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On the resonant properties of the THz metal-dielectric waveguide impedance

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Abstract. Various asymptotic representations for the longitudinal monopole impedance of a two-layer metal-dielectric cylindrical waveguide with finite-conducting external walls and a lossy internal dielectric layer are derived. Areas of their applicability are determined and their properties are investigated. New features for the resonance properties of the structure under consideration are revealed and described.

1. Introduction

Dielectric-loaded waveguides are widely used in accelerator physics and related fields. Investigations are conducted for the generation of beam-based THz radiation [1-7], two-beam acceleration [8-11], beam manipulation [12-15], beam diagnostics [16] and micro-bunching [17]. A thin internal dielectric layer approximation is also used for modeling corrugated and rough inner surfaces of accelerating structures [18-21].

Theoretical studies are usually considering a structure model, which consists of an ideally conductive external wall and a lossless internal dielectric coating. The analytical representations of the longitudinal impedance for the conducting waveguides with lo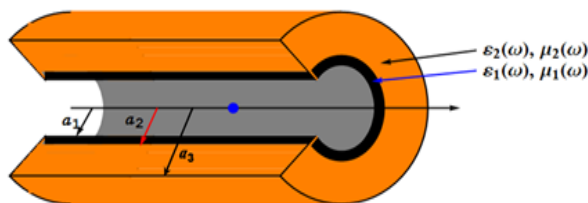ssy dielectric loads are missing. In this paper several asymptotic representations of the longitudinal impedance are derived taking into account the finite conductivity of the outer wall and the imaginary component of the dielectric constant of the inner layer. The solutions are based on the analysis of the exact solution for the longitudinal impedance of two-layer waveguides [22]. In our consideration, the dielectric constant of the inner layer is a complex quantity $\varepsilon_1 = \varepsilon_1' + j\,\varepsilon_1''$. For a better interpretation of the results, numerical examples are also given for a lossless dielectric layer $\varepsilon_1'' = 0$ and an unbounded outer copper wall with a conductivity of $\sigma_2 = 58 \cdot 10^6 \ \Omega^{-1}m^{-1}$. In the framework of our study, we assume the materials of both layers are nonmagnetic.

Figure 1. Two-layer metal-dielectric round waveguide.

2. Statement of the problem and exact solution

A point charge q moving along the axis of a cylindrical metal-dielectric waveguide (metallic waveguide with an internal dielectric layer) is considered (figure 1). In figure 1: a_1 is the inner radius of the waveguide, $a_2 > a_1$ is the inner radius of the metallic wall, $d_1 = a_2 - a_1$ is the dielectric layer thickness and a_3 is the outer radius of the waveguide. The relative dielectric constant of the conducting outer layer is given by $\varepsilon_2 = 1 + jZ_0\, \sigma_2/k$, where σ_2 is the conductivity of the outer metal wall, Z_0 is the impedance of free space, $k = \omega/c$ is the wavenumber with the frequency ω and the speed of light c.
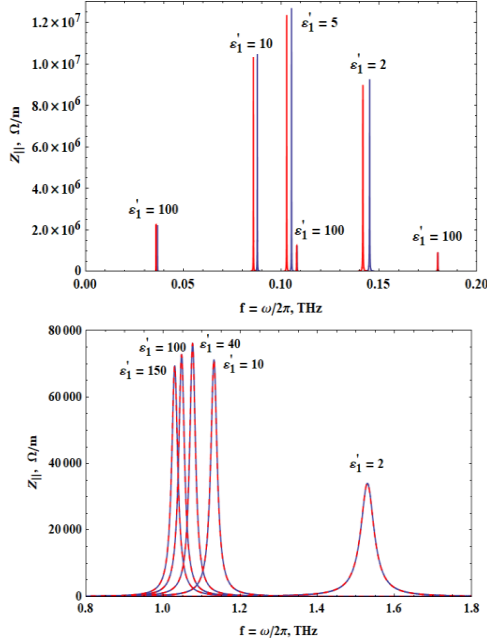
Figure 2. Longitudinal monopole impedance of the metal-dielectric waveguide for different values of the dielectric constant, calculated by the exact formula (blue) and by approximation (4) (red); $d_1 = 200\mu m$ (top) and $2\mu m$ (bottom); $a_1 = 2mm$.

