Bootstrapping the space of $4d$ $\mathcal{N} = 2, 3$ SCFTs

Madalena Lemos

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Based on:
1312.5344 w/ C. Beem, P. Liendo, W. Peelaers, L. Rastelli and B. van Rees
1511.07449 w/ P. Liendo
1702.05101 w/ M. Cornaglioitto and V. Schomerus
1 The Superconformal Bootstrap Program
2 A solvable subsector
3 $4d \mathcal{N} = 3$ SCFTs
4 Constraining the space of $\mathcal{N} = 2$ SCFTs
5 Summary and Outlook
1. The Superconformal Bootstrap Program

2. A solvable subsector

3. $4d$ $\mathcal{N} = 3$ SCFTs

4. Constraining the space of $\mathcal{N} = 2$ SCFTs

5. Summary and Outlook
What is the space of consistent SCFTs?
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→ Maximally supersymmetric theories: well known list of theories
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→ $\mathcal{N} = 2$ theories: large known list of theories
  many lacking a Lagrangian description
The Superconformal Bootstrap Program

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Can we bootstrap specific theories?
The Superconformal Bootstrap Program

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**Can we bootstrap specific theories?**

- See Pedro’s talk!
Conformal field theory defined by
Set of local operators and their correlation functions
Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

\[ \{ \mathcal{O}_{\Delta, \ell, \ldots}(x) \} \text{ and } \{ \lambda_{\mathcal{O}_i \mathcal{O}_j \mathcal{O}_k} \} \]
Conformal field theory defined by
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\{O_{\Delta,\ell,\ldots}(x)\} and \{\lambda_{O_iO_jO_k}\}

CFT data strongly constrained

- Unitarity
- Associativity of the operator product algebra
Conformal Bootstrap

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\[
\langle (\mathcal{O}_1(x_1) \mathcal{O}_2(x_2)) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle = \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_{\Delta,\ell}}^2 \mathcal{O}_{\Delta,\ell}
\]
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\begin{align*}
\langle \mathcal{O}_1(x_1)(\mathcal{O}_2(x_2)\mathcal{O}_3(x_3))\mathcal{O}_4(x_4) \rangle &= \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_{\Delta,\ell}}^2 \tilde{\mathcal{O}}_{\Delta,\ell} \\
&= \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\tilde{\mathcal{O}}_{\Delta,\ell}}^2 \tilde{\mathcal{O}}_{\Delta,\ell}
\end{align*}
Various conformal families related by action of supercharges
The Superconformal Bootstrap

- Various conformal families related by action of supercharges
- Finite re-organization of an infinite amount of data
Various conformal families related by action of supercharges

Finite re-organization of an infinite amount of data

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The Superconformal Bootstrap

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→ Yes, for $4d \mathcal{N} \geq 2$ [Beem ML Liendo Peelaers Rastelli van Rees]
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5. Summary and Outlook
Chiral algebra

Organize operators in representations of superconformal algebra

\[ \{ O_{\Delta,(j_1,j_2)} \} \]
Chiral algebra

Organize operators in representations of superconformal algebra

\[ \left\{ \mathcal{O}_{\Delta, (j_1, j_2)}, \underbrace{R}_{SU(2)_R}, \underbrace{r}_{U(1)_r}, f \right\} \]
Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta, (j_1, j_2)}, R_{SU(2)_R}, r_{U(1)_r} \}, f \} \]

Claim

→ Pick a plane \( \mathbb{R}^2 \subset \mathbb{R}^4 \),
Chiral algebra

Organize operators in representations of superconformal algebra

$$\{ O_{\Delta,(j_1,j_2)}, \underbrace{R}_{SU(2)_R}, \underbrace{r}_{U(1)_r}, f \}$$

Claim

→ Pick a plane $\mathbb{R}^2 \in \mathbb{R}^4$, $(z, \bar{z}) \in \mathbb{R}^2$
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→ Pick a plane \( \mathbb{R}^2 \in \mathbb{R}^4, (z, \bar{z}) \in \mathbb{R}^2 \)

\[ \langle \mathcal{O}_{1}^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{n}^{l_n}(z_n, \bar{z}_n) \rangle \]
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Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta, (j_1, j_2)}, R, r, f \} \]

Claim

→ Pick a plane \( \mathbb{R}^2 \in \mathbb{R}^4, \ (z, \bar{z}) \in \mathbb{R}^2 \)

→ Restrict to operators with \( \Delta = 2R + j_1 + j_2 \)

\[ \langle \mathcal{O}^{l_1}_1(z_1, \bar{z}_1) \ldots \mathcal{O}^{l_n}_n(z_n, \bar{z}_n) \rangle \]
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Organize operators in representations of superconformal algebra

\[ \{ O_{\Delta,(j_1,j_2),R^2}, r \}_{SU(2)_R, U(1)_r} \]

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\[ u_{l_1}(\bar{z}_1) \ldots u_{l_n}(\bar{z}_n) \langle O_{1,1}^{l_1}(z_1, \bar{z}_1) \ldots O_{n,n}^{l_n}(z_n, \bar{z}_n) \rangle \]
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→ Meromorphic!
Why?

