribute to the VEV of the non-local operator, but are relevant for hadron matrix elements. Note that the corresponding OPE in the tree approximation is known up to operators of dimension 8 [21].

$$\Pi^{m^4}_{\parallel\perp}(z) = \frac{C_0^{\parallel\perp}(z)}{z^4} + \frac{C_{m^2}^{\parallel\perp}(z)}{z^2}\sum_i m_i^2 + C^{\parallel\perp}_{m^4,\mathrm{di}}(z)\sum_i m_i^4 + C^{\parallel\perp}_{m^4,\mathrm{di}}(z)\sum_{i\neq j} m_i^2\, m_j^2\,, \tag{2.6}$$

where $\Pi^0_{\perp\perp}(z)$ and $\Pi^0_{\parallel\perp}(z)$ correspond to massless, purely perturbative contributions, which are known to two- and three-loop accuracy from Refs. [22] and [1], respectively. The new contribution of this work is the calculation of the CFs $C^{\perp\perp}_{m^2}$, $C^{\perp\perp}_{m^4,\mathrm{di}}$, $C^{\perp\perp}_{m^4,\mathrm{nd}}$, $C^{\perp\perp}_2$, $C^{\perp\perp}_1$, $C^{\parallel\perp}_{m^2}$, $C^{\parallel\perp}_{m^4,\mathrm{di}}$, $C^{\parallel\perp}_{m^4,\mathrm{nd}}$, $C^{\parallel\perp}_2$, $C^{\parallel\perp}_1$, and the ADs $\gamma_{\perp\perp}$, $\gamma_{\parallel\perp}$ to three-loop accuracy.

3 Calculation

We compute the bare CFs of the operators in Eq. (2.4) at the three-loop level using essentially the same techniques as in Ref. [1] and the well-known method of projectors [23, 24]. The color factors are evaluated with the help of the FORM [25] package `COLOR` [26].

Let us briefly discuss the renormalization procedure. The renormalization matrix of the operators in Eq. (2.4) has been known for a long time [27–30]. Since the CFs $C_1^{\perp\perp}$ and $C_1^{\parallel\perp}$ are non-zero already in the tree approximation, their proper renormalization requires the knowledge of the Z factors, $Z_{\perp\perp}$ and $Z_{\parallel\perp}$, at three loops. Thus, the requirement of finiteness of the CFs of the operator O_1 provides an alternative way for computing $Z_{\perp\perp}$ and $Z_{\parallel\perp}$. From our results for the bare CFs, $(C_1^{\perp\perp})_B$ and $(C_1^{\parallel\perp})_B$, we successfully construct the three-loop Z factors $Z_{\perp\perp}$ and $Z_{\parallel\perp}$ as well as the corresponding ADs. We also find full agreement with the corresponding two-loop results, first computed in Ref. [1].

4 Results

In this and next two sections, we present our results for the case of standard QCD with the SU(3) gauge group and n_f active quarks triplets. The results for the CFs are presented for the case of a spacelike unit vector v, with $v^2 = -1$, and the variable $L_z = \ln(\mu e^{\gamma_E} z/2)$ set to zero. The missing terms proportional to powers $(L_z)^i$ with $i = 1, 2, 3$ can be easily restored with the help of the corresponding evolution equations (see Sect. 6). Full results for a generic gauge group including the momentum/position dependence as well the case of $v^2 = 1$ are appended in the arxiv submission of this paper as ancillary files in a computer readable format.

