Extended Axion Electrodynamics, relic Axions and Dark Matter Fingerprints in the terrestrial electromagnetic Field

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We consider extended versions of the Einstein-Maxwell-axion model based on the Lagrangian, which is linear in the pseudoscalar (axion) field, linear or quadratic in the gradient four-vector of the axion field, linear in the Riemann tensor, and includes the four-vector of macroscopic velocity of the system. We discuss applications to four problems, which are connected with axion-photon-graviton coupling in the anisotropic Bianchi-I cosmology, optical activity in the isotropic Friedmann-type universe, non-minimal effects in the plane gravitational waves background, and evolution of terrestrial magnetic and electric fields in the axion dark matter environment.

1 Introduction

Fundamentals of the theory of axions, massive particles, which were predicted by Peccei and Quinn [1] and introduced into the high-energy physics as new light pseudo-bosons by Weinberg [2] and Wilczek [3], are well-described (see, e.g., [4]-[6]). We are interested in extension of the theory of coupling between electromagnetic and pseudoscalar fields based on axion electrodynamics [7]-[10]. Why do we think that the extension of the axion-photon coupling theory is necessary? The main argument is that natural and artificial electromagnetic waves propagate (hypothetically) in the axionic dark matter environment, which has to be considered as a quasi-medium. This axionic quasi-medium has to be chiral, since the axion-photon interactions are predicted to induce the effect of polarization rotation. This axionic quasi-medium has to be non-stationary because of cosmological (accelerated) expansion. Also, it should be spatially non-homogeneous and anisotropic, since the density of the dark matter depends on the distance to the Galactic center. But, if we deal with an axionic quasi-medium, we have to describe the axion-photon coupling in terms of axion electrodynamics of continuous media rather than in terms of vacuum electrodynamics. This means that the theory of axion-photon coupling has to involve into consideration the velocity four-vector of the system as a whole, U^i, and its covariant derivative $\nabla_i U_k$, thus predicting, respectively, the existence of dynamic and dynamo-optic phenomena in the electromagnetic system. Besides, in addition to the pseudoscalar field itself, ϕ, this theory has to include the gradient four-vector of the pseudoscalar field, $\nabla_i \phi$. The quantity $\dot{\phi} = U^i \nabla_i \phi$ is responsible for the non-stationarity of the effect of axionically induced optical activity; the spatial part of this gradient, describing inhomogeneity and anisotropy of the system, can be the reason of axionically induced birefringence.

2 On the scheme of the theory extension

The standard Einstein-Maxwell-axion theory is based on the Lagrangian

$$S = \frac{\hbar}{c}\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa}\left[R{+}2\Lambda\right] + \frac{1}{4}\left[F_{mn}F^{mn}{+}\phi F^{*}_{mn}F^{mn}\right] - \frac{1}{2}\Psi_0^2\left[\nabla_m\phi\nabla^m\phi{-}V(\phi^2)\right]\right\} .$$

The extension of the electromagnetic part of this Lagrangian can be made by the replacement

$$\frac{1}{4}\left[F_{mn}F^{mn}{+}\phi F^{*}_{mn}F^{mn}\right] \quad\Rightarrow\quad \frac{1}{4}F_{pq}F_{mn}\cdot C^{pqmn}\left[g^{ik}, U^i, \nabla_i U_k, \phi, \nabla_i\phi, R_{ikmn}\right] ,$$

where the linear response tensor C^{pqmn} depends now not only on the metric g_{ik}, and on the pseudoscalar field ϕ, but also on the macroscopic velocity U^i, its covariant derivative $\nabla_i U_k$, on the gradient four-vector $\nabla_i\phi$, and finally, on the Riemann tensor R_{ikmn} and its convolutions. Similarly, the axionic part of the Lagrangian can be extended as follows:

$$\frac{1}{2}\Psi_0^2\nabla_m\phi\nabla^m\phi \quad\Rightarrow\quad \frac{1}{2}\Psi_0^2\nabla_m\phi\nabla_n\phi\cdot\mathcal{C}^{mn}\left[g^{ik}, U^i, \nabla_i U_k, F_{pq}, R_{ikmn}\right] .$$

The tensor $\mathcal{C}^{mn}$ in principle can be a linear function of the Maxwell tensor F_{pq}. Let us consider three simplest examples in order to illustrate the idea of the model extensions.

