DESY 21-030
TTK-21-07

Expectation management

Johannes Braathen[a *], Mark D. Goodsell[b †], Sebastian Paßehr[c ‡] and Emanuelle Pinsard[d §]

[a] *Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, D-22607 Hamburg, Germany.*

[b] *Laboratoire de Physique Théorique et Hautes Energies (LPTHE), UMR 7589, Sorbonne Université et CNRS, 4 place Jussieu, 75252 Paris Cedex 05, France.*

[c] *Institute for Theoretical Particle Physics and Cosmology, RWTH Aachen University, Sommerfeldstraße 16, 52074 Aachen, Germany.*

[d] *Laboratoire de Physique de Clermont (UMR 6533), CNRS/IN2P3, Univ. Clermont Auvergne, 4 Av. Blaise Pascal, F-63178 Aubière Cedex, France*

We consider the application of a Fleischer–Jegerlehner-like treatment of tadpoles to the calculation of neutral scalar masses (including the Higgs) in general theories beyond the Standard Model. This is especially useful when the theory contains new scalars associated with a small expectation value, but comes with its own disadvantages. We show that these can be overcome by combining with effective field theory matching. We provide the formalism in this modified approach for matching the quartic coupling of the Higgs via pole masses at one loop, and apply it to both a toy model and to the μNMSSM as prototypes where the standard treatment can break down.

* e-mail: johannes.braathen@desy.de
† e-mail: goodsell@lpthe.jussieu.fr
‡ e-mail: passehr@physik.rwth-aachen.de
§ e-mail: emanuelle.pinsard@clermont.in2p3.fr

1 Introduction

The mass of the SM-like Higgs boson, discovered by ATLAS and CMS [1–3], is now an electroweak precision observable, thanks to its outstandingly accurate determination at the LHC [4–6], and it plays an important role in constraining the allowed parameter space of Beyond-the-Standard-Model (BSM) theories. On the one hand, the Higgs mass is a prediction in supersymmetric theories (see Ref. [7] and references therein for a recent review) and interestingly it depends most heavily on the electroweak couplings and scale – quantities that are already known from other observations – while it is only at loop level that a dependence on the scale of supersymmetric particles appears. This property has spurred significant developments in precision scalar-mass calculations, advanced in recent years by the KUTS initiative [8–54] as described in the report [7]. On the other hand, in non-supersymmetric theories, the Higgs mass is not a prediction by itself, but it can be used to extract the Higgs quartic coupling and, in turn, investigate the stability of the electroweak vacuum. In this context, a precise calculation is essential to produce reliable results on vacuum stability (see Refs. [55–59] for works in the SM) and to correctly appreciate the potential impact of new particles [60–65].

We refer the interested reader to Ref. [7] and references therein for an in-depth review of Higgs-mass computations, and we only recall here the main steps involved (applicable for any BSM theory). The standard calculational technique begins with the extraction of SM-like parameters – namely the electroweak and strong gauge couplings, the quark and lepton Yukawa couplings, and the Higgs vacuum expectation value (vev) – from observables. Adding then the BSM parameters to these, the Higgs (and other particle) masses can be calculated, along with any other desired predictions. The relevant observables for the electroweak sector are typically, as in calculations in the SM, either $M_Z, M_W, \alpha(0)$ or $M_Z, G_F, \alpha(0)$ where $M_{Z,W}$ are the Z and W boson masses, $\alpha(0)$ is the fine-structure constant extracted in the Thompson limit, and G_F is the Fermi constant. This latter quantity is extracted from muon three-body decays, whereas the others are related essentially to self-energies. In general, this extraction of the SM-like couplings and the Higgs vev can be performed at one-loop for any theory, but the two-loop relationships are only known for the SM and a small subset of other models in certain limits.

At the tree level, the expectation value v of the Higgs boson is related to the other parameters in the theory by the requirement that the theory be at the minimum of the potential. To be concrete, consider the Higgs potential of the SM, $V = \mu^2\,|H|^2 + \lambda\,|H|^4$; then the minimisation condition gives

$$0 = \mu^2 + \lambda\, v^2\,. \tag{1.1}$$

Since we do not have an observable for μ^2 we typically use this equation to eliminate it, giving the Higgs mass to be

$$m_h^2 = \mu^2 + 3\,\lambda\, v^2 = 2\,\lambda\, v^2\,. \tag{1.2}$$

However, once we go beyond tree level, there are several possible choices. The approach typically taken in BSM theories, and in the SM in Ref. [66], is to insist that the expectation value v is a fixed "observable", and instead keep solving for μ^2 order-by-order in perturbation theory. In this way,

$$\mu^2 = -\lambda\, v^2 - \frac{1}{v}\,\frac{\partial \Delta V}{\partial h}\bigg|_{h=0} \equiv -\lambda\, v^2 - \frac{1}{v}\, t_h\,, \tag{1.3}$$

where ΔV are the loop corrections to the effective potential, and then the Higgs pole mass M_h reads

$$M_h^2 = 2\,\lambda\, v^2 - \frac{1}{v}\, t_h + \Pi_{hh}\big(M_h^2\big) \equiv 2\,\lambda\, v^2 + \Delta M_h^2\,, \tag{1.4}$$

where $\Pi_{hh}\big(M_h^2\big)$ is the Higgs self-energy evaluated on-shell. One of the chief advantages of this approach is that tadpole diagrams do not appear in any processes, since they vanish by construction.

On the other hand, while this is in principle a straightforward procedure to follow, it is complicated by the fact that the self-energies and effective potential implicitly depend on μ^2. In Landau gauge, or the gaugeless limit, this leads to the "Goldstone Boson Catastrophe" at two loops [67–70] – its solution appears by consistently solving the above equation order by order [27, 33]. Indeed, one way to formalise this is as a finite (or possibly IR-divergent) counterterm for μ^2:

$$\mathcal{L} \supset -\left(\mu^2 + \delta\mu^2 + \lambda\, v^2\right) v\, h - \frac{1}{2}\left(\mu^2 + \delta\mu^2 + 3\,\lambda\, v^2\right) h^2 + \dots\,, \tag{1.5}$$

where $\delta\mu^2 = -\frac{1}{v}\, t_h$. Another drawback is that it manifestly breaks gauge invariance, since the loop corrections above depend on the gauge; and it also means that the expectation value v is not an $\overline{\text{MS}}$ parameter, so the renormalisation-group equations for the expectation value are no longer just given by those of μ^2 and λ, but have extra contributions [71, 72].

However, there is a further drawback to the above procedure which we wish to highlight in this paper. When considering a BSM theory with additional scalars that may have an expectation value, it is typical to take the same approach as for the scalar field in the SM and fix their expectation values, solving the additional tadpole equations for other dimensionful parameters – for example, their mass-squared parameters, or sometimes a cubic scalar coupling. To take the example of a real singlet S with mass-squared *Lagrangian parameter* m_S^2 – not to be confused with the pole mass, which we denote M_S – and expectation value v_S, this means that analagously to eq. (1.3),

$$m_S^2 = \left(m_S^2\right)^{\text{tree}} - \frac{1}{v_S}\frac{\partial \Delta V}{\partial S}. \tag{1.6}$$

If the loop corrections are not large, and v_S is not small, this is completely acceptable – so for models such as the NMSSM there is generally no problem. However, if we consider a different theory or regions of the parameter space where v_S is small, for example if $m_S \gg v$ and $v_S \propto v^2$ (as may be found in examples of EFT matching [41]) then we can easily find the case that $\delta m_S^2 > \left(m_S^2\right)^{\text{tree}}$. This makes the calculation unreliable.

The archetypal example of this problem is the case where the neutral scalar obtaining an expectation value actually comes from an $SU(2)$ triplet $\mathbf{T}$ with expectation value v_T and mass-squared m_T^2 – for example in Dirac-gaugino models [73–76]. In that case, $v_T \propto v^2/m_T^2$ multiplied by other dimensionful parameters of the theory. Moreover, we require that $v_T \lesssim 4$ GeV from electroweak-precision constraints, generally requiring $m_T \gtrsim 1$ TeV. So then

$$\delta m_T^2 \sim \frac{1}{4\ \text{GeV}} \times \frac{1}{16\pi^2} \times \mathcal{O}\!\left(\text{TeV}^3\right) \sim 2.5 \times \mathcal{O}\!\left(\text{TeV}^2\right), \tag{1.7}$$

i. e. we see that there is a severe problem whenever v_T/m_T is of the order of a loop factor.

Moreover, for such cases where v_S is small, this procedure works in the opposite way to that which we would desire. In BSM theories the scalar expectation values beyond v are not top-down inputs or tied closely to some observables, whereas we may typically want to define the masses and couplings as fixed by some high-energy boundary conditions (for example constrained or minimal SUGRA conditions where soft masses have a common origin). In this case we would like to solve the tadpole equations for v_S; even if this would typically lead to coupled cubic equations, nowadays it is almost trivial to solve them numerically, or start from an approximation.

