Mathematical Aspects of String Compactifications

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In this report we summarize the research in mathematics and theoretical physics done in project A6 of the SFB 676. The main research directions include the study of internal spaces of string compactifications, geometry of scalar manifolds and moduli spaces, heterotic/type II string duality, partial breaking of supersymmetry and supersymmetric AdS backgrounds. Typical results are the construction of a wealth of new inhomogeneous complete quaternionic Kähler manifolds based on a combination of ideas from supergravity and perturbative string theory with new mathematical tools and the derivation of low energy effective actions in physically relevant situations, among other examples.

1 Geometry of compactification spaces

One of the strands of this project has been the study of G-structures of the type occurring on internal spaces of string compactifications. From a mathematical point of view we were specially interested in Einstein metrics and, in particular, in metrics of special holonomy. From a physics point of view the study of manifolds with G-structures are of interest for two reasons. Firstly they correspond to backgrounds which can exhibit spontaneous supersymmetry breaking. This is a necessary ingredient in order to make contact with experimental observations in particle physics and cosmology where no sign of supersymmetry is so far manifest. Secondly, they appear as the mirror duals of string backgrounds with fluxes. Indeed in [1] the specific class of manifolds with $SU(3) \times SU(3)$ structure where identified as mirror duals of type II compactifications on Calabi–Yau manifolds with magnetic fluxes.

An example of the G-structures we have investigated are half-flat $SU(3)$-structures on six-dimensional manifolds, which are related to Ricci-flat metrics of holonomy a subgroup of G_2 on seven-dimensional manifolds. In fact, a half-flat $SU(3)$-structure is precisely the structure induced on a hypersurface in a manifold with a parallel G_2-structure. By considering a family of equidistant hypersurfaces one arrives at a system of evolution equations for the $SU(3)$-structure known as Hitchin's flow equations. Similar flow equations are obtained by considering foliations by equidistant hypersurfaces in ambient manifolds with a parallel $SU(3)$- or $Spin(7)$-structure. It was shown by Hitchin [2] that for a given initial half-flat $SU(3)$-structure the system has a unique maximal solution on every compact 6-manifold M by exhibiting it as a Hamiltonian system. The solution is a Riemannian metric of holonomy contained in G_2. It is defined on the product $M \times I$, where I is an interval. The Hamiltonian is defined by integration of a certain invariant over the compact manifold. Based on a different approach, in [3] we extend this theory allowing for non-compact manifolds and non-compact structure groups. When $\dim M = 6$, this includes half-flat G-structures on possibly non-compact manifolds M as initial data for metrics of holonomy contained in the compact or in the non-compact form of G_2, depending on whether the structure group $G \in \{SU(3), SU(1,2), SL(3,\mathbb{R})\}$ is compact or non-compact.

Lie groups admitting a left-invariant half-flat $SU(3)$-structure have been classified in [4–6]. For some of these groups we have determined all left-invariant half-flat $SU(3)$-structures and have solved the Hitchin flow equations with these structures as initial data [3, 7] obtaining metrics with holonomy group G_2 or its noncompact form G_2^*. A particularly interesting case analysed in [7] is the classification of all left-invariant half-flat $SU(3)$-structures on $S^3 \times S^3$. This includes nontrivial deformations of the nearly Kähler structure and Einstein half-flat $SU(3)$-structures. Classification results for certain types left-invariant G_2-structures have been obtained in [8, 9]. Solving the Hitchin flow with some of these structures as initial data, metrics with holonomy group $SU(4) \subset Spin(7)$ have been constructed in [10].

The Ricci-flat Riemannian manifolds of special holonomy obtained by solving the flow equations are in general incomplete, even if the initial manifold M is compact or homogeneous. In [11] we study under which conditions a maximal incomplete solution of the flow equations with homogeneous initial data can be completed to a complete Riemannian manifold of special holonomy. We restrict to the case when M admits a simply transitive action of a solvable Lie group preserving the initial G-structure, where $G \in \{SU(2), SU(3), G_2\}$ and $\dim M = 5, 6, 7$ respectively. We prove under certain natural assumptions in this setting that a completion as above does not exist. As a consequence, the classes of solvable Lie groups considered in our work cannot act with co-homogeneity one on a complete and non-flat Riemannian manifold with a parallel $SU(3)$-, G_2-, or $Spin(7)$-structure. These results do not apply to semi-simple Lie groups, such as $SL(2,\mathbb{C})$. For the latter group we classify all left-invariant half-flat $SU(3)$-structures which are also right-invariant under the maximal compact subgroup $SU(2) \subset SL(2,\mathbb{C})$ and solve the Hitchin flow with these structures as initial data. The solutions are G_2-holonomy metrics defined on $SL(2,\mathbb{C}) \times (a,b)$, where $-\infty < a < b < \infty$. Some of them can be completed at one boundary point of the interval (a,b) but never at both.

