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HEAVY QUARKS

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Abstract:

We discuss the results accumulated during the last five years in heavy quark physics and try to draw a simple general picture of the present situation. The survey is based on a unified point of view resulting from quantum chromodynamics.

C O N T E N T S

INTRODUCTION

Investigation of heavy quarks is a very important part of the modern high energy physics. It suffices to recall that the quark model itself became generally accepted only after the discovery of heavy quarks that has produced an immense effect to the whole subsequent development of experiment and theory. This discovery stimulated a considerable progress in such fields, disconnected originally, as Quantum Chromodynamics (QCD) and the electroweak interaction models. It is no chance that the discovery of the first heavy quark (the charmed c quark[1]) was marked by the Nobel Prize[2].

Basic information on the c quark is compiled in previous reviews, e.g.[3,4], which are presumably familiar to the reader. A useful information can be found also in a number of reviews[5-12]; but we will not suppose that the reader has studied them all.

A lot of new data was obtained during a few recent years: the charmonium family has been proliferated considerably, a heavier quark (the beauty b quark[13]), see also in Ref. 8) was discovered, direct gluon effects were investigated. In view of the recent progress we feel it is necessary to systematize the newly gained data, to expose the modern progress from a unified point of view. The general theoretical picture which will guide us through all the review is based on QCD whose abilities are well checked now.

The first particle in the newly discovered family, J/ψ, was detected eight years ago[1], but today we look at this fact as an event of ancient history. The theory has gone a long distance from the pioneering work by Appelquist and Politzer[14], revealing the nature of charmonium, to the modern highly sophisticated methods. At the early stage, some people supposed that "charmonium may well be a 'hydrogen atom' for the physics of strong interactions. Then a considerable part of the hadron physics can be related to the charmonium spectroscopy in quite the same manner, as molecular spectra are related to the hydrogen spectrum"[15]. In a sense, though not literally, the cited prediction has been confirmed.

At present, there is a quantitative discription of all aspects of heavy quarkonium. Sometimes, the accuracy of the description is better, in other cases it is worse, since we still have no solution for the confinement problem. By and large, we understand the structure of the system quite well and can use it often as a probe for the strong interaction, concurrent with colliding e^+e^- beams. The probe is unique, as it provides with a direct information on properties of the gluonic medium filling the physical vacuum, while more convention-

al probes, like γ^* and W, are coupled to quarks. Besides, heavy quarkonium is a first-class testing ground for investigation of weak interactions, including some of most nontrivial objects (Higgs particles, axions, etc., cf. for instance, Ref. 10).

Dealing with the theory of heavy quarkonium one should have in mind that in its present state it is not yet completed, as it is the case for the hydrogen atom. A number of "standard" approaches are elaborated, but nobody has succeeded in traversing the whole way from the fundamental QCD Lagrangian to real experimental quantities (masses, widths, etc.) with no supplementary assumptions. The sum rule approach, based upon the concept of a complicated nonperturbative structure of the QCD vacuum is the closest to fundamental chromodynamics. Unfortunately, this method is not completely universal. The nonrelativistic potential model, originated in fact from the pioneering work[14], is very popular. Some people use old prescriptions and models, like the quark-hadron duality, bag model etc., filling them, however, with new contents. Below we shall give brief characteristics for the theoretical tools used in the analysis of heavy quarkonium, and discuss some particular facts. Naturally enough, we dwell mainly on recent results, and try to select the most crucial points**. Of course, the selection of "main" results is a subjective matter. In particular, we discuss the physics of e^+e^- collisions, but leave aside such processes as photo-, hadro-, and neutrino-productions of heavy quarks. The interested reader may turn to other reviews[24,28]. As to theoretical approach, we do not put special emphasis on the nonrelativistic potential model that was a basis of numerous works on heavy quarks. Actually, excellent reviews written by experts are available[29] (a compilation of recent results has been given in Ref. 30).

Besides the problems relevant directly to heavy quarkonium, we are going to consider some related issues, for which the quarkonium serves rather as a spring-board for theoretical attacks.

We start the review with an exposition of main experimental facts accumulated during the past half-decade (Chapter 1). Our purpose in this chapter is to provide the reader with an information important from the point of view

**The experimental facts used here are extracted mainly from the works presented to the 1981 Symposium on Lepton and Photon Interactions, Bonn(16-24), and XXI International Conference on High Energy Physics, Paris 1982. As a rule, we quote in the text only the data which are absent in the latest edition of the PDG tables(31).

of the theory. A theoretical interpretation of this material is presented in Chapters 2 - 5, which constitute the most important part of the review.

We consider also such aspects as the mass spectrum of heavy particles, their leptonic and hadronic decays, effects due to the weak interaction. Application of QCD perturbative methods to processes involving heavy quarks is discussed in Chapter 5. The expected properties of toponium, the $t\bar{t}$-system containing the sixth, as yet not discovered quark, are considered briefly in Chapter 6.

Finally, in the last Chapter 7, we give a list of trends in the experimental and theoretical investigation, which seem to be most promising in our opinion.

1. BASIC EXPERIMENTAL FACTS

1.1. c QUARKS

The experimental study of c quarks has continued to develop intensively during the last six years. The main subjects of the investigation performed in e^+e^- collisions were as follows: (i) a detailed analysis of radiative transitions between charmonium levels; (ii) a search for missing levels (in particular*, 1^1S_0, 2^1S_0, $^1P_1$, ...); (iii) a study of radiative decays of J/ψ and search for new hadronic states, lying below J/ψ; (iv) investigation of properties of charmed hadrons.

A substantial progress in investigations of charmonium properties was made recently, owing to the neutral detector Crystal Ball (CB) operated at the SPEAR machine in Stanford. In essence, CB is a spherical shell composed of NaI(Tl) crystals; it is able to detect photons with high accuracy (the photon energy resolution is $\Delta E_\gamma/E_\gamma = 2.6\%/(E_\gamma^{1/4}(\text{GeV}))$, the angular resolution is $1\text{-}2^0$). The total statistics, accumulated by CB, amounts to about $2.2 \cdot 10^6$ J/ψ bosons and about $1.8 \cdot 10^6$ ψ' bosons. In the last year the CB detector was transported to Hamburg and is presently being used to study the Υ system at the new DORIS II e^+e^- storage ring at DESY.

A) Charmonium

Known levels of charmonium lying below the open flavor threshold and the corresponding radiative transitions are shown in Figure 1.

a) ψ mesons. Above the charm production threshold in the $e^+e^- \to$ hadrons cross section the $\psi''(3770)$ resonance is detected, which is a "D-meson factory" ($M_{\psi''} - 2M_{D0} \simeq 40$ MeV). A number of resonance structures lying higher than the ψ'' have also been observed . Parameters and supposed quantum numbers of these states are presented in Table 1. The parameters of J/ψ and ψ' states are also given here, for the sake of completeness. In particular, we would like to draw attention to precision measurements of the masses performed in Novosibirsk[32] by means of

* We use the following spectroscopic notations:
$(n_r + 1)^{(2S+1)}L_J$ where n_r is the radial quantum number (the number of nodes in the radial wave function).

an original method of the resonance depolarization of $e^{\pm}$ beams.

b) C-even states. These states were investigated in inclusive spectra of photons from decays of ψ' and J/ψ, as well as in the inclusive channels $J/\psi \to \gamma$ + hadrons, $J/\psi \to 3\gamma$, and $\psi' \to \gamma$ + hadrons, $\psi' \to 2\gamma + J/\psi$. Five states are established reliably now: three ^3P_J levels, χ_0, χ_1, χ_2,[33] and two paracharmonium states $\eta_c(2980)$[34,35] and $\eta_c'(3592)$[36]. The latter are identified as 1^1S_0 and 2^1S_0 levels, respectively. Figure 2 illustrates the inclusive photon spectrum in the radiative decay of ψ'[16] as obtained by the CB Collaboration.

c) Decays. $\psi' \to J/\psi + (\eta, \pi^0)$. A very interesting byproduct from investigation of cascade radiative transitions between ψ' and J/ψ states is observation of the decays $\psi' \to J/\psi + \eta$ and $\psi' \to J/\psi + \pi^0$. The latter decay breaks the isotopical symmetry, and the ratio of the decay widths allows one to measure the current quark masses "directly".

The CB group [36a] attempted to find the elusive $^1P_1$ ($J^{PC} = 1^{+-}$) charmonium level in the decay $\psi' \to \pi^0 + (^1P_1)$ and the cascade $\psi' \to \pi^0 + (^1P_1)_{\to \gamma\eta_c}$. The following bounds were obtained in the expected mass interval $M(1^1P_1)$ (see eq.(2.20))

$BR(\psi' \to \pi^0\, ^1P_1) < 0.42\%$,

$$BR(\psi' \to \pi^0\, ^1P_1) \cdot BR(^1P_1 \to \gamma\eta_c) < 0.20\% \ (\text{for } M(^1P_1) = 3.50 - 3.515 \text{ GeV}), \quad (1.1)$$

$$BR(\psi' \to \pi^0\, ^1P_1) < 0.55\%,\ BR(\psi' \to \pi^0\, ^1P_1) \cdot BR(^1P_1 \to \gamma\eta_c) < 0.14\% \ (\text{for } M(^1P_1) = 3.515 - 3.525 \text{GeV}) \quad (1.2)$$

B) Decays $J/\psi \to \gamma$ + light hadrons

In gluon physics, this process is an analogue of the famous e^+e^- annihilation. In terms of quarks and gluons, we say that

$$Q\bar{Q} \to gg\gamma,\ gg \to \text{light hadrons} \quad (1.3)$$

(the two-gluon system is in a colourless state). By regulating the photon energy we control the invariant mass of the hadronic system,

$$m^2 (\text{light hadrons}) = M^2 (1 - x),\ x = 2E_\gamma/M.$$

The lowest order of the QCD perturbation theory suggests (compare the graphs of Figures 3a,b) that

$$\delta_\gamma = \frac{\Gamma_{\gamma gg}}{\Gamma_{3g}} = \frac{36}{5} Q_q^2 \frac{\alpha}{\alpha_s(m^2)} \quad (1.4)$$

where Q_q is the quark electromagnetic charge. In this approximation the x distribution of photons grows practically linearly for all x. (The domain allowed kinematically is $0 \leq x \leq 1$). By identifying $\Gamma_{\gamma gg}$ and $\Gamma(J/\psi \to \gamma$ + light hadrons) we assume, in fact, the gluon-hadron duality, which is evidently broken at large x (low m^2_{hadr}). If one still accepts that $\Gamma_{\gamma gg} = \Gamma(J/\psi \to \gamma$ + light hadrons) one finds that the branching ratio of the radiative transition of J/ψ into light hadrons is related to δ_γ in the following way

$$BR(J/\psi \to \gamma + \text{light hadrons}) = \frac{\delta_\gamma}{1 + \delta_\gamma} [1 - (R + 2)\, BR(J/\psi \to e^+e^-) - BR(J/\psi \to \gamma\eta_c)], \quad (1.5)$$

where $R = \sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-)$. For $\alpha_s = 0.18$ we get $BR(J/\psi \to \gamma + \text{light hadrons}) \approx 8\%$.

Experimentally, the yield of inclusive photons is identified reliably only for $x \gtrsim 0.5$. For lower x values there are considerable uncertainties due to subtraction of the photonic contribution arising from the two-photon decays of η and π^0 mesons; the latter are copiously produced in the hadronic decays of J/ψ. The data obtained at SPEAR[39] indicate that the total yield of the γ quanta is in reasonable agreement with (1.5), while the shape of the photon spectrum deviates sharply from the prediction of the perturbation theory[38] (see Figure 4a). One sees from Figure 4 that in the intermediate region x = 0.5 - 0.7 the ratio $(dN/dx)_{exp} / (dN/dx)_{theor}$ is considerably higher than 1, which might be due to some broad resonance structures. For large x the ratio is less than 1. However, more careful measurements of the inclusive photons have shown that this domain is saturated by narrow peaks, corresponding to radiative transitions of J/ψ into the known mesons (π^0, η, η', f), as well as into new states[16,40,25] (Figure 4(B) and Table 2).

Two of them, ι and θ, are generally accepted as respectable resonances since they were observed in different channels by two groups (Table 3). Apart from these mesons whose existence is beyond any doubt a few other (probably resonance) structures were found in J/ψ radiative decays. We mean, first of all, a peak in the channel $J/\psi \to \gamma\rho^0\rho^0$ [43]. If one tries to fit it by the Breit-Wigner curve one gets[43]

$$M_{\rho\rho} = 1650 \pm 50 \text{ MeV}, \Gamma_{tot} = 200 \pm 100 \text{ MeV},$$

and, moreover,

$$BR\ (J/\psi \to \gamma\rho^0\rho^0\ ,\ m_{\rho\rho} < 2 \text{ GeV}) = (1.25 \pm 0.35 \pm 0.4) \cdot 10^{-3}.$$

It may well turn out that the peak is actually a manifestation of the θ meson. In this case the 2 ρ mode will be one of the most important decay modes of θ. The available data are insufficient to prove or rule out this hypothesis.