In this paper we will instead examine an alternative procedure, proposed by Fleischer and Jegerlehner in examining Higgs decays in the SM [77], which has the potential to solve both of these issues. Instead of taking the expectation values as fixed, we take them to be the tree-level solutions of the tadpole equations. This means that we do not work at the "true" minimum of the potential and must include tadpole diagrams in all processes. While this implies the addition of some new Feynman diagrams in the Higgs mass calculation, it is not technically more complicated than including finite counterterm insertions for μ^2. This approach has the additional advantages that, since the Lagrangian is specified in terms of $\overline{\text{MS}}$ parameters only, the result is manifestly gauge independent, and the expectation values are just the solutions to the tree-level tadpole equations. For these reasons, it has been used and advocated in the SM, in particular at two loops in Ref. [57]; and applied to certain extensions of the Two Higgs Doublet Model (THDM) when considering decays [78–81]. We also note that this approach is closely related to the various on-shell renormalisations used in *e. g.* [82–85] in the THDM and the Minimal Supersymmetric Standard Model (MSSM).

In the example of the SM at the one-loop order, this would mean

$$M_h^2 = 2\,\lambda\,v^2 - \frac{6\,\lambda\,v}{m_h^2}\,t_h^{(1)} + \Pi_{hh}^{(1)}(m_h^2)\,, \tag{1.8}$$

where the superscripts in brackets indicate the loop order, and we put the momentum in the self-energy at the tree-level Higgs mass in order to respect the order of perturbation theory. In other words, the tadpole contribution is suppressed by the mass-squared of the Higgs, although – since $m_h^2 = 2\,\lambda\,v^2$ – here we find that they have a very similar form to the previous approach. On the other hand, in the case of a heavy singlet or triplet the contributions to the singlet self-energy would be similarly suppressed by m_S^2, and we can have m_S much greater than the triplet coupling – so the corrections to the singlet mass would be well under control.

On the other hand, in the BSM context this approach was proposed by [86] for the following very different reason: by no-longer forcing the electroweak expectation value to have its observed value, we allow new physics to disturb the electroweak hierarchy. In the above approach, the contribution $-\frac{6\,\lambda\,v}{m_h^2}\,t_h^{(1)} = -\frac{3}{v}\,t_h^{(1)}$ is effectively the contribution *from a shift in* v. We can view the calculation as equivalent to counterterms for the expectation value $\delta^{(1)}v$, where

$$\mathcal{L} \supset -\left(\mu^2 + \lambda\,v^2\right) v\,h - \left(\mu^2 + 3\,\lambda\,v^2\right) \delta^{(1)}v\,h - \ldots \tag{1.9}$$

so that now

$$\delta^{(1)}v = -\frac{1}{m_h^2}\,t_h^{(1)}\,. \tag{1.10}$$

In this case, if there is heavy new physics at a scale $\Lambda \gg m_h$, then we shift the Higgs expectation value up to that new scale suppressed only by a loop factor. Indeed in Ref. [86] the proposal was to use

$$\frac{\delta m_h^2}{m_h^2} \equiv \frac{1}{m_h^2}\left[-\frac{3}{v}\,t_h^{(1)} + \Pi_{hh}^{(1)}(m_h^2)\right] \tag{1.11}$$

as *as a measure of fine-tuning of the theory.*

Another perspective on the difference between the two approaches is given by viewing the SM as an EFT. In this case, in the EFT the SM receives corrections to both μ^2 and λ at the matching scale from integrating out heavy states which can be done with $v = 0$. As discussed in Ref. [27], when expanding in v, in order to respect gauge invariance we must have:

$$\begin{aligned} \Delta V &= \Delta V_0 + \frac{1}{2}\,\Delta V_{hh}\big|_{v=0}\,v^2 + \mathcal{O}(v^4) + \ldots\,, \\ \Pi_{hh}(m_h^2) &= \Delta V_{hh}\big|_{v=0} + \mathcal{O}(v^2) \end{aligned} \tag{1.12}$$

and therefore $t_h = v\,\Delta V_{hh}|_{v=0} + \ldots$ This shows that the EFT-matching correction to μ^2, which is $\Delta V_{hh}|_{v=0}$, and the origin of the hierarchy problem, correspond to t_h/v to lowest order in v. Hence in the "standard" approach of eq. (1.4) this cancels out and leaves only corrections proportional to v^2 – whereas in the modified approach it remains and gives a large shift to the Higgs mass.

However, the reappearance of the hierarchy is a problem for the *light* Higgs mass, whereas the problem we wished to solve actually appeared in new, *heavy* states! If we wish to explore theories which may remain natural while having heavy states, such as those in Ref. [86], then the modified tadpole approach should work best. There must consequently be some trade-off between losing control of the light Higgs and losing control of the heavier states (and losing gauge invariance too). In section 2 we will set up the necessary general formalism and explore this in detail for a toy model.

However, there are *two* potential solutions to allow us to have the best of both worlds:

1. Retain counterterms for μ^2 as in eq. (1.5) for the SM Higgs, but *only* for them. This is somewhat tricky to automate, since we must make a special case of the electroweak sector, and we also lose gauge invariance.
2. For cases where the tuning of the hierarchy becomes large, use EFT pole matching [26] with the modified treatment of tadpoles. This way, the heavy states remain entirely under control, we keep the heavy masses and couplings as top-down inputs (that remain genuinely $\overline{\text{MS}}$ or $\overline{\text{DR}}'$), and we have gauge invariance built-in.

In section 4 we will adopt the second approach for the example of the general NMSSM (and apply it specifically to the variant known as the μNMSSM [63]). We establish the necessary formalism for the matching and give a detailed examination, via implementing the computation in a modified `SPheno` [87, 88] code generated from `SARAH` [13, 16, 33, 89–93].

2 Treatment of tadpoles for theories with heavy scalars

For a general renormalisable field theory, once we have solved the vacuum minimisation conditions and diagonalised the mass matrices, we can write the potential in terms of real scalar fields $\{\phi_i\}$ as

$$V = \text{const} + \frac{1}{2}\, m_i^2\, \phi_i^2 + \frac{1}{6}\, a_{ijk}\, \phi_i\, \phi_j\, \phi_k + \frac{1}{24}\, \lambda_{ijkl}\, \phi_i\, \phi_j\, \phi_k\, \phi_l\,. \tag{2.1}$$

If we take the standard approach and fix the expectation values, adjusting the mass parameters order by order in perturbation theory, then as described in [27] we can write the pole masses as

$$\big(M_i^2\big)^{(1)} = m_i^2 + \Delta_{ii} + \Pi_{ii}^{(1)}\big(m_i^2\big) \equiv m_i^2 + \Delta M_i^2\,. \tag{2.2}$$

To define the shifts Δ_{ii} in a general way, we must start from some basis of fields $\big\{\phi_i^0\big\}$ split into expectation values and fluctuations so that $\phi_i^0 \equiv v_i + \hat{\phi}_i^0$ and then diagonalise the fields via $\hat{\phi}_i^0 = R_{ij}\,\phi_i$. In the simplest case where we solve the tadpole equations for some mass-squared parameters in the original basis and where we ignore pseudoscalars, we can then write

$$\Delta_{ii} = -\sum_k R_{ki}^2\, \frac{1}{v_k}\, \frac{\partial \Delta V}{\partial \hat{\phi}_k^0}\bigg|_{\hat{\phi}_k^0=0} = -\sum_{k,l} R_{ki}^2\, R_{lk}\, \frac{1}{v_k}\, t_l^{(1)}\,. \tag{2.3}$$

The generalisation to solving for other variables (such as cubic scalar couplings) and to include pseudoscalar mass shifts is given in [33].

On the other hand, taking the modified approach and including the tadpole diagrams, the pole masses up to one loop are simply

$$\big(M_i^2\big)^{(1)} = m_i^2 - \frac{1}{m_j^2}\, a_{iij}\, t_j^{(1)} + \Pi_{ii}^{(1)}\big(m_i^2\big) \equiv m_i^2 + \hat{\Pi}_{ii}^{(1)}\big(m_i^2\big)\,, \tag{2.4}$$

where we have defined $\hat{\Pi}_{ij}\big(p^2\big)$ for later use to be the self-energies including the tadpoles. The expressions for the tadpoles and self-energies at one loop can be found *e. g.* in [27, 94]; this calculation is therefore more straightforward to automate, being purely diagrammatic in nature. An *explicitly* gauge-invariant expression for this (*i. e.* one where there are no gauge parameters present) will be given in future work.

