First order sensitivity analysis of electron acceleration in dual grating type dielectric laser accelerator structures

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Abstract. Symmetrically driven dual-grating type DLA (Dielectric Laser Accelerator) linac structures allow for in-channel electric field gradients on the order of GV/m at optical wavelengths. In this work we study the sensitivity of important final beam parameters like mean energy, energy spread and transverse emittance on DLA drive laser as well as input beam parameters. To this end a fast specialized particle tracking code (DLATracker) is used to compute the so called first order sensitivity indices based on a large number of Monte Carlo simulation runs of an exemplary external injection based DLA experiment. The results of this work point out important stability constraints on the drive laser setup and the externally injected electron beam.

1. Introduction

The concept of dielectric laser acceleration (DLA) has gained growing attention in the past years due to the high achievable electric field gradients of the order of ∼GV/m at optical wavelengths [1, 2]. Operation at micron wavelengths at the same time implies a significant size reduction of the accelerating structures by orders of magnitude compared to traditional RF technology. This reduction in size of both the device itself, but also the period of the accelerating field leads to challenging timing stability ($<$ 1 fs rms), as well as beam size and alignment ($<$ 1 micron) requirements. In this work we apply the so-called *First Order Sensitivity Analysis* technique described by Saltelli et al. [3] to the external injection and acceleration of pre-accelerated electron bunches in laser-driven dual grating-type DLA structures[1]. The goal of this analysis is to identify the individual contribution of a given number of input parameters to the stability of an output parameter of interest. This way important stability constraints on the drive laser setup and the externally injected electron bunch can be extracted. In the next section the theory behind this specific sensitivity analysis is described following the derivation given in [4].

2. Theory

In classical accelerators, the stability of beam parameters like *mean beam energy*, *beam spot size*, etc. is ultimately the result of the stability of basic input parameters like the *gun RF*

[1] See [4] for an application of this method to classical accelerators.

phase, or the *current of focusing magnets*. For the DLA-based external injection experiment important input parameters are for example the *drive laser amplitude and phase*, but also the *electron beam centroid position*, etc.. In certain cases the effect of an input parameter can be readily infered from the physics behind the specific interaction. At the same time the process or output parameter of interest might depend on multiple factors. This can lead for example to the phenomenon of jitter compensation, when certain parameters influence each other (cf. [4]).

In order to model the external injection experiment we consider a transfer function

$$P_i = F_i(\mathbf{a}), \tag{1}$$

where P_i are beam parameters of interest and $\mathbf{a}$ is a set of input parameters specific to the experiment. In the ideal case F_i would be an analytical formula, which fully describes the DLA-based acceleration process. In our case – because of the complexity of the system – F_i is not available. Instead it is necessary to rely on numerical simulations to model this *black-box*. In this study DLATracker [5] is used to perform these simulations.

2.1. Model Free Sensitivity Measures

There are multiple ways to conduct a sensitivity analysis of a given system. If the system can be analytically described, the method of so called *sigma normalized partial derivatives* (SNPD) is a useful sensitivity measure (cf. [3]). The sensitivity measure is defined by

$$\tilde{S}_{a_i} = \tilde{\sigma}_i \cdot \frac{\partial f(\mathbf{a})}{\partial a_i}, \tag{2}$$

where $\tilde{\sigma}_i = \sigma_{a_i}/\sigma_{f(\mathbf{a})}$ and σ refers to the standard deviation. It can be shown that for a linear additive model like

$$f(\mathbf{a}) = \sum_{i=0}^{M} c_i a_i \tag{3}$$

the sum of the $\tilde{S}^2_{a_i}$ is equal to one, which is a necessary requirement to be considered a comparable sensitivity measure.

If the system – as in our case – cannot be described by a simple mathematical model, so called *model free sensitivity measures* are needed. In this study the method of *averaged partial variances* is used. This method is based on a large number of Monte Carlo runs of a given model. This way also numerical models can be used. The resulting sensitivity measure is refered to as the *first order sensitivity index* of a given input parameter to the system. It is given by

$$S_i = \frac{\mathrm{V}_{a_i}(\mathrm{E}_{\sim a_i}(f(\mathbf{a})|a_i))}{\mathrm{V}(f(\mathbf{a}))} \in [0, 1], \tag{4}$$

where – sticking to the literature – V is the variance, E the expectation value and $\sim a_i$ means “all but a_i” (cf. [3]). The numerator can be read as “the variance of the expectation value of $f(\mathbf{a})$ for fixed (known) a_i”. One additional advantage of this measure is the fact that the whole configuration space is explored instead of focusing on one fixed a_i. In order to calculate the S_i, Monte Carlo data of a given output parameter is plotted vs. a given input parameter ($\rightarrow$ *scatter plot method*). Then the data is divided into slices along the abscissa and the mean of the data contained within these slices is determined. The variance of these slice averages – in the limit of infinite slices – now corresponds to the so called *first order effect* of a_i on $f(\mathbf{a})$. Eq. 4 is its normalized form. Since this model can only describe the first order effect and therefore neglects all interdependencies between the input parameters, it is implied that for a model, where the interdependencies between input parameters affect the output

$$S_T = \sum_{i=1}^{M} S_i < 1. \tag{5}$$

It is important to note here that *first order* in this scheme does not mean *linear dependence*. It means that the effect of a_i on $f(\mathbf{a})$ does not depend on the state of any other input parameter.

