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MeV-scale Seesaw and Leptogenesis

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Abstract

We study the type-I seesaw model with three right-handed neutrinos and Majorana masses below the pion mass. In this mass range, the model parameter space is not only strongly constrained by the requirement to explain the light neutrino masses, but also by experimental searches and cosmological considerations. In the existing literature, three disjoint regions of potentially viable parameter space have been identified. In one of them, all heavy neutrinos decay shortly before big bang nucleosynthesis. In the other two regions, one of the heavy neutrinos either decays between BBN and the CMB decoupling or is quasi-stable. We show that previously unaccounted constraints from photodisintegration of nuclei practically rule out all relevant decays that happen between BBN and the CMB decoupling. Quite remarkably, if all heavy neutrinos decay before BBN, the baryon asymmetry of the universe can be quite generically explained by low-scale leptogenesis, i.e. without further tuning in addition to what is needed to avoid experimental and cosmological constraints. This motivates searches for heavy neutrinos in pion decay experiments.

Contents

1 Introduction

The observation of neutrino-flavour oscillations is one of the few hints for new physics beyond the Standard Model that have been discovered to date. In fact, it is the only one found in the laboratory that has been established beyond doubt. One way of explaining these oscillations is by adding right-handed neutrinos ν_R to the Standard Model (SM) of particle physics, thus giving mass to the light neutrinos [1–6]. Such a *type-I seesaw model* with right-handed neutrino masses below the electroweak scale is a minimal and testable extension of the SM that can simultaneously explain the generation of the observed neutrino masses as well as the baryon asymmetry of the Universe. With the absolute mass scale of the SM neutrinos being bounded only from above, most studies in this context merely consider two right-handed neutrinos, thus leaving one SM neutrino massless. This drastically reduces the complexity of the problem, and in many cases serves as a good proxy for the relevant dynamics. However, based on both theoretical and experimental considerations, it is necessary to go beyond this simplification. Firstly, all other fermions in the SM come in three generations and overarching concepts such as gauging the difference of baryon and lepton number – with a possible embedding in a grand unified theory – mandate the introduction of three generations of right-handed neutrinos. Secondly, the combination of a high-dimensional parameter space together with neutrino oscillation data, constrains the theory to highly non-trivial sub-manifolds of the parameter space, where the naive intuition gained from the simplified model with only two neutrinos may fail. Finally, predicting particles at an energy scale within the reach of collider experiments, a selling point of this model is its falsifiability. To guide future experimental efforts, it is thus mandatory to map out the full range of potential observables, especially since it is well known that the inclusion of three right-handed neutrinos can significantly change the experimentally viable parameter space [7], cosmological constraints [8], and the perspectives for leptogenesis [9]. In this work we study a comparably unexplored region of parameter space in which all heavy neutrinos have masses below the pion mass, kinematically limiting their decay products to SM neutrinos, electrons, positrons, and photons.

Suppressing SU(2) indices for brevity, the most general renormalisable Lagrangian including SM fields and the right-handed neutrinos ν_R reads

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{\nu_R}_i \partial\!\!\!/ \nu_{Ri} - \frac{1}{2}\left(\overline{\nu^c_R}_i (M_M)_{ij}\nu_{Rj} + \overline{\nu_R}_i (M_M^\dagger)_{ij}\nu^c_R\right) - F_{ai}\overline{\ell_L}_a \varepsilon\phi^*\nu_{Ri} - F^*_{ai}\overline{\nu_R}_i \phi^T\varepsilon^\dagger \ell_{La}\,. \quad (1)$$

Here ℓ_L and ϕ are the left-handed lepton and Higgs doublet of the SM, respectively, F is the matrix of Yukawa couplings, and ε denotes the totally antisymmetric SU(2) tensor. The Majorana mass matrix M_M introduces a new fundamental scale in nature, which is usually referred to as the *seesaw scale*. More precisely, for n flavours of ν_R, Eq. (1) contains n new dimensionful parameters that can be identified with the eigenvalues of M_M, which roughly coincide with the physical masses M_i of the heavy neutrino mass eigenstates N_i (see Eq. (4) below). The phenomenological and cosmological implications of the ν_R's existence strongly depend on the choice of the seesaw scale(s) (cf. e.g. [10] for a review).