We have mentioned above the class of half-flat $SU(3)$-structures. The manifolds carrying these structures can be considered as generalizations of the well known Calabi–Yau three-folds, which are Ricci-flat and Kähler. The differential system satisfied by a half-flat $SU(3)$-structure does not imply the Einstein equations for the metric on the underlying 6-dimensional (real) manifold. Examples of half-flat $SU(3)$-structures which are Einstein include the so-called nearly Kähler structures. Until recently, the only known complete nearly Kähler manifolds were the homogeneous ones, classified by Butruille [12]. In [13] we show that inhomogeneous, locally homogeneous examples exist in abundance. These are obtained as quotients by a finite group of automorphisms acting freely on the simply connected 3-symmetric space

$$(SU(2) \times SU(2) \times SU(2))/SU(2) \cong S^3 \times S^3,$$

where the isotropy group $SU(2)$ is diagonally embedded. These quotients include co-homogeneity one examples. Simply connected examples of co-homogeneity one (diffeomorphic to $S^3 \times S^3$) were later constructed by Foscolo and Haskins [14].

The nearly Kähler metric on $S^3 \times S^3$ has the special property of being a left-invariant Einstein metric, a property shared by the product metric. The simply connected homogeneous Einstein manifolds in dimension 6 have been completely classified with exception of the case of left-invariant Einstein metrics on $S^3 \times S^3$, see [15] for the state of the art. In [16] we classify left-invariant Einstein metrics on $S^3 \times S^3$ under the additional assumption that the stabilizer in the group of proper isometries is neither trivial nor $\mathbb{Z}_2$. Under this assumption we find that the metric is either the left-invariant nearly Kähler metric or the product metric, which have the stabilizer $SU(2)$ and $SU(2) \times SU(2)$ respectively.

The relation of string theory with particle physics and cosmology is facilitated via the low-energy effective action which consists only of the light modes of string theory while all heavy excitations have been integrated out. String backgrounds with spontaneously broken supersymmetry correspond to a supersymmetric effective action with a scalar potential whose minimum breaks (part of the) supersymmetry. Apart from the scalar potential the metric of the scalar fields in the low-energy effective action is of prime importance. Via supersymmetry it fixes many other physical interesting terms in the action. This metric is the metric on the moduli space of the string compactifications at hand. Thus not only the compactification manifolds but almost more importantly its moduli space contains vital physical information.

Manifolds with $SU(2) \times SU(2)$ structure of dimension four, five and six have been investigated in [17,18]. Such compactifications correspond to backgrounds with spontaneously broken $\mathcal{N} = 4$ supersymmetry. In these cases we determined the moduli space and established the consistency with $\mathcal{N} = 4$ supergravity. Depending on the structure of the intrinsic torsion, anti-symmetric tensor fields can become massive in some of these cases. $\mathcal{N} = 2$ orientifolds of these background were studied in detail in [19]. Massive tensor fields and their coupling to three-forms were studied in [20]. (These research projects were also part of the three PhD-theses [21–23] and the Master-thesis [24].)

Furthermore, we studied type II backgrounds with spontaneously broken $\mathcal{N} = 2$ supergravity in [1,25,26]. In [25] we showed that the low-energy effective action of such backgrounds displays the U-duality group $E_{7(7)}$. In particular we derived $E_{7(7)}$-invariant expressions for the Kähler and hyper-Kähler potentials describing the moduli space of vector and hypermultiplets together with the Killing prepotentials defining the scalar potential. In [26] we incorporated perturbative quantum correction in this formalism.

In collaboration with project A1 we also studied heterotic backgrounds with G-structure. This is summarized in the PhD-thesis [27]. M-theory backgrounds with G-structure were studied in [28].

