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CEP dependence of signal and idler upon pump-seed synchronization in optical parametric amplifiers

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We study the effect of pump-seed timing fluctuations on the carrier-envelope phase (CEP) of signal and idler pulses emerging from an OP(CP)A. A simple analytical model is derived in order to provide an intuitive explanation of the origin of CEP fluctuations, while split-step simulations are performed to cover a broad range of different seeding schemes. Finally, we compare the simulation results with real observations of the CEP of idler pulses generated by an OPA. The quantitative model presented provides a key tool for designing the next generation of low-noise CEP-stable OP(CP)A-based sources. © 2018 Optical Society of America

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Full control of the electric field of an ultrashort laser pulse is necessary to precisely control light–matter interaction processes happening on femtosecond (fs) and attosecond (as) time scales [1–3]. To describe the time-dependence of the electric field of a laser pulse, the standard notation decomposes $E(t)$ into the product of a carrier wave and a slowly varying amplitude or pulse envelope. To fully define such field, the phase between the carrier wave and the peak of the amplitude envelope (the *carrier-envelope phase* [CEP]) needs to be specified.

Laser sources based on optical parametric amplifiers (OPAs) established themselves in several scientific research fields due to their ability to generate ultrashort pulses with durations down to few fs and energies up to several millijoules [4], covering the spectral range that spans from the midinfrared to the ultraviolet. Furthermore, the possibility of producing CEP-stable pulses via difference-frequency generation [5,6] allows achieving full control on the electric field. In the past decade, many CEP-stable OPAs were proposed and realized, some achieving

good levels of stability [7,8]. However, in order to reach high pulse energies, OP(CP)A systems have grown in size and complexity, entailing an increase in the timing jitter among the different amplification stages.

In this Letter, we intend to clarify the influence of pump-seed timing fluctuations on the CEP of both signal and idler pulses emerging from an OP(CP)A. We analyze two of the most significant schemes: generation of a CEP-stable idler and amplification of a broadband stretched signal successively compressed.

We start by considering the system of three coupled equations [9], which describes parametric amplification along the propagation direction, within the approximation of monochromatic plane waves and perfect phase matching. The parametric amplification process is maximized when the *generalized phase* $\Phi = \phi_p - \phi_s - \phi_i$ is equal to $\pi/2$ [10], in other words $\phi_i = \phi_p - \phi_s - \pi/2$. It is worth noting that the phase ϕ_k ($k = p, s, i$), represents the *absolute phase* (AP) of each wave, that is, the phase with respect to the lab reference frame and not the CEP of the corresponding pulses. To analytically determine the CEP of the signal and idler pulses generated in an OPA, it would be necessary to solve the aforementioned coupled equation system while considering the time dependency of the amplitude envelope of each pulse and the effect of dispersion. Unfortunately, no analytic solutions have been found in this case. However, within certain limits, it is still possible to describe analytically the effect of the pump-seed timing fluctuations on the CEP of signal and idler.

Consider a white-light (WL) seeded OPA or any other parametric amplifier where the seed has been produced starting from a portion of the pump pulse via a coherent process, i.e., the seed pulse has inherited the AP from the pump pulse. During the spectral broadening, an additional term is added to the phase of the seed, but we will not consider it because such phase term has no dependence on timing fluctuations but rather on intensity fluctuations, whose effects on CEP goes beyond the scope of this paper. For small pump-seed relative arrival time fluctuations ΔT compared with the pump pulse

duration ($\Delta T \ll \tau_p$), in the case that the seed pulse duration is longer than the pump pulse duration ($\tau_{\text{seed}} \gg \tau_p$), it is possible to describe pump and seed electric field (neglecting the complex conjugate) in the time reference frame of the pump pulse, peaked at $t = 0$, as

$$\begin{aligned} E_p(t) &= A_p(t)e^{i(\omega_p t + \phi_p)}, \\ E_{\text{seed}}(t - T) &= e^{i(\omega_{\text{seed}}(t-T) + \phi_p)}, \end{aligned} \quad (1)$$

