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Coherent backward transition radiation from sub-fs "pancake-like" bunches as a tool for beam diagnostics

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Abstract: Coherent transition radiation (CTR), produced by short electron bunches, is a well-known radiation mechanism which is applied for bunch length monitoring. By analyzing the CTR spectrum it is possible to reconstruct the longitudinal bunch shape. Such a technique is widely used for accelerators with electron beams possessing bunch lengths σ_z much larger than their transverse beam sizes $\sigma_{x,y}$ (with $\sigma_{x,y,z}$ the rms transverse and longitudinal beam sizes). New developments in the field of accelerator technology allow to produce and accelerate bunches with a duration of less than 1 fs with a "pancake-like" bunch shape, i.e. with $\sigma_z \ll \sigma_{x,y}$. Diagnostic schemes of this kind of bunches give rise to interesting effects, for example connected with a transverse asymmetry ($\sigma_x \neq \sigma_y$), with a tilting of the bunches (i.e. the bunch axis does not coincide with the velocity direction), and even more.

Spectral-angular distributions of coherent backward transition radiation (BTR) were simulated taking into account a possible bunch tilt. It is shown that spectral intensity measurements of coherent BTR emitted in backward direction allow to obtain information about the bunch length.

Keywords: Beam-line instrumentation (beam position and profile monitors; beam-intensity monitors; bunch length monitors); Instrumentation for FEL

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Contents

1 Introduction

The recent development of intense, phase-stabilized lasers permits generation of trains of pulses on the attosecond time scale, providing access to fundamental physical, chemical, and biological processes on a microscopic scale in the time domain [1, 2]. Besides these newly developed short pulse lasers, new 4^{th} generation light sources as free-electron lasers (FELs) provide an alternative way for generating high brightness ultrashort radiation pulses. While various schemes have been proposed for attosecond pulse generation with FELs, mainly relying on a complex manipulation of electrons in phase-space followed by an energy modulation with external lasers (see e.g. refs. [3–5]), the simplest method proposed uses short electron bunches of relatively low charge in the self-amplified spontaneous emission (SASE) mode [6, 7]. Generation and control of short electron bunches with bunch charges of a few pC is therefore mandatory. At DESY (Hamburg, Germany) a long-term dedicated accelerator research and development facility SINBAD (Short INnovative Bunches and Accelerators at DESY) is currently under construction in order to study the production of ultra-short electron bunches for ultra-fast science [8, 9].

While the generation of short electron bunches is a challenging task, the bunch length diagnostics is no less demanding. Different techniques are in use in order to resolve beam sizes in the order of a few femtoseconds, imposing a different level of complexity: a method widely applied for bunch length diagnostics is Coherent Radiation Diagnostics (CRD) [10]. Radiation is emitted coherently if the wavelength is in the order of the bunch length, i.e. information about bunch length and shape is encoded in the emission spectrum which is exploited in CRD. In principle any kind of coherent radiation can be used as a radiation source.

Electro-Optical (EO) techniques can be applied for ultra-relativistic electrons where the particle Coulomb field is purely transversal, i.e. the field strength of the non-propagating particle field is a measure of the longitudinal bunch profile. If the bunch passes close to an electro-optical crystal (ZnTe or GaP), its Coulomb field induces a change in the crystal refractive index (so called Pockels effect). The information about the longitudinal profile is therefore encoded in an refractive index change which can be converted into an intensity variation by means of a laser together with polarizers. Principle resolution limit of EO bunch lengths measurements is the existence of transverse optical phonon frequencies in the THz frequency range which show up as a resonances in the dispersion relation, thus distorting the temporal profile of the fields as they propagate through the crystal. The resonance effect is immaterial when trying to measure electron bunches $\geq$ 120 fs for ZnTe and $\geq$ 60 fs for GaP [11]. Detailed information about EO techniques and measurements can be found e.g. in ref. [12].