2 Special geometry

Another strand of this project has been the study of the scalar geometry of the low energy limit of string theory. The relevant geometries for type II string theory are governed by $\mathcal{N} = 2$ supergravity and its quantum corrections. They occur in three basic variants: projective special real geometry, projective special Kähler geometry, and quaternionic Kähler geometry. Quaternionic Kähler manifolds are examples of Einstein manifolds and have therefore been intensively studied in mathematics. Despite this fact, there are many open questions in this area and examples are scarce. The three special geometries mentioned above are intimately related by geometric constructions known as the r- and the c-map originating from the dimensional reduction of supergravity theories, respectively from 5 to 4 and from 4 to 3 space-time dimensions [29,30]. We show in [31] that the supergravity r-map and the supergravity c-map do both preserve the completeness of the underlying metrics. As a consequence, every complete projective special real manifold of dimension n gives rise (by the r-map) to a complete projective special Kähler manifold of (real) dimension $2n + 2$ and every complete projective special Kähler manifold of dimension $2n$ gives rise (by the c-map) to a complete quaternionic Kähler manifold of dimension $4n + 4$. The scalar curvature of the resulting quaternionic Kähler manifolds is always negative.

A *projective special real manifold* of dimension n is by definition a hypersurface $\mathcal{H} \subset \mathbb{R}^{n+1}$ such that there exists a homogeneous cubic polynomial h on $\mathbb{R}^{n+1}$ with the following properties:

(i) $\mathcal{H} \subset \{h = 1\}$ and

(ii) the Hessian $\partial^2 h$ is negative definite on $T\mathcal{H}$.

The manifold $\mathcal{H}$ is endowed with the canonical Riemannian metric $g_{\mathcal{H}}$ induced by the tensor field $-\frac{1}{3}\partial^2 h$. Projective special real manifolds can be intrinsically characterized by a partial differential equation satisfied by their underlying centro-affine structure [32, Theorem 2.3]. We show in [32, Theorem 2.5] that the projective special real manifolds $\mathcal{H}$ for which the metric $g_{\mathcal{H}}$ is complete are precisely those for which the subset $\mathcal{H} \subset \mathbb{R}^{n+1}$ is closed. As a corollary, we proof that every locally strictly convex component of the level set $\{h = 1\} \subset \mathbb{R}^{n+1}$ of a homogeneous cubic polynomial defines an explicit complete quaternionic Kähler metric on $\mathbb{R}^{4n+8}$. Complete projective special real manifolds and the corresponding complete quaternionic Kähler manifolds are classified in low dimensions in [31,33] and in the case of reducible polynomials h in [34]. The examples which we obtain by this method include complete quaternionic Kähler manifolds of cohomogeneity one in all dimensions ≥ 12. Further inhomogeneous complete examples (including the dimensions 4 and 8) were obtained in [35] by combining the above methods with a one-parameter deformation of the metric known as the one-loop quantum correction [36]. In fact, it is shown in [35, Theorem 27] that every quaternionic Kähler manifold associated with a complete projective special real manifold admits a canonical deformation by complete quaternionic Kähler manifolds depending on a parameter $c \geq 0$. The same is true for quaternionic Kähler manifolds associated with projective special Kähler manifolds, provided that the special Kähler manifold has regular boundary behaviour [35, Theorem 13]. These two results imply, in particular, the existence of this type of explicit deformation for all the known homogeneous quaternionic Kähler manifolds of negative Ricci curvature with exception of the simplest such homogeneous spaces, the quaternionic hyperbolic spaces. The fact that the metrics obtained by the one-loop deformation of the supergravity c-map are quaternionic Kähler was proven in [37,38] based on a geometric construction which allows to reduce the supergravity c-map to the much simpler rigid c-map. (This work was also part of the PhD project [39].) A similar construction allows to reduce the supergravity r-map to its rigid version [40]. (This work was also part of the PhD project [41].) A geometric description of the rigid r-map is given in [42].

The moduli space of complete projective special real manifolds has been systematically studied in the PhD thesis [43]. One of the main results is that the set of normal forms describing these manifolds can be parametrized by a compact convex neighborhood of zero in a finite-dimensional vector space. In particular, any two complete projective special real manifolds can be connected by connecting their normal forms in the convex set. Another important consequence are uniform curvature bounds depending only on the dimension.

In the above geometric constructions pseudo-Riemannian cones play an important role. This is due to the fact that a projective special Kähler manifold is the base of a $\mathbb{C}^*$-bundle the total space of which is a conical affine special Kähler manifold with indefinite metric of index 2. Similarly, quaternionic Kähler manifolds of negative scalar curvature are the base of a bundle the total space of which has a conical hyper-Kähler structure of index 4 (the Swann bundle). In [44] we study pseudo-Riemannian cones and their holonomy. In the Riemannian setting, a metric cone over a complete manifold is either flat or irreducible, by Gallot's theorem [45]. This is no longer true in the in the pseudo-Riemannian setting and we describe the properties of the holonomy representation of the cone and how it relates to the geometry of the base manifold.