Expanding a generic AD $\gamma(a)$ in $a = g^2/(16\pi^2)$ as

$$\gamma = \sum_{n\geq 1} (\gamma)_n\, a^n\,, \tag{4.1}$$

our results for the ADs $\gamma_{\perp\perp}$ and $\gamma_{\parallel\perp}$ read:

$$
\begin{aligned}
(\gamma_{\perp\perp})_1 &= -3\,, \\
(\gamma_{\perp\perp})_2 &= -34 + 6\,\pi^2 + \frac{13}{3}\,n_f\,, \\
(\gamma_{\perp\perp})_3 &= -\frac{899}{2} - 10\pi^2 + 18\pi^4 - 108\zeta_3 + n_f\,\left(76 + \frac{16}{3}\pi^2 - 2\pi^4 + 40\zeta_3\right) + \frac{2}{3}\,n_f^2\,, \\
(\gamma_{\parallel\perp})_1 &= 0\,, \\
(\gamma_{\parallel\perp})_2 &= 6\,\pi^2\,, \\
(\gamma_{\parallel\perp})_3 &= 8\pi^2 + 18\pi^4 + n_f\,\left(\frac{22}{3}\pi^2 - 2\pi^4\right)\,.
\end{aligned}
\tag{4.2}
$$

Expanding the CFs C_{m^2}, $C_{m^4,\mathrm{di}}$, $C_{m^4,\mathrm{nd}}$, C_2 and C_1 as

$$
C_{m^2} = \sum_{n\geq 1} (C_{m^2})_n\, a^n\,, \qquad C_{m^4,\mathrm{di}} = \sum_{n\geq 2} (C_{m^4,\mathrm{di}})_n\, a^n\,, \qquad C_{m^4,\mathrm{nd}} = \sum_{n\geq 3} (C_{m^4,\mathrm{nd}})_n\, a^n\,,
$$
$$
C_2 = \sum_{n\geq 1} g^2\,(C_2)_n\, a^n\,, \qquad C_1 = 1 + \sum_{n\geq 1} (C_1)_n\, a^n\,, \tag{4.3}
$$