A broad enhancement was observed also in the channels $J/\psi \to \gamma\eta\pi^+\pi^-$, $J/\psi \to \gamma\eta\pi^0\pi^0$ 25). The Breit-Wigner description yields

$$M_{\eta\pi\pi} = 1710 \pm 45 \text{ MeV}, \Gamma_{tot} = 530 \pm 110 \text{ MeV}.$$

The number of events in the peak $\sim 5 \cdot 10^2$, and the corresponding branching ratio amounts to

$$BR\ (J/\psi \to \gamma\eta\pi^+\pi^-) = (3.5 \pm 0.2 \pm 0.7) \cdot 10^{-3},$$
$$BR\ (J/\psi \to \gamma\eta\pi^0\pi^0) = (2.3 \pm 0.3 \pm 0.8) \cdot 10^{-3}.$$

Notice that these rates are comparable to those of most intensive radiative decays known previously,

$$BR\ (J/\psi \to \gamma\eta') \sim 4 \cdot 10^{-3},\ BR\ (J/\psi \to \gamma f) \sim 1 \cdot 10^{-3}.$$

There are some indications of other mesons - gluonium candidates - in hadronic reactions and two-photon collisions.We shall not touch upon the corresponding data. The interested reader may turn to the talk[25)] or the original papers.

Both mesons, ι and θ , are discussed in the literature as serious candidates, perhaps, opening the family of particles built from gluons - glueballs. There are arguments pro and contra such an interpretation. Some of them will be considered in Chapter 2. In any case, one should keep in mind that there are no vacancies in the lowest pseudoscalar and tensor nonets. Indeed, the 0^{-+} isoscalar states are η and η'; the corresponding places in the 2^{++} nonet are occupied by f(1270) and f'(1515).

C) Charmed hadrons

a) D-mesons ($c\bar{u}$, $c\bar{d}$). At present, they are rather well studied. There are two isodoublets; spinless D^0 and D^+, and vector D^{*0} and D^{*+}. Their properties are investigated mainly at the resonance ψ'' ($\psi'' \to D\bar{D}$ almost completely), and at the peak $\psi(4030)$. The values of D-meson masses and basic non-leptonic decays are presented in Ref. 31). New data referring to D* are compiled in Table 4. Mean multiplicities of the charged particles from D decays are

$$\langle n_{ch}\rangle_{D^0} = 2.46 \pm 0.14\ ;\ \langle n_{ch}\rangle_{D^+} = 2.16 \pm 0.16 \qquad (1.6)$$

Measurements of semi-leptonic decays of D-mesons provide very important information on the decay $c \to se^+\nu_e$. Since $\Delta T = 0$ in this transition, the widths of the semi-leptonic decays of D^0 and D^+ are obviously equal.

$$\Gamma(D^0 \to e^+ + \nu_e + X) = \Gamma(D^+ \to e^+ + \nu_e + X).$$

This fact implies that the ratio of the D^+, D^0 life-times is

$$\frac{\tau_{D^+}}{\tau_{D^0}} = \frac{BR\ (D^+ \to e^+ + \nu_e + X)}{BR\ (D^0 \to e^+ + \nu_e + X)} \qquad (1.7)$$

The evolution of the experimental situation has been rather dramatic. The value of BR ($D^+ \to e^+ + \nu_e + X$) - (19 ± 3)%,according to SLAC data, - turned out to be close to the number expected for the free quark decay, ≈ 1/5 (see Fig. 5). Meanwhile, the same experiments gave an essentially lower value for BR ($D^0 \to e^+ + \nu_e + X$), so that the ratio τ_{D^+} / τ_{D^0} turned out to be too high (to give an idea on the scale of the anomaly we quote the DELCO result[45)] $\tau_{D^+}/\tau_{D^0} > 4.3$). Such a large difference, by a factor of ~5, between the life-times of D^0 and D^+ seemed to be confirmed also by direct measurements in nuclear emulsions exposed to beams from proton accelerators, see Table 5 and Ref. 23), 24), 12). The observed anomaly τ_{D^+}/τ_{D^0} gave rise to an outburst of theoretical speculations. Forestalling the end of the story let us notice that reasonable theoretical estimates for the ratio τ_{D^+}/τ_{D^0} lead to a number close to unity, and in any case not higher than 1.5 - 2. However, under the pressure of the data, some theorists gave in and proposed a number of models "explaining" the τ_{D^+}/τ_{D^0}

anomaly and "predicting" $\tau_{D^+} / \tau_{D^0} \simeq 5 - 10$.

In 1982 the situation seems to begin to normalize after appearance of the data on the direct measurements of D^0, D^+ life-times performed at the SLAC hydrogen bubble chamber in the photoproduction experiment at $E = 20$ GeV. The result obtained, $\tau_{D^\pm} = (7.3 {+3.0 \atop -2.5}) \cdot 10^{-13}$ sec[26], is close to the emulsion data and to predictions of the free quark decay model, $\tau_c \simeq 6.5 \cdot 10^{-13}$ sec. At the same time the value of $\tau_{D^0} = (7.7 {+3.0 \atop -2.5}) \cdot 10^{-13}$ sec, found at SLAC, became considerably higher than in previous measurements, see Table 5 and Ref. 26.

Note also that the semileptonic decays are, seemingly, practically exhausted by three-particle channels, $D \to K^*(890) e\nu_e$ ((37 ± 16)%) and $D \to K e\nu_e$((55 ± 14)%)[47].

b) F-meson ($c\bar{s}$). The situation here is not yet established. The preliminary results of the DASP group[48] were interpreted at first as observation of the F-meson with mass $M_F = 2030 \pm 60$ MeV and of the F*-meson with mass $M_{F^*} = 2140 \pm 60$ MeV in e^+e^- collisions (in the reactions $e^+e^- \to F\bar{F}^*, F^*\bar{F}^*$). They have not been confirmed, however, by later experiments of the CB group[49]. The existence of the F mesons is now established reliably only at proton synchrotrons (e.g. Ref. 23), 24), 12)). It is claimed[12,26] that the emulsion data correspond to the value of $\tau_F = (2.0 {+1.8 \atop -0.8}) \cdot 10^{-13}$ sec.

More recent measurements yield a larger number,

$$\tau_{F^+} = (4 - 5) \cdot 10^{-13} \text{ sec}, \qquad (1.8)$$

true, with an enormous error (see Table 5).

One can expect that, with statistics increasing, experiment will stabilize at the level (1.8). It goes without saying that the rapprochement of τ_F and τ_D is welcome. Indeed, the value $\tau_F \sim 2 \cdot 10^{-13}$ sec, as well as that for τ_{Λ_c} quoted below, seem very suspicious - hardly admittable - from the theoretical point of view. As it will be argued in Chapter 3 the life-times of all charmed particles should coincide with the naive estimate $\tau_c \approx 6.5 \cdot 10^{-13}$ sec up to pre-asymptotic corrections ($\approx O(50\%)$).

c) Charmed baryons Λ_c(cud), Σ_c (cdd, cud, uuc). An abundant experimental information on the production of Λ_c was obtained on proton accelerators; and there are some traces of Σ_c^+, Σ_c^{++}. These measurements result in the following value of τ_{Λ_c} : $(1 - 3) \cdot 10^{-13}$ sec[26]. The charmed baryons were detected in e^+e^- collisions by a sharp increase in the inclusive yields of $p, \bar{p}$ and $\Lambda, \bar{\Lambda}$ at $W \simeq$

4.5 GeV[50]. The Λ_c mass amounts to $M_{\Lambda_c} = (2282.2 \pm 3.1)$ MeV, decay branching ratios are presented in tables[31].

By studying inclusive electron yields in the e^+e^- annihilation with production of baryons, in the energy range $W = 4.5 - 6.8$ GeV, the MARK II group has measured[51] semileptonic decays of charmed baryons,

$$BR\ (\Lambda_c \to e^+\nu_e X) = (4.5 \pm 1.7)\ \%$$

As was mentioned above, the value of BR ($\Lambda_c \to e^+\nu_e X$), as well as τ_{Λ_c}, seems to be underestimated by a factor of 3 - 4 (for details see Chapter 3).

1.2. b QUARKS

A new breakthrough in particle physics happened in May-June of 1977. The united CFS group, headed by L. Lederman, operating at the proton synchrotron at FNAL (USA), discovered a new family of heavy particles with masses near 10 GeV[13]. These particles, named Υ mesons, were found by the mass spectrum of $\mu^+\mu^-$ pairs in proton-nucleus collisions (Figure 6). The new discovery was "tried and tested in fire", since soon after the first observations of the mesons a fire flared up at the experimental set-up, and the results were reproduced only after the group had done away with the damage. With further increase of statistics the authors were able to make a statement that the data are described best of all by the hypothesis of existence of three narrow peaks in the spectrum, corresponding to mesons Υ, Υ', Υ" with the masses ~ 9.4 GeV, ~ 10 GeV, and ~ 10.4 GeV, respectively.

Immediately after the Υ meson discovery, the physics community, which was actually prepared by the preceding J/ψ episode began to speak about a new, still heavier quark. The popular hypothesis which had practically no competition read that we were witnessing the birth of a quark, its mass about 4.8 GeV, with a new quantum number, named "beauty"*, conserved in strong interactions. Correspondingly, the Υ mesons were considered as particles with hidden beauty, the 1^{--} levels of the $b\bar{b}$ quarkonium("bottonium").

As in the case of the ψ mesons, only the e^+e^- annihilation experiments could provide with a final proof of the existence of the new quark. They were necessary in order to investigate its properties in more detail. In e^+e^- colliders

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* Another name for the new quark can be also found in the literature, it is sometimes referred to as the bottom quark (i.e. the lower quark in a new quark doublet.

available at that time, energies were insufficient for the Υ searches. Immediately after the publication of the CSF results[13] a new modification of the DORIS facilities was undertaken, and already by the end of April or first days of May, 1978, two groups, PLUTO[52] and DASP-2[53] have detected a narrow peak in the e^+e^--annihilation cross section, corresponding to the Υ meson with the mass M_Υ= 9.46GeV. By the end of August, 1978, the Υ' peak with the mass $M_{\Upsilon'}$ = 10.02 GeV was observed at DORIS[54]. The observed widths of Υ and Υ' were in good agreement with the calculated energy resolution of DORIS, so that the proper widths turned out to be very small (<< 20 MeV). This was the most important argument in favour of the bottonium interpretation of the Υ mesons.

The resonances Υ and Υ' were confirmed at the CESR collider (Cornell, USA), which came into operation in 1979. The third narrow resonance Υ" whose mass $M_{\Upsilon''}$ = 10.35 GeV was first studied here[55,56]. Soon the CESR team announced a broad resonance Υ'''[57,58] with the mass $M_{\Upsilon'''}$ ≈ 10.55 GeV and the width Γ about twice as large as the energy resolution (Γ = 14 ± 5 MeV). This width is larger than two orders of magnitude, so there is apparently no Zweig - Iizuka suppression in the Υ''' decay. According to a subsequent analysis, this resonance is an analogue of ψ(3770). In other words, Υ''' represents a factory of B mesons (i.e. the mesons composed of the quark pairs $b\bar{u}$, $b\bar{d}$): Υ''' → $B\bar{B}$. No new states $\tilde{\Upsilon}$ in the region between Υ" and Υ''' were found[18] (the upper bound is $\Gamma(\tilde{\Upsilon} \to e^+e^-)$ <0.03 keV). To illustrate the situation, we reproduce in Figure 7a the cross section $e^+e^- \to$ hadrons in the Υ-resonance domain. Note that the Υ-resonance peaks are comparable to the non-resonant cross section, in sharp contrast to the case of the J/ψ meson, where the resonance-to-background ratio amounts to about two orders of magnitude. This fact reflects both a substantial smearing of the beam energy resolution ($\sigma_e \sim E_e^2$) which results in the resonance suppression, relative increase of the background, and a trivial decrease of the resonance cross section due to $(Q_b/Q_c)^2 = 1/4$[59]. Measuring the ratio R in the energy region W = 10.40-11.50 GeV one observes a typical open flavour threshold: above Υ''' the value of R becomes larger by $\overline{\Delta R} \approx 0.2$, on the average (see Fig. 7b) (the systematical errors practically cancel after the subtraction). The measurement of other characteristics of the e^+e^- annihilation - multiplicities, shape of the hadronic events, direct lepton yields - also confirms this rise of the cross section. The results are in good agreement with the assumption that the charge of the new quark is $Q_b = -1/3$, so that $\overline{\Delta R}_{theor} = 3Q_b^2 = 1/3$.

A) Properties of Υ resonances

The level structure of the $b\bar{b}$-quarkonium (bottonium) and the expected transitions are shown in Figure 8. Unlike the charmonium, only the $\Upsilon(n^3S_1)$ levels[17] and the excited P levels (2^3P_J)[25] are established reliably at present. No wonder, of course, since the world of the beauty particles is considerably younger, and besides that, the statistics accumulated here is about two orders of magnitude lower than in the case of the charmonium states.

The Υ-resonance parameters, known at present, are given in Tables 6 and 7. As it follows from Table 6, the width of the direct decays Υ → 3g → hadrons,

$$\Gamma(\Upsilon \to 3g) \approx \Gamma_\Upsilon - (3 + R(W = 10\ \text{GeV}))\ \Gamma\ (\Upsilon \to e^+e^-), \qquad (1.9)$$

turns out to be approximately equal to $\Gamma(\Upsilon \to 3g) = 27.5^{+6}_{-5}$ keV, which is considerably less than $\Gamma(J/\psi \to 3g) \approx 44$ keV. This fact indicates a decrease of the colour coupling α_s with the momentum transfer increasing and is a striking manifestation of asymptotic freedom (more details are given in Section 2.1 of Chapter 2.).

The event shapes were investigated thoroughly in direct (non-electromagnetic) decays of Υ mesons, in order to test their three-gluon nature, Υ → 3g (details see in Refs. 11, 63). In complete agreement with QCD, the distributions in eight different topological characteristics in the Υ decay agree well with the three-gluon model, while outside the resonance, these distributions correspond best of all to the quark-antiquark pair production. The confirmation of the three-jet structure in the Υ hadronic decays was the first direct manifestation of gluons. It was only because of the insufficient energy of the gluonic jets that this fact by itself was not considered as a decisive proof of the existence of gluons; their discovery was declared after the observation of bremsstrahlung gluons in the process $e^+e^- \to q\bar{q}g \to$ hadrons made at PETRA (Hamburg) at W $\gtrsim$ 30 GeV (see e.g. Ref. 11).