The most state-of-the-art instrument for bunch length measurements and even more is a *Transverse Deflecting Structure* (TDS). A TDS is an iris loaded RF waveguide structure designed to provide hybrid deflecting modes (HEM_{11}), a linear combination of TM_{11} and TE_{11} dipole modes resulting in a transverse force that acts on the synchronously moving relativistic particle bunch. The working principle resembles that of an intra beam streak camera: a single bunch inside the bunch train, traversing the structure at an appropriate RF phase experiences a vertical kick which depends linearly on time and vanishes in the bunch center. Due to the vertical deflection, the vertical position of the electrons inside this bunch are linearly correlated to their longitudinal coordinates. Besides other parameters, the TDS resolution depends on the deflecting RF frequency and on the deflecting voltage. By using an X-band structure in combination with RF pulse compression via SLED technique, at the LCLS at SLAC (Stanford, U.S.A.) it was possible to achieve a TDS resolution of 0.5 fs [13, 14].

As can be seen on the basis of this listing, a great effort is required in order to resolve bunch lengths of a few femtoseconds and less. However, for many applications it is not the precise knowledge of the bunch length which is required rather than a rough estimate. In this case a much simpler detection scheme is already sufficient. Examples for such type of diagnostics are bunch compression monitors based on pyroelectric detectors in combination with a coherent THz radiation source (typically diffraction radiation) which provide substantial information to tune the bunch compression in a FEL and which are used for beam-based feedbacks to stabilize the RF phases [15]. In the following a similar approach is presented by which accelerator physicists are given tools to tune and optimize bunch lengths in the attosecond region. It is shown that coherent transition radiation (CTR) emitted in backward direction can be used to obtain information about the bunch length.

2 Coherent transition radiation from a tilted "pancake-like" bunch

This section aims to provide a short description of the spatio-temporal coherence properties of backward transition radiation and is based on the model developed in ref. [16]. The spectral angular distribution of coherent backward transition radiation is defined in terms of the bunch form factor $F(\vec{k})$ and can be written as

$$\frac{d^2W_{\mathrm{CTR}}}{d\omega d\Omega} = \left[N + N(N-1)F(\vec{k})\right] \frac{d^2W_{\mathrm{BTR}}}{d\omega d\Omega} \,. \tag{2.1}$$

Here, N is the number of electrons in a bunch, $d^2W_{\mathrm{BTR}}/d\omega d\Omega$ the spectral-angular distribution of backward transition radiation (BTR) from a single electron, and $k = \frac{2\pi}{\lambda}$ the wave vector of the emitted photon. The bunch form factor $F(\vec{k})$ for the transition radiation process is defined via the bunch charge distribution $\rho(\vec{r})$

$$F(\vec{k}) = \left| \int \mathrm{d}\vec{r}\rho(\vec{r}) \exp\left[-i\Delta\varphi\right] \right|^2,$$
$$\Delta\varphi = \vec{k} \cdot \Delta\vec{r} - \omega\Delta t. \tag{2.2}$$

The phase $\Delta\varphi$ has to be distinguished from the conventional phase calculated for the emission process of accelerated charges because the transition radiation process is caused by dynamical polarization of the electrons inside the target material which is at rest. Taking into account the kinematics of BTR which is illustrated in figure 1 (a), both geometrical and temporal contributions to the phase $\Delta\varphi$ can be derived from geometrical considerations as shown in ref. [16]. Neglecting terms proportional to γ^{-2} with γ the Lorentz factor, the BTR phase factor can be expressed as

$$\Delta\varphi_{\mathrm{BTR}} = \frac{2\pi}{\lambda}[x\theta_x - y\theta_y + z]. \tag{2.3}$$

Here, $\theta_x, \theta_y \sim \gamma^{-1}$ are the projection angles of the emitted photon as measured with respect to the direction of specular reflection.