The exact expression for the longitudinal monopole impedance of an unbounded dielectric-loaded metallic cylindrical waveguide in the case of an ultra-relativistic point charge moving along the axis of the waveguide with speed of light c is expressed as [22]:

$$Z_{||} = j\frac{Z_0}{\pi k a_1^2}\left\{1 - \frac{2\varepsilon_1}{a_1\beta_1}G\right\}^{-1}, \quad G = \frac{\beta_2\varepsilon_1 U_4 + \beta_1\varepsilon_2 U_3\alpha}{\beta_2\varepsilon_1 U_2 + \beta_1\varepsilon_2 U_1\alpha}, \quad \alpha = \frac{K_1(a_2\beta_2)}{K_0(a_2\beta_2)} \tag{1}$$

$$U_1 = I_0(\beta_1 a_2)K_0(\beta_1 a_1) - I_0(\beta_1 a_1)K_0(\beta_1 a_2) \approx \sinh(\beta_1 d)/\beta_1\sqrt{a_1 a_2}$$

$$U_2 = I_1(\beta_1 a_2)K_0(\beta_1 a_1) + I_0(\beta_1 a_1)K_1(\beta_1 a_2) \approx \cosh(\beta_1 d)/\beta_1\sqrt{a_1 a_2}$$

$$U_3 = -I_0(\beta_1 a_2)K_1(\beta_1 a_1) - I_1(\beta_1 a_1)K_0(\beta_1 a_2) \approx -\cosh(\beta_1 d)/\beta_1\sqrt{a_1 a_2}$$

$$U_4 = I_1(\beta_1 a_1)K_1(\beta_1 a_2) - I_1(\beta_1 a_2)K_1(\beta_1 a_1) \approx -\sinh(\beta_1 d)/\beta_1\sqrt{a_1 a_2} \tag{2}$$

with the modified Bessel functions of first and second kind and zero and first order $I_{0,1}, K_{0,1}$ and the transverse wave numbers in the dielectric and the outer metallic layers $\beta_{1,2} = k\sqrt{1 - \varepsilon_{1,2}}$. For a high conductivity of the outer wall $\alpha \approx 1$.

The approximations in (2) are valid for $|\beta_1|a_1 \gg 1$.

3. General asymptotic expression for the impedance

The general asymptotic expression for the longitudinal impedance is obtained by substituting approximations (2) into (1):

$$Z_{||} = j\frac{Z_0}{\pi k a_1^2}\left\{1 + \frac{2}{a_1}\frac{\varepsilon_1}{\beta_1}\frac{\tanh(\beta_1 d_1) + u}{1 + u\tanh(\beta_1 d_1)}\right\}^{-1}, \tag{3}$$

where $u = \frac{\varepsilon_2\beta_1}{\varepsilon_1\beta_2} = -(1-j)\xi\frac{\sqrt{\varepsilon_1 - 1}}{\varepsilon_1\sqrt{2}}$ and $\xi = \sqrt{\frac{\sigma_2 Z_0}{k}}$. After substituting $u = \tanh(\eta)$ with $\eta = \text{arcth}(u) \approx -j\pi/2 + u^{-1}$ (for $\lfloor u \rfloor \gg 1$) in (3), one obtains the following representation of the monopole longitudinal impedance, expressed through a single hyperbolic cotangent:

$$Z_\parallel = j\frac{Z_0}{\pi k a_1^2}\left\{1 + \frac{2}{a_1}\frac{\varepsilon_1}{\beta_1}coth(y)\right\}^{-1}, \quad y = \beta_1 d_1 + u^{-1}. \tag{4}$$