2.1 A toy model

Let us now apply these general expressions to the simplest toy model that can illustrate the differences of prescriptions for dealing with radiative corrections to tadpoles. This consists of the abelian Goldstone model coupled to a real singlet S, and has scalar potential

$$V = \mu^2\,|H|^2 + \frac{1}{4}\,\lambda\,|H|^4 + \frac{1}{2}\,m_S^2\,S^2 + a_{SH}\,S\,|H|^2 + \frac{1}{2}\,\lambda_{SH}\,S^2\,|H|^2 + \frac{1}{6}\,a_S\,S^3 + \frac{1}{24}\,\lambda_S\,S^4 \tag{2.5}$$

with the fields

$$H \equiv \frac{1}{\sqrt{2}}\,(v + h + i\,G)\,, \quad S \equiv v_S + \hat{S}\,, \tag{2.6}$$

v and v_S denoting the Higgs and singlet vacuum expectation values (vevs), respectively. The minimisation conditions at the tree level yield the conditions

$$-\mu^2 = \frac{1}{4}\,\lambda\,v^2 + a_{SH}\,v_S + \frac{1}{2}\,\lambda_{SH}\,v_S^2\,, \tag{2.7}$$

$$\left(m_S^2 + \frac{1}{2}\,\lambda_{SH}\,v^2\right)v_S = -\frac{1}{2}\,a_{SH}\,v^2 - \frac{1}{2}\,a_S\,v_S^2 - \frac{1}{6}\,\lambda_S\,v_S^3 \tag{2.8}$$

that lead to the tree-level (squared) mass matrix for the scalars (which do not mix with the massless pseudoscalar):

$$\mathcal{M}^2_{\text{tree}} = \begin{pmatrix} \frac{1}{2}\,\lambda\,v^2 & a_{SH}\,v + \lambda_{SH}\,v\,v_S \\ a_{SH}\,v + \lambda_{SH}\,v\,v_S & m_S^2 + \frac{1}{2}\,\lambda_{SH}\,v^2 + a_S\,v_S + \frac{1}{2}\,\lambda_S\,v_S^2 \end{pmatrix}. \tag{2.9}$$

The one-particle irreducible one-loop contributions to the one- and two-point functions (see figure 1) of this toy model are given by

$$t_i^{(1)} = -\frac{1}{16\,\pi^2}\,\frac{1}{2}\,a_{ijj}\,A\big(m_j^2\big)\,, \tag{2.10}$$

$$\Pi_{ij}^{(1)}(p^2) = \frac{1}{16\,\pi^2}\left[\frac{1}{2}\,\lambda_{ijkk}\,A\big(m_k^2\big) - \frac{1}{2}\,a_{ikl}\,a_{jkl}\,B\big(p^2, m_k^2, m_l^2\big)\right] \tag{2.11}$$

with A and B denoting the scalar one-point and two-point one-loop integrals in the conventions of *e. g.* [27, 94], and p^2 denoting the external momentum. In the approach of keeping the vevs fixed, we find for the one-loop pole masses:

$$\big(M_i^2\big)^{(1)} = m_i^2 - R_{i1}^2\,\frac{1}{v}\,t_h^{(1)} - R_{i2}^2\,\frac{1}{v_S}\,t_S^{(1)} + \Pi_{ii}\big(m_i^2\big)\,, \tag{2.12}$$

where $t_h^{(1)} = \partial\Delta V/\partial h\big|_{h,\hat{S}=0}$, $t_S^{(1)} = \partial\Delta V/\partial S\big|_{h,\hat{S}=0}$. Thus the tadpole corrections suffer from the division by the vev; in particular, the mass predictions can become numerically unstable in scenarios

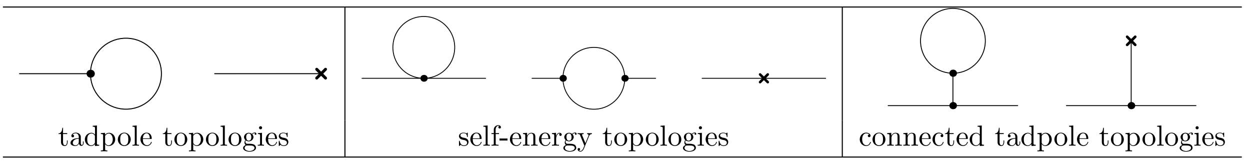

Figure 1: *left:* one-loop tadpole diagrams; *middle:* one-loop self-energy diagrams appearing in standard and modified calculation; *right:* additional self-energy diagrams in the modified approach.

with a small singlet vev. Let us see this in practice for our example when m_S^2 is large; in this case

$$v_S \sim -\frac{a_{SH}\, v^2}{2\, m_S^2}, \qquad R \sim \begin{pmatrix} 1 & -\frac{a_{SH}\, v}{m_S^2} \\ \frac{a_{SH}\, v}{m_S^2} & 1 \end{pmatrix}. \tag{2.13}$$

If we take v small and just look at the singlet mass in the limit $p^2 \to 0$ for simplicity,* we have

$$\Delta M_S^2 \approx \Pi_{SS}(0) - \frac{1}{v_S}\, t_S \supset -\frac{a_S\, m_S^2\, \kappa}{2\, v_S} \left(\overline{\log}\, m_S^2 - 1 \right) + \ldots \tag{2.14}$$

When the system is really decoupled and $v = 0$, then $v_S \sim m_S^2/a_S$ and this expressions remains well-controlled, but when $0 < v \ll m_S$ – which is the case we are interested in – we instead have

$$\Delta M_S^2 \propto \frac{a_S\, m_S^4}{16\, \pi^2\, a_{SH}\, v^2}\, \overline{\log}\, m_S^2 \tag{2.15}$$

which can be very large compared to m_S^2.

If we take the modified approach to tadpoles, then the relevant generic expression for the self-energy is

$$\hat{\Pi}^{(1)}_{ij}(p^2) = \frac{1}{16\, \pi^2} \left[\frac{1}{2}\, \lambda_{ijkk}\, A\big(m_k^2\big) - \frac{1}{2}\, a_{ikl}\, a_{jkl}\, B\big(p^2, m_k^2, m_l^2\big) - \frac{1}{2\, m_k^2}\, a_{ijk}\, a_{kll}\, A\big(m_l^2\big) \right]; \tag{2.16}$$

and for our example

$$\hat{\Pi}^{(1)}_{SS}\big(m_S^2\big) \approx \Pi_{SS}(0) - \frac{a_{SH}^2\, \kappa}{2\, m_h^2}\, A\big(m_S^2\big) - \frac{a_S^2\, \kappa}{2\, m_S^2}\, A\big(m_S^2\big) + \ldots \sim -\frac{\kappa}{2} \left(\frac{a_{SH}^2}{m_h^2} - \lambda_S \right) m_S^2\, \overline{\log}\, m_S^2\,. \tag{2.17}$$

Provided that $a_{SH} \lesssim m_h$ this is well under control, in contrast to the previous "standard" approach.

2.2 Numerical examples

We turn in this section to some numerical examples, to illustrate the different behaviours of the two approaches to tadpoles in the toy model defined in eq. (2.5). For this purpose, we present results for the one-loop pole masses M_h and M_S computed diagrammatically both in the standard approach – following eq. (2.2) – and in the modified approach of equation (2.4). For all the following figures, we set $\lambda = 0.52$, to reproduce a mass of the Higgs-like state near 125 GeV, and we also fix $\lambda_{SH} = 0$ and $\lambda_S = 1/24$.

In figure 2, we show first M_h (left side) and M_S (right side) as a function of the trilinear coupling a_{SH}, at tree level (green curves) and at one loop in the standard (red curves) and modified (blue curves) schemes for the tadpoles. We choose here a scenario with a large Lagrangian mass term $m_S = 2000$ GeV and a non-zero trilinear self-coupling $a_S = 100$ GeV for the singlet (and we also fix the renormalisation scale to be $Q = 2000$ GeV). Consequently, we find ourselves exactly in the dangerous region $0 < v \ll m_S$, *c.f.* eq. (2.15), and as expected from our theoretical discussion, we find that the standard treatment of the tadpoles breaks down. On the one hand, for M_h one can observe that the radiative corrections are larger in the standard approach and lead to larger variations of the loop-corrected mass than in the modified tadpole scheme. On the other hand, more strikingly, the results for M_S in the standard approach are manifestly spurious. Indeed, while the loop corrections in the modified scheme remain very small (the green tree-level and blue one-loop curves are almost superimposed), in the standard scheme the corrections are huge: for large $a_{SH} \gtrsim v$ – meaning not too

* This limit is not implemented in our code and serves only the more lucid presentation. In fact, an off-shell evaluation of the self-energies implies unphysical behaviour of Higgs-mass predictions [95].