First attempts were undertaken to estimate the width of the radiative decay $\Upsilon \to gg\gamma \to \gamma$ + hadrons[64]. The published upper bound $\Gamma(\Upsilon \to \gamma gg)/\Gamma(\Upsilon \to 3g) < 27\%$ (90% C.L.) is rather weak (cf. (1.4), $\delta_\gamma^\Upsilon \approx 3.5\%$).

Very important characteristics of vector bottonium are its full and purely hadronic ("three-gluon") widths. If for the ground level, Υ, these parameters are measured directly, for the excited states, Υ' and Υ", one has to rely on theoretical reconstructions[18]. Consider for definiteness Υ'. The full width is

$$\Gamma_{tot}(\Gamma') = \Gamma(\Upsilon' \to 3g) + \Gamma(\Upsilon' \to \gamma \to \ldots) + \Gamma(\gamma' \to \pi\pi\Upsilon)$$

$$+ \Gamma(\Upsilon' \to \gamma + P \text{ levels}) + \text{inessential channels.}$$

Moreover, for the first two terms one can evidently accept

$$\Gamma(\Upsilon' \to 3g) + \Gamma(\Upsilon' \to \gamma \to \ldots) = \frac{\Gamma(\Upsilon' \to e^+e^-)}{BR(\Upsilon \to e^+e^-)}$$

Besides that

$$\Gamma(\Upsilon' \to \pi\pi\Upsilon) = \Gamma_{tot}(\Upsilon') \,/\, BR(\Upsilon' \to \pi\pi\Upsilon),$$

and the widths of the radiative E1 transitions on P levels are estimated theoretically either in potential models or by a simple rescaling of the corresponding numbers for Υ'. In the case of Υ'' there are direct experimental data for $BR(\Upsilon'' \to \gamma P)$ which are, though, in good agreement with model calculations (see below). In this way we reconstruct the full widths of Υ', Υ'' and their three-gluon widths. The results are presented in Table 7.

Another recent result having far-going consequences for the theory is the measurement of the pion spectrum in the decays $\Upsilon' \to \Upsilon\pi^+\pi^-$, $\Upsilon'' \to \Upsilon\pi^+\pi^-$ [65]. If in the former decay the pion invariant mass distribution (it describes 17 events) is compatible with that in $\psi' \to J/\psi\ \pi^+\pi^-$, in the Υ'' decay the picture is drastically different. The $m_{\pi\pi}$ distribution is absolutely flat (Fig. 9), and this fact is hardly understandable within the framework of the standard theoretical constructions. We shall dwell on the issue in Chapter 2.

A few words about observation of the bottonium P levels in E1 radiative transitions. The most definite result refers to the transition to the excited P levels,

$$\Upsilon'' \to \gamma + 2^3P_J ,$$

which has been studied both in the inclusive photon spectrum $\Upsilon'' \to \gamma + \ldots$ and in the cascades $\Upsilon'' \to \gamma + 2^3P_J \to 2\gamma + \Upsilon$ [25]. The statistics available allows one to draw the following conclusions

$$M(2P,b\bar{b}) \approx 10.250 \text{ GeV}$$

$$BR(\Upsilon'' \to \gamma + 2^3P_J) = (34 \pm 3)\ \% \qquad (1.10)$$

$$BR(3S \to 2P + \gamma)\cdot BR(2P \to 2S + \gamma) = (5.9 \pm 2.1)\ \%$$

$$BR(3S \to 2P + \gamma)\cdot BR(2P \to 1S + \gamma) = (3.6 \pm 1.2)\ \%$$

Moreover, spin splittings are also detected and different spin levels are resolved (Figure 10.).

The CLEO experimentalists observe also a signal in the $\Upsilon'' \to 2\gamma\gamma$ cascade at $E_{\gamma low} \sim 410$ MeV [25]. If one accepts that the signal corresponds to the transition to the ground P state, $\Upsilon'' \to \gamma + 1^3P_J$, then [25]

$$M(1^3P_J,b\bar{b}) = 9.93 \text{ GeV};\ BR(3S \to 1P + \gamma)\ BR(1P \to 1S + \gamma) = 3.1 \pm 2.2\%$$

However, such a conclusion seems to be premature both from experimental and theoretical points of view. The experimental signal is statistically not very significant. Of course, more important for us are theoretical arguments. The existing analysis of the QCD sum rules does not admit the ground P state mass higher than 9.86 GeV. Likewise, the majority of potential calculations yield a quantity $\leq$ 9.90 GeV.

For a more detailed discussion of this question see Chapter 2.

Notice also that previously the CUSB group has been studying the radiative transitions to the P levels using the fact that, unlike $\Upsilon'^{(\prime\prime)} \to 3g$, $^3P_{0,2}$ levels decay predominantly into two-gluon state. This should result in an admixture of two-jet events in Υ', Υ'' due to cascades $\Upsilon(nS) \to \gamma + ((n-1)^3\ P_{0,2})$.

Although in such statistical procedure there are some ambiguities - it is clear from the very beginning - it still yielded the following important information [18]:

$$BR(\Upsilon\ (2S) \to \gamma(1^3P_{0,2})) = (8 \pm 2)\ \%,$$

$$BR(\Upsilon\ (3S) \to \gamma(2^3P_{0,2})) = (20 \pm 3)\ \%. \qquad (1.11)$$

Adding transitions to $^3P_1$ levels, with equal weights, we get the values of $BR\ (\Upsilon(ns) \to \gamma\ (n^3P))$ by a factor of 1.5 higher than that in (1.11).

B) Beautiful hadrons

Mesons and baryons which contain a single b quark are called beautiful. Most accessible for investigations are B mesons ($\bar{B}^0 = b\bar{d}$, $B^- = b\bar{u}$) since they are copiously produced in the Υ''' resonance. Recently the CLEO group observed B mesons in a direct way for the first time having reconstructed it in the following modes:

$$B^- \to D^0\pi^-,\ \bar{B}^0 \to D^0\pi^+\pi^-,\ \bar{B}^0 \to D^{*+}\pi^-,\ B^- \to D^{*+}\pi^-\pi^- + \text{C.C.}$$

All the branching ratios are of the order of few percent. Besides that, there exists a rather rich indirect information.

a) Mass. M_B satisfies the following inequality

$$M_{\Upsilon''} < 2 M_B < M_{\Upsilon'''} \quad .$$

Experimentally it is established[17,18] that the $\Upsilon''' \to \bar{B}^*B$ mode constitutes only a small fraction of the full Υ''' width, so that the $\Upsilon''' \to B\bar{B}$ channel is the dominant decay mode. The modern values of the B-meson masses are:

$$M_{B^0} = 5274.5 \pm 1.4 \text{ MeV}$$
$$M_{B^+} = 5274.0 \pm 2.1 \text{ MeV}. \qquad (1.12)$$

b) Yields of K mesons. Weak decays of the B mesons, which have finally confirmed the nature of the new b quark, were found by a substantial increase in the yield of K mesons and inclusive leptons in the Υ''' resonance. An important information on the structure of weak interaction is obtained from these data. The whole set of the data, including the topological characteristics of hadronic events, indicates that the production and subsequent decay of the $B\bar{B}$ takes place, the particle mass is about 5 GeV, and the transition $b \to c_s + W^-$ dominates for b quarks[171] (see Figure 11a)), as it was expected in the framework of the standard six-quark model. In particular, such a decay should result in a large number of K mesons in the final state because of the transition $B\bar{B} \to D\bar{D}X \to K\bar{K} + \ldots$. As it was shown by means of the Monte-Carlo simulation, the ratio of the $(K^\pm + K^0)$ yield in the $B\bar{B}$ events to that in the non-resonance region, $\rho_K = \sigma_{B\bar{B}}(K) / \sigma_{off}(K)$, must be, approximately, 1.8 for the $b \to cW^-$ transition (Figure 11a), and about 1 for the case $b \to uW^-$ (Figure 11b). Experimentally $\rho_{K\ exp} = 1.9 \pm 0.3$[62].

The fact that the transition $b \to uW^-$ is small, is confirmed also by the shape of the electron spectrum in semileptonic decays $B \to e\nu_e X$.

c) Semileptonic decays. In the Υ''' resonance the electron yield increases sharply. By measuring the electron (muon) yield one studies semileptonic B-meson decays.

The inclusive leptonic events are in agreement with the assumption that the dominant role in the semileptonic B decays belongs to three-particle channels $B \to D(D^*)l\nu_l$.

The observed shape of the leptonic spectra corresponds to the decay of the free b quark with production of the hadronic system $X = D + \ldots$ with mass ~ 2 GeV. The possibility of smaller masses, $M_X \lesssim 1$ GeV, is ruled out experimentally. This is more evidence in favour of the dominance of the $b \to cW^-$ transition, since for $b \to uW^-$ the X system mass might be noticeably lower than 2 GeV.

Quantitatively[26])

$$BR(B \to e\nu_e X_u) \,/\, BR(B \to e\nu_e X_c) \leq 0.093 \qquad (90\% \text{ C.L.})$$

which implies, in turn (see chapter 4)

$$|V_{bu} \,/\, V_{bc}| \leq 0.21 \qquad (1.13)$$

Data in the Υ''' resonance enabled one to determine mean multiplicities of charged particles in purely hadronic and semileptonic decays of the B meson,

$$\langle n^{ch}\rangle^h_B = 6.3 \pm 0.3 \pm 0.3 \ ; \quad \langle n^{ch}\rangle^{s.l.}_B = 4.1 \pm 0.35 \pm 0.2 \qquad (1.14)$$

It is seen that the multiplicity in the B-meson decay is rather large, so that enormous difficulties in observation of particular exclusive channels become clear.

The probabilities of the B-meson semileptonic decays, measured by various groups, are collected in Table 8. Within the errors they are in mutual agreement, and are lower than the prediction based on the free quark decay (Fig. 11), which amounts to $BR(b \to c + e^- + \bar{\nu}_e) \approx 16\%$, with account of the phase space effects, see e.g. Ref. 67*. One may think that with time the experimental values of $BR(b \to c + e^- + \bar{\nu}_e)$ will come to agreement with this quantity.

* An enhancement of the non-leptonic Hamiltonian due to hard gluon exchanges inessentially lowers this number, down to $\approx 15\%$, see text-book(7).

Note that measuring yields of inclusive leptons with momenta larger than those allowed in the decay $B \to De\nu_e$, one will be able to get unambiguous information on the transition $b \to uW^-$.

The CLEO collaboration have announced recently [26] their attempts to detect the decay

$$B \to J/\psi + X$$

expected in the standard model. The J/ψ's have been identified by the e^+e^- or $\mu^+\mu^-$ modes. The following upper bound has been obtained

$$BR(B \to J/\psi + X) < 1.4\% \quad (90\% \text{ C.L.}) .$$

We shall return to theoretical interpretation of the fact later on.

d) B-meson lifetime. The JADE group, working at PETRA, has obtained a lower bound for the B-meson lifetime [68,26].

$$(\tau_B)_{exp} < 1.4 \times 10^{-12} \text{sec.} \ (95\% \text{ C.L.}) \qquad (1.16)$$

The authors managed to bound the B-meson decay length in events with inclusive muons. Thus one gets a new lower bound on the quark mixing angles (see Section 4.1 of Chapter 4). Note that starting from existing information on the mixing angles we expect

$$\tau_B = 10^{-13} - 10^{-14} \text{sec.} \qquad (1.17)$$

e) Non-standard models. All the available data on the B-meson decays agree with the standard six-quark model of weak interaction (see Section 4.1 of chapter 4). Besides, there is a number of clear-cut experimental arguments [17,20] against various non-standard and exotic models.

Finally, the B-meson decays rule out the existence of charged Higgs bosons H^+ with the mass in the interval 2 GeV $< m_H < m_b$. In fact, the data on the inclusive lepton yields and the measurement of the energy carried away by charged hadrons in the b-meson decay do not admit the cascades

$$b \to qH^- \quad , \qquad (1.18)$$
$$\quad H^- \hookrightarrow \tau^- \bar{\nu}_\tau, c s^-$$

which were expected to be dominant if $H^\pm$ existed in the indicated mass interval.

2. THEORY OF HEAVY QUARKONIA

2.1. SPECTRUM

A) Quarks and their masses

As it was explained in Chapter 1, the existence of two heavy quarks, c and b, is established reliably. Their electric charges are + 2/3 and - 1/3, and they form the charmonium and bottonium families, respectively. People believe that there must exist a third heavy quark, t, not yet found experimentally, with the electric charge + 2/3.

The most important characteristic of the heavy quark is its mass. Since the quarks are confined, it is impossible to weigh an isolated quark, as one does weigh, for instance, the muons. Nevertheless, one can introduce a notion of the so called current quark - an object which is essentially deprived of its gluonic cloud. (To be more exact, only soft gluons are eliminated; hard gluons result in logarithmic corrections which are readily accounted for.)