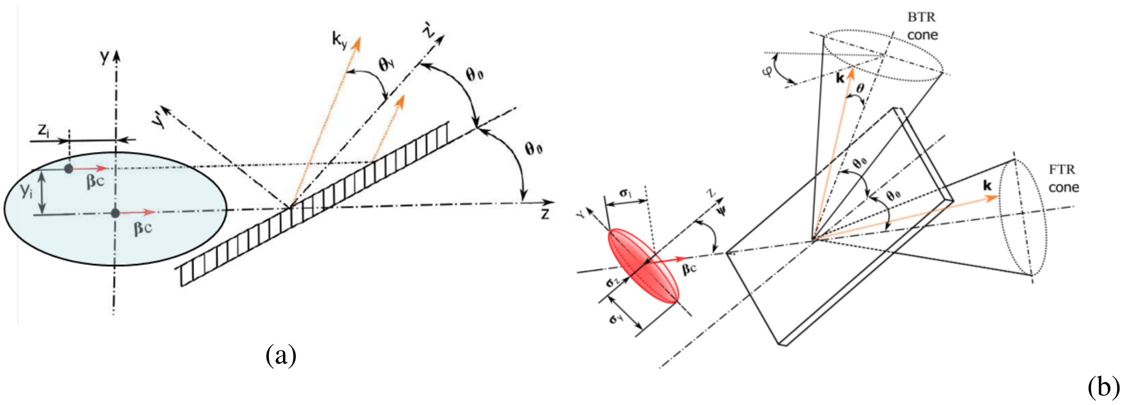

Figure 1. (a) Scheme of BTR generation from an inclined target which is rotated by θ_0 about the x-axis. For the BTR phase calculation both spatial and temporal phase contributions have to be taken into account for a bunch particle with respect to the reference particle in the bunch center. (b) Geometry of transition radiation generation for a pancake-like bunch which is tilted by ψ with respect to the velocity direction. Besides the BTR emission, transition radiation is also emitted in forward direction (FTR cone).

Assuming a three-dimensional Gaussian charge distribution with longitudinal bunch length σ_z and transverse bunch sizes σ_x, σ_y, the bunch has the form of an ellipsoid. In this case the integration in eq. (2.2) can be carried out analytically, resulting in the expression

$$F(\lambda, \theta_x, \theta_y) = \exp\left\{-4\pi^2\left(\frac{\sigma_z}{\lambda}\right)^2 \cdot \left[\left(\frac{\sigma_y}{\sigma_z}\right)^2 \sin^2\psi + \cos^2\psi + \left(\frac{\sigma_x}{\sigma_z}\right)^2 \theta_x^2 + \ldots\right.\right.$$
$$\left.\left. + \left(\left(\frac{\sigma_y}{\sigma_z}\right)^2 - 1\right)\theta_y \sin 2\psi + \left(\left(\frac{\sigma_y}{\sigma_z}\right)^2 \cos^2\psi + \sin^2\psi\right)\theta_y^2\right]\right\} \tag{2.4}$$

for the bunch form factor. In the derivation of eq. (2.4) it was assumed that the bunch is tilted by an angle ψ with respect to the velocity direction, i.e. that the minor ellipsoid axis does not coincide with the velocity direction as illustrated in figure 1 (b). As can be seen from this equation, F depends on the ratio between wavelength and bunch length λ/σ_z, on the ratio between transverse beam sizes and bunch length $\sigma_{x,y}/\sigma_z$, and on the bunch tilt angle ψ. In the standard configuration of bunches with $\sigma_z \gg \sigma_{x,y}$ and $\psi = 0$, eq. (2.4) is reduced to the conventional expression $F(\lambda) = \exp(-4\pi^2\sigma_z^2/\lambda^2)$ where the effect of spatial coherence proportional to $\sim \exp(-4\pi\sigma_{x,y}^2\theta_{x,y}^2/\lambda^2)$ is negligible small.

The form factor eq. (2.4) possesses a Gaussian-like shape which is reasonable since in the classical representation a form factor is the Fourier transform of the bunch shape. For pancake-like bunches and tilt angles $\psi \ll 1$ the form factor has its maximum value at the BTR emission angle $\theta_y \approx -\psi$. While the width of F is determined by both ratios λ/σ_z and $\sigma_{x,y}/\sigma_z$, its amplitude is strongly dominated by the λ/σ_z dependency.

The spectral angular distribution of BTR from an inclined target as depicted in figure 1 (a) and for ultra relativistic beam energies ($\gamma \gg 1$) can be written as [17]

$$\frac{d^2W_{\mathrm{BTR}}}{d\omega d\Omega} = N\frac{\alpha\hbar}{\pi^2}\frac{\theta_x^2+\theta_y^2+\theta_y\tan\theta_0\left(\gamma^{-2}+\theta_x^2+\theta_y^2\right)}{\left(\gamma^{-2}+\theta_x^2+\theta_y^2\right)^2}. \tag{2.5}$$

As it was shown in ref. [18], the ultra relativistic approximation eq. (2.5) agrees within an accuracy of better than γ^{-1} (corresponding to better than 0.5% in the case under consideration) with the exact formula.