3. First Order Sensitivity Analysis

Since Eq. 4 is based on the limit of infinite slices, a large number of runs is needed in order to converge [4]. For this study 15000 DLATracker runs were performed. In each of these runs eight input parameters were varied according to a Gaussian distribution. Table 1 summarizes the input parameters and how they were varied in the course of the Monte Carlo experiment. We assume a phase-locked acceleration scheme, as presented in [6]. In this scheme the drive laser is split in order to illuminate both the upper and the lower part, resulting in an incoming common laser phase and two individual relative phases of the two arms. Furthermore, we consider a case where the laser pulse is much longer than the time needed for the electrons to traverse DLA period, i.e. a quasi steady state case. As output parameters *energy gain* (ΔE), *rms energy spread* (σ_E), as well as *normalized horizontal rms emittance* ($\epsilon_{n,x}$) were chosen. Table 2 shows the most important parameters of the DLATracker input file used as the template for the Monte Carlo runs.

In order to ensure validity of the calculated first order sensitivity indices, a convergence test was carried out. Figure 1 shows the results of the sensitivity analysis for different numbers of data slices. It can be seen that due to the large number of model runs 50 slices are already enough to reach convergence.

Based on this convergence test the analysis was carried out for all of the output parameters mentioned above. Figure 2 exemplarily shows the raw data for the output parameter $\epsilon_{n,x}$. From the scatter plots and the corresponding slice analysis (as discussed above, see solid lines) the sensitivity towards certain input parameters can already be seen qualitatively by eye. In the following the quantitative results are shown and discussed.

3.1. Results and Discussion

Table 3 summarizes the the result of the first order sensitivity analysis for all input and output parameters. The first interesting observation is that in the case of the *energy gain* $\sum S_i$ actually

Table 1. Input parameters chosen to be varied for each Monte Carlo run. Given are the center μ and the RMS width σ of the corresponding Gaussian distribution.

Name	μ_i	σ_i
a_1: Laser Phase (Common)	0.0 rad	0.01 rad
a_2: Laser Amplitude Scaling	1.0 a.u.	0.01 a.u.
a_3: Laser Wavelength	2.0 um	0.01 um
a_4: Electron σ_z	0.1 um	0.01 um
a_5: Electron Bunch σ_x	0.1 um	0.01 um
a_6: Electron Bunch E_0	100.0 MeV	100.0 keV
a_7: $\Delta\phi$ Upper Grating	0.0 rad	0.01 rad
a_8: $\Delta\phi$ Lower Grating	0.0 rad	0.01 rad

Table 2. Key parameters used in the base input file for DLATracker.

Parameter	Value
Macro Particles	10000
RMS Emittance (x,y)	50 nm
Grating Type	Rectangular Dual Grating (hor.)
Spatial Harmonics	20
E_{sync}	100 MeV
Period	$\beta_{\text{sync}} \cdot 2.0$ um
Channel Width	1.5$\cdot$ Period
Number of Periods	1

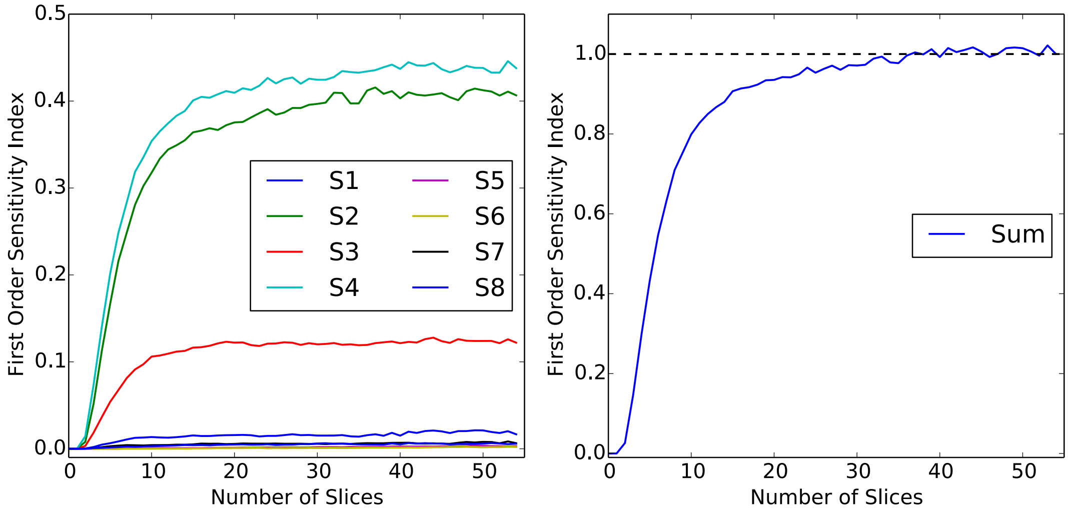

Figure 1. Convergence test for 15000 model runs. Shown are the first order sensitivity indices S_i and $\sum S_i$ vs. the number of data slices used in the analysis. The output parameter here is the *energy gain* (ΔE).