The current quark mass depends on a normalization point and is involved in all the calculations based on fundamental chromodynamics. For the c and b quarks it was determined [69,70] from the QCD sum rules:

$$m_c \approx 1.40 \text{ GeV} \quad , \quad m_b \approx 4.80 \text{ GeV} \qquad (2.1)$$

We give here the numbers referring to the so called on-shell mass, a gauge invariant quantity, well-defined in the perturbation theory. (Note that a somewhat lower value of $m_b \approx 4.71$ GeV was obtained in Ref. 71). The Euclidean mass, also often mentioned in the literature [69], does, on the contrary, depend on the gauge condition. For instance, in the Landau gauge one has

$$M(p^2 = -m^2) = m\,(p^2 = m^2) \left\{1 - \frac{2\,\alpha_s \ln 2}{11\,\pi} + \ldots\right\} ,$$

and the coefficient of α_s varies in proceeding to other gauges. At present, there exist many independent estimates for the Euclidean mass of charmed quarks [69,72-74]. They all agree with each other and with Eq. (2.1):

$$m_c \approx 1.25 \text{ GeV} \quad (p^2 = -m_c^2 \text{ , Landau gauge}). \qquad (2.2)$$

Unfortunately, the situation with b quarks is worse. According to Refs. 74 and 71, m_b (Euclidean) $\approx$ 4.26 GeV, and this quantity is, seemingly, too small: it should be larger by approximately 150 MeV in order to be in agreement with Eq. (2.1). Evidently, the problem deserves a further analysis.

People who work with constituent quarks usually get considerably larger masses. This is rather natural, of course, since the quarks they deal with incorporate some gluon cloud.

It is noteworthy that

$$M_{J/\psi} > 2m_c \quad , \tag{2.3}$$

while

$$M_\Upsilon < 2m_b \quad , \quad 2m_b - M_\Upsilon \approx 130 \text{ MeV } ^{70)} \tag{2.4}$$

The situation with J/ψ seems quite natural, but Eq. (2.4) might be surprising. Actually, effects due to confinement tend to increase the resonance mass as compared with the quark mass doubled. Why then $M_\Upsilon < 2m_b$? The Coulomb attraction becomes numerically important for the Υ family; it overcompensates a positive mass shift due to the confinement forces.

As for the hypothetical t quark, its absence in the PETRA experiments[20,21] implies that

$$m_t > 18.5 \text{ GeV} \ . \tag{2.5}$$

Now, to understand the quarkonium spectrum, one should know, apart from the quark masses, the nature of the binding forces. According to the modern views, quarks exist in a complicated medium - the non-perturbative QCD vacuum, populated densely by longrange fluctuations of gluon fields. These non-perturbative fluctuations lower the vacuum energy density as compared with its perturbative value. If a $Q\bar{Q}$ pair is injected into the vacuum, the colour quark field somewhat freezes the fluctuations in an adjacent domain[75], and an effective attraction between Q and $\bar{Q}$ results.

One should have in mind that in real systems, like J/ψ or Υ, the attraction force is not described, generally speaking, by a static potential. Indeed, the impact on the gluonic medium can be represented by a potential only if the medium has time enough to tune itself and follow the (slow) quark motion. In other words, the condition under which a potential can be introduced is

$$\omega_{quark} \ll \omega_{glue} \ , \tag{2.6}$$

where ω is a characteristic frequency. For the charmonium and bottonium families, the order of magnitude of the characteristic frequencies is the following

$$\omega_{quark} \sim M_{\psi'} - M_{J/\psi} \sim M_{\Upsilon'} - M_\Upsilon \sim 0.6 \text{ GeV} \quad , \tag{2.7}$$

and the frequencies specific for the gluonic medium are approximately the same. Moreover, the validity of the multipole expansion (see below) implies that $\omega_{glue} < \omega_{quark}$. If this is the case one may expect substantial deviations from the potential picture, especially for charmonium. This expectation is confirmed, in a sense, by the recent analysis of various relativistic effects[77].

B) Gluonic condensate

Peculiar properties of the QCD vacuum which are responsible for the formation of spectrum, are not yet understood completely, but some gross features are known. For instance, the net effect of long-range gluon fluctuations is measured by the vacuum expectation value of the gluon field squared,

$$\langle vac | G^a_{\mu\nu} G^a_{\mu\nu} | vac \rangle \neq 0. \tag{2.8}$$

On one hand, this parameter is reduced, in a straightforward way, to the vacuum energy density [73],

$$\varepsilon_{vac} = \frac{1}{4} \langle vac | \theta_{\mu\mu} | vac \rangle \approx - \frac{9}{32} \frac{\alpha_s}{\pi} \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle \ , \tag{2.9}$$

where $\theta_{\mu\nu}$ is the energy-momentum tensor. Here we have used the fact that in QCD $\theta_{\mu\mu}$ is determined by the so-called triangle anomaly[73],

$$\theta_{\mu\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_q m_q \bar{q} q \approx - \frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^a_{\mu\nu} \ .$$

On the other hand, the gluonic condensate plays a special role in the heavy quarkonium physics. What is the reason? The $Q\bar{Q}$ pair, forming a quarkonium state, is colorless, so its coupling to the vacuum field is of the dipole type

$$H_{int} = - \frac{1}{2} g \, (t^a_1 - t^a_2) \boldsymbol{r} E^a \quad , \tag{2.10}$$

where E^a is the chromoelectric field and $t^a_{1,2}$ stand for the color SU(3) generators acting on the quark and antiquark indices, respectively. For transitions between colorless states, the first-order term in H_{int} vanishes, and the dominant effect is due to the second-order term, proportional to

$$<E^aE^a> = -\frac{1}{4} <G^a_{\mu\nu} G^a_{\mu\nu}> ,$$

plus higher iterations.

The vacuum expectation value (2.8) was introduced in Ref. 79 where its magnitude was estimated from the charmonium sum rules:

$$<vac| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | vac> \approx 1.2 \cdot 10^{-2} \text{ GeV}^4 \quad . \tag{2.11}$$

More recent investigations[70,71,80,81], based on similar principles but involving more data, indicate that the vacuum expectation value can be, in fact, larger by a factor of ~1.5.

Comparing Eqs. (2.11) and (2.9), we see that ϵ_{vac} is negative. This is in perfect agreement with the fact that in the confining theory non-perturbative fluctuations should necessarily lower the vacuum energy density.

C) Pre-Coulomb behaviour

In one particular case, the information written down in Eq. (2.11) is sufficient to build a genuine and exhaustive theory of the quarkonium levels. If the quark mass is large enough, the quarks are bound essentially by the Coulomb force at distances of order of k_n^{-1} where k_n is given by the following relation:

$$k_n = \frac{m}{n} \frac{2}{3} \alpha_s(k_n) \tag{2.12}$$

(n is the principal quantum number and m is the quark mass). For large k_n the orbit radius is small, compared to the charcteristic wave length of the vacuum fluctuations, hence

$$|r_\mu D_\mu G_{\alpha\beta}| << |G_{\alpha\beta}| \tag{2.13}$$

and higher-order non-perturbative effects can be neglected. The life of the quarks becomes simple: they form a Coulomb system which feels, however, an influence of an external constant field. This field is (a) weak, (b) random, (c) chromoelectric (corrections due to the chromomagnetic field are suppressed

by two powers of the coupling $\alpha_s(k_n)$).

Being stated in this manner, the problem has an elegant and exact solution (based on the operator product expansion) described originally by Voloshin and Leutwyler[76,82-84]. They succeeded in finding an analytical expression for the level shifts, namely

$$M_{nl} = 2m - \frac{k_n^2}{m} \left\{1 - \frac{m^2}{k_n^6} n^2 a_{nl} < \frac{\pi\alpha_s}{18} G^2 > \right\} \quad , \tag{2.14}$$

where l is the orbital angular momentum and a_{nl} is a known coefficient function of order of unity; say, $a_{10} \approx 1.65$, $a_{20} \approx 1.78$, etc. (m stands for the so-called on-shell mass, see the above discussion). This formula will be useful for the $t\bar{t}$ phenomenology; it is even more important from the point of view of the theory, since it provides with a quantitative answer to the question at which quark masses the Coulomb-like picture sets in.

Evidently, the expansion parameter in Eq. (2.14) is

$$\frac{m^2}{k_n^6} n^2 < \frac{\pi\alpha_s}{18} G^2 > ,$$

and it is of order unity for the $b\bar{b}$ systems. (We mean the ground state, $k_1(b\bar{b}) \approx 0.96$ GeV with $\alpha_s(1\text{GeV}) = 0.3$, i.e. $\Lambda_{\overline{MS}} = 100$ MeV.) For lighter quarks the binding force has nothing to do with the Coulomb interaction, and the latter is quite negligible. On the contrary, heavier quarks form almost perfect Coulomb levels, the deviations are very small. The upsilon family lies somewhere in between: here the Coulomb terms are competing with non-perturbative ones.

It is instructive also to examine the n-dependence which turns out to be very sharp. Already the first excited level in the $b\bar{b}$ system is completely non-Coulombian. For n = 2 the lower boundary of the Coulomb domain is shifted to $m \gtrsim 20$ GeV. For such quark mass the number of the excited levels below the continuum threshold is rather large[85],

$$n \approx 2(m_t / m_c)^{\frac{1}{2}} \gtrsim 7 - 8,$$

and one can enjoy a rich spectrum of dynamical scenarios in one and the same quarkonium family.

Of great practical interest is the result for the electronic width $\Gamma(1^3S_1 \to e^+e^-)$,

$$\Gamma = \Gamma_{Coul}\ |1 + \frac{A_Z}{A_\gamma}|^2 \left\{1 + \frac{m^2}{k_1^6}\, 4.93 < \frac{\pi\alpha_s}{18} G^2 > \right\} \qquad (2.15)$$

The expression presented here is taken from the work of Voloshin, that obtained by Leutwyler is somewhat different; Γ_{Coul} is a purely Coulombic width,

$$\Gamma_{Coul} = 4\pi\ (Q_q)^2 \frac{\alpha^2}{m^2} \frac{k_1^3}{\pi} (1 - \frac{16\alpha_s(m)}{3\pi}) , \qquad (2.16)$$

Q_q is the quark charge, and the correction factor $|1 + A_Z / A_\gamma|^2$ is due to the Z-boson contribution. The curve for the reduced width,

$$\Gamma_{red} \equiv \Gamma(1^3S_1 \to e^+e^-)/Q_q^2\ |1 + \frac{A_Z}{A_\gamma}|^2 \qquad (2.17)$$

is displayed in Figure 12. It is surprisingly flat in the domain $m \gtrsim 10$ GeV, where the result is reliable.

It is interesting that within the framework of the same approach one can estimate spin splittings as well.

Say, the splitting for the ground state (n = 1) is the following[88]:

$$M(1^3S_1) - M(1^1S_0) = \frac{32\pi}{9} \frac{\alpha_s(m)}{m^2} \mid \psi_1(o) \mid^2$$
$$+ < \frac{\pi\alpha_s}{18} G^2 > (4mk_1^2)^{-1} \frac{688}{153} \qquad (2.18)$$

where $\psi_1(o)$ is the wave function at the origin, containing in turn two distinct terms,

$$|\psi_1(o)|^2 = \frac{k_1^3}{\pi} \left\{1 + 4.93\ \frac{m^2}{k_1^6} < \frac{\pi\alpha_s}{18} G^2 > \right\} \qquad (2.19)$$

Substituting here the b-quark mass, $m_b = 4.8$ GeV, one gets 90 MeV, approximately. Unfortunately, the expansion parameter is equal to unity, and we have no reasons to believe that higher-order non-perturbative terms do not change the answer.

As it was noted in Ref. 89, the dominant effect is just a renormalization of $|\psi_1(o)|^2$, while other non-perturbative contributions are limited under reasonable assumptions ($\lesssim$ 5 MeV for the $b\bar{b}$ system). On the other hand, $|\psi_1(0)|^2$ is known from phenomenology, from $\Gamma(\Upsilon \to \mu^+\mu^-)$. In this way we get

$$M_\Upsilon - M_{\eta_b} \approx 8\Gamma_{\mu\mu}(\Upsilon)\ \frac{\alpha_s(m_b)}{\alpha^2}\ (1 + 6.1\ \frac{\alpha_s(m_b)}{\pi})$$
$$+ < \frac{\pi\alpha_s}{18} G^2 > \ (4mk_1^2)^{-1}\ \frac{688}{153} \approx 36 \text{ MeV} \qquad (2.20)$$

where the $O(\alpha_s)$ corrections, found in Fig. 88, are also included. Amusingly, this formula does also work for J/ψ (the resulting splitting is 60 + 30 = 90 MeV), although, of course, one can hardly justify theoretically such a distant extrapolation.

It goes without saying, that with such a small difference it is extremely difficult to detect η_b experimentally.

Formulae for the hyperfine splittings in 1P and 2P states, analogous to (2.18), have been obtained in Ref. 90. Unfortunately, as it was mentioned above, in proceeding from L=0 to L=1 the domain of the applicability of the method is shifted sharply to higher masses. We do not present here the analytical expressions for the mass splittings $\Delta_1 = M(^1P_1) - M(^3P_0)$, $\Delta_2 = M(^1P_1) - M(^3P_1)$, $\Delta_3 = M(^3P_2) - M(^1P_1)$, that were obtained in Ref. 90; they are rather lengthy. The approximation becomes reasonable for $m \gtrsim 40$ GeV; and for quarks with mass 40 - 50 GeV, occupying the lowest P levels, one has $\Delta_1 \approx 9$ MeV, $\Delta_2 \approx 2$ MeV, $\Delta_3 \approx 3$ MeV.

D) Charmonium and Bottonium

The charmonium and bottonium size is too large to apply the technique described above directly. A very powerful alternative approach are the QCD sum rules which allow one to get accurate predictions for the lowest-lying levels with different quantum numbers in terms of fundamental parameters.