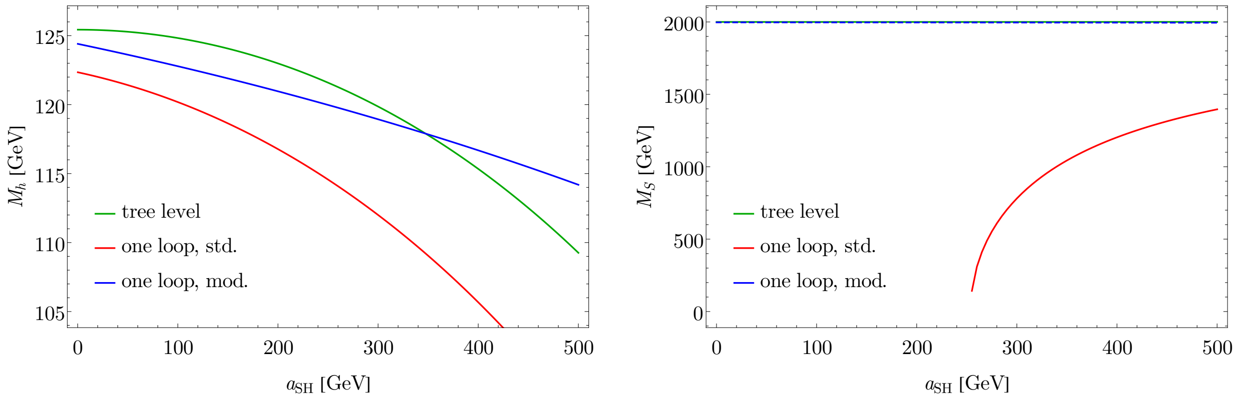

Figure 2: M_h (*left*) and M_S (*right*) as a function of a_{SH}. $m_S = Q = 2000\,\text{GeV}$, $a_S = 100$ GeV, $\lambda = 0.52$, $\lambda_{SH} = 0$, $\lambda_S = 1/24$. The tree-level values are shown with the green curves, while the red and blue curves correspond to the one-loop results using respectively the standard (eq. (2.2)) and modified (eq. (2.4)) treatments of tadpoles.

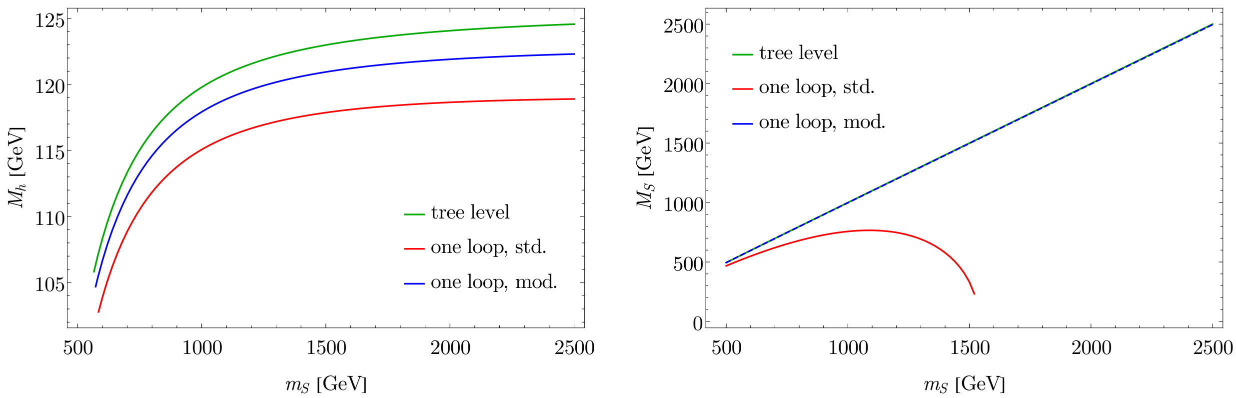

Figure 3: M_h (*left*) and M_S (*right*) as a function of m_S. $Q = m_S$, $a_{SH} = 150$ GeV, $a_S = 100$ GeV, $\lambda = 0.52$, $\lambda_{SH} = 0$, $\lambda_S = 1/24$. The colours for the different curves are the same as in figure 2.

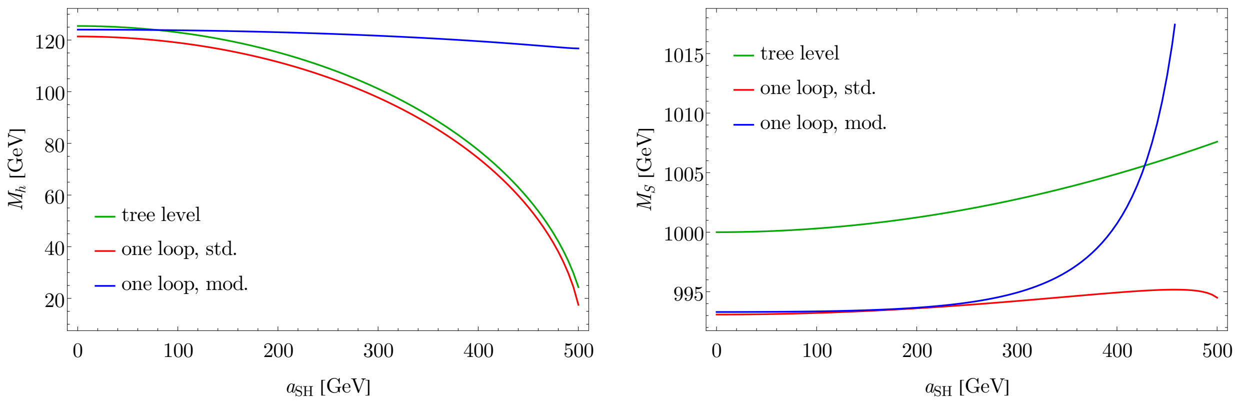

Figure 4: M_h (*left*) and M_S (*right*) as a function of a_{SH}. $m_S = 1000$ GeV, $Q = 5000$ GeV, $a_S = 0$ GeV, $\lambda = 0.52$, $\lambda_{SH} = 0$, $\lambda_S = 1/24$. The colours for the different curves are the same as in figure 2.

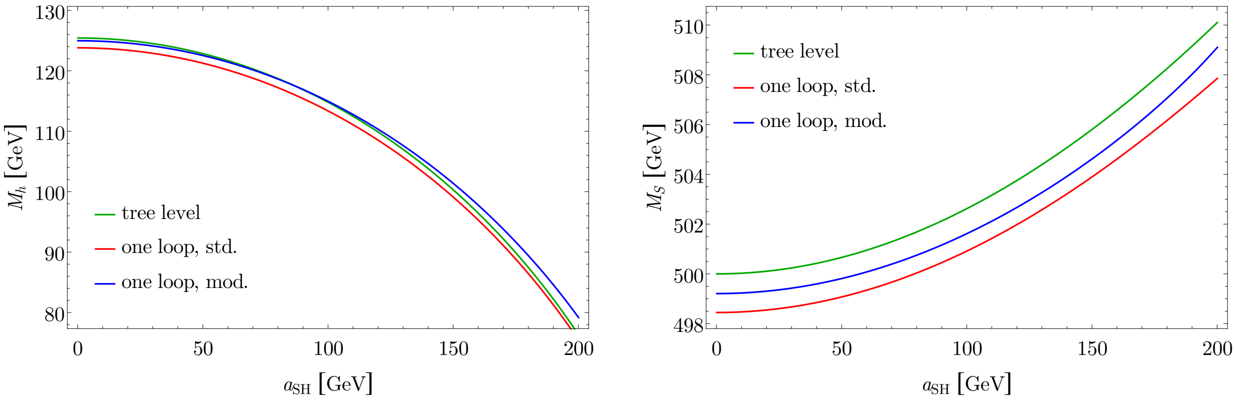

Figure 5: M_h (*left*) and M_S (*right*) as a function of a_{SH}. $m_S = Q = 500\,\text{GeV}$, $a_S = 0$ GeV, $\lambda_{SH} = 0$, $\lambda = 0.52$, $\lambda_S = 1/24$. The colours for the different curves are the same as in figure 2.

small values of the singlet vev v_S – they already amount to several hundred GeV, and if one decreases a_{SH} (thereby increasing ΔM_S^2, *c.f.* eq. (2.15)) the singlet pole mass becomes tachyonic below $a_{SH} = v$.