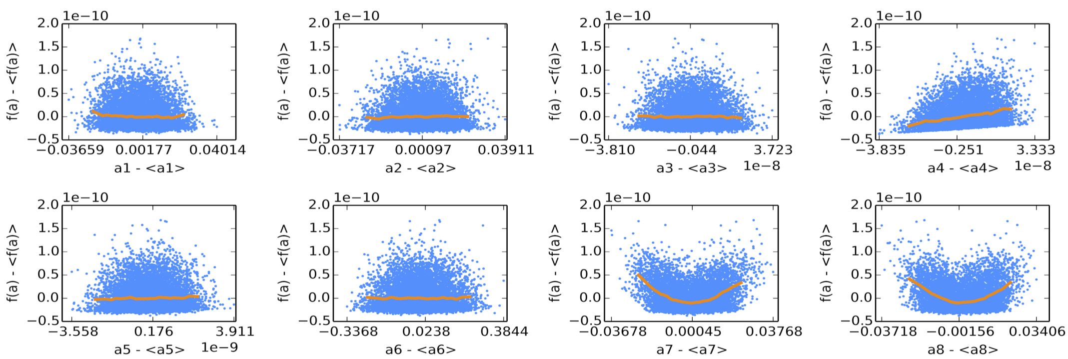

Figure 2. Scatter plots of the raw data obtained from 15000 Monte Carlo model runs. The output parameter $f(\mathbf{a})$ is the *normalized horizontal rms emittance* ($\epsilon_{n,x}$). The Input parameters a_i are specified in Table 1. **Scatter plot**: Raw data. **Solid line**: Average of the output data within a data slice. Note the different scaling of the axes.

Table 3. First order sensitivity indices for the slice based scatter plot analysis using DLATracker simulations.

	S_1	S_2	S_3	S_4	S_5
$\epsilon_{n,x}$	0.006	0.002	0.002	0.07	0.003
ΔE	0.017	0.407	0.122	0.438	0.003
σ_E	0.002	0.014	0.01	0.932	0.002
	S_6	S_7	S_8		ΣS_i
$\epsilon_{n,x}$	0.001	0.206	0.182		0.471
ΔE	0.002	0.006	0.006		1.000
σ_E	0.002	0.002	0.002		0.967

converges to 1.0. This means that there is no interdependency between the S_i. From the obtained S_i it becomes clear that for realistic σ_i, S_2 and S_4 are the most crucial parameters with values > 0.4. S_2 here corresponds to the laser amplitude scaling. It is intuitive that this parameter is important for the mean energy gain. S_4, the electron bunch length, is the most important parameter for the mean energy gain. This also makes sense, since for longer bunches significant parts might already be decelerated or not accelerated at all. The third highest contribution to the overall effect is S_3, the laser wavelength, which in other words describes the mismatch between the structure and the drive laser and effectively translates into a laser to electron phase error (cf. [7], spatial phase Ψ). Note that this mismatch jitter can also be caused e.g. by laser beam pointing jitter, as the incident angle determines the effective/projected grating period. Hence the relatively large σ_3 in this study. This is an interesting observation, as it constitutes a phase jitter source even for the assumed phase locked scheme. All other input parameters have negligible contributions. This essentially means that the accelerating fields in the 100 MeV ($\beta \approx 1$) case are sufficiently constant across the channel.

The results for the *rms energy spread* are not suprising as the most important input parameter is by far S_4, the electron bunch length, with a value of > 0.9. There seem to be small interdepencies between the S_i, as the sum does not fully converge to 1.0.

The *normalized horizontal rms emittance* is dominated by S_7 and S_8, which correspond to the phase errors of the two individual drive laser arms respectively. The values are both ≈ 0.2. If the phase between the two drive lasers differs, the DLA fields get asymetric across the channel, which influences the transverse emittance of the electron beam. As the transverse forces are also phase dependent, the rms bunch length is again also an important parameter. Here $S_4 = 0.07$. In the case of $\epsilon_{n,x}$ there are clearly interdependencies between the S_i. The sum converges to 0.47, which means that less than half of the effect on $\epsilon_{n,x}$ can be attributed to the a_i alone.

4. Conclusion

We have performed a first order sensitivity analysis of the process of the external injection of pre-accelerated electron bunches into a dual grating type DLA structure based on a large number of Monte Carlo runs of DLATracker simulations. From the results it can be seen that both ΔE and σ_E can be described mostly by first order effects of the given input parameters. $\epsilon_{n,x}$ on the other hand is cleary influenced either by higher order interdependencies of the a_i, or by input parameters not considered in this study. Therefore a higher order analysis as described in [3] and [4] is necessary here.

Further studies could for example focus on the analysis of the staging of multiple DLA structures and the implied complicated drive laser distribution system.

Finally it has to be noted that this kind of analysis does not have to be based on simulations.

It can also be performed on experimental data. If the data aquisition of all relevant machine parameters is time synchronized, recorded data then corresponds to the Monte Carlo runs of a given model.

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