Let us consider, for instance, the vector channel of charmonium. The definition of the spectral density R_c is standard,

$$R_c = \sigma(e^+e^- \to \text{charm}) \ /\ \sigma(e^+e^- \to \mu^+\mu^-),$$

where the cross section $\sigma(e^+e^- \to \text{charm})$ includes J/ψ , higher resonances, and the charmed continuum. As well-known, at present QCD does not resolve separate resonance structures, only the smeared cross section is predicted. Thus, formally speaking, one has to recede a step back as compared to the theory presented above (Subsect. C). Instead of a particular level we shall consider now the weighted sums over many levels.

However, if the weight function is steep enough, the sum may be in fact saturated by the lowest-lying state, and we get a (quasi-) theory of such states.

In other words, everything depends on our abilities to calculate integrals

$$\int R_c(s) f(s) ds$$

with steep weight functions (of power or exponential type). The first systematic investigation of such integrals in QCD has been undertaken in Ref. 91 - 93. During the last years we were witnessing a considerable progress in this direction. At present, the QCD sum rules are successfully exploited not only in heavy quarkonium but also in such problems as the spectrum of light mesons[73] and baryons[94], form factors at intermediate momentum transfers[95], three-particle coupling constants, for instance, $\rho\omega\pi$ [96] etc.

The basic theoretical steps are simple. We start with a two-point function with the appropriate quantum numbers. For instance, in order to analyse R_c we take

$$\Pi_{\mu\nu} = i \int dx e^{iqx} \langle 0 | T \{ \bar{c}\gamma_\mu c(x), \bar{c}\gamma_\nu c(0) \} | 0 \rangle . \qquad (2.21)$$

The simplest graph contributing to $\Pi_{\mu\nu}$ is shown in Fig. 13a.) Moreover, because of the famous asymptotic freedom, it is the only one which survives in the deep Euclidean domain of Q^2. A trivial smooth behaviour of $R_c(s)$ at high s is in one-to-one correspondence with asymptotic freedom.

When we proceed from the Euclidean domain towards the physical region, the interactions become important bringing in an additional mass scale. Respectively, the smooth curve for $R_c(s)$ becomes less smooth at lower s, and resonance structures do appear.

The interaction which is the first to manifest itself and turns out to be the most important is depicted in Figure 13b . The crosses on the gluon lines indicate that they are non-perturbative, and the graph is reduced to the vacuum expectation value (2.8), times a known function of Q^2 [79,72-74].

On the other hand $\Pi_{\mu\nu}$ can be expressed in terms of R_c by means of the general dispersion relation. Thus the resonance properties are correlated with the fundamental vacuum parameters.

We do not dwell on exposition of the technicalities, they can be found in the original papers[71-74]. Just to illustrate the specific features of the method, we will reproduce a plot from one of the early works[73] (Figure 14). It displays the ratio of the moments,

$$r_n = \int \frac{R_c(s)}{s^{n+1}} ds \Big/ \int \frac{R_c(s)}{s^n} ds =$$

$$= \frac{n^2-1}{n^2+\frac{3}{2}n} \frac{1}{4m_c^2} \left\{ 1 - \frac{(6n+14)n(n+1)(n+2)}{(2n+3)(2n+5)} \langle \frac{4}{9} \pi\alpha_s G^2 \rangle \frac{1}{(4m_c^2)^2} + \ldots \right\} \qquad (2.22)$$

as a function of n. For large n all the contributions, except that of J/ψ , die away, and $r_n \to M_{J/\psi}^{-2}$. Unfortunately, in the present-day theory we cannot consider infinitely large n, since the non-perturbative terms blow up. However, for n = 5 - 6 (from the physical point of view these numbers may be considered as large), the term proportional to $\langle G^2 \rangle$ is still under control. Meanwhile, for such n the J/ψ contribution exceeds 95%. Thus, the J/ψ mass is expressed in terms of the quark mass to a one-percent accuracy.

In fact, the investigations proceed in the reverse order: the quark mass and $\langle G^2 \rangle$ were fitted to reproduce the experimental value of $M_{J/\psi}$. With these parameters in hand , one can predict masses of the lowest states with other quantum numbers. The results for the P levels of charmonium, obtained in Ref. 74, are given in Figure 15. In all cases one has a nice stability plateau (this fact is due to some technical improvements developed in Ref. 74); its position is in perfect agreement with the experimental mass value. The authors have also predicted the position of the elusive $^1P_1$ level:

$$M(1^1P_1) = 3.51 \pm 0.01 \text{ GeV}. \qquad (2.23)$$

Seemingly, this is the most accurate and reliable estimate available today; we believe it will be confirmed when the level will be discovered*.

Perhaps it is worth recalling a dramatic story of the η_c discovery. It was first found at a wrong place, 2.83 GeV, while the sume rules imply[72]

$$M_{\eta_c} = 3.00 \pm 0.03 \text{ GeV}.$$

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*Recall (Section 1.1 of Chapter 1) that the state $^1P_1$ ($J^{PC} = 1^{+-}$) cannot be observed as a resonance in the direct channel of the e^+e^- annihilation, neither in the decay $\psi' \to \gamma^1P_1$. The cascade $\psi' \to \gamma\chi_2 \to \gamma\gamma^1P_1$ is expected to be suppressed because of a small mass difference between χ_2 and $^1P_1$. Search for the $\psi' \to \pi^0\, ^1P_1$ transition[37] was not successful thus far. Moreover, the upper bound for $BR(\psi' \to \pi^0\, ^1P_1)$ is already somewhat lower than the existing, though very rough, theoretical estimates, for details see Ref. 37. The cascade $\psi' \to \gamma\eta_c' \to \gamma\gamma^1P_1$ is probably the most promising way of observation on the $^1P_1$ level.

The discovery of the 2.98 GeV state at SLAC was thus one of the major successes of the QCD-based ideas*.

An analogous analysis for the bottonium family is hampered by the necessity of accounting for the Coulomb interaction. All the difficulties were overcome, one by one, in Refs. 70, 98 where a non-relativistic variant of the Borel summation technique was used. First, the 1^- channel was considered, and an accurate value of the b-quark mass was extracted, that was quoted above. Then a more difficult problem is investigated, namely, that of the 1S-P level splitting. The final result for M(P) is[70]

$$M(1P,b\bar{b}) = 9.83 \pm 0.03 \text{ GeV}. \tag{2.24}$$

A close number, 9.80 GeV, has been obtained by Bertlmann in a somewhat different way[71]. It is noteworthy that various potential models provide with a spectrum of predictions, ranging from 9.86 up to 9.94 GeV (the most typical number is 9.90, see below).

The superfine splitting in bottonium has also been studied within the sum rule approach, and it was obtained[99]

$$M_\Upsilon - M_{\eta_b} \approx 30 \text{ MeV} \tag{2.25}$$

This result is in very good agreement with Eq. (2.20) and, thus, the corresponding prediction should be considered as reliable.

(Later on, exactly the same method was used in Ref. 74 but with a different result, $\Delta M \approx 60$ MeV. Probably, this number is overestimated, because the Coulomb corrections were not taken into account in this work.)

The QCD sum rules are insensitive to the position of the radial excitations. As far as ^3P_J-level splittings are concerned, they can be, in principle, extracted from the sum rules. However, thus far, this has not been done. Thus, in both cases, one has to invoke the potential model. An exhaustive analysis of the bottonium family accounting for the relativistic corrections $\sim(v/c)^2$ is undertaken in Ref. 77. The authors use, in essence, the potential of Eichten et al[29] (the funnel-like potential). In order to account for the spin dependence they construct the Breit-Fermi Hamiltonian in which the Coulomb singularity at the

*The state $\chi(2.83)$ was shady from the very beginning bcause of the expected large width of the $J/\psi \to \gamma X$ transition, stemming from a naive estimate of the M1 transition (e.g. see Ref. 97)

origin is smeared out artificially at distances $\lesssim m_Q^{-1}$. The energy eigenvalues and eigenfunctions are determined by virtue of a numerical integration of an equation with the Breit-Fermi Hamiltonian. The authors assume that the linear term in the potential is scalar under the Lorenz transformations - only in this case they manage to reproduce the charmonium spectrum. All free parameters are fixed by fitting charmonium where theoretical results deviate from the experimental ones by at most 20 - 30 MeV. Then, the predictions for the bottonium family are formulated (Table 9). For 2^3S_1 and 2^3P_J the agreement with experiment is very good. It seems that one may expect the same accuracy ($\sim$10 - 20 MeV) for 1P levels as well; their center of gravity is located, according to [77] at 9.90 GeV.

This last result, and <u>especially</u> the sum rule prediction (2.24) is a forcible argument questioning the fact of observation of $\Upsilon'' \to (1P,b\bar{b}) + \gamma$ with $E_\gamma \approx 410$ MeV (the corresponding 1P state mass amounts to 9.95 GeV, see Chapter 1).

Notice that calculations of Ref. 77 imply a too large superfine splitting $1^3S_1 - 1^1S_0$, approximately three times larger than the correct answer (cf. eq. (2.20)). The authors of Ref. 77 confess that their predictions for superfine splittings are not quite reliable since they depend on the details of smearing of the singularity at the origin. As it is common to all potential calculations one fails[77] to reproduce the leptonic widths of n^3S_1 levels (see Subsection 2.2.).

2.2. LEPTONIC AND PHOTONIC DECAYS

A) Leptonic widths

The same sum rules which are so useful in the spectroscopy, give simultaneously leptonic widths of the ground levels. All the relativistic effects, renormalizations etc. are taken into account automatically. The result for J/ψ is known for several years now[69,72,73]. It is in excellent agreement with the experiment; evidently, no further comments are necessary. As for the Υ particle, the situation became clear quite recently. Because of large Coulomb factors appearing in the sum rules this decay rate is very sensitive to the value of α_s. The theory requires[70]

$$\Gamma(\Upsilon \to \mu^+\mu^-) = 1.15 \pm 0.20 \text{ keV},$$

and

$$\alpha_s(1 \text{ GeV}) = 0.30 \pm 0.03, \tag{2.26}$$

corresponding to $\Lambda_{\overline{MS}} \approx 100 - 150$ MeV. It is impossible to go beyond these limits, at least if our basic concepts are true. The world average for the leptonic width is $\Gamma(\Upsilon \to \mu^+\mu^-)_{exp} = 1.17 \pm 0.05$ keV (Table 6).

As to the quark-gluon coupling constant, the latest data welcome Eq. (2.26). A set of results referring to deep-inelastic lepton-photon and photon-photon scattering, properties of the three-jet events in the e^+e^- annihilation, and a series of other characteristics (details can be found, say, in Ref. 100) show that the world average is*

$$\Lambda_{\overline{MS}} = 160 \begin{smallmatrix} +100 \\ -80 \end{smallmatrix} \text{ MeV}. \tag{2.27}$$

The branching ratio BR $(\Upsilon \to \mu^+\mu^-)$ is also in agreement with this number, BR $(\Upsilon \to \mu^+\mu^-) = (3.3 \pm 0.5)$ % (a detailed discussion is given in Ref. 99). It is worth recalling that the QCD sum rules[73] have never admitted a value of α_s(1GeV) considerably higher than 0.3. (A few years ago the commonly accepted magnitude was $\Lambda \approx 500 - 700$ MeV)

In principle, the sum rule method enables one to determine also other leptonic widths, e.g. $\Gamma(\eta_c \to 2\gamma)$. Technically, this is a more complicated problem, as compared to the case of the electronic widths, since instead of the two-point functions one has to analyse the three-point functions of the type

*For a definition of the parameter $\Lambda_{\overline{MS}}$ see Ref. 101.

$$\langle 0|T\{\bar{c}(0)\,\gamma_5 c(0),\ j_\mu^{em}(x),\ j_\nu^{em}(y)\}|0\rangle . \tag{2.28}$$

In order to get a reliable prediction, one has, of course, to take into account the gluon condensate effects, as well as the usual gluon exchanges. The first steps along these lines have been done in Ref. 102, where G^2 terms in the three-point function (2.28) were found. The non-perturbative effects are shown to be substantial, and the obtained width of the $\eta_c \to 2\gamma$ decay, 4.5 keV, is considerably less than the naive potential model estimates that give 6.0 - 6.5 keV. The former number, 4.5 keV, is quite close to the old estimate[69], obtained from the sum rules in their primitive form (i.e. without the gluon condensate). Of course, without the gluon condensate, one cannot estimate the accuracy of the prediction reliably. Results of Ref. 102 permit one to achieve the same extent of reliability as in the problem of electronic width.

As was already mentioned the sum rules are insensitive to radial excitations and do not fix with sufficient accuracy their masses and coupling constants. Likewise, the potential models cannot provide with accurate estimates of $|\psi(0)|^2$ - the quantity determining $\Gamma(n^3S_1 \to e^+e^-)$ and $\Gamma(n^1S_0 \to 2\gamma)$. In particular, in Ref. 77 $|\psi(0)|^2$ for Υ, Υ' and Υ'' systematically exceeds, by a factor of 2, experimental values. Moreover, according to Ref. 77 the superfine interaction enhances $|\psi(0)|^2_{1^1S_0}$ by 3-4 times (!) as compared to $|\psi(0)|^2_{1^3S_1}$ - a result which is in sharp disagreement with the sum rule predictions, see the previous paragraph. However, non-potential effects seem to cancel in the ratios $\Gamma(n^3S_1 \to e^+e^-) / \Gamma(1^3S_1 \to e^+e^-)$ (Table 10).

B) Other methods, other trends

We have already mentioned the potential model in connection with various aspects of quarkonium physics. The conventional local duality[104] is also used often; its starting assumption is

$$\int_{s_0-\Delta s}^{s_0+\Delta s} \sigma_{physical}(s)ds = \int_{s_0-\Delta s}^{s_0+\Delta s} \sigma_{bare\ quarks}(s)ds \tag{2.29}$$

However, one should not demand from these models more than they can really give.

