Duration of Classicality of Homogeneous Condensates with Attractive Interactions

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DOI: http://dx.doi.org/10.3204/DESY-PROC-2017-02/sikivie_pierre

Dark matter axions and other highly degenerate bosonic fluids are commonly described by classical field equations. In a recent paper [1] we calculated the duration of classicality of homogeneous condensates with attractive contact interactions and of self-gravitating homogeneous condensates in critical expansion. According to their classical equations of motion, such condensates persist forever. In their quantum evolution parametric resonance causes quanta to jump in pairs out of the condensate into all modes with wavevector less than some critical value. We estimated in each case the time scale over which the condensate is depleted and after which a classical description is invalid.

This contribution to the Proceedings of the 13th Patras Workshop on Axions, WIMPs and WISPs (Thessaloniki, May 15 to 19, 2017) is a summary of our recent paper ”Gravitational interactions of a degenerate quantum scalar field” [1]. We quote extensively from the Introduction to that paper, and then state the paper’s main results on the duration of classicality of homogeneous condensates with attractive interactions.

One of the leading candidates for the dark matter of the universe is the QCD axion. It has the double virtue of solving the strong CP problem of the standard model of elementary particles [2, 3] and of being naturally produced with very low velocity dispersion during the QCD phase transition [4], so that it behaves as cold dark matter from the point of view of structure formation [5]. Several other candidates, called axion-like particles (ALPs) or weakly interacting slim particles (WISPs), have properties similar to axions as far as the dark matter problem is concerned [6]. ALPs with mass of order \(10^{-21}\) eV, called ultra-light ALPs (ULALPs), have been proposed as a solution to the problems that ordinary cold dark matter is thought to have on small scales [7]. Axion dark matter has enormous quantum degeneracy, of order \(10^{61}\) or more. The degeneracy of ULALP dark matter is even higher [9]. In most discussions of axion or ALP dark matter, the particles are described by classical field equations. The underlying assumption appears to be that huge degeneracy ensures the correctness of a classical field description.

However it was found in refs. [8, 10, 11, 12] that cold dark matter axions thermalize, as a result of their gravitational self-interactions, on time scales shorter than the age of the universe after the photon temperature has dropped to approximately one keV. When they thermalize, all the conditions for their Bose-Einstein condensation are satisfied and it is natural to assume that this is indeed what happens. Axion thermalization implies that the axion fluid does not obey classical field equations since the outcome of thermalization in classical field theory is a UV catastrophe, wherein each mode has average energy \(k_B T\) no matter how high the mode’s oscillation frequency, whereas the outcome of thermalization of a Bosonic quantum field is to produce a Bose-Einstein distribution. On sufficiently short time scales, the axion fluid does
obey classical fields equations. It behaves then like ordinary cold dark matter on all length scales longer than a certain Jeans length \[13, 14\]. However, on longer time scales, the axion fluid thermalizes. When thermalizing, the axion fluid behaves differently from ordinary cold dark matter since it forms a Bose-Einstein condensate, i.e. almost all axions go to the lowest energy state available to them. Ordinary cold dark matter particles, weakly interacting massive particles (WIMPs) and sterile neutrinos do not have that property.

Axion thermalization has implications for observation. It was found \[10\] that the axions which are about to fall into a galactic potential well thermalize sufficiently fast that they almost all go to their lowest energy state consistent with the total angular momentum they acquired from tidal torquing. That state is one of rigid rotation in the angular variables (different from rigid body rotation but similar to the rotation of water going down a drain), implying that the velocity field has vorticity \((\vec{\nabla} \times \vec{v} \neq 0)\). In contrast, ordinary cold dark matter falls into gravitational potential wells with an irrotational velocity field \[15\]. The inner caustics of galactic halos are different in the two cases. If the particles fall in with net overall rotation the inner caustics are rings whose cross-section is a section of the elliptic umbilic catastrophe, called caustic rings for short \[16, 17\]. If the particles fall in with an irrotational velocity field, the inner caustics have a tent-like structure \[15\] quite distinct from caustic rings. Observational evidence had been found for caustic rings. The evidence is summarized in ref. \[18\]. It was shown \[19, 20\] that axion dark matter thermalization and Bose-Einstein condensation explains the evidence for caustic rings of dark matter in disk galaxies in detail and in all its aspects, i.e. it explains not only why the inner caustics are rings and why they are in the galactic plane but it also correctly accounts for the overall size of the rings and the relative sizes of the several rings in a single halo. Finally it was shown that axion dark matter thermalization and Bose-Einstein condensation provide a solution \[20\] to the galactic angular momentum problem \[21\], the tendency of galactic halos built of ordinary cold dark matter (CDM) and baryons to be too concentrated at their centers. An argument exists therefore that the dark matter is axions, at least in part. Ref. \[20\] estimates that the axion fraction of dark matter is 35% or more.

The above claimed successes notwithstanding, axion thermalization and Bose-Einstein condensation is a difficult topic from a theoretical point of view. Thermalization by gravity is unusual because gravity is long-range and, more disturbingly, because it causes instability. Bose-Einstein condensation means that a macroscopically large number of particles go to their lowest energy state. But if the system is unstable it is not clear in general what is the lowest energy state. The idea that dark matter axions form a Bose-Einstein condensate was critiqued in refs. \[22, 23, 24\]. It was concluded in ref. \[24\] that “while a Bose-Einstein condensate is formed, the claim of long-range correlation is unjustified.”