- Subsector $= \text{Cohomology of nilpotent } \mathbb{Q}$
Chiral algebra

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- Subsector = Cohomology of nilpotent $\mathbb{Q} \sim Q + S$
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  - commutes with $\mathbb{Q}$
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- twisted translations $u_I(\bar{z})$
Why?

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On plane \( sl_2 \times \bar{sl}_2 \) commutes with \( Q \) does not

\[ u_I(\bar{z}) \]

-diagonal subalgebra \( \bar{sl}_2 \times su(2)_R \) is \( Q \) exact
Chiral algebra

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- On plane $\mathfrak{sl}_2 \times \overline{\mathfrak{sl}}_2$
  - Commutes with $\mathbb{Q}$
  - Does not
- Twisted translations $u_I(\bar{z})$
- Diagonal subalgebra $\mathfrak{sl}_2 \times \mathfrak{su}(2)_R$ is $\mathbb{Q}$ exact
- Anti-holomorphic dependence drops out
Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]
Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

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Chiral algebra

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\[\rightarrow q(z, \bar{z})\tilde{q}(0) \sim \bar{z} \tilde{Q}^*(z, \bar{z}) \tilde{Q}(0) \sim \frac{\bar{z}}{z\bar{z}}\]
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4d $\mathcal{N} \geq 2$ SCFT $\rightarrow$ chiral algebra

Which operators are in the cohomology?

$\rightarrow$ Stress tensor $T_{\mu\nu}$
Which operators are in the cohomology?
→ Stress tensor $T_{\mu\nu} \rightsquigarrow$ superdescendant
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu} \rightsquigarrow$ superdescendant
→ Stress tensor supermultiplet
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$\rightarrow$ Stress tensor supermultiplet

$$T(z)T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots ,$$
4d $\mathcal{N} \geq 2$ SCFT $\rightarrow$ chiral algebra

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$\rightarrow$ Stress tensor supermultiplet $\Rightarrow$ 2d stress tensor

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$\leftrightarrow$ Global $\mathfrak{sl}_2$ enhances to Virasoro

$\leftrightarrow c_{2d} = -12c_{4d}$
Which operators are in the cohomology?

→ Theory with flavor symmetry
4d $\mathcal{N} \geq 2$ SCFT $\rightarrow$ chiral algebra

Which operators are in the cohomology?

$\rightarrow$ Theory with flavor symmetry

$\rightarrow$ Multiplet containing flavor current
Which operators are in the cohomology?

→ Theory with flavor symmetry
→ Multiplet containing flavor current
← Affine Kac Moody current algebra

\[ J^a(z)J^b(0) \sim -\frac{k_{4d}/2\delta^{ab}}{z^2} + if^{abc} \frac{J^c(0)}{z} + \ldots , \]
Which operators are in the cohomology?

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\[ J^a(z)J^b(0) \sim -\frac{k_{4d}}{2z^2} + if^{abc} \frac{J^c(0)}{z} + \ldots , \]

← \[ k_{2d} = -\frac{k_{4d}}{2} \]
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→ \ldots
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5 Summary and Outlook
$\mathcal{N} = 3$ Chiral algebra

- $4d \, \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
\[ \mathcal{N} = 3 \text{ Chiral algebra} \]

- \(4d \mathcal{N} \geq 3\): some of the extra supercharges commute with \(Q\)
  \(\Rightarrow 4d \mathcal{N} = 4 \Rightarrow 2d \) “small” \(\mathcal{N} = 4\) chiral algebra
\( \mathcal{N} = 3 \) Chiral algebra

- 4d \( \mathcal{N} \geq 3 \): some of the extra supercharges commute with \( Q \)
  - \( 4d \mathcal{N} = 4 \Rightarrow 2d \) “small” \( \mathcal{N} = 4 \) chiral algebra
  - \( 4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2 \) chiral algebra \[ \text{[Nishinaka, Tachikawa]} \]
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
  - $4d \mathcal{N} = 4 \Rightarrow 2d$ “small” $\mathcal{N} = 4$ chiral algebra
  - $4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

- $2d$ stress tensor promoted to supermultiplet
$\mathcal{N} = 3$ Chiral algebra

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$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$
\( \mathcal{N} = 3 \) Chiral algebra

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  - \( 4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2 \) chiral algebra \([\text{Nishinaka, Tachikawa}]\)

- 2d stress tensor promoted to supermultiplet

2d \( \mathcal{N} = 2 \) Stress tensor \( \mathcal{J} \)

→ Present in any local \( \mathcal{N} = 3 \) SCFT
**$\mathcal{N} = 3$ Chiral algebra**

- $4d \, \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
  
  $\implies 4d \, \mathcal{N} = 4 \implies 2d$ “small” $\mathcal{N} = 4$ chiral algebra

