Stronger QCD in the early Universe to suppress axion isocurvature perturbations

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A stronger QCD in the early Universe renders the axion heavy, and thus can suppress the isocurvature perturbations of axion dark matter, relaxing the constraint on the inflation scale. Supersymmetry provides a natural framework for this because there is a flat direction involving Higgs fields. Stabilized at a large value during inflation, the Higgs flat direction leads to that the QCD interaction becomes strong at a high energy scale, making high scale inflation scenarios compatible with axion dark matter.

1 Introduction

The Peccei-Quinn (PQ) extension of the standard model (SM) includes the axion, which provides a natural solution to the strong CP problem through spontaneously broken PQ symmetry [1, 2]. The axion potential is generated by non-perturbative QCD instanton effects, dynamically relaxing the CP phase in QCD. This implies that coherent oscillations of the axion are necessarily induced and contribute to cold dark matter via the misalignment mechanism [2]. The axion can thus explain both a tiny CP violation in QCD and the dark matter of the Universe. However, the axion dark matter generally puts a strong constraint on the inflation scale because its quantum fluctuations during inflation induce isocurvature perturbations in the CMB spectrum. If the axion is the main component of dark matter in the Universe, the observed CMB spectrum constrains the inflation scale to be [3, 4, 5]

$$H_{\rm inf} < 0.87 \times 10^7 {\rm GeV} \left(\frac{f_a}{10^{11}{\rm GeV}} \right)^{0.408}, \qquad (1)$$

at 95% CL, where f_a is the axion decay constant. Hence high scale inflation scenarios such as the chaotic inflation [6] are in tension with the axion dark matter.

The isocurvature constraint can be avoided if the PQ symmetry is restored during or after inflation since the axion appears after PQ symmetry breaking [7, 8]. However, in this case, the axion properties are constrained because topological defects such as cosmic strings and domain walls are generated. To avoid overclosure of the Universe, the PQ sector should be such that the domain wall number is equal to one and f_a is less than about 10^{10} GeV. Alternatively one can consider models where the coefficient of the axion kinetic term is larger during inflation than present so that axion quantum fluctuations can be suppressed [7, 9, 10].

In this work, we propose another interesting way to relax the isocurvature constraint on the inflation scale in the axion dark matter scenario. The idea is to make the QCD stronger during inflation so that the axion becomes heavy enough to have quantum fluctuations significantly suppressed at superhorizon scales. A stronger QCD can be achieved when the Higgs field has a sufficiently large value during inflation because the quarks then decouple at a high energy scale while making the QCD coupling run faster at lower scales.

2 Suppressing axion isocurvature perturbations

The Higgs field can naturally have a large field value during inflation in the supersymmetry SM since they compose flat directions. Let us consider a supersymmetry axion model where the Higgs fields H_u and H_d are neutral under the PQ symmetry. We are interested in the case that the $H_u H_d$ flat direction is stabilized at a large value during inflation so that the QCD confines at a high scale. This is realized when it obtains a tachyonic mass from a Planck-suppressed coupling with the inflaton in the Kähler potential. The Higgs flat direction can then be fixed at a large value through the interplay between the Hubble-induced tachyonic mass and a higher order superpotential term. We will assume that it is stabilized around or above the GUT scale, $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV.

2.1 Axion potential

Let us first examine how the axion potential is generated in the early Universe. If all the quarks become heavier than the effective QCD scale Λ_h for the flat direction fixed around $M_{\rm GUT}$, they can be integrated out in a supersymmetric way. The low energy theory is a pure supersymmetric $\mathrm{SU}(3)_c$ with gauge kinetic function given by [11]

$$f_h = (\text{constant}) - \frac{n}{8\pi^2} \ln S - \frac{N_f}{8\pi^2} \ln \phi, \tag{2}$$

where S is the axion superfield, and ϕ^2 parameterizes the $H_u H_d$ flat direction. The constant n counts the number of PQ charged quarks when the PQ symmetry is linearly realized, and $N_f = 6$ is the flavor number of the SM quarks. For Λ_h larger than $H_{\rm inf}$, the gluino condensate is formed during inflation, and consequently induces the non-perturbative superpotential [12]

$$W_{\rm np} = N_c \Lambda_0^3 \propto e^{-8\pi^2 f_h / N_c}, \tag{3}$$

with $N_c = 3$. Here Λ_0 denotes the condensation scale, which is equal to the QCD scale Λ_h in the present case.