where T represents the relative arrival time difference (ATD) between the seed and the pump pulse. The seed envelope is considered as a (unitary) constant because it varies on a time scale that is much longer than the duration of the pump envelope. During the amplification, the amplitude envelopes of signal and idler pulses will experience the highest gain at the peak of the pump envelope, regardless of the precise arrival time of the longer seed pulse. This effect locks the envelopes of the three pulses together in the crystal until the pump is significantly depleted and backconversion starts to occur [10]. The amplified seed (named signal) can then be written as the product between seed carrier and pump envelope, while the idler pulse, generated via DFG, can be written as the product between seed carrier and pump pulse:

$$\begin{aligned} E_s(t) &= \alpha_s A_p(t) e^{i(\omega_{\text{seed}}(t-T) + \phi_p)} \quad \text{and} \\ E_i(t) &= \alpha_i A_p(t) e^{i(\omega_p t + \phi_p - \omega_{\text{seed}}(t-T) - \phi_p - \pi/2)} \\ &= \alpha_i A_p(t) e^{i(\omega_i t + \omega_{\text{seed}} T - \pi/2)}, \end{aligned} \quad (2)$$

where α_s and α_i accounts for the conversion efficiencies, the $-\pi/2$ term comes from the aforementioned generalized phase, and we have defined $\omega_i = \omega_p - \omega_{\text{seed}}$. By looking at each phase for $t = 0$, we can now obtain the CEPs of signal and idler, respectively, $\Psi_s = -\omega_{\text{seed}} T + \phi_p$ and $\Psi_i = \omega_{\text{seed}} T - \pi/2$. If the relative ATD T is kept constant from one pulse to the next, meaning that $\Delta T = T_N - T_{N-1} = 0$, the CEP of the idler pulses is stabilized, while the CEP of the signal still depends on ϕ_p . If T varies by π/ω_{seed} , the CEP of both signal and idler changes by π (with opposite signs), that is $\Delta\Psi_{s,i} = \pm\omega_{\text{seed}}\Delta T$.

Following the same line of argumentation, it is now possible to obtain the CEP in the hypothesis that $\tau_p \gg \tau_{\text{seed}}$. In this case, pump and seed can be written as

$$\begin{aligned} E_p(t - T) &= A_p(t) e^{i(\omega_p(t-T) + \phi_p)} \quad \text{and} \\ E_{\text{seed}}(t) &= A_{\text{seed}}(t) e^{i(\omega_{\text{seed}} t + \phi_p)}, \end{aligned} \quad (3)$$

where we introduced the time delay on the pump pulse for simplicity (T is a relative delay). This time the envelope of the pump is considered as a constant because it varies in time much slower than the seed's one. In the reference frame of the seed, the electric fields after the OPA are

$$\begin{aligned} E_s(t) &= \alpha_s A_p A_{\text{seed}}(t) e^{i(\omega_{\text{seed}} t + \phi_p)} \quad \text{and} \\ E_i(t) &= \alpha_i A_p A_{\text{seed}}(t) e^{i(\omega_p(t-T) + \phi_p - \omega_{\text{seed}} t - \phi_p - \pi/2)} \\ &= \alpha_i A_p A_{\text{seed}}(t) e^{i(\omega_i t - \omega_p T - \pi/2)}. \end{aligned} \quad (4)$$

The CEPs then become $\Psi_s = \phi_p$ for the signal pulse and $\Psi_i = -\omega_p T - \pi/2$ for the idler pulse. Differently from the former case, the CEP of the signal beam no longer depends on T but only on the CEP of the pulse that drives the seed generation, the pump pulse in our case.

The idler's CEP instead still depends on T , but now the CEP varies by $-\pi$ for $\Delta T = \pi/\omega_p$, that is $\Delta\Psi_i = -\omega_p\Delta T$.