In the spirit of Gallot's theorem, we prove that a metric cone over a compact and complete pseudo-Riemannian manifold is either flat or indecomposable. Matveev and Mounoud have later given another proof, which does also apply to incomplete compact manifolds [46].

Homogeneous pseudo-Riemannian manifolds of index 4 with a compatible almost hyper-complex or almost quaternionic structure are classified in [47, 48] in the case of $\mathbb{H}$-irreducible isotropy representation. (This work was also part of the PhD project [49].) We prove that the resulting spaces are always locally symmetric if the dimension is at least 16 and give counterexamples in dimension 12. In [50] we classify homogeneous locally conformally Kähler manifolds under the assumption that the normalizer of the isotropy group is compact.

The special geometry of Euclidean supersymmetry with eight real supercharges has been systematically developed in a collaboration with Mohaupt and his group initiated in [51, 52]. A common feature is the appearance of para-complex and para-quaternionic structures replacing the complex and quaternionic structures present in the standard Minkowskian theories. Time-like and space-like reductions relating the scalar geometries of various Euclidean and Minkowskian theories of supergravity in space-time dimensions $d \in \{3, 4, 5\}$ have been worked out in [53–55]. The twistor spaces of para-quaternionic Kähler manifolds are studied in [56]. Also the geometric construction of the c-map obtained in [37, 38] admits a generalization to the Euclidean setting, as shown in [57].

The rigid limit of $\mathcal{N} = 2$ supergravity coupled to vector and hypermultiplets is somewhat subtle. In [58] we showed how the respective scalar field spaces reduce to their global counterparts. In the hypermultiplet sector we focused on the relation between the local and rigid c-map.

For some further geometric aspects related to the themes discussed so far see [59–64].

3 Second quantized mirror symmetry

Apart from the standard perturbative mirror symmetry, which relates two Calabi–Yau manifolds with reversed Hodge numbers, there also is a non-perturbative duality which relates type II Calabi–Yau compactifications to heterotic K3 compactifications. This duality is fairly well understood in the vector multiplet sector of $\mathcal{N} = 2$ supergravity but poorly understood in the hypermultiplet sector.

In [65] we revisited this duality and considered the heterotic string theory compactified on K3 $\times T^2$ and type IIA compactified on a Calabi–Yau threefold X in the hypermultiplet sector. We derived an explicit map between the field variables of the respective moduli spaces at the level of the classical effective actions. We determined the parametrization of the K3 moduli space consistent with the Ferrara–Sabharwal form. From the expression of the holomorphic prepotential we were led to conjecture that both X and its mirror must be K3 fibrations in order for the type IIA theory to have an heterotic dual. We then focused on the region of the moduli space where the metric is expressed in terms of a prepotential on both sides of the duality. Applying the duality we derived the heterotic hypermultiplet metric for a gauge bundle which is reduced to 24 point-like instantons. This result is confirmed by using the duality between the heterotic theory on T^3 and M-theory on K3. We finally studied the hyper-Kähler metric on the moduli space of an $SU(2)$ bundle on K3.

In [66] we continued this investigation and predicted the form of the quaternion-Kähler metric on hypermultiplet moduli space when K3 is elliptically fibered, in the limit of a large fiber and even larger base. The result is in general agreement with expectations from Kaluza–

Klein reduction, in particular the metric has a two-stage fibration structure, where the B-field moduli are fibered over bundle and metric moduli, while bundle moduli are themselves fibered over metric moduli.

A different insight coming out of mirror symmetry has been suggested in [67]. Given a maximally degenerating family of Calabi–Yau varieties of the general kind studied in [68, 69], there is a canonical basis of sections ("generalized theta functions") of the polarizing line bundle of the family. Under homological mirror symmetry this basis of sections is dual to intersection points of a pair of canonical isotopy classes of Lagrangian sections of the mirror SYZ fibration. Generalized theta functions are built by counting tropical versions of holomorphic cylinders connecting two SYZ fibres and in such a way capture tree-level information of the mirror SYZ geometry. The general properties of generalized theta functions have been comprehensively studied in [70]. By generalizing the monomial basis of toric local Calabi–Yaus in the crystal melting picture, we expect that generalized theta functions actually also encode the higher genus and non-perturbative information necessary for second-quantized mirror symmetry for compact Calabi–Yaus. Work is currently under way to prove the mirror enumerative meaning of theta functions [71, 72].