A particularly intriguing feature of this model is the fact that the same ν_R that give masses to the light neutrinos can also explain the observed matter-antimatter asymmetry in the early universe, which is believed to be the origin of all baryonic matter that is present today.[1] This is realized via the process of *leptogenesis* [12], which is feasible for a very wide range of possible M_i (see Ref. [13] for a recent review). For M_i above the electroweak scale, the asymmetry is typically generated during the freeze-out and decay of the heavy neutrinos [12] ("freeze-out scenario"), while for M_i below the electroweak scale, it is instead generated during their production [14–16] ("freeze-in scenario").[2] It is well-known that leptogenesis is in principle feasible with M_i in the range of a few MeV [18]. However, in this mass range, the model parameter space is strongly constrained by laboratory experiments, cosmology, and astrophysics.

Constraints on the properties of heavy neutrinos are conveniently expressed in terms of the mixing angles θ_{ai}, (cf. Eq. (3) below). In fact, for given M_i, the values of θ_{ai} determine the thermal N_i production rate in the early universe, the N_i lifetime, the N_i contribution to the generation of light neutrino masses, and the N_i production cross-section in experiments. For masses below ~ 100 MeV and values of θ_{ai} that are small enough to satisfy exclusion bounds from various laboratory experiments, the heavy neutrinos tend to have lifetimes larger than 0.1 s. This means that their presence in the primordial plasma and their decay may affect cosmological observables, such as the abundances of light elements that are produced during big bang nucleosynthesis (BBN), or the anisotropies in the cosmic microwave background (CMB). The resulting constraints on M_i and θ_{ai} have e.g. been summarised in [19].[3] Usually, these limits can be avoided for sufficiently small values of θ_{ai}. However, since the mixing angles also govern the size of the light neutrino masses, there exist additional lower bounds on different combinations of θ_{ai} from the requirement to explain the observed light neutrino oscillation parameters. These lower bounds depend on the number n of right-handed neutrino flavours and the mass m_{lightest} of the lightest neutrino (cf. [22] for a recent discussion). In the minimal model with $n = 2$ and $m_{\text{lightest}} = 0$, the seesaw mechanism necessarily enforces that all N_i reach thermal equilibrium if their masses are below ~ 100 MeV [23]. In combination with bounds from direct searches, this practically rules out the entire mass range below ~ 100 MeV (~ 350 MeV) for normal (inverted) ordering of the light neutrino masses [24].[4]

In the next-to-minimal model with $n = 3$ considered here, one of the N_i – which we may call N_1 without loss of generality – can have small enough mixings θ_{a1} to avoid equilibration in the early universe and hence is no longer constrained by the lower bound on M_i if $m_{\text{lightest}} \lesssim 10^{-3}$ eV [8].[5]

[1] The evidence for a matter-antimatter asymmetry in the observable universe and its connection to the origin of matter are e.g. discussed in ref. [11].

[2] The statement that the freeze-out scenario works for M_i above the electroweak scale and the freeze-in scenario works for M_i below the electroweak scale should be thought of as a rule of thumb. In fact, both mechanisms overlap between roughly ~ 5 GeV and the TeV scale [17].

[3] The authors in [19] ruled out lifetimes longer than the CMB decoupling time by rescaling the CMB bounds on decaying Dark Matter particles found in [20]. These were obtained under the assumption that the particles have a lifetime that exceeds the age of the universe and can therefore strictly speaking not be applied in all of the parameter space considered here. However, it turns out that the parameter region where this rescaling is not applicable is ruled out by the results obtained in [21], so that we can safely apply the bounds presented in [19] here.

[4] The authors of [24] assumed a mass degeneracy among the N_i. However, since both, the lifetime bound from BBN and constraints from direct searches in good approximation apply to each N_i individually, this can at most introduce a factor 2 in the upper bound on the mixing (if the two N_i cannot be distinguished kinematically), which will not change these conclusions.

[5] We do not consider the small window of M_1 in the eV range that was reported in [8] because the scenario of a eV

This leaves three distinct regions of parameter space for models with $n = 3$ and all M_i below the pion mass. In **scenario I)** all three N_i decay before BBN. A global fit of direct and indirect experimental constraints in this region has recently been performed in [7]. In **scenario II)** two of the N_i decay before BBN. The third one never reaches thermal equilibrium and decays between BBN and the decoupling of the CMB. In **scenario III)** two of the N_i decay before BBN. The third one is quasi-stable and contributes to the Dark Matter [27,28]. This scenario corresponds to the well-known *Neutrino Minimal Standard Model* (νMSM) [15, 29].