Inserting eqs. (2.4) and (2.5) in eq. (2.1), it is possible now to calculate the spectral angular distribution of backward emitted CTR. In figure 2, spectral angular distributions of incoherent and coherent BTR are shown together with the bunch form factor. The parameters for the calculation are chosen such that they correspond to the SINBAD-ARES-linac which is presently under construction at DESY [8].

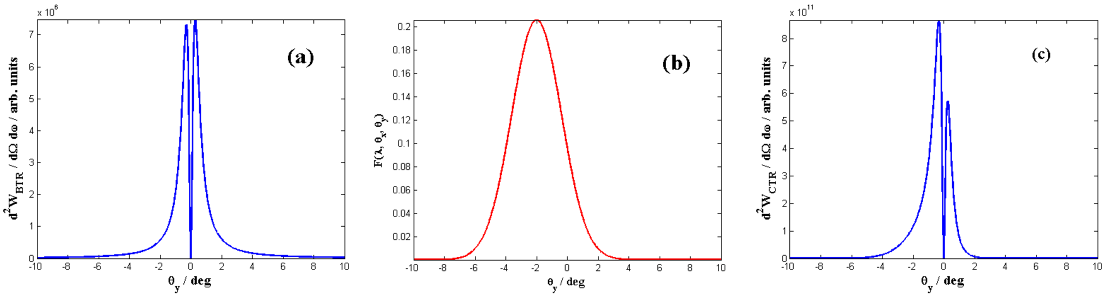

Figure 2. (a) Spectral angular distribution of incoherent BTR for $\gamma = 200$, $N = 10^6$ and a target inclination angle $\theta_0 = 45$ deg. (b) Calculated bunch form factor for a bunch tilt angle $\psi = 2$ deg and the ratios $\lambda/\sigma_z = 5$, $\sigma_{x,y}/\sigma_z = 20$. (c) Resulting CTR spectral angular distribution.

While the position of the bunch form factor maximum is determined by the bunch tilt angle ψ, its amplitude by the ratio λ/σ_z, and the width by the ratios λ/σ_z and $\sigma_{x,y}/\sigma_z$, in principle it is possible to perform a dedicated manipulation of the incoherent spectrum by choosing the bunch parameters such that a coherent spectrum with appropriate emission characteristics is generated.

3 Coherent transition radiation for bunch length measurements

In this section an application is described to use the CTR emission properties as diagnostics for attosecond bunch length tuning and optimization. The calculations presented are again based on the parameters of the 100 MeV SINBAD-ARES-linac which will serve as a test facility for the production of ultra-short electron bunches for ultra-fast science [8].

Figure 3 shows calculated CTR emission spectra as function of the ratio λ/σ_z. The spectral characteristics is determined by the bunch tilt angle ψ, the ratio $\sigma_{x,y}/\sigma_z$, and the angular acceptance $\Delta\theta_{x,y}$. For a given — but unknown — bunch length σ_z, the CTR intensity varies widely depending on the observation wavelength λ. In other words, this intensity variation can be used as basis for a diagnostics scheme: while measuring the CTR intensity at different wavelengths, it is possible to determine σ_z.