It is important to realise that the potential describing the spectra of charmonium and bottonium is nothing else but an effective potential. The true static energy might reveal itself in highly excited levels (slightly below the flavor threshold). Here the level spacings are small and the quark frequencies are substantially less than those specific for the gluon medium. In other words, in this case the potential approach is quite justified from the theoretical point of view.

The potential model is indispensable for orientation, and it gives a nice general picture; however, it cannot (and should not) answer all subtle questions, such as the superfine splittings, precise determination of the leptonic decay rates and so on.

The naive duality relations like (2.29) are usually exploited in order to extract the couplings of mesons to various currents. It is known that the amplitudes

$$\langle 0|\bar{c}\gamma_\mu c|J/\psi\rangle, \langle 0|\bar{c}\gamma_\mu c|\psi'\rangle, \ldots$$

are reproduced well in this way.

The origin of the duality is quite clear within the semi-classical treatment of the Schroedinger equation[105]; actually, it is of a more general nature.

The true reason explaining the local duality is the fact that the interaction switches off at short distances. If at short distances the theory approaches the asymptotically free limit, the local duality at high energies is guaranteed.

Consider a virtual photon of a high energy E, which converts into a $Q\bar{Q}$ pair. The conversion takes place at distances $\sim 1/E$, and its probability is proportional to $\sigma(e^+e^- \to \text{bare quarks})$. Only at much larger distances, $\sim E/\Lambda^2$, confinement effects switch on in full. They play the role of a large box which makes the spectrum discrete. Evidently, the sum over a few adjacent discrete levels reproduces the cross section $\sigma(e^+e^- \to \text{bare quarks})$ up to terms $\sim (1/E)$ to a positive power.

Thus, the local duality is a rather trivial fact for highly excited states. As for the lowest state, J/ψ, its validity, confirmed by the sum rules, seems more surprising, at least, at first sight. One could hardly foresee beforehand that the non-perturbative effects would be so modest and would not blow up in the domain where the current correlation functions are practically saturated by the J/ψ. The fact that this is indeed the case shows that the characteristic quark frequencies in J/ψ are numerically larger than those inherent to the

gluon medium. In other words, the J/ψ size is smaller than the confinement radius. The expansion parameter reduces to $R_{J/\psi}/R_{conf} \approx 1/2$ to a rather large positive power (2 or, even, 4).

With quark mass increasing the accuracy of the relations (2.29) becomes worse. Actually, in a purely Coulombic situation the Coulomb poles should be added "by hand" to the right-hand side of Eq.(2.29), so the procedure becomes almost senseless.

One can show that the standard duality is invalid also for mesons with high spins, $J \geq 4$[106].

In many cases QCD suggests a more refined version, instead of the naive formula (2.29),

$$\int_{\text{threshold}}^{\infty} d s \sigma_{\text{physical}}(s) f(s) = \int_{4m_c^2}^{\infty} d s \sigma_{\text{quark}}(s) f(s) \qquad (2.30)$$

with specific weight functions $f(s) \sim s^{-n}$. Equations of this type are based on the asymptotic freedom and dispersion relations[93,69]. The resonance masses and the position of the continuum threshold are put in by hand, and the resonance coupling constants are the desirable output. Ref. 107 is devoted to the radiative transitions $\psi' \to \chi + \gamma$ and $\chi \to J/\psi + \gamma$. Three-point functions

$$\int dx dy\, e^{-i(kx + qy)} \langle 0|T\{j_1(0)\, j_\mu^{em}(x)\, j_2(y)\}|0\rangle \qquad (2.31)$$

were introduced, where j_1 and j_2 are external charmed quark currents with appropriate quantum numbers, say, $j_1 = \bar{c}c$ and $j_\mu^{em} = \bar{c}\gamma_\mu c$. In the Euclidian domain two alternative expressions exist for the matrix element (2.31) (Fig. 16), so one has an overdetermined set of equations. Approximate solutions of these equations are displayed in Table 11.

For comparison, this table contains also the experimental numbers and those obtained in the potential model[77]. Relativistic corrections are especially large for the transitions $\psi' \to \chi_0 + \gamma$ and $\psi' \to \chi_1 + \gamma$.

C) Photon transitions

Photon transitions play a distinguished role revealing the world of C-even charmonium levels. As for the electric dipole transitions, nothing dramatic has happened on the theoretical scene during the last years. The results obtained in the framework of the potential models, dispersion approach, and from the nonrelativistic sum rules[108] (Thomas - Reiche - Kuhn) coexist peacefully, waiting for a future development. Some of them are compiled in Tables 11 and 12 which contain also the experimental data. Note that according to the data of the CB group, the quantity $\Gamma(\psi' \to \gamma\chi_J)_{exp}/E_\gamma^3\ (2J+1)$, is approximately identical for all the χ_J-states, namely, 1.00 ± 0.07; 1.05 ± 0.08; 1.37 ± 0.09 for $J = 0,1,2$[109]. The analogous data of the CUSB group in the bottonium family are[25] $\Gamma(\Upsilon'' \to 2^3P_J + \gamma)\ /\ E_\gamma^3\ (2J+1) = 1.03 \pm 0.5\ (J = 0)$; $1 (J = 1)$; $0.95 \pm 0.3\ (J = 2)$. Such a behaviour is natural for the non-relativistic quarkonium model. One should keep in mind, however, that relativistic effects are rather large, especially in charmonium, so that, perhaps, we are dealing with a numerical coincidence.

A few words about the M1 transitions. The allowed decays, like $J/\psi \to \eta_c\gamma$ must be described accurately enough by the simplest formula, like

$$\Gamma(1^3S_1 \to 1^1S_0 + \gamma) = \frac{16}{3}\mu^2\omega^3 \tag{2.32}$$

where μ is the Dirac magnetic moment, μ = (quark charge) $\sqrt{\alpha}/2m$. Sometimes an adhoc assumption is made according to which c and b quarks may have large anomalous magnetic moments; in this case Eq. (2.32) would be invalid, of course. However, this assumption is not true. The derivation of Eq. (2.32) is perfectly controlled by the theory; moreover, corrections to Eq. (2.32) are calculable and small. Actually, one can show that[110,111]

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{2}{9}\frac{\Gamma(\eta_c \to 2\gamma)}{\Gamma(J/\psi \to e^+e^-)}\,\alpha\,\frac{M_\psi^4}{M_{\eta_c}^3}\left(1 - \frac{M_{\eta_c}^2}{M_\psi^2}\right)^2(1 - 0.28\alpha_s) \tag{2.33}$$

where

$$\Gamma(\eta_c \to 2\gamma)\ /\ \Gamma(J/\psi \to e^+e^-) = 3Q_q^2\,(1 + O(\alpha_s, \mu_{non\text{-}pert}/m)) \tag{2.34}$$

Similar relations hold for Υ. It is easy to understand why there are no large corrections to the magnetic moment in the transitions like $J/\psi \to \eta_c\gamma$. Let us consider the amplitude $\eta_c \to 2\gamma$ and represent it in the form of the dispersion integral in one of the photons. The dispersion integral is dominated by the J/ψ contribution. Other states are separated by a large gap, $\delta^2 = 2M\Delta M$ (where $\Delta M \sim M_{\psi'} - M_\psi$) and their contribution is small, $O(\alpha_s(\delta^2))$. In this way we get Eq. (2.33), and $O(\alpha_s)$ corrections are determined, essentially, by short distances[110,111].

A naive reasoning suggests that the ratio $\Gamma(\eta_c \to 2\gamma)\ /\ \Gamma(J/\psi \to e^+e^-)$ would be close to 4/3, then $\Gamma(J/\psi \to \eta_c\gamma) \approx 2.5$ keV. Perturbative effects tend to increase the ratio of the widths, resulting in $1.12 \cdot 4/3$ instead of 4/3; however, non-

perturbative effects draw to the opposite direction (see Subsect. A). Seemingly, the minimal value permitted by the modern theory is $\Gamma(\eta_c \to 2\gamma)\ /\Gamma(J/\psi \to e^+e^-) = 0.8$. Then $\Gamma(\eta_c \to 2\gamma) = 4$ keV and $\Gamma(J/\psi \to \eta_c\gamma) \approx 1.5$ keV. On the other hand, according to Ref. 25, $\Gamma(J/\psi \to \eta_c\gamma)_{exp} = (0.76^{+0.23}_{-0.20})$ keV. The experimental result is lower than the theoretical prediction, which seems to be absolutely reliable, by 3 standard deviations. In order to reproduce 0.8 keV theoretically one should assume that $\Gamma(\eta_c \to 2\gamma)$ is close to 2 keV, not 4 keV as was expected. Such mysterious suppression, incredible by itself, would immediately lead to troubles in other places[69], in particular, to an enormous violation of the Appelquist - Politzer recipe that cannot be understood in any way.

In the $b\bar{b}$ family, the small mass difference $M_\Upsilon - M_{\eta_b}$ hampers searches for the decay. The numbers give reasons for pessimism, since $\Gamma(\Upsilon \to \eta_b\gamma) \approx 2$ eV, and $BR(\Upsilon \to \eta_b\gamma) \sim 10^{-4}$. Unfortunately, it would not be easier to attain η_b' starting from Υ'.

Decays like $\Upsilon' \to \eta_b\gamma$ or $\psi' \to \eta_c\gamma$ are forbidden in the non-relativistic limit. The ψ' decay has been seen experimentally with the width $\Gamma(\psi' \to \eta_c\gamma) = 0.6 \pm 0.2$ keV[16,35]. Thus deviations from the non-relativistic approximation must be substantial. What does the theory say on that?

In Ref. 111 it was argued that the $\psi' \to \eta_c\gamma$ transition is mainly due to a gluon admixture in the ψ' wave function. The arguments are as follows. If the local duality holds, one can substitute the amplitude

$$A(\gamma \to \psi')A(\psi' \to \gamma\,\eta_c), \tag{2.35}$$

in the sense of duality, by another amplitude, namely,

$$A(\gamma \to \text{quarks, gluons})A(\text{quarks, gluons} \to \gamma). \tag{2.36}$$

The latter product is easily calculated, of course, fixing, in turn, the amplitude $A(\psi' \to \eta_c\gamma)$. The theoretical result is compatible with $\Gamma(\psi' \to \eta_c\gamma)_{exp}$. A more important fact is that the contribution to (2.36), dominating numerically, is due to the intermediate state $c\bar{c}g$, to be interpreted, naturally, as the gluon admixture in ψ'. At the same time, a rather large width,

$$\Gamma(\psi'' \to \eta_c\gamma) \sim 1 \text{ keV}, \tag{2.37}$$

is predicted, considerably higher than in the standard potential models where the $\psi'' \to \eta_c\gamma$ decay is strongly forbidden. Unfortunately, it is not easy to check the latter estimate experimentally, because the corresponding branching ratio does not exceed $5 \cdot 10^{-5}$.

The Υ' decay rate is further suppressed as compared to $\psi' \to \eta_c\gamma$, by at least the following factor

$$\frac{1}{4}\left(\frac{M_\psi}{M_\Upsilon}\right)^2 \left(\frac{\alpha_s(\Upsilon)}{\alpha_s(\psi)}\right)^2 \cdot 0.8 \sim 1/100 \qquad (2.38)$$

The ratio of the coupling constants characterizes here the gluon admixture, and the factor 0.8 is due to the phase space. Combining this factor with $\Gamma(\Upsilon' \to \eta_c\gamma)_{exp}$, one gets

$$\Gamma(\Upsilon' \to \eta_b\gamma)_{theor} \lesssim 8 \text{ eV}. \qquad (2.39)$$

The corresponding branching ratio is expected to be smaller than 5×10^{-4}.

2.3. HADRONIC DECAYS

A) Heavy quarkonia and the Old World

The issues discussed up to now refer mostly to heavy quarks and their relations with the vacuum environment. Now we proceed to another fundamental aspect - coupling of heavy quarkonium to old hadrons. Theoretical and experimental investigations in this field give information on the structure of the $Q\bar{Q}$ systems, glueballs, and traditional old hadrons. In many cases, the information is unique, since it is impossible to get it in any other way.

B) Inclusive hadronic decays

The famous Appelquist - Politzer recipe[14] prescribes to calculate the elementary processes $Q\bar{Q} \to 2g$, $3g$, or $q\bar{q}g$, instead of summing over a large number of exclusive channels. This brilliant invention is applicable, beyond any doubt, to asymptotically heavy $Q\bar{Q}$ states. We are interested, however, in charmonium and bottonium, and various preasymptotic (non-perturbative) corrections may be of importance here.

The Appelquist - Politzer prescription assumes an ideal gluon - hadron duality. For light quarks the quantity 9 GeV^2 ($\approx M^2_{\eta_c}$) is indeed an asymptotical domain, where the hadron cross section coincides with the quark cross section. Is this statement true also for gluons?

The onset of the asymptotic behaviour is determined by the non-perturbative effects - one will scarcely doubt this fact today. Meanwhile, these effects are drastically different for the quark and gluon channels. Gluonic currents are coupled to vacuum fields much stronger than the quark currents (Fig. 17, further details can be found in Ref. 112), therefore the asymptotic regime sets in at larger energies for gluons.

It is not a simple task to find out quantitatively what is the critical energy. Still some estimates do exist in the literature. It was shown[112], in particular, that the boundary of the asymptotic domain is

$$(s_0)_{\text{two gluons},\ J^P = 0^-} = 6 - 16 \text{ GeV}^2. \qquad (2.40)$$

If this is true, the charmonium family is in a dangerous vicinity of the critical zone, or, perhaps, even inside it. Therefore one would be hardly surprised by some (moderate) deviations from the perturbative formulae for the $c\bar{c}$ annihilation. On the other hand, the $b\bar{b}$ annihilation should be described by these formulae to a good accuracy.