In the present work, we present two new results regarding these scenarios. Firstly, we demonstrate that scenario II) is ruled out when combining previously unaccounted constraints from photodisintegration after BBN with constraints from CMB anisotropies [21] and the ionisation of the intergalactic medium [20]. Secondly, we find that the baryon asymmetry generated in scenario I) generically is of the right order of magnitude to explain the observed matter-antimatter asymmetry. This surprising result indicates that within the highly constrained region of parameter space where all experimental constraints are satisfied, no or little additional tuning is needed for successful leptogenesis. These results extend the previous parameter scan of scenario I) in [9] to smaller masses, $M_i \gtrsim 50$ MeV. Finally, let us note that we do not consider baryogenesis in scenario III) and instead refer the reader to [18,30] for a comprehensive overview and to [17,24,31–34] for recent updates on the viable parameter space in this model. The remainder of this article is organised as follows. In Sec. 2, we summarise the existing laboratory and cosmological constraints, before introducing our new bound from photodisintegration after BBN in Sec. 2.3. We comment on the supernova bound in Sec. 2.4, which could rule out the entire scenario I) but comes with some uncertainties. We summarise all constraints in Sec. 2.5, demonstrating that the neutrino oscillation data can be accounted for in the remaining parameter space. Sec. 3 is dedicated to the study of leptogenesis in scenario I), followed by a brief conclusion in Sec. 4.

2 Laboratory, cosmological and astrophysical constraints

2.1 Laboratory constraints

The strongest experimental constraints on the heavy neutrino properties come from the requirement to explain the light neutrino oscillation data. If the eigenvalues of the Majorana mass matrix M_M are at least a few eV in magnitude, there exist two distinct sets of mass eigenstates after electroweak symmetry breaking, which can be represented by the flavour vectors of Majorana spinors

$$\nu \simeq U_\nu^\dagger \left(\nu_L - \theta \nu_R^c\right) + c.c. \quad , \quad N \simeq U_N^\dagger \left(\nu_R + \theta^T \nu_L^c\right) + c.c. \,. \tag{2}$$

Here *c.c.* denotes the c-conjugation which e.g. acts as $\nu_R^c = C\overline{\nu_R}^T$ with $C = i\gamma_2\gamma_0$, U_ν is the standard light neutrino mixing matrix, and U_N is its equivalent among the heavy neutrinos. The mixing between left- and right-handed neutrinos is quantified by the entries of the matrix

$$\theta = vFM_M^{-1} \,, \tag{3}$$

seesaw [25] is meanwhile even more disfavoured by cosmological date [26].

with the Higgs field expectation value v, and the mass matrices for ν and N are given by

$$m_\nu = -\theta M_M \theta^T \ , \ M_N = M_M + \frac{1}{2}(\theta^\dagger \theta M_M + M_M^T \theta^T \theta^*)\,. \tag{4}$$

The squares of the physical masses m_i and M_i of ν_i and N_i, are given by the eigenvalues of the matrices $m_\nu^\dagger m_\nu$ and $M_N^\dagger M_N$. Here we work at tree level and expand all expressions to second order in the small mixing angles θ_{ai}. The ν_i can be identified with the well-known light neutrinos, while the N_i are new heavy (almost) sterile neutrinos. Their masses M_i coincide with the eigenvalues of M_M up to $\mathcal{O}(\theta^2)$ corrections in Eq. (4). Within the pure seesaw model in Eq. (1), the N_i interact with the SM only through their mixing with the doublet fields ν_L in Eq. (2), which practically leads to a θ-suppressed weak interaction.

The requirement to explain the observed light neutrino mass splittings $m_i^2 - m_j^2$ as well as the mixing angles in the matrix U_ν imposes constraints on the matrix m_ν, and therefore on F and M_M. At low energies, this leads to restrictions on the relative size of the heavy neutrino mixing with individual SM flavours [7, 24, 31, 35–39], i.e. on the quantities U_{ai}^2/U_i^2 with $U_i^2 = \sum_a U_{ai}^2$ and

$$U_{ai}^2 = |\Theta_{ai}|^2 \quad \text{with} \quad \Theta = \theta U_N^*\,. \tag{5}$$

There also is a lower bound on the different U_i^2 from neutrino oscillation data [36, 37] which roughly reads $U_i^2 > m_{\rm lightest}/M_i$ (cf. [22] for a recent discussion).