Next, in figure 3, we fix the trilinear coupling $a_{SH} = 150$ GeV and now consider M_h (left) and M_S (right) as a function of the Lagrangian mass term m_S. We also set $Q = m_S$ and $a_S = 100$ GeV. Once again, with our choice of a non-zero singlet trilinear self-coupling a_S and relatively small a_{SH} – hence also a small singlet vev – we expect the standard approach to exhibit instabilities. For M_h (left side of figure 3) both approaches behave relatively well and no instability seems to occur, although the radiative corrections are significantly larger in the standard scheme. However, for M_S the calculation in the standard approach (red curve) once again breaks down when m_S is increased – equivalently for small v_S – while the loop corrections to M_S in the modified approach (blue curve) remain minute.

In figure 4, we illustrate the behaviour of eq. (2.17). We plot once more M_h (left) and M_S (right) as a function of the trilinear coupling a_{SH}, but now for a scenario where $a_S = 0$ (in order to avoid large corrections ΔM_S^2 in the standard scheme), and with $m_S = 1000$ GeV and $Q = 5000$ GeV so as to increase the size of the logarithms $\overline{\log}\, m_S^2$. For small values of a_{SH}, both schemes (red and blue curves) produce very similar results, however, as a_{SH} becomes larger the radiative corrections to M_h as well as M_S increase significantly in the modified tadpole scheme, leading to less reliable predictions (especially for $a_{SH} \gtrsim 300$–400 GeV).

Finally, we present in figure 5 an example of scenario in which both ways to treat the tadpole contributions give reliable results. We take a small singlet mass parameter $m_S = 500$ GeV, set $a_S = 0$ and maintain $a_{SH} < 200$ GeV. We observe here that the radiative corrections to M_h and M_S remain well behaved in both approaches.

3 Pole mass matching with tadpole insertions

When matching two theories via pole masses, care must be taken that subleading logarithms are correctly subtracted. The best way to do this is to expand the expressions on both sides of the matching relation in terms of the same parameters; the most efficient way to do this is to use those of the high-energy theory (HET) even though this adds a layer of complication because it is the SM parameters that we know from the bottom-up observations. To this end we require the shifts in the vacuum expectation value as well as gauge, Yukawa and of course quartic couplings.

The most straightforward way to match the vacuum-expectation value of the Higgs is via matching the Z mass, which gives (see *e. g.* [26, 29, 41]):

$$v_{\text{SM}}^2 = v_{\text{HET}}^2 + \frac{4}{g_Y^2 + g_2^2}\left[\hat{\Pi}_{ZZ}^{\text{HET}}(0) - \hat{\Pi}_{ZZ}^{\text{SM}}(0)\right] + \mathcal{O}\big(v^4\big)\,. \tag{3.1}$$

If we match the one-loop Higgs mass in the SM to the HET, where the light Higgs mass at tree level is m_0, then we have

$$2\,\lambda_{\rm SM}\,v_{\rm SM}^2 + \hat{\Pi}_{hh}^{\rm SM}\big(2\,\lambda_{\rm SM}\,v_{\rm SM}^2\big) = m_0^2 + \hat{\Pi}_{hh}^{\rm HET}\big(m_0^2\big) \tag{3.2}$$
$$\lambda_{\rm SM} = \frac{1}{2\,v_{\rm HET}^2}\left\{m_0^2 + \hat{\Pi}_{hh}^{\rm HET}\big(m_0^2\big) - \hat{\Pi}_{hh}^{\rm SM}\big(m_0^2\big) - \frac{4\,m_0^2}{v_{\rm HET}^2\,(g_Y^2+g_2^2)}\Big[\hat{\Pi}_{ZZ}^{\rm HET}(0) - \hat{\Pi}_{ZZ}^{\rm SM}(0)\Big]\right\}.$$

It should be noted that – in order to preserve gauge invariance, and cancel large logarithms exactly without introducing spurious subleading ones – the matching of the quartic coupling should be performed according to this equation, as opposed to performing some iteration, matching eigenvalues of the mass matrices, or separately matching the expectation values and Higgs mass (as performed in some codes) [32, 51]. With the prescription of including tadpole diagrams, this leads to

$$\hat{\Pi}_{hh} \equiv \Pi_{hh} - a^{hhk}\,\frac{1}{m_k^2}\,t_k\,, \qquad \hat{\Pi}_{ZZ} \equiv \Pi_{ZZ} - g^{ZZk}\,\frac{1}{m_k^2}\,t_k\,. \tag{3.3}$$

In the SM with $\mathcal{L} \supset -\lambda_{\rm SM}\,|H|^4$ we have

$$\hat{\Pi}_{hh}^{\rm SM} \equiv \Pi_{hh}^{\rm SM} - \frac{6\,\lambda\,v}{m_h^2}\,t_h^{\rm SM} = \Pi_{hh}^{\rm SM} - \frac{3}{v}\,t_h^{\rm SM}\,, \qquad \hat{\Pi}_{ZZ}^{\rm SM} \equiv \Pi_{ZZ}^{\rm SM} - \frac{2\,M_Z^2}{v\,m_h^2}\,t_k^{\rm SM}\,, \tag{3.4}$$

and so

$$\hat{\Pi}_{hh}^{\rm SM} - \frac{m_h^2}{M_Z^2}\,\hat{\Pi}_{ZZ}^{\rm SM} = \Pi_{hh}^{\rm SM} - \frac{m_h^2}{M_Z^2}\,\Pi_{ZZ}^{\rm SM} - \frac{3}{v}\,t_h^{\rm SM} + \frac{2}{v}\,t_h^{\rm SM} = \Delta M_{\rm SM}^2 - \frac{m_h^2}{M_Z^2}\,\Pi_{ZZ}^{\rm SM}\,, \tag{3.5}$$

where the $\Delta M^2_{\rm SM}$ is now just the standard set of vacuum conditions as in eqs. (1.4) or (2.2). So what we have shown is that the modified treament of tadpoles cancels out exactly in the matching of the light Higgs, *for the SM part.* Of course, the shift in the matching condition should only depend on the Lagrangian parameters, which are not affected by the treatment of tadpoles, so the same is true for the matching in the HET part *up to terms of higher order in* v.

We have already implicitly shown how the change in scheme affects the matching of the gauge bosons; now for fermions we have

$$\Gamma_{F_iF_j}(p) = i\,(\not{p} - m_F)\,\delta_{ij} + i\left[\not{p}\left(P_L\,\hat{\Sigma}_{ij}^L(p^2) + P_R\,\hat{\Sigma}_{ij}^R(p^2)\right) + P_L\,\hat{\Sigma}_{ij}^{SL}(p^2) + P_R\,\hat{\Sigma}_{ij}^{SR}(p^2)\right]. \tag{3.6}$$

For fermions at one loop we can write the mass-matrix corrections as

$$\delta m_F = -\Sigma^{SL} - \frac{1}{2}\left(\Sigma^R\,m + m\,\Sigma^L\right). \tag{3.7}$$

This means that our tadpole shift just affects

$$\delta\Sigma^{SL} = \delta\Sigma^{SR} = \frac{1}{m_k^2}\,y^{ijk}\,\frac{\partial V}{\partial\phi_k}\,, \tag{3.8}$$

where y^{ijk} are the Yukawa couplings, that can be written in terms of Weyl spinors $\{\psi_i\}$ as

$$\mathcal{L} \supset -\frac{1}{2}\,y^{ijk}\,\psi_i\,\psi_j\,\phi_k\,. \tag{3.9}$$

To match the Yukawa couplings via the pole masses of the quarks, the matching of the electroweak expectation value must also be included; working in the basis with diagonalised Yukawa couplings, we can match the diagonal elements as (using $Y^F \equiv y^{FFh}$ for h the SM Higgs and a general fermion F)

$$
\begin{aligned}
M_F &= v\, Y^F - \Sigma^{SL} - \frac{1}{2}\left(\Sigma^R m + m\,\Sigma^L\right), \\
Y^F_{\rm SM} &= Y^F_{\rm HET} + \frac{1}{v_{\rm HET}}\left[(\delta m_F)^{\rm HET} - (\delta m_F)^{\rm SM} - \frac{1}{m_k^2}\, y^{FFk}_{\rm HET}\, t_k + \frac{1}{m_h^2}\, Y^F_{\rm SM}\, t_h^{\rm SM}\right] \\
&\quad - \frac{Y^F_{\rm HET}}{2\,M_Z^2}\left[\hat{\Pi}^{\rm HET}_{ZZ}(0) - \hat{\Pi}^{\rm SM}_{ZZ}(0)\right] \\
&= Y^F_{\rm HET} + \frac{1}{v_{\rm HET}}\left[(\delta m_F)^{\rm HET} - (\delta m_F)^{\rm SM} - \frac{1}{m_k^2}\, y^{FFk}_{\rm HET}\, t_k\right] - \frac{Y^F_{\rm HET}}{2\,M_Z^2}\left[\hat{\Pi}^{\rm HET}_{ZZ}(0) - \Pi^{\rm SM}_{ZZ}(0)\right],
\end{aligned}
\tag{3.10}
$$

where we once again see that the shift in the tadpole scheme cancels out exactly in the SM part. This procedure is particularly important since the shift to the expectation value arising in eq. (3.1) is very large, as discussed in the introduction. In this case, since the corrections to μ^2 – and therefore also to v^2 – are very large, it becomes impractical in an implementation to actually use the "correct" value of v^2 in the high-energy theory. Indeed, this can even become impossible, if $\delta\mu^2$ is such that μ^2 would become positive in the SM! Instead, provided we take v much less than the matching scale, we can just treat it as perturbation parameter to extract the SM values. In our numerical calculation in the next section we do exactly this: we just use the SM value of v in both high- and low-energy theories, but use the correct shifts of the expectation values in the matching of the parameters. This is very similar to a standard EFT calculation, which assumes *e. g.* in split supersymmetry that the heavy Higgs masses are tuned according to the mixing angle given as an input, and takes $v = 0$ explicitly, since we are not interested in corrections to Lagrangian parameters of order v^2/M^2 where M is the matching scale.