  $\implies 4d \, \mathcal{N} = 3 \implies 2d \, \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

- $2d$ stress tensor promoted to supermultiplet

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**$2d \, \mathcal{N} = 2$ Stress tensor $\mathcal{J}$**

- Present in any local $\mathcal{N} = 3$ SCFT

- A trivial statement in $2d$:
  
  $\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$
Space of $\mathcal{N} = 3$ SCFTs

$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

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- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d} \mathcal{O}_{2d}$$
Space of $\mathcal{N} = 3$ SCFTs

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- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \rightarrow \lambda_{\mathcal{O}_{2d}}^2$$
Space of $\mathcal{N} = 3$ SCFTs

$2d$ $\mathcal{N} = 2$ Stress tensor $\mathcal{I}$

$\langle \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} \rangle$ is fixed in terms of $c_{2d}$

- $2d$ Superblock decomposition:

$$
\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \rightarrow \mathcal{O}_{2d}
$$

$\rightarrow \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2$

assumptions: interacting theory, unique stress tensor
2d $\mathcal{N} = 2$ **Stress tensor** $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- **2d Superblock decomposition:**

  \[
  \sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \rightarrow \mathcal{O}_{2d}
  \]

  $\xrightarrow{\lambda_{\mathcal{O}_{2d}}^2} \lambda_{\mathcal{O}_{4d}}^2 \geq 0$

  assumptions: interacting theory, unique stress tensor

$4d$ unitarity
Space of $\mathcal{N} = 3$ SCFTs

$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

\[
\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}} \rightarrow \lambda^2_{\mathcal{O}_{2d}} \sum_{\mathcal{O}_{4d}} \lambda^2_{\mathcal{O}_{4d}} \geq 0 \Rightarrow \text{New unitarity bound}
\]

4d unitarity

assumptions: interacting theory, unique stress tensor
Space of $\mathcal{N} = 3$ SCFTs

$2d \, \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \xrightarrow{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{4d}}^2 \geq 0 \implies \text{New unitarity bound}$$

assumptions: interacting theory, unique stress tensor

$$c_{4d} \geq \frac{13}{24} \quad \text{[Cornagliotto, ML, Schomerus]}$$
Space of $\mathcal{N} = 3$ SCFTs

$2d$ $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$c_{4d} \geq \frac{13}{24}$ [Cornagliotto, ML, Schomerus]

→ Not saturated by any known SCFT
2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

\[ c_{4d} \geq \frac{13}{24} \]  
[Cornagliootto, ML, Schomerus]

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smallest interacting known theory: \[ c_{4d} = \frac{15}{12} \]
Space of $\mathcal{N} = 3$ SCFTs

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smallest interacting known theory: $c_{4d} = \frac{15}{12}$

$\rightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]
Space of $\mathcal{N} = 3$ SCFTs

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Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

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$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct 4d operators appearing in $\mathcal{J} \mathcal{J}$

$\rightarrow$ Inconsistent with an *interacting* 4d SCFT
2d $\mathcal{N} = 2$ **Stress tensor $J$**

\[ c_{4d} > \frac{13}{24} \]  

[Cornaglione, ML, Schomerus]

$\rightarrow$ Not saturated by any known SCFT

smallest interacting known theory: $c_{4d} = \frac{15}{12}$

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$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct 4d operators appearing in $J J$

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1. The Superconformal Bootstrap Program
2. A solvable subsector
3. $4d$ $\mathcal{N} = 3$ SCFTs
4. **Constraining the space of** $\mathcal{N} = 2$ SCFTs
5. Summary and Outlook
$4d \mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$\langle TTTT \rangle, \langle J^a J^b J^c J^d \rangle, \langle TT J^a J^b \rangle$
$4d \mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle TTJ^a J^b \rangle$

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]
1. The Superconformal Bootstrap Program
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New constraints on the space of allowed $\mathcal{N} > 1$ SCFTs
Summary and Outlook

New constraints on the space of allowed $\mathcal{N} > 1$ SCFTs

$\rightarrow$ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$
Summary and Outlook

New constraints on the space of allowed $\mathcal{N} > 1$ SCFTs

→ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$

← can we improve on this bound analytically?
What are the conditions for a chiral algebra to correspond to a 4$d$ SCFT?
Summary and Outlook

New constraints on the space of allowed $\mathcal{N} > 1$ SCFTs

→ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$
  → can we improve on this bound analytically?
  What are the conditions for a chiral algebra to correspond to a 4d SCFT?

→ Can the numerical bootstrap complement these?
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Numerically solving theories?

▶ Pedro’s talk for the “simplest” $\mathcal{N} = 2$ SCFT
Thank you!
Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

$E_6$ flavor symmetry

[Beem, ML, Liendo, Peelaers, Rastelli, van Rees] [ML, Liendo] [Beem, ML, Liendo, Rastelli, van Rees]