The presence of weak interactions implies that a wide range of experiments is sensitive to the existence of the heavy neutrinos. An updated overview of the existing constraints that we are aware of can be found in [7]. Broadly speaking, one can distinguish between direct and indirect searches. Direct searches are experiments in which the N_i appear as real particles. If kinematically allowed, the N_i production cross-section is roughly given by $\sigma_{N_i} \sim \sum_a U_{ai}^2 \sigma_{\nu_a}$, with σ_{ν_a} being the production cross-section for a SM neutrino ν_a. Hence, direct searches always impose upper bounds on the different U_{ai}^2. For sub-GeV masses this mainly includes beam dump experiments and peak searches. Indirect searches include precision tests or searches for rare processes in the SM that are indirectly affected by the existence of the heavy neutrinos, e.g. through the modification of the light neutrinos' interactions via the mixing θ. In the mass range considered here, direct searches strongly dominate,[6] in particular from PIENU [41, 42], KEK [43], LBL [44], SIN [45], TRIUMF [46] and CHARM [47] (cf. also [48–50]). All of these constraints are summarized in the grey regions in Fig. 1. The only indirect constraint that is relevant in this region comes from neutrinoless double β-decay ($0\nu\beta\beta$). However, the rate of the $0\nu\beta\beta$-decay can be suppressed even for mixing angles that are orders of magnitude larger than the ones considered here if one requires that the Lagrangian in Eq. (1) approximately conserves a generalisation of the SM lepton number L (more precisely, the difference between baryon number B and L) under which the heavy neutrinos are charged [51, 52],[7] and the current bound on the $0\nu\beta\beta$-lifetime rules out

[6] For a more complete listing see the pdgLive page on HNLs [40].

[7] In Ref. [52] it was pointed out that imposing a generalised $B - L$ symmetry can lead to a parametric suppression of all lepton number violating observables. This suppression indeed happens for the $0\nu\beta\beta$-decay and for the (Majorana) masses m_i of the light neutrinos (where it is necessarily needed to allow for mixings $U_i^2 \gg \sqrt{\sum_j m_j^2}/M_i$ without tuning).