Because in an OPA it is always true that $\omega_p > \omega_{\text{seed}}$, we can infer that a given time fluctuation will produce a smaller CEP variation in the case of $\tau_{\text{seed}} \gg \tau_p$. The results obtained so far are summarized in Fig. 1.

Although useful, in our opinion, to develop an intuitive understanding of the process, the developed model lacks in generality. In many OPAs, indeed the pulse duration of the pump and of the seed is similar.

To investigate the intermediate regime ($\tau_p \approx \tau_{\text{seed}}$) and, at the same time, validate the previous model, we performed 1D split-step numerical calculations of the coupled wave equations solved via the fourth-order Runge–Kutta method, which takes into account the OPA process (in absence of material absorption), the dispersion of the nonlinear crystal and the self-phase modulation for pump, signal, and idler pulses. In all the simulations, we varied the seed pulse duration while keeping the pump duration fixed to 150 fs FWHM. In order to avoid backconversion, the peak intensity of the seed pulse is kept fixed for any pulse duration. To simulate a pump-seed timing jitter, we run the OPA simulation for various initial ATDs. In all our simulations, the self-phase modulation affected the CEP of each pulse by $<2^\circ$, and pump pulse energy fluctuations of 0.5% led to CEPs variations of $<0.5^\circ$. We started simulating a narrowband OPA (BBO, Type II, $\theta = 25.9^\circ$, 2.35 mm thickness) configured to generate a CEP stable idler as the difference between a pump pulse, centered at 800 nm, and a phase-locked (as for WL) Gaussian seed centered at 1300 nm. The seed pulse duration is varied from 10 to 300 fs, while keeping it transform-limited (TL). The change induced in the CEPs of signal and idler pulses are shown in Fig. 2. As expected, the CEP change introduced is directly proportional to the pump-seed ATD variation and resides in between $\omega_{\text{seed}}\Delta T$ and $\omega_p\Delta T$. For seed pulse durations that exceed the pump duration, $\Delta\Psi$ of both signal and idler quickly approach the analytical value, while, when the seed becomes shorter, the CEP variations tend toward the corresponding

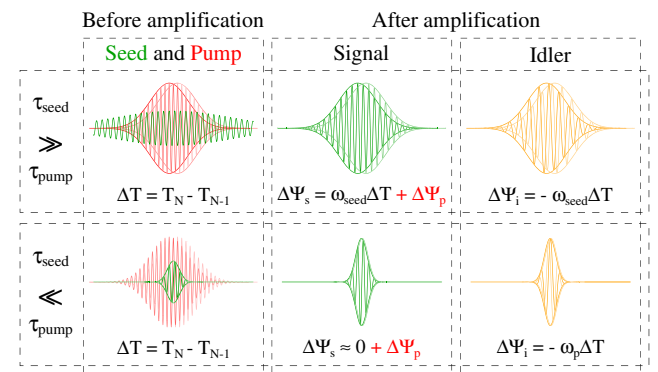


Fig. 1. Left column: representation of the electric field of pump and seed pulses before the OPA process begins. Central and left columns: $E(t)$ of signal and idler pulses after the OPA process. In the upper case, the envelope of the pump (solid red line) is shorter than the seed's one, and it is imprinted on signal and idler envelopes. In the lower case, the envelope of the seed (solid green line) is shorter than the pump one, and it is imprinted on envelopes of signal and idler pulses. The corresponding CEP variations, induced by the pump-seed ATD fluctuation ΔT , are reported. The contributions due to pump CEP variations are shown in red.