Experiment indicates the following. The Appelquist - Politzer recipe seems to be valid in charmonium and bottonium in the channels 1^- and 2^+ and is invalid for C-even charmonium levels with $J^P = 0^{\pm}$. The anomalously tardy onset of asymptotics just in the channels with zero spin has been predicted theoretically[112]. The reason lies in a very strong interaction of gluons with direct instanton fluctuations in the vacuum which takes place only if the total spin is vanishing.

Let us demonstrate, first of all, that $BR(\Upsilon \to \mu^+\mu^-)_{exp}$ confirms, among other things, the Appelquist - Politzer prescription.

Indeed, starting from $\Gamma(\Upsilon \to e^+e^-) = 1.17 \pm 0.05$ keV and $BR(\Upsilon \to \mu^+\mu^-) = 3.3 \pm 0.5$ %, we get $\Gamma_{tot}(\Upsilon) = 35.5 \pm 7$ keV, or $\Gamma_{direct\ hadr}(\Upsilon) = 27 \pm 6$ keV. Suppose that the hadronic and gluonic widths are equal, and use the formula

$$\frac{\Gamma_{3g}(\Upsilon)}{\Gamma_{\mu\mu}(\Upsilon)} = \frac{10}{81}\,\frac{\pi^2 - 9}{\pi}\,\frac{\alpha_s^3\,(m_b)}{\alpha^2 Q_b^2}\left(1 + (1.1 \pm 0.5)\frac{\alpha_s}{\pi}\right) \qquad (2.41)$$

where the $O(\alpha_s)$ correction (in the $\overline{MS}$ scheme) has been found in Ref. 113. The corresponding magnitude of $\alpha_s(m_b) = 0.156 \pm 0.013$. The standard renormalization group expression implies now that $\alpha_s(m_c) = 0.210 \pm 0.028$, which results, respectively, in the ratio

$$\Gamma_{3g}\,(J/\psi)\ /\ \Gamma_{\mu\mu}\,(J/\psi) = 15.5 \pm 6. \qquad (2.42)$$

Agreement with the experimental value, 9.2 ± 2.4, is quite satisfactory.

On the other hand, the data on the hadronic widths of η_c and χ_0 are now available (Table 13). Their photonic widths are to an extent fixed by the theory[69,110,102] (4.5 and 4.5 - 5.5 keV, respectively), and one can compare the ratios $\Gamma_{hadr}\,(\eta_c)\ /\ \Gamma(\eta_c \to 2\gamma)$ and $\Gamma_{hadr}\,(\chi_0)\ /\ \Gamma(\chi_0 \to 2\gamma)$ with

$$\Gamma(c\bar{c} \to 2g)\ /\ \Gamma(c\bar{c} \to 2\gamma). \qquad (2.43)$$

A formidable work has been done in order to account for the first-order perturbative correction in ratios like (2.43)[113-115]. For instance, in the case of η_c it was found[114] that

$$\frac{\Gamma(c\bar{c} \to 2g)}{\Gamma(c\bar{c} \to 2\gamma)} = \frac{9}{8}\left(\frac{\alpha_s(m_c)}{\alpha}\right)^2 \left(1 + 9.2\,\frac{\alpha_s}{\pi}\right), \qquad (2.44)$$

where m_c is the quark mass, and the coefficient of α_s depends, in fact, on the

renormalization procedure (Eq. (2.44) is written for the $\overline{MS}$ scheme).

With $\alpha_s(m_c) = 0.2$, the $O(\alpha_s)$ correction amounts to 50 %, a substantial, but not dangerous level; we mean that the perturbative series is, seemingly, under theoretical control, and does not blow up.

Substituting the two-photon widths, we obtain $\Gamma_{hadr}(\eta_c) = 6$ MeV; the correspondponding result for χ_0 is 6.2 - 7.6 MeV. The experimental data, quoted in Table 13, exceed these numbers systematically, by a factor of 2. Note that if $\Gamma(\eta_c \to 2\gamma) \approx 2$ keV, as it seems from $\Gamma(J/\psi \to \eta_c\gamma)$ (see above), the discrepancy amounts to a factor of 4 in the case of η_c - the possibility hardly imaginable. Finally, the Appelquist - Politzer recipe works perfectly for the tensor channel (i.e. χ_2) yielding $\Gamma_{hadr}(\chi_2) = 1.7 - 2.3$ MeV, while the experimental width is $\Gamma(\chi_2)_{exp} = 2.1^{+1.0}_{-0.7}$ MeV (see Table 13).

Thus, the anomaly takes place only for spin zero. As was already noted, the possibility of such a situation was predicted theoretically[112].

Turning back to technical aspects, we feel it would be in order to present a few expressions[114,115]; much computational efforts were expended to derive these formulae,

$$\frac{B(0^{-+})}{B(0^{++})} = \begin{cases} 1 + 0.9\alpha_s/\pi\ (c\bar{c}) \\ 1 + 2.1\alpha_s/\pi\ (b\bar{b}) \end{cases};\quad \frac{B(2^{++})}{B(0^{++})} = \begin{cases} 1 + 6.5\alpha_s/\pi\ (c\bar{c}) \\ 1 + 4.0\alpha_s/\pi\ (b\bar{b}) \end{cases} \qquad (2.45)$$

where $B(J^{PC}) = \Gamma(J^{PC} \to 2\gamma)\ /\ \Gamma(J^{PC} \to \text{gluons})$. These ratios are convenient because they are independent of the renormalization scheme. The following recent result[113] is of great practical importance,

$$\Gamma(Q\bar{Q}, 1^- \to 3g) = \Gamma_0 \begin{cases} 1 - (3.8 \pm 0.5)\ \alpha_s/\pi & (c\bar{c}) \\ 1 - (4.2 \pm 0.5)\ \alpha_s/\pi & (b\bar{b}) \end{cases} \qquad (2.46)$$

Further details and numerical examples can be found in the paper by Barbieri et al[88] which gives a nice review of the whole subject.

C) Hadronic transitions between quarkonium levels

Decays like

$$\psi' \to J/\psi + \pi\pi \text{ or } \quad \Upsilon'' \to \eta\Upsilon \qquad (2.47)$$

and others of this type probe the gluonic content of usual hadrons. The transitions (2.47) can be considered as two-step processes: first, emission of soft gluons by heavy quarks at relatively short distances; second, conversion of the gluons into light hadrons at relatively large distances. So long as the quarkonium size is small as compared to the sizes of the "old" hadrons, one can use consistently the familiar multipole expansion to describe the gluon emission[116-119].

The factorization alone (plus symmetry properties of the transition amplitudes) yields a lot of predictions for the relative rates, for instance[118]

$$d\Gamma(2^3S_1 \to 1^3S_1 + 2\pi) = d\Gamma(2^1S_0 \to 1^1S_0 + 2\pi),$$
$$d\Gamma(1^3D_3 \to 1^3S_1 + 2\pi) = d\Gamma(1^1D_2 \to 1^1S_0 + 2\pi), \ \ldots \qquad (2.48)$$

More intriguing is a unique possiblity to test the QCD low-energy theorems. Within the framework of the multipole expansion, one has the following decompositions[120]

$$A(n_i\ {}^3S_1 \to n_f\ {}^3S_1 + \pi\pi) = C_1 <0|\ \vec{E}^a\vec{E}^a|\pi\pi> + \text{higher multipoles},$$
$$A(n_i\ {}^3S_1 \to n_f\ {}^3S_1 + \ \eta) = C_2 <0|\ \vec{E}^a x\vec{H}^a|\eta> + \text{higher multipoles}, \qquad (2.49)$$

where $\vec{E}^a$ and $\vec{H}^a$ stand for the chromoelectric and chromomagnetic fields, respectively, and the coefficient functions C_1 and C_2 contain information on the heavy quarkonium. These coefficients are proportional to each other and cancel in the ratio of the amplitudes; moreover in particular quarkonium models they can be found explicitly [121].

At first sight one would suppose that it is impossible to calculate such nontrivial matrix elements as

$$<0|\vec{E}^a\vec{E}^a|\ \pi\pi> \quad , \quad <0|\vec{E}^a x\vec{H}^a|\ \eta> \ ,$$

which represent conversion of gluons to mesons at large distances. Surprisingly, one can do that, starting just from first principles. These matrix elements are related to the so-called triangle anomalies in the trace of the energy-momentum tensor and in the divergence of the axial-vector current. The answers look so attractive that we feel it is worth illustrating them by a few examples. For instance, $<0|\vec{E}^a\vec{E}^a|\pi\pi>$ is reduced to a combination of the following quantities[120,122] $m^2_{\pi\pi}$, B, $\rho^G(\mu)$, $\alpha_s(\mu)$ (here B is the first coefficient in the Gell-Mann - Low function, ρ^G is the gluonic share of the pion momentum, an analogous quantity for the nucleon is measured in deep inelastic scattering; μ is a normalization point, its order of magnitude is that of the inverse quarkonium radius).

The ratio $\Gamma(\psi' \to J/\psi + \pi\pi)\ /\Gamma(\psi' \to J/\psi + \eta)$ was found in Ref. 120 in perfect agreement with experiment. The prediction for bottonium is

$$\frac{\Gamma(\Upsilon' \to \Upsilon\eta)}{\Gamma(\Upsilon' \to \Upsilon\pi\pi)} \approx 4 \cdot 10^{-3} \left(\frac{M_{\Upsilon'} - M_{\Upsilon} - M_\eta}{10\ \text{MeV}}\right)^3,$$
$$\frac{\Gamma(\Upsilon'' \to \Upsilon\eta)}{\Gamma(\Upsilon'' \to \Upsilon\pi\pi)} \approx 2 \cdot 10^{-2} \qquad (2.50)$$

The shape of the pion is also well described by theory[122]. Namely,

$$\frac{d\,\Gamma}{dq^2} \sim \{\ [q^2 - \kappa(\Delta M)^2\ (1 + \frac{2m_\pi^2}{q^2})]^2 + \frac{\kappa^2}{5}\ [(\Delta M)^2 - q^2]^2 (1 - \frac{4m_\pi^2}{q^2})^2\} \qquad (2.51)$$

where $q^2 \equiv m^2_{\pi\pi}$, $\Delta M = M(Q\bar{Q})_i - M(Q\bar{Q})_f$, and $\kappa = \frac{B}{6\pi}\ \alpha_s(\mu)\ \rho^G(\mu) \approx 0.2$, for charmonium. The second term, $\sim \kappa^2/5$ in Eq.(2.51) is due to a D-wave contribution; thus a suppression of the D-wave ($\kappa^2/5 \approx 1/125$) is explained by the theory. Note that the magnitude of κ is not universal: this parameter is expected to be smaller by a factor of 1.5 - 2 for the decay $\Upsilon' \to \Upsilon\pi\pi$.

The Υ'' size is rather large, and therefore the accuracy of the approach in the $\Upsilon'' \to \Upsilon\pi\pi$ transition should be somewhat worse. Still, one may expect that $\kappa \sim$ 1/5. Then, neglecting small corrections in Eq.(2.50), we get

$$\{d\Gamma/dq^2\}^{1/2} \sim q^2 - cm_\pi^2 \ , \ c = 3 - 4 \qquad (2.51)$$

The linear rise in q^2, up to $q^2 \approx 0.3$ GeV^2, is observed in $\psi' \to J/\psi\pi^+\pi^-$ and $\Upsilon' \to \Upsilon\pi^+\pi^-$ decays, in full accordance with Eq. (2.51). If the multipole expansion does work, just the same behaviour of $\{d\Gamma/dq^2\}^{1/2}$ should take place in this q^2 region in the $\Upsilon'' \to \Upsilon\pi^+\pi^-$ transition as well. Looking at Figure 9 we see that experiment rather favours a flat q^2 dependence, $\{d\Gamma/dq^2\}^{1/2} \approx$ const. Such a behaviour of the pion spectrum is hardly understandable from the theoretical point of view. A possible explanation has been suggested by Voloshin who has noticed the following. The pion spectrum in the $\Upsilon'' \to \Upsilon\pi\pi$ decay would be indeed flat if there existed a bottonium level of the molecular type - call it X - with mass close to $M_{\Upsilon''}$ and quantum numbers I = 1, $J^P = 1^+$. Then the decay would proceed as a cascade

$$\Upsilon'' \to \underbrace{X_{virtual}\ + \pi}_{\longrightarrow\ \Upsilon + \pi}$$

Due to vicinity of the X pole the multipole expansion would be inapplicable. Is this explanation satisfactory from the experimental point of view? The question is still open.

For very heavy quarks forming the Coulombic levels, the quarkonium coefficients C_1 and C_2 are calculable, so that the absolute rates are fixed unambiguously. However, the real c and b quarks are not heavy enough, and one has to invoke models. One of them has been developed in Ref. 121. Perhaps, the most interesting finding of that work is an unexpected suppression of the $\Upsilon'' \to \Upsilon\pi\pi$ transition rate, due to cancellations in C_1. It was found that $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ must be less than $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$, in spite of the increased phase space.

As was discussed in Section 1.2 of Chapter 1, the averaged value of $BR(\Upsilon'' \to \Upsilon\,\pi^+\pi^-) = (4.9 \pm 1.1)$ %. In other words, $\Gamma(\Upsilon'' \to \Upsilon\,\pi\pi) \sim 1$ keV, cf. with $\Gamma(\Upsilon' \to \Upsilon\,\pi\pi)_{exp} = 5.2 \pm 1.1$ keV. A suppression is evident, and one sees that the model[121] is not bad, at least at the qualitative level.

The most important lesson, supported by the data, is the applicability of the multipole expansion.