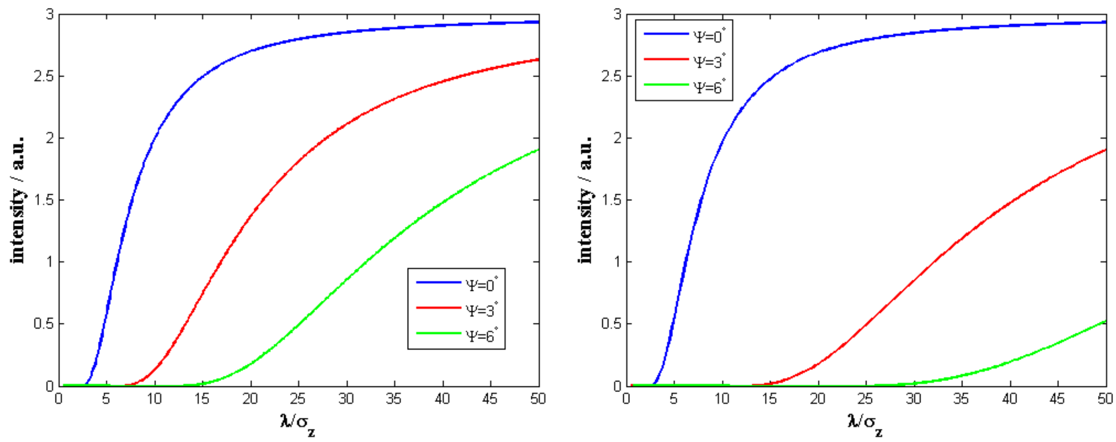

Figure 3. CTR intensity as function of λ/σ_z for (left) $\sigma_{x,y}/\sigma_z = 50$ and (right) $\sigma_{x,y}/\sigma_z = 100$. The angular integration was performed over the acceptance angle $\Delta\theta_{x,y} = \pm 0.5\gamma^{-1}$ with $\gamma = 200$.

As can be concluded from the comparison of both plots in figure 3 with different $\sigma_{x,y}/\sigma_z$, for this bunch length determination it is necessary to know additionally the transverse beam sizes $\sigma_{x,y}$ and the bunch tilt angle ψ. However, according to this figure it is advantageous to use upright standing bunches with $\psi = 0$ deg: firstly, the transverse beam size influence is strongly reduced and secondly, the sensitivity on small bunch length variations is much higher as can be seen from the steeper slope in the vicinity of the coherent threshold. However, it has to be noted that the advantage of a steeper slope sets a limitation on the dynamic range of beam size variations.

In the following the wavelength region will be discussed which can be utilized for attosecond bunch length diagnostics. In order to apply CTR for diagnostics purposes it must be ensured that the intensity is sufficient for a detection scheme, especially having in mind the small bunch charges down to $Q_b = 1$ pC in order to achieve these small bunch lengths. The number of CTR photons emitted in backward direction is calculated according to

$$N_{\mathrm{phot}} = \frac{\alpha}{\pi^2} \int_{\Delta\Omega} \mathrm{d}\Omega \int_{\Delta\lambda} \frac{\mathrm{d}\lambda}{\lambda} \left[N + N(N-1) F(\lambda, \theta_x, \theta_y) \right] \frac{\theta_x^2 + \theta_y^2 + \theta_y \tan\theta_0 \left(\gamma^{-2} + \theta_x^2 + \theta_y^2 \right)}{\left(\gamma^{-2} + \theta_x^2 + \theta_y^2 \right)^2} \tag{3.1}$$

with $\Delta\Omega$ the solid angle covered by an aperture defining the angular acceptance and $\Delta\lambda$ the spectral bandwidth of a wavelength filter.

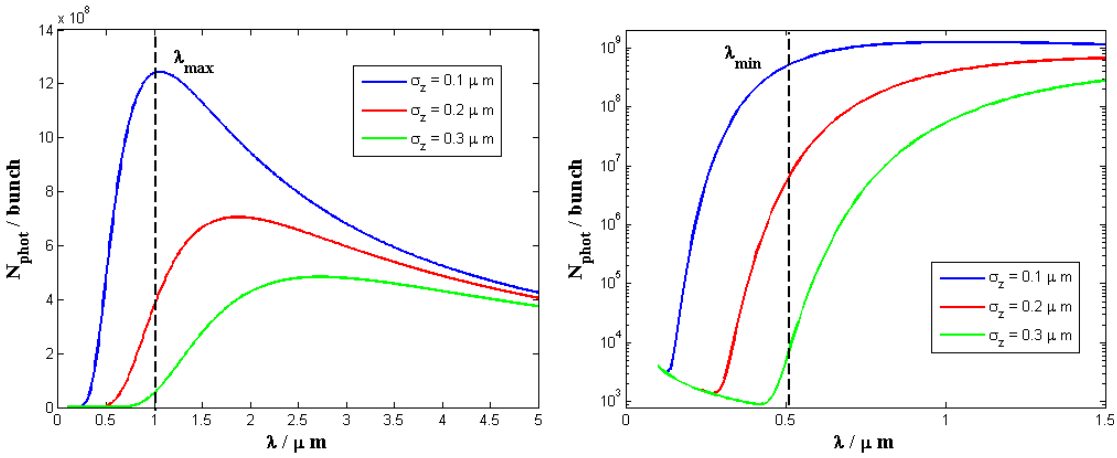

Figure 4. Calculated number of CTR photons in linear (left) and logarithmic (right) representation. Parameters of calculation: $\gamma = 200$, N = 6×10^6, $\sigma_{x,y} = 15\,\mu$m, $\Delta\theta_{x,y} = \pm\gamma^{-1}$, $\Delta\lambda = \pm 50$ nm, and $\psi = 0$ deg.