4 Application in the μNMSSM

In the introduction, we explained that the modified treatment of tadpoles can be useful for stability under perturbation theory of heavy scalar masses when they are associated with a small expectation value. In section 2 we showed how it worked in practice in a toy model. In section 3 we described how, for theories where the new scalars are substantially above the electroweak scale, it can be practically applied via EFT matching of the pole masses. Here, we shall apply this technique to a real test case, the μNMSSM.

4.1 NMSSM, μNMSSM and GNMSSM

The superpotential of the most general form of the NMSSM – the GNMSSM – is [96, 97]

$$
W_{\rm GNMSSM} = Y_u\, Q\cdot H_u\, U - Y_d\, Q\cdot H_d\, D - Y_e\, L\cdot H_d\, E + \frac{1}{3}\,\kappa\, S^3 + (\mu + \lambda\, S) H_u\cdot H_d + \xi\, S + \frac{1}{2}\,\mu_S\, S^2
$$

and the supersymmetry-breaking terms in the Higgs sector are

$$
\begin{aligned}
V_{\rm soft} \supset\; & m_S^2\,|S|^2 + m_{H_u}^2\,|H_u|^2 + m_{H_d}^2 |H_d|^2 \\
& + \left(B_\mu\, H_u\cdot H_d + T_\lambda\, S\, H_u\cdot H_d + \frac{1}{3}\, T_\kappa\, S^3 + \frac{1}{2}\, B_S\, S^2 + \xi_S\, S + \text{h. c.}\right).
\end{aligned}
\tag{4.1}
$$

Once the singlet develops an expectation value, we can write effective terms

$$
\mu_{\rm eff} \equiv \mu + \frac{1}{\sqrt{2}}\,\lambda\, v_S\,, \qquad B_{\rm eff} \equiv B_\mu + \frac{1}{\sqrt{2}}\, T_\lambda\, v_S + \lambda\left(\xi + \frac{1}{\sqrt{2}}\,\mu_S\, v_S + \frac{1}{2}\,\kappa\, v_S^2\right), \tag{4.2}
$$

and the tadpole equations become

$$
\begin{aligned}
0 &= -B_{\text{eff}} \cot\beta + m_{H_u}^2 + \mu_{\text{eff}}^2 - \frac{M_Z^2}{2}\, c_{2\beta} + \frac{1}{2}\,\lambda\, c_\beta^2\,, \\
0 &= -B_{\text{eff}} \tan\beta + m_{H_d}^2 + \mu_{\text{eff}}^2 + \frac{M_Z^2}{2}\, c_{2\beta} + \frac{1}{2}\,\lambda\, s_\beta^2\,, \\
0 &= v_S\left(B_S + m_S^2 + \mu_S^2 + 2\,\kappa\,\xi\right) + \frac{1}{\sqrt{2}}\, v_S^2\,(T_\kappa + 3\,\kappa\,\mu_S) + \kappa^2\, v_S^3 \\
&\quad + \sqrt{2}\,\mu_S\,\xi + \sqrt{2}\,\xi_S + \frac{1}{2\sqrt{2}}\, v^2\Big(2\,\lambda\,\mu_{\text{eff}} - (T_\lambda + 2\,\kappa\,\lambda\, v_S + \mu_S\,\lambda)\, s_{2\beta}\Big)\,. \qquad (4.3)
\end{aligned}
$$

The first two lines are essentially modified versions of the MSSM tadpole equations with an extra term from the λ coupling. The third line, however, is the crucial one for our discussion. In a general non-supersymmetric theory, we can redefine singlet fields to remove their tadpole terms. However, in the GNMSSM, which has tadpole parameters ξ in the superpotential and ξ_S in the soft-breaking terms, we can only remove one of these, or the combination $\sqrt{2}\,\mu_S\,\xi + \sqrt{2}\,\xi_S$.

Clearly in the GNMSSM, it is most logical to choose a linear combination of the singlet tadpole terms ξ and ξ_S (or just one) as the variable to be eliminated by the tadpole equations. However, this is not possible in the NMSSM or μNMSSM, since these terms vanish by the assumption of (at least partial) $\mathbb{Z}_3$ symmetry. Then aside from $(m_{H_u}^2, m_{H_d}^2)$ or (μ, B_μ), the dimensionful parameter that we can now choose for elimination via the singlet tadpole equation is one of $\{m_S^2, \mu_{\text{eff}}, T_\lambda, T_\kappa\}$.

Let us consider the case that the singlet is rather heavier than the SM-like Higgs, so that $v^2/m_S^2 \ll 1$. In the μNMSSM, neglecting all terms that break the $\mathbb{Z}_3$ symmetry except for μ and B_μ, we find

$$
v_S \simeq -v^2 \left(\frac{2\,\lambda\,\mu - T_\lambda\, s_{2\beta}}{2\sqrt{2}\, m_S^2}\right), \qquad (4.4)
$$

where the true value can be found numerically.

The logical choice for this case is to solve for T_λ. In this case we have

$$
\Delta T_\lambda = -\frac{2\sqrt{2}}{v^2\, s_{2\beta}}\,\frac{\partial \Delta V}{\partial v_S}\,, \qquad (4.5)
$$

and the terms in the mass matrix become

$$
\mathcal{M}^2_{h_u^0 h_u^0} \supset -\frac{v_S\, t_S^{(1)}}{v^2\, s_\beta\, c_\beta} + \ldots \propto \frac{t_S^{(1)}}{m_S^2}\,, \qquad \mathcal{M}^2_{h_u^0 s_R} = -\frac{m_S^2\, v_S + t_S^{(1)}}{v\, s_\beta} + \ldots\,. \qquad (4.6)
$$

Note that this is in the "flavour basis" before we diagonalise the fields at tree level, so the contributions to the light Higgs and heavy singlet masses are $\propto t_S^{(1)}/m_S^2$.

On the other hand, this choice leads to a (potentially very) large quantum correction to T_λ. Suppose we want to investigate gauge-mediation scenarios where trilinears are small (nearly vanishing), or are otherwise specified by the top-down inputs – this would be completely inappropriate. Furthermore, we have to not only take into account shifts in the masses but also the *couplings* – this is moderately cumbersome to implement at one-loop, but much more so if we want to compute the two-loop corrections. Indeed, it is not included in the algorithm to generate "consistent vacuum equations" of [27, 70], which assumes that the parameters that we solve the tadpole equations for only affect scalar masses.

To solve both of these issues the simplest choice is to solve for m_S^2, and this leads to exactly the same problem as in the toy model, that the corrections to the singlet mass scale as $t_S^{(1)}/v_S$ leading to numerical instabilities for tiny v_S. Hence this model is an excellent prototype for comparing the different approaches to solving the tadpole equations.

4.2 Numerical comparison of tadpole schemes

We present in this section numerical investigations of several scenarios of the μNMSSM illustrating the differences between the two approaches to the treatment of tadpoles. For this, we compare results obtained using the original version of `SPheno` code obtained directly from `SARAH` (for the model `SMSSM`), as well as with a version of the `Fortran` output extensively modified according to the prescriptions described in section 3.[†] We have devised three types of scenarios:

- Scenario 1: large singlet vev and intermediate λ;
- Scenario 2: small singlet vev and small λ;
- Scenario 3: small singlet vev but large λ.

Table 1 summarises the values taken for the BSM input parameters relevant for `SPheno` – note that we have adjusted the soft terms m_0 (scalar mass) and A_0 (scalar trilinear coupling) in order to obtain a mass for the lightest Higgs boson within the interval [123 GeV, 127 GeV]. We should also emphasise that the numbers in table 1 are given to `SPheno` as high-scale inputs (as this only requires a limited set of values). We then convert these into low-scale input parameters using the standard version of the μNMSSM `SPheno` code, and the plots presented in the following are obtained by varying one of the low-scale inputs. In light of the analytic expressions in the previous section, we can expect the two approaches to the tadpoles to give relatively similar results in scenario 1, where the singlet vev is large. However, in scenarios 2 and 3, the singlet vev is taken to be small, so that the differences between the two schemes should be more pronounced. Scenario 3 furthermore allows us to investigate the effect of increasing the coupling λ.