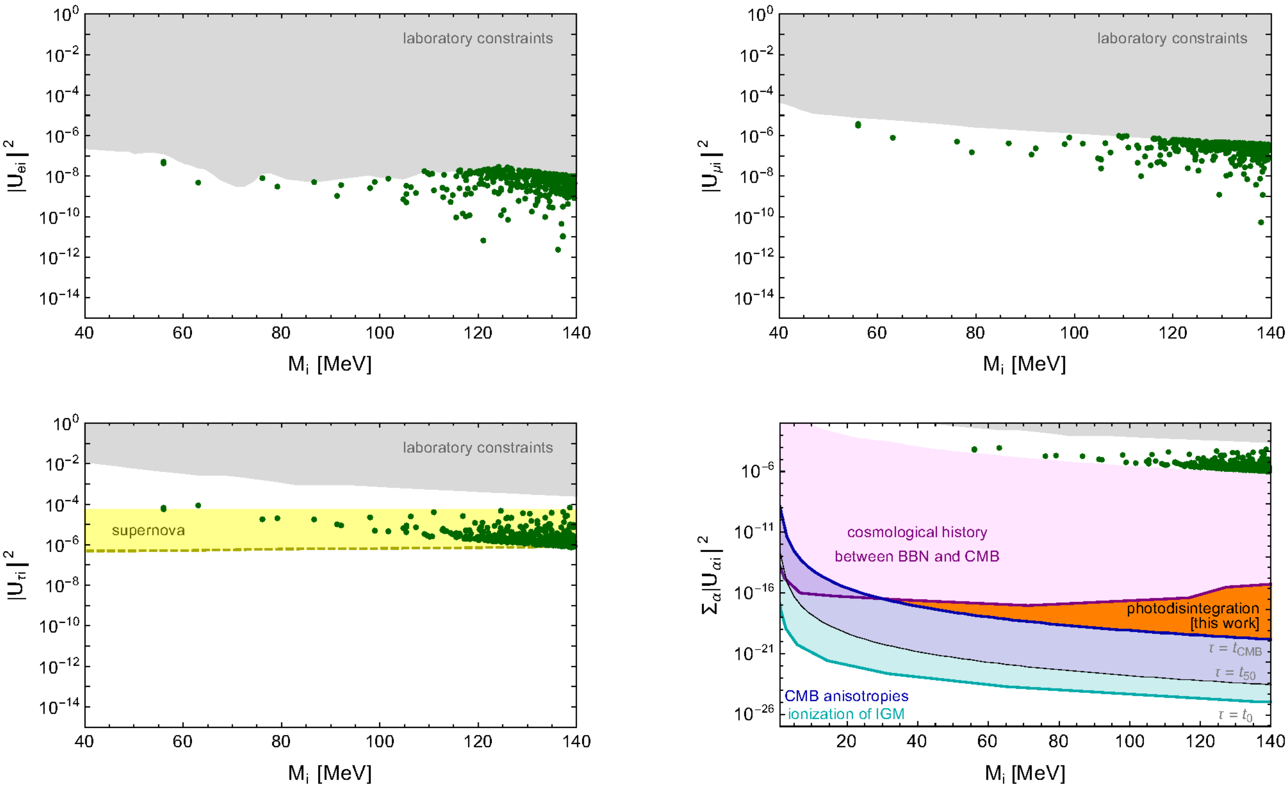

Figure 1: Constraints on the sterile neutrino mixing with SM neutrinos. The shaded regions are excluded by laboratory constraints (grey, see Sec. 2.1), cosmological constraints from Refs. [19] (pink), [21] (blue) and [20] (cyan) (see Sec. 2.2) and by the photodisintegration bound (orange, this work, Sec. 2.3). The supernova constraint from Ref. [55] is indicated by the yellow shaded region (see Sec. 2.4). The green dots show realisations of neutrino masses and mixings which reproduce the neutrino oscillation data (see Sec. 2.5). Every parameter point is represented by a triplet (one point for each N_i).

only a small fraction of the leptogenesis parameter space [9, 31, 54].

2.2 Summary of previously known cosmological constraints

Sterile neutrinos in the $\mathcal{O}(10-100)$ MeV mass range can alter our cosmological history and are hence strongly constrained by observations related to BBN and the CMB. Here we distinguish three cases, depending on the lifetime τ_i of the sterile neutrino N_i [56],

$$\tau_i^{-1} \approx 7.8\,\mathrm{s}^{-1} \left(\frac{M_i}{10\text{ MeV}}\right)^5 \left[1.4\, U_{ei}^2 + U_{\mu i}^2 + U_{\tau i}^2\right] . \tag{6}$$

(i) Short-lived N_i. If N_i decays significantly before BBN, its decay products are fully thermalised and merely lead to a shift in the overall temperature of the thermal bath, which only shifts the onset of BBN. Hence, the highly constrained process of nucleosynthesis as well as the post-BBN cosmic history remain largely unaltered. This condition results in an upper bound on the lifetime of N_i of $\mathcal{O}(0.2-1)$ s for 30 MeV$\leq M_i \leq$140 MeV [57–61]. Such short lifetimes require a sizeable mixing with SM neutrinos (above the region labeled '*cosmological history between BBN and CMB*' in Fig. 1), which leads to a

However, in the mass range considered here, this symmetry does not suppress lepton number violating signatures in collider based experiments [53].

non-trivial interplay with the laboratory constraints discussed above (gray region in Fig. 1).[8]

(ii) Long-lived N_i. Heavy neutrino decays during BBN would directly alter the formation of light elements. If N_i decays after BBN (but before CMB decoupling) it can impact the post-BBN cosmological history. In extreme cases, the non-relativistic sterile neutrinos can even come to dominate the energy budget of the Universe. Moreover, their decay leads to an entropy injection into the SM thermal bath. This leads to an upper bound on the mixing $\sum_\alpha |U_{\alpha i}|^2$ of N_i with the SM neutrinos ν_α [19] (region labeled '*cosmological history between BBN and CMB*' in Fig. 1).[9] A further constraint arises from the effective number of relativistic degrees-of-freedom $N_{\rm eff}$ at the time of BBN. As was demonstrated in Ref. [8], at least two out of the three sterile neutrinos temporarily reach thermal equilibrium - and consequently a sizeable abundance - in the early Universe. This leads to a significant contribution to $N_{\rm eff}$ during BBN if the sterile neutrino is relativistic at decoupling. We find the resulting upper bound on the mixing between active and sterile neutrinos to be weaker than the constraint derived in [19] in the parameter space of interest.