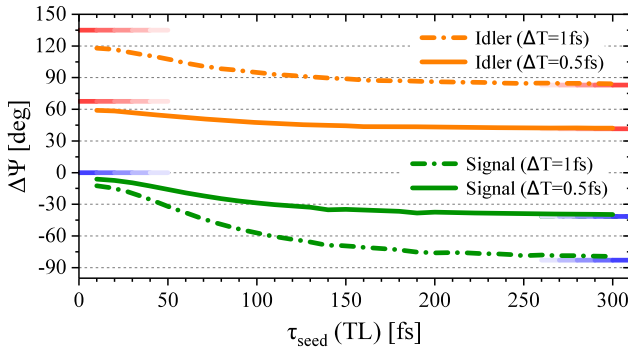


Fig. 2. CEP variations of signal (green lines) and idler (orange lines) induced by a pump-seed timing fluctuation of 0.5 fs (solid lines) and 1 fs (dashed lines), as a function of the seed pulse duration, while τ_p is fixed to 150 fs. The corresponding $\tau_{\text{seed}} \ll \tau_p$ and $\tau_{\text{seed}} \gg \tau_p$ analytic limits are shown by the blue-colored gradients for the signal and by the red-colored gradients for the idler.

analytical limits without reaching them completely. Such discrepancy has to be attributed to the narrowband nature of the phase-matching of this particular OPA; even if the seed duration gets extremely short, not all its bandwidth is amplified, limiting the effective signal pulse duration.

To further verify accuracy of the analytic model, we now consider a broadband degenerate OPA configuration (BBO, Type I, $\theta = 19.8^\circ$, 0.92 mm thickness), still pumped with 800 nm pulses, generating both signal and idler around 1.6 μm [11]. As shown in Fig. 3 (dashed lines), the agreement between numeric and analytic values improves in the broadband case, where the seed bandwidth is maintained through the amplification even for short seed durations.

We conclude that, in the case of designing a narrowband passively CEP-stable OPA, in order to minimize the influence of timing jitter on the CEP stability of the idler, it is advantageous to seed with $\tau_{\text{seed}} > \tau_p$. Furthermore, it is evident that the pump-seed jitter plays a major role in defining the CEP stability of the idler emerging from an OP(CP)A. For example, in the most favorable case of seeding with $\tau_{\text{seed}} > \tau_p$, a timing fluctuation of 100 fs causes a CEP change of 145 mrad for a 2080 nm idler pumped by 800 nm pulses or 106 mrad for a 2450 nm idler pumped by 1030 nm pulses.

Because the conventional scheme to generate ultrashort pulses via OP(CP)As consists of amplifying a broadband and chirped seed and compressing it afterward, in the following we focus on how the pump-seed jitter is affecting the CEP stability of the signal once a broadband and chirped pulse is employed to seed the DOPA. We started considering a 10 fs TL seed pulse, and we gradually introduced a quadratic chirp (GDD) in order to increase its pulse duration. As we can see from Fig. 3, in the chirped broadband seed case, the behavior remains similar to the broadband TL seed case.

The $\Delta\Psi$ behavior upon timing fluctuations changes completely after the signal pulses are compressed. In the simulations, after the amplification took place, we then introduced an additional chirp, of opposite sign of the one applied initially to stretch the seed. The results are shown in Fig. 4. Once half of the initial GDD is compensated (dashed line), the CEP fluctuations are already significantly suppressed. After full compression, the signal pulses experience extremely small CEP shifts,

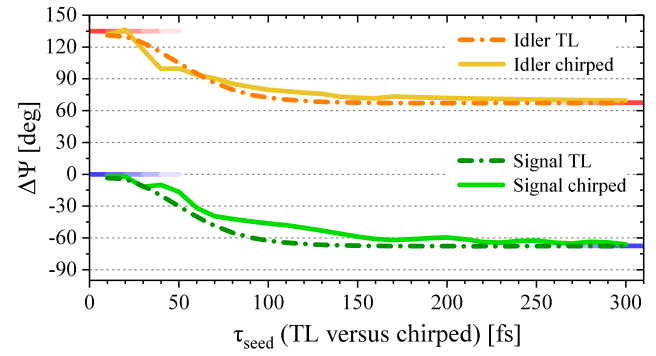


Fig. 3. Comparison between the CEP fluctuations of signal and idler introduced by a ΔT of 1 fs in a broadband OPA in the case of a TL seed (dashed lines) and broadband chirped seed (solid lines) as a function of their pulse durations, while τ_p is fixed to 150 fs.

in the order of $0.5\text{--}2^\circ/\text{fs}$ in our simulation, in agreement with the $0.8/90 \text{ rad/fs} \approx 0.5^\circ/\text{fs}$ reported in former study [12].