2.1 The model linear in the axion field and its gradient four-vector

The simplest model is linear in the pseudoscalar field ϕ and its gradient four-vector, $\nabla_i\phi$. As it was proved in [11], all possible variants of this type can be reduced to the model with the linear response tensor of the form

$$C^{ikmn} = \frac{1}{2}\left(g^{im}g^{kn}{-}g^{in}g^{km}\right){+}\frac{1}{2}\epsilon^{ikmn}\left(\phi + \nu\ U^i\nabla_i\phi\right)$$

with one new coupling constant ν introduced phenomenologically (here ϵ^{ikmn} is the absolutely skew-symmetric Levi-Civita tensor). In this model only the structure of the tensor of magneto-electric cross-effect, ${\nu_p}^m$, is changed:

$${\nu_p}^m = \epsilon_{ikpq}U^qC^{ikmn}U_n = -\Delta^m_p\left(\phi + \nu\dot{\phi}\right) , \qquad \Delta^m_p = \delta^m_p - U_pU^m ,$$

and the new term $\nu\dot{\phi}$ describes a non-stationarity in the polarization rotation effect induced by the axionic dark matter [11].

2.2 The model quadratic in the gradient four-vector of the axion field

This model is the two-parameter one [12], and it is described by the linear response tensor

$$C^{ikmn} = \frac{1}{2}\left(g^{im}g^{kn}{-}g^{in}g^{km}\right)(1{+}\lambda_1\nabla_p\phi\nabla^p\phi){+}\frac{1}{2}\phi\epsilon^{ikmn}{+}\frac{\lambda_2}{2}\left(g^{i[m}\nabla^{n]}\phi\nabla^k\phi{+}g^{k[n}\nabla^{m]}\phi\nabla^i\phi\right) .$$

The corresponding dielectric permittivity tensor

$$\varepsilon^{im} = \Delta^{im}\left[1 + \lambda_1\overset{\perp}{\nabla}_q\phi\overset{\perp}{\nabla}{}^q\phi + \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\dot{\phi}^2\right] + \frac{1}{2}\lambda_2\overset{\perp}{\nabla}{}^i\phi\overset{\perp}{\nabla}{}^m\phi ,$$

and the tensor of magnetic impermeability

$$\left(\mu^{-1}\right)_{im} = \Delta_{im}\left[1+\lambda_1\dot{\phi}^2+\left(\lambda_1+\frac{1}{2}\lambda_2\right)\overset{\perp}{\nabla}_q\phi\overset{\perp}{\nabla}{}^q\phi\right]-\frac{1}{2}\lambda_2\overset{\perp}{\nabla}_i\phi\overset{\perp}{\nabla}_m\phi$$

become anisotropic, when the spatial part of the gradient four-vector $\overset{\perp}{\nabla}_i\phi = \Delta_i^k\nabla_k\phi$ is non-vanishing. This means that axionic dark matter is (in general case) a birefringent quasi-medium. As for the tensor of magneto-electric cross-effects

$$\nu^{pm} = -\phi\Delta^{pm}+\frac{1}{2}\lambda_2\dot{\phi}\ \epsilon^{pmkq}U_q\overset{\perp}{\nabla}_k\phi\,,$$

it demonstrates the possibility of a new type of optical activity, which can be realized, when the axionic dark matter is non-stationary and non-homogeneous simultaneously.