(iii) Quasi-stable N_i. N_i lifetimes beyond the time of CMB decoupling ($\sim 10^{12}$ s) are highly constrained by CMB observations [21,62] (region labeled '*CMB constraints*' in Fig. 1), the impact of their decays on the intergalactic medium (IGM) [20,63] (region labeled '*IGM constraints*' in Fig. 1), and the produced X-rays [64]. The constraints can be avoided for sufficiently small θ_{ai}, which suppresses both the thermally produced abundance and the N_i decay rate. In the mass range considered here, such a long lifetime requires mixing angles that are so tiny that the amount of thermally produced N_i is negligible for all practical purposes (white region at the bottom of Fig. 1).[10]

This leads to the three distinct regions I)-III) of the parameter space which survive both the laboratory and cosmological constraints. In scenario I) all three sterile neutrinos belong to population (i). They have relatively large mixing angles, and thermalise and decay before BBN. This region is found by applying the bound on the lifetime of sterile neutrinos from Refs. [59–61]. In scenario II), two of the heavy neutrinos N_2 and N_3 belong to population (i). The third heavy neutrino N_1 features significantly smaller mixings θ_{a1} with the SM states and belongs to population (ii). N_1 avoids thermalisation [8] and obeys the bounds derived in Ref. [19]. More precisely, we use the bound depicted in Fig. 2 of Ref. [19], which leaves open a window for $M_1 \gtrsim 50$ MeV and $|\theta_{ai}|^2 < 10^{-14}$. However, as we will see in the following Sec. 2.3, this window is closed if the effect of N_i decays on photodisintegration of nuclei is taken into account. Scenario III) is similar to scenario II), but N_1 has even smaller mixings and is part of population (iii).

[8]Note that these cosmologically "short-lived" N_i are still classified as "long-lived particles" from the viewpoint of accelerator-based experiments. Their decay length e.g. exceeds the size of the LHC main detectors.

[9]Note that the setup of Ref. [19] contains only one sterile neutrino which couples exclusively to ν_e. Taking into account the actual flavour structure in the couplings, we impose the bound derived in Ref. [19] on the mixing summed over all SM flavours.

[10] For masses in the keV range, Eq. (6) permits mixing angles that are large enough that thermally produced N_i can make up a considerable fraction of the DM and the bounds summarised in [65,66] should be applied.

2.3 Additional constraints from photodisintegration of nuclei

Further, the N_i decay can also disintegrate nuclei in the primordial plasma after BBN. The resulting bound strongly depends on the hadronic branching ratio of the decay, which vanishes for the mass range considered here, and does not affect any of the points in our sample [67]. Hence, we only have to take into account electromagnetic decay channels, which we incorporate via the procedure described in [68].[11] The corresponding code will be published in [69]. On that note, we first determine the non-thermal photon/electron-spectra by solving the full cascade equation [70] with the appropriate source terms $\propto n_i/\tau_i$. The resulting spectra are then used to determine the late-time modifications of the nuclear abundances via photodisintegration by solving the appropriate non-thermal Boltzmann equation. Finally, we compare the resulting abundances with the most recent set of observations [71, 72]. Specifically, we use

$$\mathcal{Y}_{\mathrm{p}} = (2.45 \pm 0.03) \times 10^{-1}\,, \tag{7}$$

$$\mathrm{D}/{}^{1}\mathrm{H} = (2.547 \pm 0.025) \times 10^{-5}\,, \tag{8}$$

$${}^{3}\mathrm{He}/\mathrm{D} = (8.3 \pm 1.5) \times 10^{-1}\,. \tag{9}$$

The resulting constraints are shown in the lower right panel of Fig. 1 (orange) and we find that these additional limits are particularly important for closing the region of parameter space between the solid blue and purple line, i.e. the region that is otherwise neither excluded by CMB observations nor by a modified cosmological history between BBN and CMB.

2.4 Supernovae bound

The detection of SN 1987A neutrinos arriving over an interval of about 10 s, in agreement with the predictions of a core-collapse supernova with the standard cooling scenario, imposes constraints on the existence of light BSM particles which would constitute an additional channel of energy-loss, shortening the duration of the neutrino burst [73]. This has in particular been used to constrain axions [73], dark photons [74], and sterile neutrinos of different mass ranges [55, 57, 58, 75–78]. The constraints are particularly relevant for the mixing with ν_τ, since the laboratory constraints are weakest in this case. We indicate the constraints found in [55] by the yellow shaded area in the bottom left panel of Fig. 1.

However, as it has been recently pointed out in Ref. [79], these bounds rely on the standard core-collapse supernova model. If instead the supernova is modelled by a collapse-induced thermo-nuclear explosion [80], the observed neutrino signal could stem from the accretion disk and would be insensitive to the cooling rates. With this in mind, we do not apply the supernova bounds of Ref. [55] in our main analysis, but emphasize that this region of parameter space can be fully probed in the near future - both by laboratory and astrophysical observations.