The axion potential is generated from the non-perturbative superpotential $W_{\rm np}$ through supersymmetry breaking with the dominant contribution from the Higgs F-term:

$$V = |\partial_\phi(\Delta W + W_{\rm np})|^2 + \cdots = c N_f H_{\rm inf} \Lambda_0^3 \cos\left(\frac{a}{N_c f_a} + \theta_0\right) + \cdots, \tag{4}$$

where the constant c determines the overall size of the Hubble-induced mass term for ϕ and is generally order unity, and ΔW is the higher order superpotential term which competes with the Hubble-induced mass term to stabilize the flat direction. The above relation holds when the condensation scale lies in the range

$$H_{\rm inf} < \Lambda_0 < f_a, \tag{5}$$

where the upper bound is needed not to destabilize the saxion, while the lower bound comes from the requirement that the gauge-mediated gluino mass associated with Higgs F-term $F_\phi/\phi \sim cH_{\rm inf}$ should be small since otherwise the gluino would decouple at a scale above Λ_0. Note also that the non-perturbative superpotential does not modify significantly the potential of ϕ if

$$H_{\rm inf}\phi_0^3 \gg \Lambda_0^3. \tag{6}$$

On the other hand, in the case that there are n_f quarks lighter than Λ_h, they form meson fields and are stabilized by their mass term m and the Affleck-Dine-Seiberg superpotential. Integrating out the meson fields leads to the gluino condensation with $\Lambda_0^{3N_c} = \Lambda_h^{3N_c - n_f}\det(m)$. One thus obtains the axion potential given by (4) using Λ_0 defined this way.

From the axion potential (4) induced by the non-perturbative superpotential and supersymmetry breaking, one finds that the axion mass is given by

$$m_a^2 = \frac{\hat{c} H_{\rm inf}\Lambda_0^3}{f_a^2}, \tag{7}$$

during inflation, where $\hat{c} \equiv cN_f/N_c^2$ is generally order unity.

2.2 Stronger QCD

The QCD confines at a higher scale during inflation if the quarks become heavier, which is realized when the Higgs fields are stabilized at a large value. It is obvious that such effect is more efficient if there exist extra colored particles that obtain larger masses during inflation. Let us introduce N_Ψ pairs of PQ-singlet fields that belong to $\mathbf{5}+\bar{\mathbf{5}}$ of SU(5) and obtain masses from

$$W = \left(M_\Psi + \frac{\phi^2}{M}\right)\Psi\Psi^c, \tag{8}$$

where M is an effective cut-off scale. The coupling to H_uH_d $(=\phi^2)$ can make $\Psi + \Psi^c$ heavier during inflation for the flat direction stabilized at a large field value. Using the property that the universal gauge coupling constant at $M_{\rm GUT}$ is $g^2_{\rm GUT} \simeq 0.5$ in the MSSM, we find that the QCD scale reads

$$\Lambda_h \simeq 1.3 \times 10^7 {\rm GeV}\left(\frac{M_{\rm GUT}}{M_\Psi}\right)^{N_\Psi/9}, \tag{9}$$

for ϕ_0 and M around $M_{\rm GUT}$. Here we have taken into account the hierarchy in the SM quark Yukawa couplings, because of which the condensation scale Λ_0 is slightly smaller than Λ_h if Λ_h is larger than about $10^{-5}M_{\rm GUT}$. One can see that the QCD scale is about 10^7 GeV in the MSSM for $\phi_0 \sim M_{\rm GUT}$, but it is raised higher in the presence of extra colored particles which couple to H_uH_d.