When we consider the applications to homogeneous cosmological models (anisotropic model of the Bianchi-I type with magnetic field, or isotropic model of the Friedmann type, see [12, 11]), we obtain the following expressions for the permittivity scalars and refraction index, respectively:

$$\varepsilon(t) = 1+\left(\lambda_1+\frac{1}{2}\lambda_2\right)\dot{\phi}^2\,,\quad \frac{1}{\mu(t)} = 1+\lambda_1\dot{\phi}^2\,,\quad n^2(t)=\varepsilon(t)\mu(t)=\frac{1+\left(\lambda_1+\frac{1}{2}\lambda_2\right)\dot{\phi}^2}{1+\lambda_1\dot{\phi}^2}\,.$$

Keeping in mind that the quantity $\dot{\phi}^2$ can be expressed using the mass density of the axionic cold dark matter $\rho_{(\mathrm{DM})}$ as $\dot{\phi}^2 = c^2\Psi_0^{-2}\rho_{(\mathrm{DM})}$ (see [12]), we obtain useful phenomenological formulas for the refraction index of the axionic dark matter as a function of cosmological time, and of the phase velocity $v_{\mathrm{ph}}=\frac{c}{n(t)}$ of the electromagnetic waves, which propagate in the axionic dark matter quasi-medium. The obtained function $n^2(t)$ can be (in principle) negative during some epoch, and because of axion-photon coupling in this (unlighted) epoch the electromagnetic waves do not propagate in the Universe, do not scan its internal structure and can not bring information to observers (see [13] for details).

One of the most important consequences of the gradient-type model of the axion-photon coupling is the following. Backreaction of the electromagnetic field on the pseudoscalar field evolution is shown to produce inflationary-type growth of the number of axions in the early Universe, and these axions become relic at present time and form now the axionic dark matter.

2.3 Non-minimal models

Non-minimal Einstein-Maxwell-axion models can be obtained, if the linear response tensor

$$\mathcal{C}^{ikmn} = \frac{1}{2}\left(g^{im}g^{kn}-g^{in}g^{km}\right)+\frac{1}{2}\phi\epsilon^{ikmn}+\mathcal{R}^{ikmn}+\frac{1}{2}\phi\left[\chi^{*ikmn}_{(\mathrm{A})}+^{*}\chi^{ikmn}_{(\mathrm{A})}\right]$$

is equipped by the terms $\mathcal{R}^{ikmn}$ and $\chi^{*ikmn}_{(\mathrm{A})}$ linear in the Riemann tensor R_{ikmn}, Ricci tensor R_{ik} and Ricci scalar R (see [14, 15] for details). The analysis of such models in application to the problem of gravitational wave action on the axion-photon system shows, that when the axion field is constant $\phi = \phi(0)$, and thus is hidden from the electrodynamic point of view, the coupling to curvature removes the degeneracy with respect to hidden pseudoscalar field, providing visualization of the effects of birefringence and optical activity induced by axion-photon coupling.

3 Outlook

Analysis of the extended models of axion electrodynamics shows that there is a chance to verify experimentally the predictions of the model of interaction of terrestrial magnetic and electric fields with the relic axion background (see [16] for details). The idea, based on the corresponding exact solutions, is the following. Relic dark matter axions produce in the terrestrial electrodynamic system oscillations of a new type, which belong to the class of longitudinal magneto-electric clusters. These oscillations have the following specific feature: the electric and magnetic fields are parallel to one another and are coupled by axion field; in the absence of axions such oscillations decouple. Electric and magnetic fields of this type are correlated, and the oscillations are characterized by identical frequencies. Generally, there are two sets of hybrid frequencies of longitudinal oscillations, which belong to the range $\nu_{\rm A} \simeq 10^{-3} - 10^{-5}$Hz. Optimistic estimations of the dimensionless parameter ξ, which characterizes the ratio between the amplitude of axionically induced electromagnetic oscillations and amplitude of their (electric or magnetic) sources, give the value of the order $\xi \simeq 10^{-7}$ for the mass density of the dark matter in the Solar system of the order $\rho_{\rm (DM)} \simeq 0.033\ M_{\rm (Sun)}{\rm pc}^{-3}$ and for the axion-photon coupling constant $g_{{\rm A}\gamma\gamma} \simeq 10^{-9}{\rm GeV}^{-1}$. We are waiting for the first results of the experiments with infra-low frequency variations of the terrestrial electric and magnetic fields, which are organized in the Vladimir University (Russia), and hope to find fingerprints of relic axions in the correlated components of the signals of longitudinal electric and magnetic field variations.

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