In case of a chirped seed, the pump envelope AT determines the amplification that different spectral components undergo but does not significantly influence the spectral phase of the amplified seed pulse because $\tau_p \gg 2\pi/\omega_{p,s,i}$. Moreover, the AT of the amplified pulse at the compression point depends exclusively on its chirp and the dispersion of the compressor. Therefore, the CEP of the amplified pulses is only marginally affected by pump timing fluctuations at the compression point.

This result is particularly interesting for multistage high-energy OP(CP)As, where the pump for the high-energy stages is frequently obtained by a different laser amplifier with respect to the one that drives the seed generation. Provided that the jitter between the different laser amplifiers is small compared to the pump duration (e.g., $\Delta T \approx 10$ fs for a $\tau_p \approx 1$ ps), and that the signal pulses are subsequently compressed close to their TL, their CEP stability should not be significantly affected.

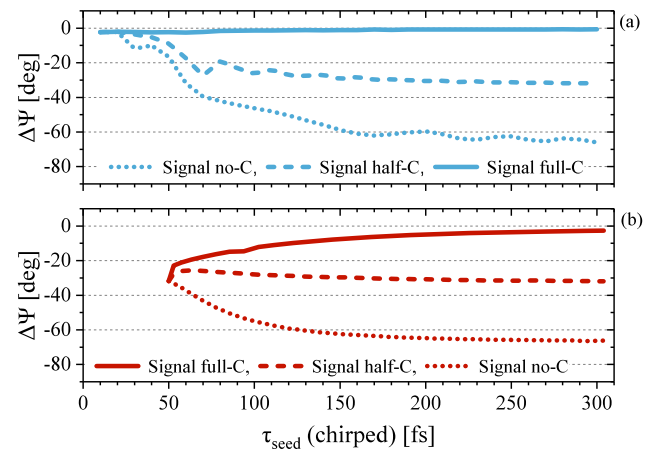


Fig. 4. Comparison of the CEP variation ($\Delta T = 1$ fs) for the signal pulse produced by a broadband stretched seed pulse in the case of no compression (dotted lines), half compression (dashed lines), and full compression (solid line). In (a) the TL pulse duration is 10 fs, while in (b) it is 50 fs. The CEP fluctuations of the compressed signal goes quickly to zero for the broadband seed pulse, while they reach zero only for large amounts of chirping/compressing for the narrower seed.

Based on those insights, we built a narrowband CEP-stable OPA, aimed to drive multiple CEP-stable WLs that seed the different channels of a parametric waveform synthesizer. This OPA is able to deliver high-energy stability [10], which makes it a perfect candidate for driving CEP-stable WLs capable of seeding over more than two octaves [13]. The OPA consists of two sequential amplification stages, pumped by a commercial Ti:sapphire laser producing 20 mJ, 150 fs pulses centered at 800 nm with 1 kHz repetition rate. The seed pulses, centered around 1300 nm, are obtained by WL generation. Because the WL broadening process maintains the AP of the driving laser pulse, the idler pulse train theoretically exhibits a stable CEP, even if the driver laser does not. After the first amplification, the signal pulses that seed the second amplifier are filtered by a 3 nm bandpass filter to ensure that $\tau_{\text{seed}} \gg \tau_p$. To verify the predictions of our model, we introduced a controlled delay in the path of the pump beam of the second-stage amplifier and recorded the CEP of the emerging idler pulses via two independent f-2f interferometers, as shown in Fig. 5. In both f-2f setups, the fringes are obtained by frequency doubling the 1.2 μm region of a WL filament driven by the second harmonic (1040 nm) of the idler (2080 nm) of the OPA, beating with the 600 nm component of the WL filament itself. To obtain the corresponding CEP for the idler pulse, we then multiplied the measured phase by a factor of $(600/520) \cdot 2$, where the first term comes from the fact that the f-2f fringes are measured around 600 nm and not around 520 nm, which would be the second-harmonic of the WL driver, and the factor of 2 accounts for the fact that we are driving the WL with the second-harmonic (SH) of the idler beam, thus doubling the original idler CEP. Experiment and theory agree within the measurement error, proving that the formulas in Fig. 1, at least in this case, establish a valid model. In real-world experiments, the timing fluctuation originates from mechanical vibrations of the guiding mirrors, thermal drifts, and air density fluctuations.