We show first in figure 6 the behaviour of the lightest Higgs mass M_{h_1} (left side) and of the additional CP-even Higgs-boson masses M_{h_2} and M_{h_3} (right side) as a function of the superpotential coupling λ. The tree-level values are shown in green, while the one-loop results using the standard and the modified treatments of tadpoles are in red and blue respectively. Among the two BSM states h_2 and h_3, the former is singlet-like while the latter is doublet-like, in this figure. As can be seen in the right-hand side plot of figure 6, the heavy Higgs bosons receive only minute mass corrections in either of the approaches for tadpoles. For the lightest scalar mass M_{h_1}, the results in the two schemes are also in excellent agreement. However, we have cut off the plot before $\lambda = 0.19$ because beyond this value perturbativity is lost: in the *standard approach* the singlet-like pseudoscalar Higgs becomes tachyonic at one loop (from a tree-level mass of 750 GeV!). If we continued the plot into this regime

[†] This private code is not intended for public release, although it is available on request from the authors. The new functionality should eventually be made available in a future release of `SARAH`.

Scenario	1	2	3
m_0 [GeV]	2000	1500	1500
λ	$0.1^\dagger$	0.01	0.15
κ	0.005	0.05	0.05
T_λ [GeV]	1000	$1000^\dagger$	$7500^\dagger$
v_S [GeV]	3000	$1.0^\dagger$	$1.0^\dagger$
μ [GeV]	500	200	200
μ_S [GeV]	0	–200	–200
ξ [GeV^2]	$1.0 \cdot 10^8$	$1.7 \cdot 10^6$	$5.0 \cdot 10^4$
B_μ [GeV^2]	$2.0 \cdot 10^5$	$1.0 \cdot 10^6$	$4.0 \cdot 10^5$

Table 1: Definitions of the input parameters in the considered μNMSSM scenarios. Some of the BSM parameters are not modified, and remain the same for the three scenario. Namely, we take: $\tan\beta = 10$, $m_{12} = 2$ TeV, $A_0 = 3$ TeV, $B_0 = 0$, $m_A = 500$ GeV, $T_\kappa = -0.5$ GeV. The renormalisation scale is kept at $Q = 3$ TeV for all computations. Finally, the numbers marked with a "†" are varied for some of the parameter scans.

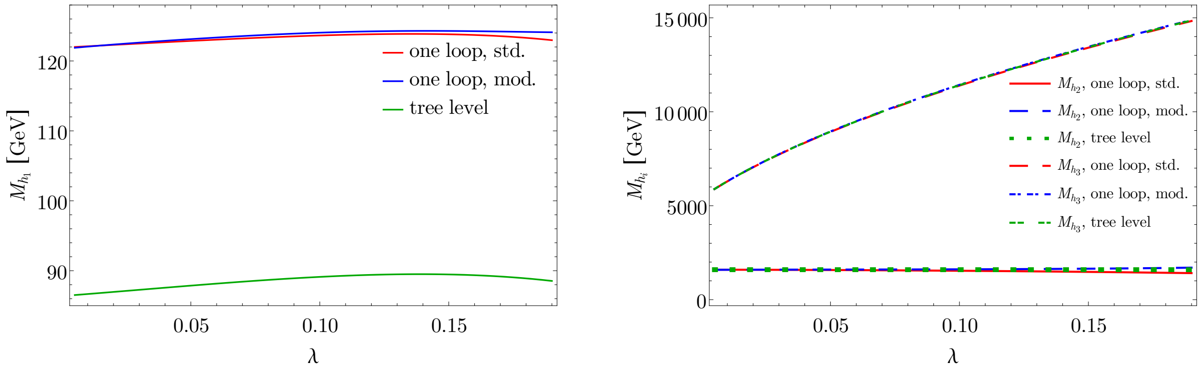

Figure 6: M_{h_1} (*left*) and M_{h_2} and M_{h_3} (*right*) as a function of λ, in scenario 1. The other inputs are taken as in table 1. Tree-level values are shown with green curves, while the red and blue curves correspond to the pole masses computed at one loop, respectively with the standard and modified approaches to the tadpoles. The colour coding of the figures remains the same for all figures in this section.

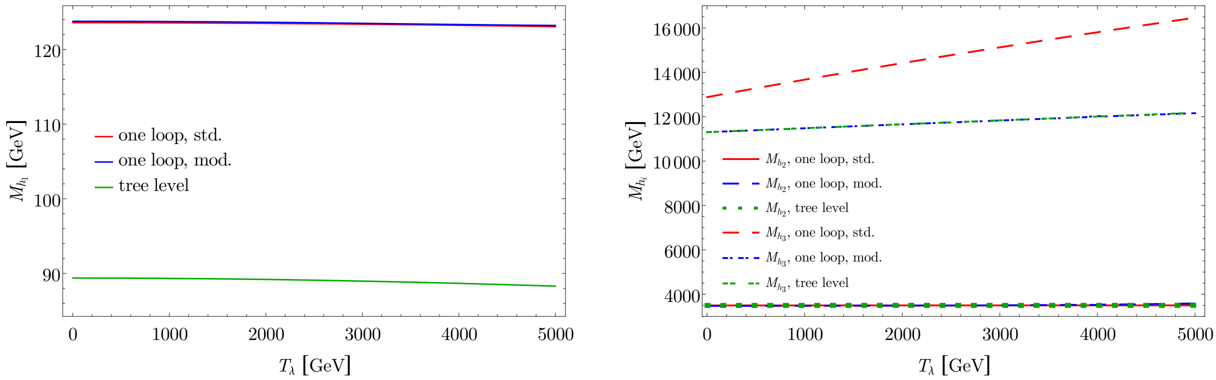

Figure 7: M_{h_1} (*left*) and M_{h_2} and M_{h_3} (*right*) as a function of the soft trilinear coupling T_λ, in scenario 2. The values of the other BSM parameters are taken as in table 1.

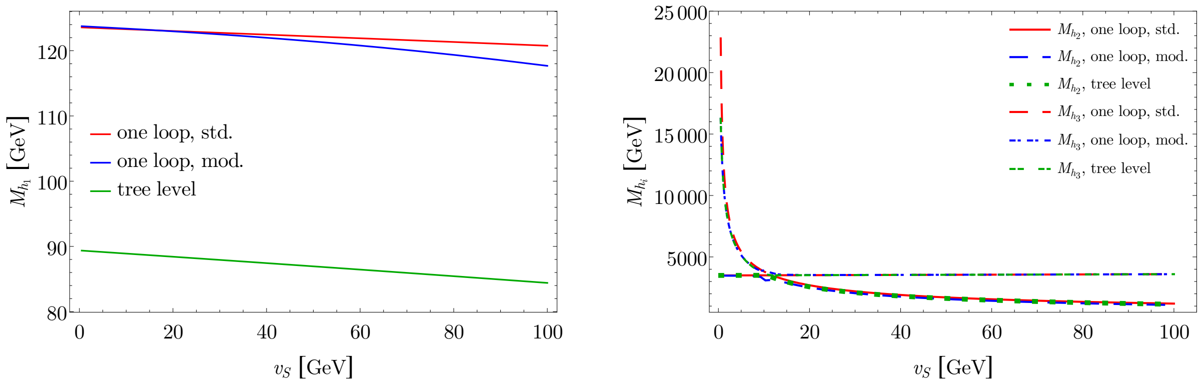

Figure 8: M_{h_1} (*left*) and M_{h_2} and M_{h_3} (*right*) as a function of v_S, in scenario 2. Input values for the other BSM parameters are given in table 1.

we would see the predictions diverging, with the standard approach predicting ever decreasing masses and the modified approach increasing ones for larger λ (compare 104 GeV and 138 GeV respectively for $\lambda = 0.3$).

Next, we turn to scenario 2, *i. e.* we consider a small $\lambda = 0.01$ and small singlet vev $v_S = 1$ GeV. Figure 7 shows the behaviour of the CP-even masses as a function of the soft trilinear coupling T_λ, at tree level and one loop (the colouring of the curves is the same as previously explained). We should emphasise that we have made sure to fulfill constraints from vacuum stability (and the absence of a charge-breaking minimum) on T_λ – see Ref. [63] – and the tree-level mass of the charged Higgs boson remains positive for the entire range of T_λ investigated here. While for M_{h_1} (left side) and M_{h_2} (lower curves of the right-side plot) it seems essentially impossible to distinguish the two approaches to the tadpole treatment, the radiative corrections to M_{h_3} – the mass of the singlet-like scalar – are clearly much larger with the standard method, and the result of the modified scheme is certainly more reliable. As a concrete comparison, we have for the intermediate value $T_\lambda = 2$ TeV a one-loop correction to M_{h_3} of 2752 GeV (*i. e.* 24% of the tree-level result) in the standard approach, but only of –4.5 GeV in the modified scheme.