The condition $|u| \gg 1$ is satisfied, when the usual upper limit $k \ll Z_0\sigma_2$ is imposed on the frequency. In the presence of the dielectric coating, this limitation takes the form $k \ll \left|(\varepsilon_1 - 1)\,\varepsilon_1^{-2}\right|\sigma_2 Z_0$. For an adequate description of the main resonance with the help of expression (4), the corresponding resonance frequency must satisfy the condition $|\beta_1|a_1 \gg 1$. With an ideally conducting external wall ($\sigma_2 \to \infty$) and in the absence of losses in the dielectric coating ($\varepsilon_1'' = 0$), the fundamental resonant frequency is given by [23]:

$$k_{res} = \sqrt{2/(1 - \varepsilon_1'^{-1})a_1 d_1}. \tag{5}$$

This expression can be used to estimate the main resonance frequency of a waveguide with a high but finite conductivity of the outer wall ($|u| \gg 1$) and low losses in the dielectric ($\varepsilon_1' \gg \varepsilon_1''$). Taking into account Eq. (5), the condition $|\beta_1|a_1 \gg 1$ takes the simple form:

$$d_1/a_1 \ll 2\varepsilon_1'. \tag{6}$$

Thus, the degree of coincidence of the asymptotic expression (4) with the exact solution for the impedance (1) depends on the smallness of the ratio d_1/a_1 with respect to the dielectric constant of the inner coating. Figure 2 illustrates this statement. It gives cases with a relatively thick ($d_1/a_1 = 0.1$, top) and a thin ($d_1/a_1 = 0.001$, bottom) dielectric layer for different values of the dielectric constant. In the first case, the deviations of the resonance peaks, obtained by approximation (4), from the exact results are noticeable. The discrepancy is observed at a low dielectric constant ($\varepsilon_1' = 2$). In the second case, the exact and approximate solutions coincide completely for all presented values of the permittivity.

4. Single-resonance asymptotic

The single-resonance nature of the impedance manifests when two conditions are simultaneously satisfied: the high conductivity of the outer metal wall ($|u| \gg 1$) and the small thickness of the inner dielectric layer ($|\beta_1|d_1 \ll 1$), which leads to a small argument y in (4) ($|y| \ll 1$) and the validity of the approximation $coth(y) \approx 1/y$ in (4).

For the fundamental resonance frequency the condition $|\beta_1|d_1 \ll 1$ is then modified to

$$d_1/a_1 \ll 1/(2\varepsilon_1'). \tag{7}$$

In terms of the characteristic distance $s_0 = (2a_1^2/Z_0\sigma_2)^{1/3}$ and the resonance wave number $\tilde{k}_0 = \sqrt{2\varepsilon_1/(\varepsilon_1 - 1)a_1 d_1}$ (complex in general) for the impedance of the resistive waveguide with an internal lossy dielectric coating the approximation (4) can be simplified as:

$$Z_\parallel = j\frac{Z_0}{\pi k a_1^2}\left\{\frac{2(1+j)}{k^{3/2}s_0^{3/2} - 2(1+j)k^2/\tilde{k}_0^2} + 1\right\}^{-1}. \tag{8}$$

For the special case of $d_1 = 0$ or $\varepsilon_1 = 1$ $\left(\tilde{k}_0 \to \infty\right)$ approximation (8) transforms to the well-known expression of the longitudinal monopole impedance of an unbounded resistive pipe [24]:

$$Z_\parallel = \frac{Z_0 s_0}{2\pi a_1^2}\left\{\frac{1+j}{\sqrt{\kappa}} - j\frac{\kappa}{2}\right\}^{-1} \tag{9}$$

with the dimensionless wavenumber $\kappa = k s_0$. In the case of $s_0 \to 0$ ($\sigma_2 \to \infty$) approximation (8) converts to the impedance of a structure with perfectly conducting outer wall and lossy dielectric channel:

$$Z_{\parallel} = Z_{diel} = j\frac{Z_0}{\pi k a_1^2}R^{-1}, \; R = 1 - \frac{\tilde{k}_0^2}{k^2} = R_1 + jR_2, \tag{10}$$

$$R_1 = 1 - \frac{2}{k^2 a_1 d_1}\frac{e_1}{e_0}, R_2 = \frac{2}{k^2 a_1 d_1}\frac{\varepsilon_1''}{e_0}, \; e_0 = (\varepsilon_1'-1)^2 + \varepsilon_1''^2 \text{ and } e_1 = (\varepsilon_1'-1)\varepsilon_1' + \varepsilon_1''^2.$$