we find the coefficients appearing in Eq. (4.3) to be

$$
\begin{aligned}
\left(C_{m^2}^{\perp\perp}\right)_2 &= 128\,, \\
\left(C_{m^2}^{\perp\perp}\right)_3 &= \frac{15872}{3} + \frac{256}{3}\pi^2 + \frac{128}{45}\pi^4 + 768\zeta_3 - \frac{1024}{9}\,n_f\,, \\
\left(C_{m^2}^{\parallel\perp}\right)_2 &= 0\,, \\
\left(C_{m^2}^{\parallel\perp}\right)_3 &= 3840 + 768\pi^2 - \frac{128}{3}\pi^4 - \frac{512}{3}\,n_f\,, \\
\left(C_{m^4,\mathrm{di}}^{\perp\perp}\right)_2 &= -\frac{28}{3}\,, \\
\left(C_{m^4,\mathrm{di}}^{\perp\perp}\right)_3 &= \frac{3650}{27} - \frac{176}{9}\pi^2 - \frac{16}{135}\pi^4 - \frac{4384}{9}\zeta_3 + n_f\,\left(-\frac{412}{9} + \frac{128}{3}\zeta_3\right)\,, \\
\left(C_{m^4,\mathrm{di}}^{\parallel\perp}\right)_2 &= \frac{20}{3}\,, \\
\left(C_{m^4,\mathrm{di}}^{\parallel\perp}\right)_3 &= -\frac{11566}{27} + \frac{800}{9}\pi^2 - \frac{16}{135}\pi^4 - \frac{6880}{9}\zeta_3 + n_f\,\left(-\frac{68}{3} + \frac{128}{3}\zeta_3\right)\,, \\
\left(C_{m^4,\mathrm{nd}}^{\perp\perp}\right)_2 &= 0\,, \\
\left(C_{m^4,\mathrm{nd}}^{\perp\perp}\right)_3 &= \frac{32}{9}\,, \\
\left(C_{m^4,\mathrm{nd}}^{\parallel\perp}\right)_2 &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
\left(C^{\parallel\perp}_{m^4,\mathrm{nd}}\right)_3 &= -\frac{544}{9}\,,\\
\left(C^{\perp\perp}_{2}\right)_1 &= -\frac{8}{9}\,,\\
\left(C^{\perp\perp}_{2}\right)_2 &= -\frac{175}{27} - \frac{52}{9}\pi^2 - 2\,n_f\,,\\
\left(C^{\perp\perp}_{2}\right)_3 &= -\frac{36190}{81} + \frac{56222}{243}\pi^2 - \frac{346}{15}\pi^4 + \frac{8996}{27}\zeta_3 - \frac{848}{3}\pi^2\zeta_3 + 1400\zeta_5\\
&\quad + n_f\left(-\frac{7654}{243} + \frac{8}{5}\pi^4 - \frac{928}{9}\zeta_3\right) + n_f^2\left(-\frac{1198}{729} + \frac{32}{9}\zeta_3\right)\,,\\
\left(C^{\parallel\perp}_{2}\right)_1 &= \frac{4}{9}\,,\\
\left(C^{\parallel\perp}_{2}\right)_2 &= \frac{725}{27} - \frac{28}{3}\pi^2 - \frac{62}{27}\,n_f\,,\\
\left(C^{\parallel\perp}_{2}\right)_3 &= \frac{176317}{81} - \frac{235214}{243}\pi^2 - \frac{3254}{81}\pi^4 + 4620\zeta_3 + \frac{1904}{3}\pi^2\zeta_3 - 3320\zeta_5\\
&\quad + n_f\left(-\frac{54658}{243} + 24\pi^2 + \frac{8}{5}\pi^4 - \frac{2176}{9}\zeta_3\right) + n_f^2\left(\frac{458}{729} + \frac{32}{9}\zeta_3\right)\,,\\
\left(C^{\perp\perp}_{1}\right)_1 &= \frac{27}{2}\,,\\
\left(C^{\perp\perp}_{1}\right)_2 &= \frac{5737}{12} + 54\pi^2 - 144\zeta_3 - \frac{160}{9}\,n_f\,,\\
\left(C^{\perp\perp}_{1}\right)_3 &= \frac{755000}{27} + \frac{4295}{2}\pi^2 + \frac{3436}{15}\pi^4 - 16176\zeta_3 - 1176\pi^2\zeta_3 + 1800\zeta_5\\
&\quad + n_f\left(-\frac{321653}{108} - \frac{866}{9}\pi^2 - \frac{848}{45}\pi^4 + \frac{20446}{27}\zeta_3 - 16\pi^2\zeta_3 + 600\zeta_5\right)\\
&\quad + n_f^2\left(\frac{5542}{81} - \frac{16}{3}\zeta_3\right)\,,\\
\left(C^{\parallel\perp}_{1}\right)_1 &= \frac{21}{2}\,,\\
\left(C^{\parallel\perp}_{1}\right)_2 &= \frac{1321}{4} + 61\pi^2 - 144\zeta_3 - \frac{41}{3}\,n_f\,,\\
\left(C^{\parallel\perp}_{1}\right)_3 &= \frac{69761}{3} + \frac{9125}{3}\pi^2 + \frac{1242}{5}\pi^4 - 18168\zeta_3 - 1968\pi^2\zeta_3 + 6120\zeta_5\\
&\quad + n_f\left(-\frac{820793}{324} - \frac{1240}{9}\pi^2 - \frac{928}{45}\pi^4 + \frac{21082}{27}\zeta_3 - 16\pi^2\zeta_3 + 600\zeta_5\right)\\
&\quad + \frac{15025}{243}\,n_f^2\,.
\end{aligned}
\tag{4.4}
$$