We can confirm that the large difference between the two treatments of the tadpoles arises from the small value of the singlet vev v_S. Indeed, in figure 8, we present the same three CP-even scalar masses for v_S varying between 0.5 and 100 GeV. One can observe that the results using both approaches for all three masses are in good agreement for large values of the singlet vev. A short comment should be made for M_{h_1}: indeed, as v_S increases the results from the two schemes seem to grow apart, and it is somewhat difficult to determine which one should be trusted more in this case. We note that the radiative corrections to M_{h_1} keep increasing with v_S in the standard approach while their size remains relatively stable in the modified scheme. On the other hand, if we consider the situation for $v_S \gtrsim 0.5$ GeV, the breakdown of the standard approach for small singlet vevs becomes obvious. Indeed, considering the different results for the mass M_{h_3} of the CP-even singlet-like scalar at $v_S = 0.5$ GeV, the one-loop corrections in the standard scheme amount to 6.5 TeV – in other words, 40% of the tree-level result – compared to only –3.3 GeV (–0.02% of the tree-level mass) in the modified scheme.

Finally, we consider the third type of scenario, *i. e.* what happens if we keep a small singlet vev $v_S = 1$ GeV but increase the coupling λ to 0.15. In figure 9, we present the CP-even scalar masses as a function of T_λ – having once again made sure to maintain vacuum stability [63]. Considering first the masses of the two doublet-like scalars h_1 and h_2, we observe an excellent agreement of the results from the two tadpole schemes for low to intermediate values of T_λ – for $0 \leq T_\lambda \lesssim 4$ TeV. However, as T_λ becomes larger, the corrections to M_{h_1} and M_{h_2} in the modified approach start growing out of control. This appears similar to the loss of accuracy of the modified scheme that we encountered in the toy model of section 2 when increasing the trilinear coupling a_{SH}, which plays the same role as T_λ – see eq. (2.17) and figure 4. Turning however to the singlet-like mass M_{h_3} we find (as in figure 7 for scenario 2) that the radiative corrections are huge with the standard treatment of tadpoles, but remain well-behaved with the modified one. Interestingly, having increased the value of λ has not made the breakdown of the standard calculation for the singlet-like mass more severe than in scenario 2. Nevertheless, the one-loop result M_{h_3} using the modified tadpole scheme is undoubtedly more reliable here.

Finally, we present in figure 10 the behaviour of the CP-even scalar masses as a function of the singlet vev v_S – restricting our attention to the low range 0.5 GeV $\leq v_S \leq 5$ GeV. As can be read from table 1, we have chosen for this figure a large value of the soft trilinear coupling $T_\lambda = 7.5$ TeV, which corresponds to the right parts of the plots in figure 9. Therefore, it is not surprising that we observe some discrepancy between the results of the two tadpole schemes for all three masses, as discussed above. More interestingly, we can compare the size of the loop corrections to M_{h_3} in the two approaches, as we vary v_S. On the one hand, in the standard approach, the one-loop corrections increase from 2.3 TeV (19% of the tree-level result) for $v_S = 2.5$ GeV to as much as 9 TeV (40% of the tree-level mass) for $v_S = 0.75$ GeV, for instance. On the other hand, in the modified scheme, the effects remain minute and vary from –46 GeV for $v_S = 2.5$ GeV to –3.6 GeV for $v_S = 0.75$ GeV (this amounts to –0.38% and –0.02% of the results at tree level, respectively).

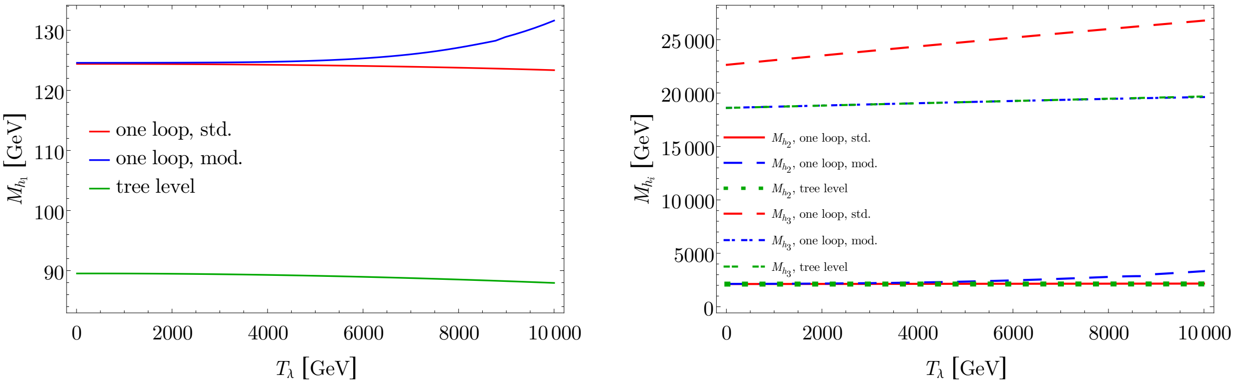

Figure 9: M_{h_1} (*left*) and M_{h_2} and M_{h_3} (*right*) as a function of T_λ, in scenario 3. The other BSM inputs are taken as in table 1.

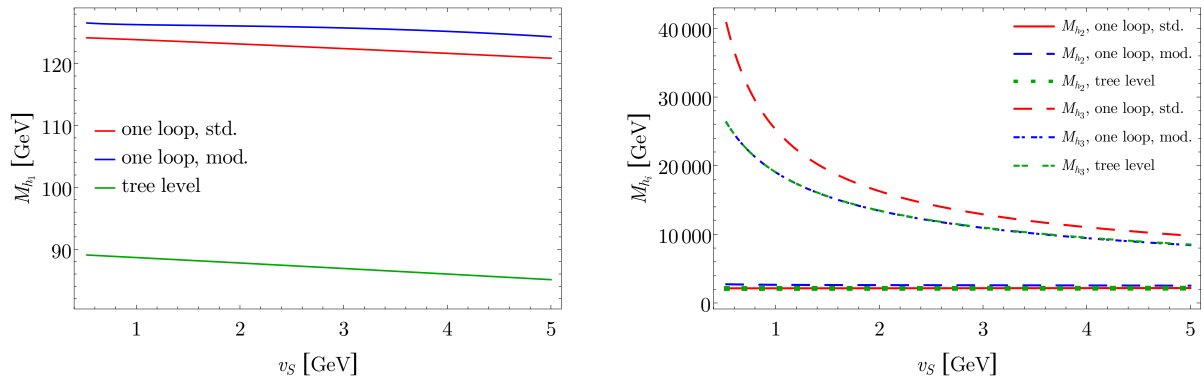

Figure 10: M_{h_1} (*left*) and M_{h_2} and M_{h_3} (*right*) as a function of v_S, in scenario 3. The values of the other relevant inputs are given in table 1.

5 Conclusions

We have shown the advantages and limitations of taking a different prescription for the solution of tadpole equations. In contrast to previous applications of this technique, in the SM or as a measure of fine-tuning, we have shown that it can be very useful when new scalars having a small expectation value are present in the theory, and in the case that they are much heavier than the electroweak scale, it is best employed via the matching of pole masses in an EFT approach. While this technique offers the advantages of perturbative stability for the heavy scalar masses, easy generalisability (the corrections are simply computed diagramatically rather than via taking derivatives of the tadpole equations) and gauge invariance, it can also lead to numerical instabilities in extracting the *light* Higgs mass, and the loss of the ability to match the electroweak expectation value.

In future work, other than a general numerical implementation in `SARAH`, it would be interesting to explore a hybrid approach (along the lines of option 1 described at the end of the introduction), where only the electroweak expectation value is fixed by appropriate counterterms. On the other hand, we intend to consider the corrections at two loops in this approach, and we shall also provide general expressions for the one-loop self-energies which are explicitly gauge independent.

Acknowledgements

We would like to thank Sven Heinemeyer and Pietro Slavich for interesting discussions. MDG acknowledges support from the grant "HiggsAutomator" of the Agence Nationale de la Recherche (ANR) (ANR-15-CE31-0002). JB is supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2121 "Quantum Universe" – 390833306. The work of SP is supported by the BMBF Grant No. 05H18PACC2. This project received support from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860881-HIDDeN.

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