Let us now examine how much the QCD scale can be raised in the supersymmetric axion model. An important constraint comes from the perturbativity condition because the gauge coupling at $M_{\rm GUT}$ increases if one adds extra colored fields:

$$\frac{1}{g^2_{\rm GUT}} \simeq 2 - \sum_{i=\Phi,\Psi}\frac{N_i}{8\pi^2}\ln\left(\frac{M_{\rm GUT}}{M_i}\right) > \frac{1}{4\pi}, \tag{10}$$

where $\Phi + \Phi^c$ are PQ quarks which belong to $\mathbf{5} + \bar{\mathbf{5}}$ and obtain masses $M_\Phi \sim f_a$ from the superpotential term $S\Phi\Phi^c$. The PQ quarks are added to make S couple to the QCD anomaly, which is required to solve the strong CP problem via the PQ mechanism. However, these PQ quarks play no role in raising the QCD scale since their masses are the same during and after inflation. Combined with the relation (9), the above requirement places an upper bound on the QCD scale,

$$\Lambda_h < 2.7 \times 10^{14}\text{GeV} \left(\frac{f_a}{M_{\rm GUT}}\right)^{N_\Phi/9}, \tag{11}$$

which is insensitive to M_Ψ. For instance, taking $f_a \geq 10^9$ GeV and $N_\Phi = 1$, the QCD scale can be as high as 10^{13} GeV.

2.3 Axion isocurvature perturbations

A higher QCD scale and large axion mass during inflation provide a simple way to relax the isocurvature constraint in the axion dark matter scenario. The axion is stabilized at the minimum of the potential (4) during inflation if its mass (7) is heavier than the inflation scale:

$$H_{\rm inf} < 10^{12}\text{GeV} \times \hat{c} \left(\frac{\Lambda_0}{f_a}\right)^3 \left(\frac{f_a}{10^{12}\text{GeV}}\right), \tag{12}$$

and then quantum fluctuations are highly suppressed at superhorizon scales. This shows that a high QCD scale can relax significantly the isocurvature constraint compared to the conventional scenario.

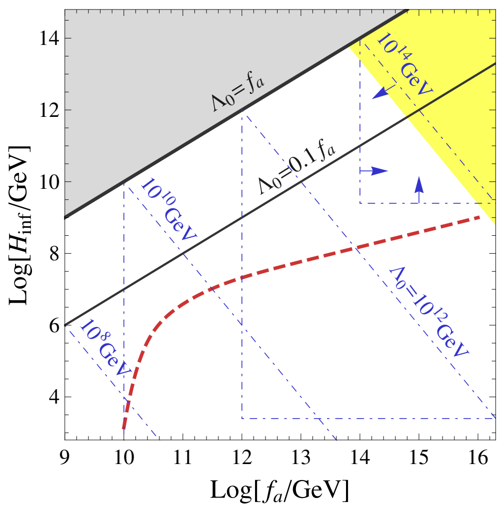

Figure 1: Inflation scale compatible with the axion cold dark matter.

Figure 1 shows the region where the axion isocurvature fluctuations are suppressed by a stronger QCD leading to $m_a > H_{\rm inf}$. Here we have taken $\hat{c} = 1$ and Λ_0 as a free parameter. The upper-right shaded region (yellow) is excluded by the perturbativity limit of the gauge interactions up to the GUT scale, while the upper-left region (gray) leads to saxion destabilization as $\Lambda_0 < f_a$. The thick (thin) solid line corresponds to the upper bound on $H_{\rm inf}$ for $\Lambda_0 = f_a$ ($\Lambda_0 = 0.1 f_a$). For fixed Λ_0, the suppression mechanism works inside a right-angled triangle, whose base, height, and hypotenuse come from the requirement $\Lambda_0 < f_a$, $H_{\rm inf} > \Lambda^3/\phi_0^2$ with $\phi_0 \sim M_{\rm GUT}$, and $m_a > H_{\rm inf}$, respectively. For instance, the upper-right one with blue dot-dashed sides is obtained for $\Lambda_0 = 10^{14}$ GeV. Other cases with $\Lambda_0 = 10^8$, 10^{10}, 10^{12} GeV are also shown from left to right. For comparison, we present the conventional isocurvature bound in the dashed red line, taking into account the anharmonic effect [13, 14].

We close this section by noting that f_a appearing in the relation (12) is the axion decay constant during inflation, which we have simply assumed to be the same as the present value. One may consider the case where f_a is around 10^{12} GeV at present but it has a larger value during inflation. Then the right amount of axion dark matter is obtained through the misalignment mechanism for an initial misalignment angle of order unity, while allowing high inflation scales around 10^{13-14} GeV.

