PHYSICAL REVIEW D **96,** 116007 (2017)

Addendum to "Charm and bottom quark masses: An update"

Konstantin G. Chetyrkin,[1] Johann H. Kühn,[2] Andreas Maier,[3] Philipp Maierhöfer,[4] Peter Marquard,[5]
Matthias Steinhauser,[2] and Christian Sturm[6]

[1]*II. Institute for Theoretical Physics, Hamburg University, 22761 Hamburg, Germany*
[2]*Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany*
[3]*IPPP, Department of Physics, University of Durham, Durham DH1 3LE, United Kingdom*
[4]*Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, 79104 Freiburg, Germany*
[5]*Deutsches Elektronen-Synchrotron, DESY, 15738 Zeuthen, Germany*
[6]*Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Emil-Hilb-Weg 22, D-97074 Würzburg, Germany*

(Received 11 October 2017; published 8 December 2017)

We update the experimental moments for the charm quark as computed in [J. H. Kühn, M. Steinhauser, and C. Sturm, Nucl. Phys. **B778**, 192 (2007)] and used in [K. G. Chetyrkin, J. H. Kühn, A. Maier, P. Maierhöfer, P. Marquard, M. Steinhauser, and C. Sturm, Phys. Rev. D **80**, 074010 (2009), K. Chetyrkin, J. H. Kühn, A. Maier, P. Maierhöfer, P. Marquard, M. Steinhauser, and C. Sturm, Theor. Math. Phys. **170**, 217 (2012)] for the determination of the charm-quark mass. The new value for the $\overline{\mathrm{MS}}$ charm-quark mass reads $m_c(3\ \mathrm{GeV}) = 0.993 \pm 0.008$ GeV.

DOI: 10.1103/PhysRevD.96.116007

In Refs. [1–3] the $\overline{\mathrm{MS}}$ charm- and bottom-quark masses have been determined using relativistic sum rules that relate theoretically calculated moments of the photon vacuum polarization function to experimentally measured moments. The latter are determined from measurements of the R-ratio and properties of the narrow resonances. The moments of the vacuum polarization function can be computed in perturbative QCD. In this note we update the experimental input and reevaluate the corresponding moments. New results for the charm-quark mass m_c are presented that are about 35% more precise than those of our previous determination.

For convenience we briefly present the formalism that is used in order to obtain m_c. The nth theory moment is obtained from

$$\mathcal{M}_n^{\mathrm{th}} = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)|_{q^2=0}, \tag{1}$$

where $\Pi_c(q^2)$ is the vector-current correlator with virtual charm-quark loops that can be cast into the form

$$\Pi_c(q^2) = Q_c^2 \frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n z^n, \tag{2}$$

with $z = q^2/(4m_c^2)$. Here $m_c = m_c(\mu)$ is the $\overline{\mathrm{MS}}$ heavy quark mass at the scale μ and $Q_c = 2/3$ is the electric charge of the charm quark in units of the elementary charge. The results that we use for the first four coefficients $\bar{C}_n$ are known up to four-loop accuracy analytically [4]. For applications and calculational techniques of the determination of the related massive tadpole diagrams we refer to the review [5]. Equating the theory moments $\mathcal{M}_n^{\mathrm{th}}$ with the experimentally measured moments,

$$\mathcal{M}_n^{\mathrm{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s), \tag{3}$$

where $R_c = \sigma(e^+e^- \to c\bar{c})/\sigma(e^+e^- \to \mu^+\mu^-)$, leads to

$$m_c = \frac{1}{2}\left(\frac{9Q_c^2}{4}\frac{\bar{C}_n}{\mathcal{M}_n^{\mathrm{exp}}}\right)^{\frac{1}{2n}}, \tag{4}$$

which can be used in order to extract the charm-quark mass. The experimental moments $\mathcal{M}_n^{\mathrm{exp}}$ receive contributions from the narrow resonances, the charm-threshold region, and the continuum region above a center of mass energy $\sqrt{s}$ of about 5 GeV. Even for small values of n the contributions from the J/Ψ and $\Psi(2S)$ resonances are dominant.

There is essential new input from measurements of the electronic decay width Γ_{ee} of the J/Ψ [6] and $\Psi(2S)$ [7] resonances from BES III that is used in the following. The latter value is incorporated into the latest PDG result [8], whereas $\Gamma_{ee}(J/\Psi)$ of Ref. [6] is not included. We thus combine the results from Refs. [6,8] and obtain the updated resonance input parameters as listed in Table I. We also update the mass values for the resonances using the recent PDG values [8]. Note, however, that their improvement has no influence on the results for the moments. Furthermore we update the value of the strong coupling constant and use

TABLE I. Updated input values for the resonance parameters.

	J/Ψ	$\Psi(2S)$
M_Ψ (GeV) [8]	3.096900(6)	3.686097(25)
Γ_{ee} (keV) [6,8]	5.57(8)	2.34(4)
$(\alpha/\alpha(M_\Psi))^2$	0.957785	0.95554

$\alpha_s(M_Z) = 0.1181 \pm 0.0011$ [8] [instead of $\alpha_s(M_Z) = 0.1189 \pm 0.002$ as in Refs. [1–3]].

For the moments we obtain

	$\mathcal{M}_n^{\rm res}$	$\mathcal{M}_n^{\rm cc}$	$\mathcal{M}_n^{\rm cont}$	$\mathcal{M}_n^{\rm exp}$	$\mathcal{M}_n^{\rm np}$
n	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$
1	0.1191(14)	0.0318(15)	0.0645(10)	0.2154(23)	−0.0001(3)
2	0.1169(15)	0.0178(8)	0.0143(3)	0.1490(17)	−0.0002(5)
3	0.1165(15)	0.0101(5)	0.0042(1)	0.1308(16)	−0.0004(8)
4	0.1176(16)	0.0058(3)	0.0014(0)	0.1248(16)	−0.0006(12)

and the updated table for the charm-quark mass reads

n	$m_c(3\text{ GeV})$	exp	α_s	μ	$\text{np}_{\rm LO}$	total	$m_c(m_c)$
1	0.993	0.007	0.004	0.002	0.001	0.008	1.279
2	0.982	0.004	0.007	0.005	0.001	0.010	1.269
3	0.982	0.003	0.008	0.006	0.001	0.010	1.269
4	1.003	0.002	0.005	0.028	0.001	0.029	1.288

where $\text{np}_{\rm LO}$ indicates that we use the leading order (LO) approximation for the gluon condensate contribution (see also the discussion in Refs. [1,3]).

One observes a noteworthy reduction of the uncertainty in the experimental moments. As compared to the results from Ref. [1] there is an increase in the charm-quark mass by 7 MeV for $n = 1$, by 6 MeV for $n = 2$, by 4 MeV for $n = 3$ and a decrease by 1 MeV for $n = 4$. For $n = 1$, which constitutes our final result, the uncertainty decreases from 13 to 8 MeV. Within the uncertainty all results in the above table are consistent with each other and with the results obtained in Ref. [1]. Our final result for the $\overline{\rm MS}$ charm-quark mass reads $m_c(3\text{ GeV}) = 0.993 \pm 0.008$ GeV and $m_c(m_c) = 1.279 \pm 0.008$ GeV.

We thank Achim Denig for clarifications concerning Refs. [6,7] and Ruth Van De Water for questions initiating this new analysis. This work is supported by the Bundesministerium für Bildung und Forschung (BMBF) through Grants No. 05H15VKCCA and 05H15GUCC1.

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