Figure 4 shows the calculated number of CTR photons per bunch according to eq. (3.1) as function of the wavelength of observation for bunch lengths between 0.1 μm and 0.3 μm (resp. 330 as and 1 fs) in linear (left) and logarithmic (right) representation. A reasonable upper limit $\lambda_{\max}$ is imposed by the maximum of the curve for $\sigma_z = 0.1\,\mu$m in order to avoid ambiguities in the curve progression, a lower limit $\lambda_{\min}$ by the curve for $\sigma_z = 0.3\,\mu$m in order to be above the coherence threshold. To be more specific, for $\sigma_z = 0.3\,\mu$m the number of photons per bunch at the boundaries of the wavelength region of interest are $N_{\mathrm{phot}}(\lambda = 0.6\,\mu\mathrm{m}) = 1.7 \times 10^5$ and $N_{\mathrm{phot}}(\lambda = 1.0\,\mu\mathrm{m}) = 5.3 \times 10^7$ which is by far sufficient for single bunch diagnostics.

In figure 5 the proposed experimental scheme is depicted. CTR in backward direction is generated at an inclined transition radiation target and collimated by means of an aperture which defines the solid angle $\Delta\Omega$. A dispersive optical element (optical grating) transforms the spectral distribution into a spatial one which is recorded by a spatial resolving detector (CCD or line array). The spatial distribution is affected by the detector quantum efficiency which has to be taken into account as additional correction.

To simplify the analysis it is sufficient to perform a measurement for only two different wavelengths and to form the intensity ratio

$$R(\sigma_z) = \frac{N_{\mathrm{phot}}(\lambda_i)}{N_{\mathrm{phot}}(\lambda = 1\,\mu\mathrm{m})}\ . \tag{3.2}$$

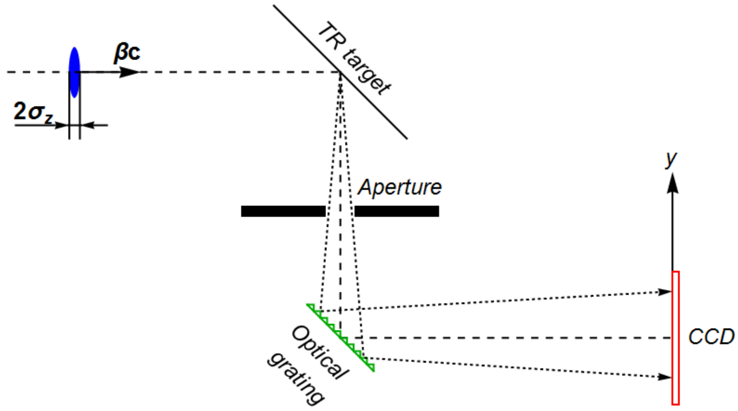

Figure 5. Sketch of the proposed experimental scheme for attosecond bunch length diagnostics.

In order to determine the most suitable wavelength λ_i, figure 6 (a) shows a comparison of intensity ratios according to eq. (3.2). From there it can be concluded that the best sensitivity could be achieved for the smallest wavelength $\lambda_i = 0.6\,\mu$m because of the steepest slope at small bunch lengths. As demonstrated in figure 6 (b), the sensitivity could even be enhanced by using a smaller aperture of $\Delta\theta_{x,y} = \pm 0.5\gamma^{-1}$. Even in this case the intensity would still be sufficient for single bunch diagnostics, the calculated number of CTR photons per bunch amounts to $N_{\text{phot}}(\lambda = 0.6\,\mu\text{m}) = 3.1 \times 10^4$ and $N_{\text{phot}}(\lambda = 1.0\,\mu\text{m}) = 8.2 \times 10^6$.