3 Conclusions

The QCD axion not only provides a natural solution to the strong CP problem, but also can account for the dark matter of the Universe if its isocurvature perturbations are sufficiently suppressed to be consistent with the observation. We have shown that a stronger QCD in the early Universe can significantly relax the isocurvature constraint on the inflation scale, making high scale inflation models compatible with the axion dark matter. This is naturally realized within a supersymmetric axion model. For the Higgs flat direction stabilized at a large value during inflation, the QCD confines at a high energy scale, and consequently the axion becomes heavy and has no sizable quantum fluctuations at superhorizon scales.

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References

[1] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," Phys. Rev. Lett. **38**, 1440 (1977); "Constraints Imposed by CP Conservation in the Presence of Instantons," Phys. Rev. D **16**, 1791 (1977).

[2] For a review, see J. E. Kim, "Light Pseudoscalars, Particle Physics and Cosmology," Phys. Rept. **150**, 1 (1987); H. Y. Cheng, "The Strong CP Problem Revisited," Phys. Rept. **158**, 1 (1988); J. E. Kim and G. Carosi, "Axions and the Strong CP Problem," Rev. Mod. Phys. **82**, 557 (2010); A. Ringwald, "Exploring the Role of Axions and Other WISPs in the Dark Universe," Phys. Dark Univ. **1** (2012) 116; M. Kawasaki and K. Nakayama, "Axions : Theory and Cosmological Role," arXiv:1301.1123 [hep-ph].

[3] P. A. R. Ade *et al.* [Planck Collaboration], "Planck 2013 results. XXII. Constraints on inflation," arXiv:1303.5082 [astro-ph.CO].

[4] D. Seckel and M. S. Turner, "Isothermal Density Perturbations in an Axion Dominated Inflationary Universe," Phys. Rev. D **32**, 3178 (1985).

[5] D. H. Lyth, "A Limit On The Inflationary Energy Density From Axion Isocurvature Fluctuations," Phys. Lett. B **236**, 408 (1990).

[6] A. D. Linde, "Chaotic Inflation," Phys. Lett. B **129**, 177 (1983); M. Kawasaki, M. Yamaguchi and T. Yanagida, "Natural chaotic inflation in supergravity," Phys. Rev. Lett. **85**, 3572 (2000) [hep-ph/0004243]; R. Kallosh and A. Linde, "New models of chaotic inflation in supergravity," JCAP **1011**, 011 (2010) [arXiv:1008.3375 [hep-th]]; K. Nakayama, F. Takahashi and T. T. Yanagida, "Polynomial Chaotic Inflation in the Planck Era," Phys. Lett. B725, **111** (2013) [arXiv:1303.7315 [hep-ph]].

[7] A. D. Linde and D. H. Lyth, "Axionic domain wall production during inflation," Phys. Lett. B **246**, 353 (1990).

[8] D. H. Lyth and E. D. Stewart, "Constraining the inflationary energy scale from axion cosmology," Phys. Lett. B **283**, 189 (1992).

[9] A. D. Linde, "Axions in inflationary cosmology," Phys. Lett. B **259**, 38 (1991).

[10] S. Folkerts, C. Germani and J. Redondo, "Axion Dark Matter and Planck favor non-minimal couplings to gravity," arXiv:1304.7270 [hep-ph].

[11] G. F. Giudice and R. Rattazzi, "Extracting supersymmetry breaking effects from wave function renormalization," Nucl. Phys. B **511**, 25 (1998) [hep-ph/9706540]; N. Arkani-Hamed, G. F. Giudice, M. A. Luty and R. Rattazzi, "Supersymmetry breaking loops from analytic continuation into superspace," Phys. Rev. D **58**, 115005 (1998) [hep-ph/9803290].

[12] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking in Supersymmetric QCD," Nucl. Phys. B **241**, 493 (1984).

[13] D. H. Lyth, "Axions and inflation: Sitting in the vacuum," Phys. Rev. D **45**, 3394 (1992).

[14] T. Kobayashi, R. Kurematsu and F. Takahashi, "Isocurvature Constraints and Anharmonic Effects on QCD Axion Dark Matter," arXiv:1304.0922 [hep-ph].