Numerically, for $n_f = 3$, we obtain

$$
\begin{aligned}
\left(C^{\perp\perp}_{m^2}\right)_{n_f=3} &= 8\,a^2\left[1 + a\ (13.656 + 10.000 L_z)\right] ,\\
\left(C^{\|\perp}_{m^2}\right)_{n_f=3} &= 105.496\ a^3 ,\\
\left(C^{\perp\perp}_{m^4,\mathrm{di}}\right)_{n_f=3} &= -\frac{7}{12}\,a^2\left[1 - 3.429 L_z + 3.429\,L_z^2\right.\\
&\quad \left. +\ a\ \left(17.099 - 41.523 L_z + 13.000\,L_z^2 + 19.429\,L_z^3\right)\right] ,\\
\left(C^{\|\perp}_{m^4,\mathrm{di}}\right)_{n_f=3} &= \frac{5}{12}\,a^2\left[1 - 4.800\,L_z^2 + a\ \left(-14.837 + 89.515 L_z - 62.600\,L_z^2 - 41.600\,L_z^3\right)\right] ,\\
\left(C^{\perp\perp}_{m^4,\mathrm{nd}}\right)_{n_f=3} &= a^3\ (0.056 - 2.000 L_z) ,\\
\left(C^{\|\perp}_{m^4,\mathrm{nd}}\right)_{n_f=3} &= a^3\ (-0.944 - 2.000 L_z) ,\\
\left(C^{\perp\perp}_{2}\right)_{n_f=3} &= -\frac{2}{9}\,a\left[1 - 3.000 L_z + a\ \left(19.549 - 26.875 L_z - 4.500\,L_z^2\right)\right.\\
&\quad \left. +\ a^2\ \left(132.651 - 104.864 L_z - 155.226\,L_z^2 - 13.500\,L_z^3\right)\right] ,\\
\left(C^{\|\perp}_{2}\right)_{n_f=3} &= \frac{1}{9}\,a\left[1 + 6.000 L_z + a\ \left(-40.586 + 70.250 L_z + 27.000\,L_z^2\right)\right.\\
&\quad \left. +\ a^2\ \left(-277.555 + 95.216 L_z + 622.451\,L_z^2 + 121.500\,L_z^3\right)\right] ,\\
\left(C^{\perp\perp}_{1}\right)_{n_f=3} &= 1 + a\ (3.375 - 3.000 L_z) + a^2\ \left(49.038 + 6.617 L_z - 2.250\,L_z^2\right)\\
&\quad +\ a^3\ \left(425.570 + 350.427 L_z + 5.519\,L_z^2 - 4.500\,L_z^3\right) ,\\
\left(C^{\|\perp}_{1}\right)_{n_f=3} &= 1 + 2.625\ a + a^2\ (44.887 + 18.617 L_z)\\
&\quad +\ a^3\ \left(399.885 + 494.172 L_z + 83.776\,L_z^2\right) .
\end{aligned}
\tag{4.5}
$$

5 RG improvements of $\mathbf{\Pi^0_{\perp\perp}}$ and $\mathbf{\Pi^0_{\|\perp}}$

In general, a multiplicatively renormalizable structure function Π depends on both a renormalization prescription, or scheme, and a normalization scale μ. It is convenient to deal with the scheme and scale invariant version of Π, which we denote as $\hat{\Pi}$. Given the RG equation for Π,

$$
\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\Pi(a,\mu) \equiv \left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(a)a\frac{\partial}{\partial a}\right)\Pi(a,\mu) = \gamma(a)\,\Pi(a,\mu) , \tag{5.1}
$$

a formal solution for $\hat{\Pi}$ reads

$$
\hat{\Pi}(a) = \frac{\Pi(a,\mu)}{f(a)} , \qquad f(a) = \exp\int^a \frac{\mathrm{d}x}{x}\,\frac{\gamma(x)}{\beta(x)} . \tag{5.2}
$$

Through the order of interest here, we have

$$f(a) = (a)^{\bar{\gamma}_1} \Big\{ 1 + (\bar{\gamma}_2 - \bar{\beta}_2\bar{\gamma}_1) a \\ + \frac{1}{2}\left[(\bar{\gamma}_2 - \bar{\beta}_2\bar{\gamma}_1)^2 + \bar{\gamma}_3 + \bar{\beta}_2^2\bar{\gamma}_1 - \bar{\beta}_2\bar{\gamma}_2 - \bar{\beta}_3\bar{\gamma}_1 \right] a^2 + \mathcal{O}(a^3) \Big\} \,, \tag{5.3}$$

where $\bar{\gamma}_i = \gamma_i/\beta_1$, $\bar{\beta}_i = \beta_i/\beta_1$ $(i = 1, 2, 3)$, and the coefficients β_i of the beta function are defined as

$$\beta(a) = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \ln a = \sum_{n=1}^{\infty} \beta_n a^n \,, \tag{5.4}$$

with $\beta_1 = -11 + \frac{2}{3} n_f$, etc.