[11] Decays into SM neutrinos do not lead to photodisintegration and therefore can be neglected.

2.5 Viable parameter space

The photodisintegration bound introduced in Sec. 2.3 excludes all points of type (ii) in the mass range considered here and therefore rules out scenario II). As already stated in the introduction, the phenomenology of scenario III) corresponds to that of the much-studied νMSM and shall not be further investigated here. This leaves us with scenario I). A priori it is not clear whether there are any viable parameter choices for which all bounds can be fulfilled *simultaneously*. This is non-trivial because neutrino oscillation data restricts the flavour mixing pattern, i.e. the range of allowed values for U_{ai}^2/U_i^2, meaning that it may not be possible to fit all three N_i into the allowed (white) parameter regions in Fig. 1. It is well-known that this considerably constrains the range of allowed masses M_i below the kaon mass in scenario III) [24].[12] For scenario I) this question has been studied in [7], where it was found that the combination of all experimental and cosmological bounds indeed leaves a sizeable region of viable parameter space with M_i below the pion mass. However, the analysis in that work did not include the supernova bound discussed in Sec. 2.4. A complete scan of the allowed parameter region is numerically extremely expensive because of the high dimensionality of the parameter space (18 free parameters) and the complicated shape that the sub-manifolds defined by the various experimental constraints in the mass region considered here form in this space. Instead, we perform a limited scan with randomised parameter choices. We use the radiatively corrected [82] Casas-Ibarra parameterisation [83]. For the mass splittings and the complex angles in the Casas-Ibarra parameterisation, we alternate between drawing our parameters from a linear versus a logarithmic distribution, as in Ref. [9]. We apply all experimental and cosmological constraints summarised above. For the experimental bounds, we use the simple strategy adapted in Refs. [24, 84] and interpret the exclusion regions published by the experimental collaborations as hard cuts (rather than using full likelihood functions as in Ref. [7]), which is sufficient for the purpose of this work. For the lifetime constraints from BBN we use the results from [59].[13] We show a representative set of viable parameter points (indicated by green dots) that are consistent with all experimental and cosmological constraints in Fig. 1. These all correspond to the normal ordering of the SM neutrinos. Taking into account that each parameter point is represented by a triplet of points in Fig. 1 (one for each sterile flavour), applying the supernova bound from Ref. [55] would exclude all points shown. However, as pointed out in Sec. 2.4, this bound strongly relies on the underlying model for the supernova explosion.

3 Baryogenesis

We now proceed to compute the baryon asymmetry for all viable parameter points found in our scan, using the set of quantum kinetic equations given in Sec. 2 of Ref. [9] to describe the evolution of the heavy neutrino abundances and lepton asymmetries in the early universe.[14]

[12] The constraints on N_2 and N_3 in this scenario are practically identical to those in the model with only two heavy neutrinos because N_1 cannot make a measurable contribution to the seesaw mechanism [81]. Bearing in mind the caveat already pointed out in footnote 4 the results found in section 2 of [24] can be applied to scenario III).

[13] The more recent bounds from [60] agree with those. In [61] only M_i above the pion mass were considered, but the authors indicate that an upcoming analysis will provide stronger bounds below the pion mass.

[14] The momentum dependent sets of kinetic equations derived in Refs. [85, 86] are more accurate than the momentum averaged equations used in Ref. [9], but require a much larger numerical effort. Since the results are typically comparable

We assume that the radiation dominated epoch of the cosmic history started with a matter-antimatter symmetric primordial plasma in which all SM particles were in thermal equilibrium at a temperature T_R that was much hotter than the temperature $T_{\rm sph} \simeq 131$ GeV [88], above which electroweak sphalerons efficiently convert L and B into each other [89]. In inflationary cosmology, this is expected because pre-inflationary asymmetries would be diluted very efficiently by the cosmic expansion. We moreover take the initial abundance of the heavy neutrinos to be negligible. The Lagrangian in Eq. (1) then contains all the necessary ingredients to generate the baryon asymmetry of our Universe: The heavy neutrinos are generated from thermal interactions in the plasma through their Yukawa couplings. In this out-of-equilibrium situation, the interplay of coherent neutrino oscillations and decoherent scatterings mediated by the CP-violating Yukawa couplings can generate a lepton asymmetry that is partially converted into a baryon asymmetry by the sphalerons. For the M_i under consideration here, and in view of the experimental constraints on the U_i, this process happens very slowly. If at least one heavy neutrino has not reached thermal equilibrium at $T \sim T_{\rm sph}$, then the baryon asymmetry is preserved ("frozen in") at lower temperatures. This *freeze-in leptogenesis* mechanism, also known as Akhmedov-Rubakov-Smirnov (ARS) leptogenesis [14], has been studied by many authors, a review is e.g. given in Ref. [90]. Our goal is to study the question of whether the observed baryon asymmetry of the Universe can be explained in scenario I) while respecting the constraints discussed in Sec. 2 if all M_i are smaller than the pion mass.