4 Partial supersymmetry breaking

Spontaneous breaking of $\mathcal{N} = 2$ supersymmetry is known to be possible only under very special circumstances. In [73] we used the embedding tensor formalism to give the general conditions for the existence of $\mathcal{N} = 1$ vacua in spontaneously broken $\mathcal{N} = 2$ supergravities. We indeed confirmed the necessity of having both electrically and magnetically charged multiplets in the spectrum, but also showed that no further constraints on the special Kähler geometry of the vector multiplets arise. The quaternionic field space of the hypermultiplets instead must have two commuting isometries. As an example we discussed the special quaternionic-Kähler geometries which appear in the low-energy limit of type II string theories. For these cases we found the general solution for stable Minkowski and AdS $\mathcal{N} = 1$ vacua, and determine the charges in terms of the holomorphic prepotentials. We further found that the string theory realisation of the $\mathcal{N} = 1$ Minkowski vacua requires the presence of non-geometric fluxes, whereas they are not needed for the AdS vacua.

In [74] we derived the low-energy effective action below the scale of partial supersymmetry breaking and computed the $\mathcal{N} = 1$ couplings in terms of the $\mathcal{N} = 2$ input data. We then showed that this effective action satisfies the constraints of $\mathcal{N} = 1$ supergravity in that its sigma-model metric is Kähler, while the superpotential and the gauge kinetic functions are holomorphic. As an example we discussed the $\mathcal{N} = 1$ effective supergravity of type II compactifications.

In [75] we made the construction of the effective $\mathcal{N} = 1$ theory mathematically rigorous. Specifically we proved that, given a certain isometric action of a two-dimensional Abelian group A on a quaternionic Kähler manifold M which preserves a submanifold $N \subset M$, the quotient $M' = N/A$ has a natural Kähler structure. We verified that the assumptions on the group action and on the submanifold $N \subset M$ are satisfied for a large class of examples obtained from the supergravity c-map. In particular, we found that all quaternionic Kähler manifolds M in the image of the c-map admit an integrable complex structure compatible with the quaternionic structure, such that $N \subset M$ is a complex submanifold. Finally, we discussed how the existence of the Kähler structure on M' is required by the consistency of spontaneous $\mathcal{N} = 2$ to $\mathcal{N} = 1$ supersymmetry breaking.

In [76] we gave explicit examples of gauged $\mathcal{N} = 2$ supergravities which arise in the low-energy limit of type II string theories and which exhibit spontaneous partial supersymmetry breaking. Specifically, for the so called quantum STU model we derived the scalar field space and the scalar potential of the $\mathcal{N} = 1$ supersymmetric low-energy effective action. We also studied the properties of the Minkowskian $\mathcal{N} = 1$ supersymmetric ground states for a broader class of supergravities including the quantum STU model. (This project was also part of the Master-thesis [77].)

In [78] we generalized the scope and studied $\mathcal{N} = 2$ vacua in spontaneously broken $\mathcal{N} = 4$ electrically gauged supergravities in four space-time dimensions. We argued that the classification of all such solutions amounts to solving a system of purely algebraic equations. We then explicitly constructed a special class of consistent $\mathcal{N} = 2$ solutions and studied their properties. In particular we found that the spectrum assembles in $\mathcal{N} = 2$ massless or BPS supermultiplets. We showed that (modulo U(1) factors) arbitrary unbroken gauge groups can be realized provided that the number of $\mathcal{N} = 4$ vector multiplets is large enough. Below the scale of partial supersymmetry breaking we calculated the relevant terms of the low-energy effective action and argue that the special Kähler manifold for vector multiplets is completely determined, up to its dimension, and lies in the unique series of special Kähler product manifolds. (This project was also part of the PhD-thesis [79].)

5 Supersymmetric AdS backgrounds

Anti-de Sitter (AdS) backgrounds of string theory and supergravity are of interest for two reasons. On the one hand that serve as an intermediate step in phenomenological investigations before "uplifting" to a de Sitter background. On the other hand, AdS backgrounds feature prominently in the AdS/CFT correspondence and determine properties of strongly coupled gauge theory living on the boundary of the AdS space. In [80] we initiated the study of the structure of the supersymmetric moduli spaces of AdS backgrounds in supergravity theories. This was continued in all space-time dimensions with all possible supercharges in [81–86].