Our results for the scheme invariant functions $\hat{\Pi}^0_{\perp\perp}$ and $\hat{\Pi}^0_{\parallel\perp}$ in position space are given by

$$\hat{\Pi}^0_{\perp\perp}(z > 0, v^2 = -1) = \frac{128}{z^4} a^{1+6/\beta_1} \left[1 + \sum_{n=1}^{2} \left(\hat{F}_{\perp\perp} \right)_n a^n \right] \,, \\ \hat{\Pi}^0_{\parallel\perp}(z > 0, v^2 = -1) = -\frac{128\, a}{z^4} \left[1 + \sum_{n=1}^{2} \left(\hat{F}_{\parallel\perp} \right)_n a^n \right] \,. \tag{5.5}$$

In phenomenological applications, one usually has $n_f = 0, 2, 3$. The CFs in Eq. (5.5) then take the values

$$\begin{aligned}
\left(\hat{F}_{\perp\perp}\right)_1^{n_f=0} &= \frac{9151}{363} + \frac{56}{11}\pi^2 + 10\, L_z \,, \\
\left(\hat{F}_{\perp\perp}\right)_2^{n_f=0} &= \frac{265666457}{263538} + \frac{808070}{3993}\pi^2 + \frac{798}{121}\pi^4 - \frac{10206}{11}\zeta_3 \\
&\quad + L_z \left(\frac{326492}{363} + \frac{1792}{11}\pi^2 \right) + 160\, L_z^2 \,, \\
\left(\hat{F}_{\perp\perp}\right)_1^{n_f=2} &= \frac{190177}{7569} + \frac{152}{29}\pi^2 + \frac{22}{3}\, L_z \,, \\
\left(\hat{F}_{\perp\perp}\right)_2^{n_f=2} &= \frac{102303720467}{114579522} + \frac{45401582}{219501}\pi^2 + \frac{6042}{841}\pi^4 - \frac{71998}{87}\zeta_3 \\
&\quad + L_z \left(\frac{16534820}{22707} + \frac{12160}{87}\pi^2 \right) + \frac{880}{9}\, L_z^2 \,, \\
\left(\hat{F}_{\perp\perp}\right)_1^{n_f=3} &= \frac{677}{27} + \frac{16}{3}\pi^2 + 6\, L_z \,, \\
\left(\hat{F}_{\perp\perp}\right)_2^{n_f=3} &= \frac{1216447}{1458} + \frac{16982}{81}\pi^2 + \frac{68}{9}\pi^4 - \frac{2330}{3}\zeta_3 \\
&\quad + L_z \left(\frac{5800}{9} + 128\pi^2 \right) + 72\, L_z^2 \,,
\end{aligned}$$