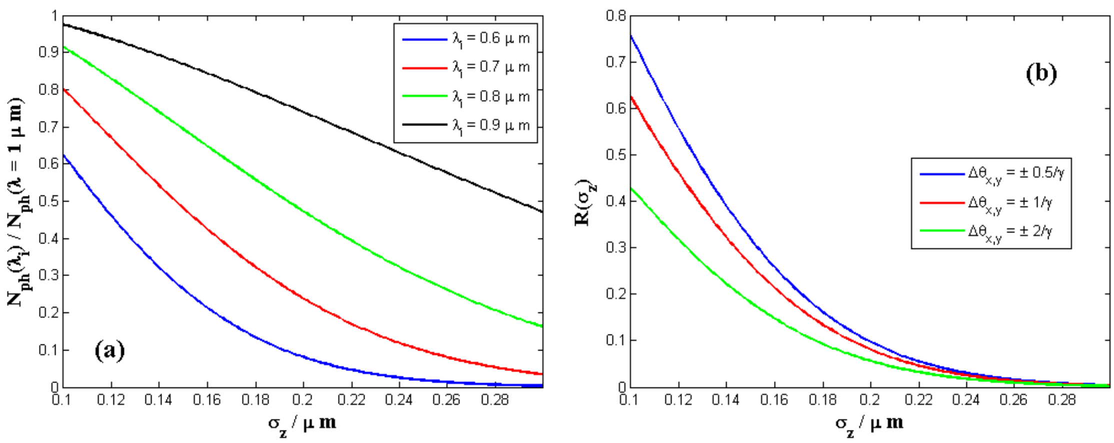

Figure 6. Sensitivity on bunch length determination. (a) Determination of the lower reference wavelength λ_i, calculation for $\Delta\theta_{x,y} = \pm\gamma^{-1}$. (b) Influence of the angular aperture size, calculation for $\lambda_i = 0.6\,\mu$m. The remaining parameters are the same than for figure 4.

In order to estimate the achievable accuracy for such a measurement, the dependency of the measured ratio $R(\sigma_z)$ on the bunch length σ_z as shown in figure 6 (b) is parametrized by an exponential function according to

$$R(\sigma_z) \approx A_0 \exp(-\sigma_z/\sigma_0)\,. \tag{3.3}$$

With the fitted parameters $\sigma_0 = 0.05\,\mu$m and $A_0 = 5.8\ (4.8, 3.4)$ for $\Delta\theta_{x,y} = \pm 0.5/\gamma\ (\pm 1/\gamma, \pm 2/\gamma)$, the accuracy between the calculated and fitted curves shown in figure 6(b) is better than 8%. Using eq. (3.3) it is possible to estimate the error in bunch length determination $\Delta\sigma_z$ assuming an experimental error ΔR by applying error propagation,

$$\Delta\sigma_z \approx \left|\sigma_0 \frac{\Delta R}{R}\right|. \tag{3.4}$$

As a worst case scenario it is supposed that the relative error in the measurement of the ration R will be 100%. Evidently, even in this case the accuracy in bunch length determination will amount to $\Delta\sigma_z = 0.05\,\mu$m which is a rather small value for such a simple experimental scheme.

4 Summary and conclusion

In this paper the spectral angular distribution of coherent BTR from pancake-like bunches is described. It is demonstrated that the emission properties can be controlled by the ratios λ/σ_z, $\sigma_{x,y}/\sigma_z$ and by the bunch orientation ψ. Based on the CTR emission characteristics, a simple bunch length measurement scheme using collimated CTR in combination with a spectral resolving detection scheme is proposed by which it is possible to optimize and tune bunch lengths in the order of attoseconds. However, as a constraint the measurement scheme requires the knowledge of the transverse beam sizes σ_x, σ_y and of the bunch tilt angle ψ.

For the case under study of symmetrical pancake-like bunches with $\sigma_x = \sigma_y \gg \sigma_z$ and $\sigma_z \leq 0.3\,\mu$m, the possibility is shown to perform bunch length diagnostics using the measured photon yield ratio for two wavelengths $\lambda_1 = 0.6\,\mu$m and $\lambda_2 = 1.0\,\mu$m. In this case the CTR intensity will be sufficient to perform single bunch diagnostics.

For the new SINBAD-ARES-linac which is presently under construction at DESY, a test experiment for studying the CTR emission characteristics in view of possible applications for bunch length diagnostics will be proposed.

Acknowledgments

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