In [80] we studied the structure of the supersymmetric moduli spaces of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity theories in AdS_4 backgrounds. In the $\mathcal{N} = 1$ case, the moduli space cannot be a complex submanifold of the Kähler field space, but is instead real with respect to the inherited complex structure. In $\mathcal{N} = 2$ supergravity the same result holds for the vector multiplet moduli space, while the hypermultiplet moduli space is a Kähler submanifold of the quaternionic-Kähler field space. These findings are in agreement with AdS/CFT considerations.

In [81] we determined the supersymmetric AdS_7 backgrounds of seven-dimensional half-maximal gauged supergravities and showed that they do not admit any deformations that preserve all 16 supercharges. We compared this result to the conformal manifold of the holographically dual $(1, 0)$ superconformal field theories and showed that accordingly its representation theory implies that no supersymmetric marginal operators exist.

In [82] we classified the $\mathcal{N} = 4$ supersymmetric AdS_5 backgrounds that arise as solutions of five-dimensional $\mathcal{N} = 4$ gauged supergravity. We expressed our results in terms of the allowed embedding tensor components and identify the structure of the associated gauge groups. We showed that the moduli space of these AdS vacua is of the form $SU(1, m)/(U(1) \times SU(m))$ and discussed our results regarding holographically dual $\mathcal{N} = 2$ SCFTs and their conformal manifolds.

In [83] we determined the conditions for maximally supersymmetric AdS_5 vacua of five-dimensional gauged $\mathcal{N}=2$ supergravity coupled to vector-, tensor- and hypermultiplets charged under an arbitrary gauge group. In particular, we showed that the unbroken gauge group of the AdS_5 vacua has to contain an $U(1)_R$-factor. Moreover we proved that the scalar deformations which preserve all supercharges form a Kähler submanifold of the ambient quaternionic Kähler manifold spanned by the scalars in the hypermultiplets.

In [84] we studied maximally supersymmetric AdS backgrounds in consistent $\mathcal{N}=2$ truncations of type IIB supergravity compactified on the Sasaki–Einstein manifold $T^{1,1}$. In particular, we focused on truncations that contain fields coming from the nontrivial second and third cohomology forms on $T^{1,1}$. These give rise to $\mathcal{N}=2$ supergravity coupled to two vector- and two hypermultiplets or one vector- and three hypermultiplets, respectively. We found that both truncations admit AdS_5 backgrounds with the gauge group always being broken but containing at least an $U(1)_R$ factor. Moreover, in both cases we showed that the moduli space of AdS vacua is nontrivial and of maximal dimension. Finally, we explicitly computed the metrics on these moduli spaces.

In [85] we studied fully supersymmetric AdS_6 vacua of half-maximal $\mathcal{N}=(1,1)$ gauged supergravity in six space-time dimensions coupled to n vector multiplets. We showed that the existence of AdS_6 backgrounds requires that the gauge group is of the form $G' \times G'' \subset SO(4,n)$ where $G' \subset SO(3,m)$ and $G'' \subset SO(1,n-m)$. In the AdS_6 vacua this gauge group is broken to its maximal compact subgroup $SO(3) \times H' \times H''$ where $H' \subset SO(m)$ and $H'' \subset SO(n-m)$. Furthermore, the $SO(3)$ factor is the R-symmetry gauged by three of the four graviphotons. We further showed that the AdS_6 vacua have no moduli that preserve all supercharges. This is precisely in agreement with the absence of supersymmetric marginal deformations in holographically dual five-dimensional superconformal field theories.

Finally, in [86] we studied maximally supersymmetric AdS_D solutions of gauged supergravities in dimensions $D \geq 4$. We showed that such solutions can only exist if the gauge group after spontaneous symmetry breaking is a product of two reductive groups $H_R \times H_{\mathrm{mat}}$, where H_R is uniquely determined by the dimension D and the number of supersymmetries N while H_{mat} is unconstrained. This resembles the structure of the global symmetry groups of the holographically dual SCFTs, where H_R is interpreted as the R-symmetry and H_{mat} as the flavor symmetry. Moreover, we discussed possible supersymmetry preserving continuous deformations, which correspond to the conformal manifolds of the dual SCFTs. Under the assumption that the scalar manifold of the supergravity is a symmetric space we derived general group theoretical conditions on these moduli. Using these results we determined the AdS solutions of all gauged supergravities with more than 16 real supercharges. We found that almost all of them do not have supersymmetry preserving deformations with the only exception being the maximal supergravity in five dimensions with a moduli space given by $SU(1,1)/U(1)$. Furthermore, we determined the AdS solutions of four-dimensional $\mathcal{N}=3$ supergravities and showed that they similarly do not admit supersymmetric moduli.

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