$$\left(\hat{F}_{\|\perp}\right)_1^{n_f=0} = \frac{28}{3} + \frac{56}{11}\pi^2 + 22\,L_z\,,$$
$$\left(\hat{F}_{\|\perp}\right)_2^{n_f=0} = \frac{9011}{18} + \frac{23299}{363}\pi^2 + \frac{798}{121}\pi^4 - 1026\zeta_3 + L_z\left(\frac{1844}{3} + 224\pi^2\right) + 484\,L_z^2\,,$$
$$\left(\hat{F}_{\|\perp}\right)_1^{n_f=2} = \frac{88}{9} + \frac{152}{29}\pi^2 + \frac{58}{3}\,L_z\,,$$
$$\left(\hat{F}_{\|\perp}\right)_2^{n_f=2} = \frac{74591}{162} + \frac{567709}{7569}\pi^2 + \frac{6042}{841}\pi^4 - \frac{2798}{3}\zeta_3 + L_z\left(\frac{14348}{27} + \frac{608}{3}\pi^2\right) + \frac{3364}{9}\,L_z^2\,,$$
$$\left(\hat{F}_{\|\perp}\right)_1^{n_f=3} = 10 + \frac{16}{3}\pi^2 + 18\,L_z\,,$$
$$\left(\hat{F}_{\|\perp}\right)_2^{n_f=3} = \frac{2627}{6} + \frac{2185}{27}\pi^2 + \frac{68}{9}\pi^4 - 886\zeta_3 + L_z\left(488 + 192\pi^2\right) + 324\,L_z^2\,. \tag{5.6}$$

Numerically, the same results read

$$\left(\hat{F}_{\perp\perp}\right)_{n_f=0} = 1 + a\,(18.864 + 2.500L_z) + a^2\left(158.283 + 156.705L_z + 10.000\,L_z^2\right),$$
$$\left(\hat{F}_{\perp\perp}\right)_{n_f=2} = 1 + a\,(19.214 + 1.833L_z) + a^2\left(164.958 + 131.729L_z + 6.111\,L_z^2\right),$$
$$\left(\hat{F}_{\perp\perp}\right)_{n_f=3} = 1 + a\,(19.428 + 1.500L_z) + a^2\left(169.120 + 119.235L_z + 4.500\,L_z^2\right),$$
$$\left(\hat{F}_{\|\perp}\right)_{n_f=0} = 1 + a\,(14.895 + 5.500L_z) + a^2\left(33.950 + 176.591L_z + 30.250\,L_z^2\right),$$
$$\left(\hat{F}_{\|\perp}\right)_{n_f=2} = 1 + a\,(15.377 + 4.833L_z) + a^2\left(48.713 + 158.228L_z + 23.361\,L_z^2\right),$$
$$\left(\hat{F}_{\|\perp}\right)_{n_f=3} = 1 + a\,(15.659 + 4.500L_z) + a^2\left(56.719 + 148.935L_z + 20.250\,L_z^2\right). \tag{5.7}$$

6 RG improvements of the CFs

The $\mathcal{O}(1/z^4)$ and $\mathcal{O}(1/z^2)$ terms in Eq. (2.5) satisfy a simple evolution equation of the form[†]

$$\mu^2\frac{d}{d\mu^2}\,\Pi_{\perp\perp}^{m^2} = \gamma_{\perp\perp}\,\Pi_{\perp\perp}^{m^2}, \tag{6.1}$$

[†]In this section, we consider only CFs corresponding to the invariant function $\Pi_{\perp\perp}$. The corresponding equations for $\Pi_{\|\perp}$ emerge by replacing $\perp\perp$ with $\|\perp$.

where

$$\mu^2 \frac{d}{d\mu^2} \equiv \mu^2 \frac{\partial}{\partial \mu^2} + \beta(a)\, a \, \frac{\partial}{\partial a} + \gamma_m \, m_i \, \frac{\partial}{\partial m_i} \,,$$
$$\Pi^{m^2}_{\perp\perp} = \frac{C_0^{\perp\perp}}{z^4} + \frac{C_{m^2}^{\perp\perp}}{z} \sum_i m_i^2 \,, \tag{6.2}$$

with $\gamma_m = -4\, a + (-\frac{202}{3} + \frac{20}{9}\, n_f)\, a^2 + \cdots$ being the quark mass anomalous dimension.