In the absence of losses in the dielectric layer ($\varepsilon_1'' = 0$), the resonance frequency is determined by (5) and the impedance (10) diverges at this resonant frequency. Otherwise, for $\varepsilon_1'' > 0$ the resonant frequency is determined from the equation $R_1 = 0$ and is equal to $k_{res} = \sqrt{2e_1/a_1 d_1 e_0}$. At this frequency, the imaginary part of the impedance (10) vanishes, and its real part acquires the maximum value: $Z_{max} = Z_0(\pi a_1 \varepsilon_1'')^{-1}\sqrt{d_1 e_0 e_1/2a_1}$.

Expression (8) can be represented as a parallel connection of two impedances:

$$Z_{\parallel} = \left\{Z_{diel}^{-1} + Z_{res}^{-1}\right\}^{-1} \tag{11}$$

where Z_{diel} (10) gives the impedance of an ideal waveguide with an internal dielectric coating, and

$$Z_{res} = j\frac{Z_0}{\pi \tilde{k}_0 a_1^2}\left\{\frac{\tilde{k}_0}{k}\frac{k^{3/2}s_0^{3/2}}{k^{3/2}s_0^{3/2}-2(1+j)k^2/\tilde{k}_0^2}\right\}^{-1} \tag{12}$$

is the contribution from the finite conductivity of the outer wall material ($Z_{res} \to \infty$ at $s_0 \to 0$).

A further refinement of the asymptotic Eq. (8) is obtained using the first two terms of the expansion of the hyperbolic cotangent ($\coth(y) \approx 1/y + y/3$) in (4):

$$Z_{\parallel} = j\frac{Z_0}{\pi k a_1^2}\left\{1 + \frac{2}{a_1}\frac{\varepsilon_1}{\beta_1}\left(\frac{1}{\beta_1 d_1 + u^{-1}} + \frac{\beta_1 d_1 + u^{-1}}{3}\right)\right\}^{-1} \tag{13}$$

Here it is also possible to represent the total impedance in the form (11) with

$$Z_{res} = j\frac{Z_0}{2\pi k a_1^2}\left\{\frac{\varepsilon_1}{a_1 \beta_1 u}\left(\frac{1}{3} - \frac{1}{\beta_1 d_1}\frac{1}{\beta_1 d_1 + u^{-1}}\right)\right\}^{-1} \tag{14}$$

$$Z_{diel} = j\frac{Z_0}{\pi k a_1^2}\left\{1 + \frac{2}{a_1}\frac{\varepsilon_1}{\beta_1}\left(\frac{1}{\beta_1 d_1} + \frac{\beta_1 d_1}{3}\right)\right\}^{-1} = j\frac{Z_0}{\pi k a_1^2}R^{-1},$$

$$R_1 = f - \frac{k_0^2}{k^2}, \; R_2 = \frac{2}{3}\frac{\varepsilon_1''}{a_1 d_1}\left(d_1^2 + \frac{3}{e_0 k^2}\right), \; k_0 = \sqrt{\frac{2}{a_1 d_1 f}}\sqrt{\frac{e_1}{e_0}}, \; f = 1 + \frac{2}{3}\frac{d_1 \varepsilon_1'}{a_1} \tag{15}$$

The resonant frequency of Z_{diel} is determined from the condition $R_1 = 0$ and equal to k_0 (15). The case $d_1/a_1 \ll 1$ (thin inner layer) and $\varepsilon_2'' = 0$ (lossless dielectric) corresponds to the resonant frequency (5) with a diverging amplitude of Z_{diel}. For fixed $\varepsilon_1'' > 0$ the resonance amplitude of the impedance Z_{diel} (14) reaches its absolute maximum value when the roots of equations $R_1 = 0$ and $\partial(kR_2)/\partial k = 0$ coincide, which is satisfied when

$$d_1/a_1 = 3/\left\{2\left(\varepsilon_1'^2 - 2\varepsilon_1' + \varepsilon_1''^2\right)\right\}. \tag{16}$$