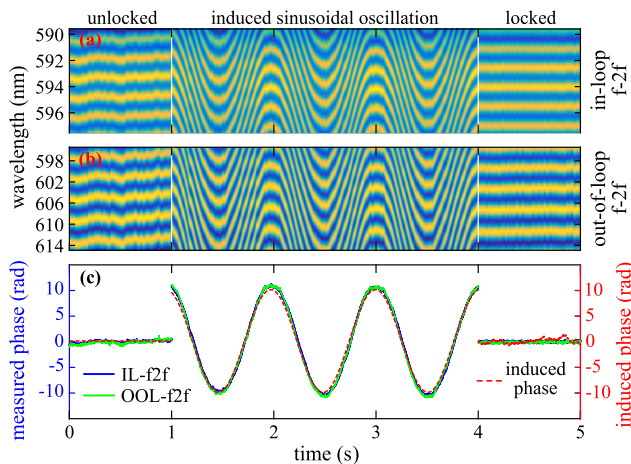


Fig. 5. Single-shot and every-shot CEP measurement of the SH of the idler obtained via two independent f-2f setups: in-loop (a) and out-of-loop (b). (c) Extracted phase: during the first second, the CEP is freely drifting; afterward, a sinusoidal ATD, with a peak-to-valley amplitude of (14.2 ± 1.4) fs, is introduced by a delay line in the pump path; in the last second, the CEP is locked to a fixed value. The measured peak-to-valley CEP variation, after calibration of the two f-2f setups, is (22.0 ± 2.4) rad, leading to (1.55 ± 0.17) rad/fs or $(88.8 \pm 9.8)^\circ/\text{fs}$, whereas the analytic model predicts $83.0^\circ/\text{fs}$.

To stabilize ΔT and minimize the CEP fluctuations, it is indeed possible to install an actively controlled delaying element, such as a piezo-driven mirror, either in the optical path of the pump or of the seed. By doing so, it is also possible to gain control on the CEP of the generated pulses. It is worth noting that, if the controlled delay is introduced in the seed path, then the CEP variation of signal and idler is caused by a variation of the AP of the seed (and of the signal); meanwhile, the *absolute arrival time* (AAT) of the signal and idler pulses is not changing. On the contrary, when the controlled delay is introduced in the beam path of the pump, the CEP variation is associated with a variation in the AAT of signal and idler pulses; meanwhile, the AP of the seed (and signal) does not change. This distinction, which is irrelevant in many cases, has to be taken into consideration once the CEP-controlled pulses generated by an OPA need to be further synchronized to other pulses originating from the same laser system, as in a multistage OPA amplifier, an OPA synthesizer, or even a pump-probe setup.

In conclusion, we have presented a simple model that describes the CEP fluctuations caused by timing jitter between the seed and the pump in an OP(CP)A. We derived few criteria that allow us to design OP(CP)As able to generate or amplify pulses with minimal CEP noise. The results presented in this article, given their general nature, can be applied to any pump pulse duration so their validity extends both on OPA and OPCPA.

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