The evolution equations for the remaining CFs are more complicated due to mixing between the operators O_1, O_2 and combinations quartic in quark masses [27, 30, 31]. The mixing is described by

$$\mu^2 \frac{d}{d\mu^2} O_1 = \gamma_{11}\, O_1 + \gamma_{12}\, O_2 + 4a \frac{\partial}{\partial a} \gamma_0 \,,$$
$$\mu^2 \frac{d}{d\mu^2} O_2 = -4\, \gamma_0 \,, \tag{6.3}$$

where

$$\gamma_{11} = -a \frac{\partial}{\partial a} \beta \,, \qquad \gamma_{12} = 4a \, \frac{\partial}{\partial a} \gamma_m \,, \tag{6.4}$$

and γ_0 is the anomalous dimension of the vacuum energy [31–33],

$$\gamma_0(a, m) = \gamma_0^{\rm di}(a) \sum_i m_i^4 + \gamma_0^{\rm nd}(a) \sum_{i \neq j} m_i^2 m_j^2 \,, \tag{6.5}$$

with

$$\gamma_0^{\rm di}(a) = -\frac{3}{16\pi^2} \left[1 + \frac{16}{3} a + \left(\frac{626}{9} - \frac{32}{3} \zeta(3) - \frac{20}{3} n_f \right) a^2 \right] \,,$$
$$\gamma_0^{\rm nd}(a) = \frac{6}{\pi^2} a^2 \,. \tag{6.6}$$

The resulting evolution equations for the CFs $\mathbb{C}_1 \equiv \frac{g^2}{12} C_1$, C_2, $C_{m^4,{\rm di}}$, and $C_{m^4,{\rm nd}}$ read:

$$\mu^2 \frac{d}{d\mu^2} \, \mathbb{C}_1^{\perp\perp} = (\gamma_{\perp\perp} - \gamma_{11})\, \mathbb{C}_1^{\perp\perp} \,,$$
$$\mu^2 \frac{d}{d\mu^2} \, C_2^{\perp\perp} = \gamma_{\perp\perp} \, C_2^{\perp\perp} - \gamma_{12} \, \mathbb{C}_1^{\perp\perp} \,,$$
$$\mu^2 \frac{d}{d\mu^2} \, C_{m^4,{\rm di}}^{\perp\perp} = (\gamma_{\perp\perp} - 4\, \gamma_m)\, C_{m^4,{\rm di}}^{\perp\perp} - \left(4a \, \frac{\partial}{\partial a} \, \gamma_0^{\rm di} \right) \, \mathbb{C}_1^{\perp\perp} + 4\, C_2^{\perp\perp} \, \gamma_0^{\rm di} \,,$$
$$\mu^2 \frac{d}{d\mu^2} \, C_{m^4,{\rm nd}}^{\perp\perp} = (\gamma_{\perp\perp} - 4\, \gamma_m)\, C_{m^4,{\rm nd}}^{\perp\perp} - \left(4a \, \frac{\partial}{\partial a} \, \gamma_0^{\rm nd} \right) \, \mathbb{C}_1^{\perp\perp} + 4\, C_2^{\perp\perp} \, \gamma_0^{\rm nd} \,. \tag{6.7}$$

We checked[‡] that our results do satisfy Eqs. (6.1) and (6.7).

7 Conclusions

We considered the non-local operator consisting of two gluon field strength tensors connected by a straight Wilson line and studied its VEV, which is determined by two nonperturbative Lorentz scalar functions. For each of the latter, we performed an OPE through mass dimension four, which, besides a purely perturbative structure function, involves the VEVs of three local operators, and calculated the AD of the structure function and the CFs of the three local operators through three loops, order a^3, in the modified minimal-subtraction ($\overline{\rm MS}$) scheme. In our recent work [1], the structure functions were derived through three loops and the ADs through two loops. Using the ADs, we also performed a RG improvement of the structure functions and presented their renormalization scheme invariant counterparts thus resulting, again through three loops. These results should be of interest to the lattice QCD community in studies of the QCD vacuum structure and serve as normalization factors in calculations of gluon PDFs within the qPDF approach, eliminating the need for nonperturbative subtractions of linear divergences.

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