When condition (16) is satisfied, the resonance amplitude of Z_{diel} reaches an absolute maximum at $k_{max} = 2\left(\varepsilon_1'^2 - 2\varepsilon_1' + \varepsilon_1''^2\right)/\sqrt{3e_0}a_1$ and is equal to $Z_{diel}^{max} = \sqrt{3e_0}Z_0/4\pi a_1\varepsilon_1''$, which goes to infinity at $\varepsilon_1'' = 0$. In this case, with a high conductivity of the outer wall, the amplitude of the structure impedance (13) is equal to $Re\{Z_{res}\}_{k=k_{max}}$ which does not diverge and is close to the maximum value of the total impedance.
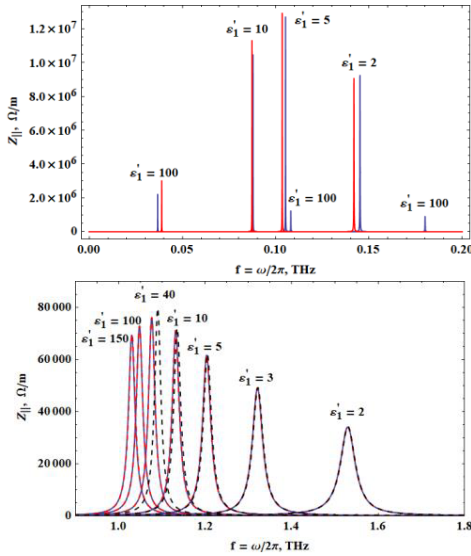
The presence of an absolute maximum of the longitudinal impedance of a two-layer metal-dielectric waveguide is confirmed by the exact solution (1) (figure 2, blue curves) and the asymptotic solution (4) (figure 2, red curves).

Figure 3 shows the longitudinal impedances given by asymptotic representations (8) and (13) and the exact solution. For a thick coating ($d_1/a_1 = 0.1$, figure 3, top), inequality (7) is not satisfied and the asymptotic (8) gives unreliable results (not shown in the figure). Nevertheless, it gives quite satisfactory results for a thin coating for low permittivity values ($\varepsilon_1' = 2, 3, 5, 10$, figure 3, bottom). The use of asymptotic (13) does not require inequality (7): it correctly reflects the situation with a large thickness and demonstrates complete agreement with the exact solution in the case of small thicknesses at any values of ε_1'. In particular, it correctly reflects the appearance of the maximum resonance amplitude under condition (16). For $d_1/a_1 = 0.1$ (figure 3, top), condition (16) corresponds to a dielectric constant of $\varepsilon_1' = 5$ (figure 3, top), and for $d_1/a_1 = 10^{-3}$ to a dielectric constant of $\varepsilon_1' = 40$ (figure 3, bottom). As follows from figure 3, the condition (16) obtained in the framework of the representation (15) adequately determines the resonant frequency which corresponds to the maximum amplitude of the impedance (13) predicted by the exact solution (1).

Figure 3. Longitudinal monopole impedance of the metal-dielectric waveguide for different values of dielectric constant, calculated by the exact formula (blue) and by approximations (13) (red) and (8) (black, dashed); $d_1 = 200\mu m$ (top) and $2\mu m$ (bottom); $a_1 = 2mm$.

5. Conclusion

Three different asymptotic solutions of the longitudinal monopole impedance of a two-layer metal-dielectric cylindrical waveguide are derived. The first is the asymptotic representation (4) of the impedance valid both for multimode and single-mode structures. It fully describes the frequency distribution of ultra-relativistic particle radiation in a waveguide and covers various combinations of the waveguide geometric and electromagnetic characteristics. The next asymptotic representation (8) is obtained for a single-resonant structure, which is valid for structures with a thin dielectric layer and comparatively low dielectric constant. The third asymptotic representation (13) describes the impedance of the first resonance and the maximum amplitude.

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