The aim of our recent paper \[1\] was to clarify aspects of Bose-Einstein condensation that appear to cause confusion, at least as far as dark matter axions are concerned. One issue is whether a Bose-Einstein condensate needs to be homogeneous (i.e. translationally invariant as is a condensate of zero momentum particles). We answer this question negatively. A Bose-Einstein condensate can be, and generally is, inhomogeneous. Nonetheless, merely by virtue of being a Bose-Einstein condensate, it is correlated over its whole extent, and its extent can be arbitrarily large compared to its scale of inhomogeneity.

A second question is whether Bose-Einstein condensation can be described by classical field equations. We state the following to be true. The behavior of the condensate is described by classical field equations on time scales short compared to its rethermalization time scale. However when the condensate rethermalizes, as it must if situated in a time-dependent background or if it is unstable, it does not obey classical field equations. A phenomenon akin to
Bose-Einstein condensation does exist in classical field theory when a UV cutoff is imposed on the wave-vectors, i.e. all modes with wavevector \( k > k_{\text{max}} \) are removed from the theory. \( k_{\text{max}} \) is related to the critical temperature \( T_{\text{crit}} \) for Bose-Einstein condensation in the quantum field theory. We emphasize however that the relationship \( k_{\text{max}} \) and \( T_{\text{crit}} \) necessarily involves a constant, such as \( \hbar \), with dimension of action. Furthermore, if we replace the quantum axion field by a cutoff classical field, even if a phenomenon similar to Bose-Einstein condensation does occur, there is no proof or expectation that the cutoff classical theory reproduces the other predictions of the quantum theory. In particular, the phenomenology of caustic rings cannot be reproduced in the classical field theory, with or without cutoff, because vorticity (the circulation of the velocity field along a closed curve) is conserved in classical field theory. In contrast, the production of vorticity and the appearance of caustic rings is the expected behavior of the quantum axion fluid.

A broadly relevant question is the following: over what time scale is a classical description of a highly degenerate but self-interacting Bosonic system valid? We call that time scale the duration of classicality of the system. In ref. [1] we calculated the duration of classicality of a homogeneous condensate, initially at rest but with attractive \( \lambda \phi^4 \) interactions (\( \lambda < 0 \)). According to its classical equations of motion, such a condensate persists indefinitely. According to its quantum evolution, quanta jump in pairs out of the condensate into all modes with wavevector less than

\[
k_J = \sqrt{\frac{\lambda n_0}{2m}}
\]

where \( m \) is the particle mass and \( n_0 \) is the condensate density. We find that the condensate is depleted over the time scale

\[
t_{c,\lambda} \sim \frac{2m}{k_J^2} \ln \left( \frac{32\pi^\frac{5}{2} n_0}{k_J^3} \right),
\]

which is its duration of classicality. We also calculated the duration of classicality of a homogeneous self-gravitating condensate in critical expansion, i.e. forming a matter dominated universe which is at the boundary of being open or closed. The condensate is initially described by the wavefunction [9]

\[
\Psi_0(\vec{r}, t) = \sqrt{n_0(t)} e^{\frac{1}{2} m H(t) \vec{r}^2}
\]

where \( H(t) \) is the Lemaître-Hubble expansion rate and

\[
n_0(t) = \frac{1}{6\pi G m t^2}
\]

is the density. Again, according to its classical equation of motion, the condensate lasts forever. According to its quantum evolution, quanta jump in pairs out of the condensate into all modes with wavevector less than

\[
\ell_J(t)^{-1} = \left( 16\pi G n(t) m^3 \right)^{\frac{1}{2}}.
\]

The condensate is depleted after a time of order

\[
t_c \sim \frac{t_\ast}{(G m^2 \sqrt{mt_\ast})^{\frac{3}{2}}}
\]

where \( t_\ast \) is the initial time when all particles were assumed to be in the condensate. A classical description is invalid after time \( t_c \).
Although we only analyze the behavior of homogeneous condensates in [1], we expect our conclusions to apply to inhomogeneous condensates as well. Indeed, a homogeneous condensate can be seen as a limiting case of inhomogeneous condensates. Since homogeneous condensates are depleted by parametric resonance, the same must be true for inhomogeneous condensates, at least in the limit of small deviations away from homogeneity. In fact in simulations of a five oscillator toy model [10, 25] we find that the condensates which persist forever according to their classical evolution are the condensates with the longest duration of classicality in their quantum evolution. We explained this result on the basis of analytical arguments [1]. By analogy with the behavior of the five oscillator toy model, we expect inhomogeneous condensates in quantum field theory to have shorter durations of classicality than homogeneous ones.

Related topics were discussed in two recent papers [26, 27]. Inter alia, ref. [26] solves the classical equations of motion for an initially almost homogeneous condensate with attractive contact interactions numerically on a lattice. If it were strictly homogeneous, the condensate would persist forever. Perturbations are introduced to mimic quantum fluctuations. As the perturbations grow, the condensate is depleted in a manner which appears qualitatively consistent with our quantum field theory treatment. Ref. [27] discusses, as we do, the duration of classicality of the cosmic axion fluid. The conclusions of ref. [27] differ from ours.

1 Acknowledgments

PS gratefully acknowledges the hospitality of the Theoretical Physics Group at the University of Oxford, the Center for Axion and Precision Physics in Daejeon, Korea, and the Institute for the Physics and Mathematics of the Universe in Tokyo. This work was supported in part by the U.S. Department of Energy under grant DE-FG02-97ER41209 and by the Heising-Simons Foundation under grant No. 2015-109.

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