Our results are shown in Fig. 2. Remarkably, if we consider the population where all three sterile neutrinos decay before BBN, the predicted baryon asymmetry is generically in the correct ball-park to explain the observed value or larger. This is far from trivial since it is well known that marginal changes in the model parameters can lead to drastic changes in the resulting baryon asymmetry, due to the fine balance between generation and wash-out of the asymmetries.

4 Discussion and conclusion

We study the type-I seesaw model with three heavy neutrinos N_i with masses below the pion mass. This part of the parameter space is relatively little studied, but in fact, experimental searches allow for relatively large mixing angles due to the absence of hadronic decay channels. There are three scenarios that are allowed by previously published constraints: I) all three N_i decay before BBN, II) two N_i decay before BBN and the third one decays between BBN and the CMB decoupling, and III) two N_i decay before BBN and the third one has a lifetime that greatly exceeds the age of the universe. We focus on scenarios I) and II); scenario III) resembles the νMSM, which has been studied in great detail in the literature. We find that scenario II) is ruled out by the effect that photons produced in a cascade from the long-lived N_i decay would have on the disintegration of light elements in the IGM. In scenario I) a representative randomised parameter scan shows that there are viable parameter values for which the N_i can avoid all constraints from experiments and cosmology for $M_i > 50$ MeV. All these points can potentially be ruled out by the observed neutrino flux from the supernova 1987a, but this conclusion depends on the modelling of the supernova explosion. Quite surprisingly most of the viable

[87] we opted for the simpler approach in the present work.

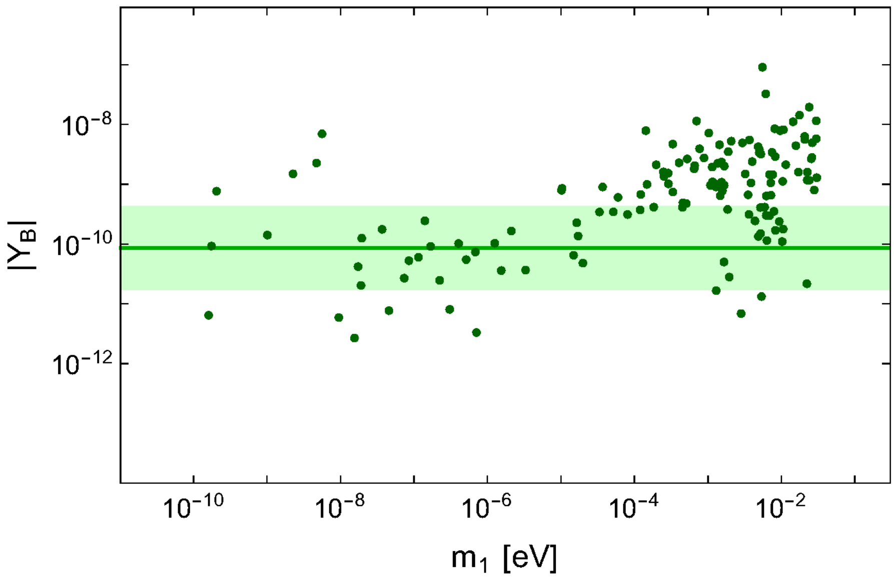

Figure 2: Predicted baryon asymmetry as a function of the lightest SM neutrino mass in scenario I). The green line indicates the observed value, the shaded region indicates an order of magnitude variance.

points give a final baryon asymmetry in the correct ball-park to explain the observed value. Given the well-known strong sensitivity of the relevant Boltzmann equations to small changes in the parameters, this is a highly non-trivial result. Our results provide a proof-of-existence for this viable leptogenesis scenario, but this is by no means an exhaustive study. Due to the high-dimensional parameter space, this requires more sophisticated numerical techniques, but we hope that the results presented here will trigger further work in this direction. This will be crucial in guiding experimental effort in fully testing freeze-in leptogenesis as the mechanism to generate the baryon asymmetry of our Universe.

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