If the reader is not entirely convinced by the facts presented above it is worth adding a few words about the $\Upsilon' \to \Upsilon\pi\pi$ decay. In the framework of the multipole expansion one has

$$C_1 \sim \langle n_f\,{}^3S_1 \,|(t_1^a - t_2^a)\, r_i G_{(8)}(\epsilon_n)\, r_i (t_1^a - t_2^a)|\, n_i\,{}^3S_1\rangle$$

where $G_{(8)}$ is the non-relativistic Green function for the color-octet state. In other words, one should expect that

$$\frac{\Gamma(\Upsilon' \to \Upsilon\,\pi\pi)}{\Gamma(\psi' \to J/\psi\pi\pi)} \approx \left\{\frac{\langle r^2\rangle_\Upsilon}{\langle r^2\rangle_{\psi'}}\right\}^2 \approx 1/16 \tag{2.52}$$

or $\Gamma(\Upsilon' \to \Upsilon\pi\pi) \approx 6.8$ keV. This expectation agrees reasonably with the data (see above).

Hadronic transitions are promising also in another respect. The cascade

$$\Upsilon'' \to 1^1P_1 + \pi\pi, \quad 1^1P_1 \to 1^1S_0 + \gamma$$

is, probably, the best way to discover at once two elusive $b\bar{b}$ levels: $^1P_1$ and $^1S_0$. A "bottle-neck" of this chain is the first decay; its branching ratio is not large. According to Ref. 121, it is about 1%, but one has to expand some more efforts in order to avoid a theoretical uncertainty present in that work. Once the $^1P_1$ level is produced, the problem of η_b would be solved: almost every second decay of $^1P_1$ is $^1S_0 + \gamma$.

Our last remark concerns the decays $\psi' \to J/\psi + \pi^0$ and $\Upsilon' \to \Upsilon\pi^0$, violating the isotopical symmetry. They measure the current quark masses directly. To be more exact, the following theorem holds[123]

$$\frac{\Gamma(\psi' \to J/\psi\pi^0)}{\Gamma(\psi' \to J/\psi\eta)} = \left(\frac{3\sqrt{3}}{4}\,\frac{m_d - m_u}{m_s}\right)^2 \left(\frac{|\vec{p}|_\pi}{|\vec{p}|_\eta}\right)^3 (1 + O(m_q/\mu)) \tag{2.53}$$

where m_q stands for the mass of u, d, or s quark, μ is a characteristic scale of strong interactions (a few hundred MeV). Of course, an analogous relation is valid for Υ.

According to the data of the CB group[33] the ratio of widths in the l.h.s. of Eq. (2.53) amounts to $(4.1 \pm 1.5) \cdot 10^{-2}$, which, in turn, leads to

$$\frac{m_s}{m_d - m_u} = 29 \pm 6.$$

This result, evidently, does not contradict the conventional magnitudes of the quark masses: $m_d = 7.5$ MeV, $m_u = 4$ MeV, $m_s = 150$ MeV (for these values one has $m_s/(m_d - m_u) \approx 42$).

D) $J/\psi \to \gamma$ + light hadrons, $\Upsilon \to \gamma$ + light hadrons

As it was mentioned in Section 1.1 of Chapter 1 (Subsection B), the observed shape of the photon spectrum from the decay $J/\psi \to \gamma$ + light hadrons deviates from the predictions of the quark-gluon model (see Figure 3b), while one has an approximate agreement in the integral probability.

The foremost questions are i) "which states do actually saturate the total probability?", ii) "what is the value of $m^2_{\text{light hadrons}}$, at which the parton-like regime sets in?". The latter parameter, the boundary of the asymptotic domain, is an important dynamical characteristic. For light quarks, as we know from the e^+e^- annihilation, it is about 1.5 GeV^2, but there are good reasons to suppose that this scale is not universal. Arguments were presented[112] that the boundary is shifted to higher energies in the gluonic sector, $s_0 \gtrsim 6$ GeV^2 (see Eq. (2.40)). If so, the genuine glueball continuum can hardly be investigated in the radiative decays of J/ψ, and this problem will be rather a prerogative of the Υ-family physics ($x < 0.94$).

On the other hand, the resonance production can be studied much easier, starting with J/ψ. It is generally believed that the gluon pair materializes in the form of a glueball, the 2^+ glueball predominantly. This conclusion is, evidently, based on the perturbative analysis of Ref. 124. It is noteworthy that in the exclusive decays of the type

$$J/\psi \to \gamma + \text{a meson} \qquad (2.54)$$

the situation is far from being so simple. In fact, direct non-perturbative fluctuations mix effectively the quark and gluon degrees of freedom in the $0^{\pm}$ channels, so that the quark-meson production is not at all suppressed[112].

For the 2^+ state the non-perturbative mixing is small; however, another effect does exist which is sometimes forgotten. Gluons are emitted in the annihilation process at distances $\sim 1/m$. In other words, the gluon source is just $\theta^G_{\mu\nu}(m)$, where m stands for the normalization point and $\theta^G_{\mu\nu}$ is the gluonic piece of the energy-momentum tensor. On the other hand, the specific off-shellness of the meson wave function is of order R^{-1}_{conf} (a few hundreds MeV), and one should account for the evolution of the momentum scale from m down to R^{-1}_{conf}. As a result, the standard logarithmic mixing arises,

$$\theta^G_{\mu\nu}(m) \Rightarrow \theta^G_{\mu\nu}(R^{-1}_{conf}) + \varepsilon\, \theta^q_{\mu\nu}\ (R^{-1}_{conf}) \qquad (2.55)$$

and the mixing parameter ε is of order unity (Figure 18). This explains, in particular, why the classical quark meson f is produced in the reaction (2.54) without a noticeable suppression.

Still, beyond any doubt, the final hadronic system in this decay should be enriched by various unusual states.

The states $\iota(1440)$ and $\theta(1640)$ are debated now as possible glueball candidates (data are compiled in Table 2). Let us discuss them in turn. There are certain arguments in favor of the gluonium interpretation of ι[16,125]: absence of vacancies in the lowest pseudoscalar quark-meson nonet, a large $J/\psi \to \iota\gamma$ width (as compared to that for the usual mesons, except η') and the dominance of the decay channel $\iota \to \delta\,\pi$.

All these arguments do not seem to us to be convincing (see below) and we are not inclined to accept ι as a glueball since the sum rules predict[112] that the pseudoscalar gluonium should lie higher, $M(0^-,\text{glue}) = 2 - 2.5$ GeV. If so, the ι might be a radial excitation of η'.

A relatively large width $\Gamma(J/\psi \to \iota\gamma)$ seems natural within the framework of the standard duality. Indeed,

$$\frac{\Gamma(J/\psi \to \iota\gamma)}{\Gamma(J/\psi \to \eta'\gamma)} = \left|\frac{\langle o|j_P|\iota\rangle}{\langle o|j_P|\eta'\rangle}\right|^2 \left(\frac{|\vec{P}_\iota|}{|\vec{P}_{\eta'}|}\right)^3 \qquad (2.56)$$

where

$$j_P = \frac{3\alpha_s}{8\pi}\, G^a_{\mu\nu}\, G^a_{\alpha\beta}\, \varepsilon_{\mu\nu\alpha\beta}$$

If one introduces the following correlation function

$$P(q^2) = i\int d\,x e^{iqx} \langle 0|\, T\{j_P(x),\, j_P(0)\}|\, 0\rangle$$

induced by the gluonic current, then one can find that the η' is dual to the interval (0 - 5 GeV^2) in the corresponding spectral density[112],

$$|\langle o|\, j_P\, |\eta'\rangle\, |^2 = \frac{1}{\pi}\int_0^5 \text{Im}\, P_o\,(s)\, ds; \ \text{Im}\, P_o(s) = \frac{9s^2}{8\pi^3}\,\alpha_s^{\ 2}\,(s).$$

Even if the ι is dual to an adjacent interval of 1 GeV^2 length (i.e. the duality interval stretches from 5 to 6 GeV^2), even in this case its coupling to j_P is approximately the same as that of η', and the ratio (2.56) is close to unity.

As far as $\theta(1640)$ is concerned, the status of the tensor gluonium for this meson would not contradict the QCD sum rules. Estimates for $M(2^+,\text{glue})$ yield 1.3 to 2 GeV.

True it is worth noting a serious problem emerging in connection with the $\theta \to 2\pi$ and $\theta \to 2\eta$ decays. Assuming that θ is a unitary singlet and that its decay amplitudes are SU(3) symmetric, we get

$$\Gamma(\theta \to \pi^+\pi^- + \pi^0\pi^0) \approx 4 \cdot 3 \cdot \Gamma(\theta \to \eta\eta) \qquad (2.57)$$

where the factor 4 on the right hand side is due to the phase space and D-wave nature of the decays. Experimentally, $\Gamma(\theta \to 2\pi) \lesssim \Gamma(\theta \to 2\eta)$, see Table 3. Both starting assumptions leading to Eq. (2.57) are now rejected, actually. According to Ref. 126, due to a small mass splitting between θ and f'(1515), one can not neglect the θ-f' mixing, which effectively results in a certain strange quark admixture in the θ wave function. Besides that, the transitions into a pair of Goldstone mesons(π,η) do not obey $SU(3)_{fl}$. The $\theta\eta\eta$ amplitude should be enhanced due to a noticeable gluon admixture in the η wave function; the gluons penetrate in η via $\eta\eta'$ mixing[127]. Due to the same reason the $\theta \to \eta\eta'$ decay should be also very essential.

In general, proving the gluonium nature of this or that meson turns out to be a very complicated task. All tests suggested thus far in the literature:

a) small total width;

b) copious production in $J/\psi \to \gamma X$;

c) $SU(3)_{fl}$ singlet structure of the wave function and unitary symmetry of the decay amplitudes

are ruled out by counterexamples, for details see Ref. 25.

Among other decay modes of J/ψ or Υ it is worth mentioning $\gamma\eta$ and $\gamma\eta'$. The ratio of the corresponding widths can be written as (see Ref. 128)

$$\frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma\ )} = \left| \frac{\langle 0|\alpha_s\ G^a_{\mu\nu}\ \tilde{G}^a_{\mu\nu}|\eta'\rangle}{\langle 0|\alpha_s\ G^a_{\mu\nu}\ \tilde{G}^a_{\mu\nu}|\eta\ \rangle} \right|^2 \times \left(\frac{|\vec{P}_{\eta'}|}{|\vec{P}_\eta\ |}\right)^3 .$$

Here the denominator is fixed by symmetry properties only, while the numerator contains a highly non-trivial information on the gluon coupling to η'. Several models suggest answers of their own for the matrix element $\langle 0|\alpha_s\ G^a_{\mu\nu}\ \tilde{G}^a_{\mu\nu}|\ \eta'\rangle$ (Ref. 128-133). The experimental result of the CB group, quoted in Table 2, is $\Gamma_{\eta'\gamma}\ /\ \Gamma_{\eta\gamma} = 4.7 \pm 0.6$; it implies that

$$\langle 0|\ \frac{3\alpha_s}{4\pi}\ G^a_{\mu\nu}\ \tilde{G}^a_{\mu\nu}|\ \eta'\rangle\ = M^2_{\eta'}\ (120\ -\ 140\ \mathrm{MeV}) \qquad (2.58)$$

in agreement with QCD-based estimates[112].

3. MESONS WITH OPEN FLAVOUR

Until now we have discussed systems of the type $Q\bar{Q}$, where Q is a heavy quark, c, b, or t. In these mesons, the quark flavour is hidden, as it is screened by the corresponding antiquark. It is quite evident from the theoretical point of view (and confirmed experimentally) that hadrons with open flavour should exist. As to mesons containing at least one heavy quark, there are twelve essentially different combinations. They are

$$(c\bar{u},c\bar{d},c\bar{s}),(b\bar{u},b\bar{d},b\bar{s}),(t\bar{u},t\bar{d},t\bar{s}),(c\bar{b},c\bar{t},b\bar{t}). \qquad (3.1)$$

In the latter three cases, where we deal with a bound state of two heavy quarks, the theory of the quarkonium state is not essentially different from that of the $Q\bar{Q}$ state. Any question referring to $c\bar{b}$ or $c\bar{t}$ gets a solution to the extent it is solved for $c\bar{c}$, and one learns practically nothing new. Of course, the estimates are quite different numerically, and if the reader is interested in some particular quantity, he/she should repeat the calculations, using one of the methods described above (in chapter 2). We shall not dwell on this subject, it is sufficient to remark that the $b\bar{t}$ system is, probably, very interesting from the point of view of electroweak effects.

On the contrary, analysis of $Q\bar{q}$ type mesons can yield a new important information on the structure of the QCD vacuum. In the hierarchy of hadrons, these mesons occupy an intermediate position between old traditional hadrons and heavy-quark systems like charmonium and bottonium. Their structure is simpler than that of light hadrons, since the heavy quark plays the role of a static center, so that some problems get trivial solutions. However, they probe some effects which are inaccessible (or almost inaccessible) in charmonium and bottonium; we mean, for instance, the quark condensate in the QCD vacuum.

Theoretical description of the $Q\bar{q}$ systems has certain peculiarities. What are the problems which seem to be most interesting in connection with these mesons ?

First of all, the spectrum, splittings of the levels with $J^P = 0^\pm,\ 1^\pm$, etc. As we shall see below, the lightest mesons are pseudoscalar - the situation which we had already got used to. Thus the pseudoscalar mesons D and B decay only due to the weak interaction. Calculation of their total widths is a junction where weak decays of quarks and strong interaction effects, especially substantial for $c\bar{q}$, are interconnected. The asymptotic formulae ($m_Q \to \infty$) are simple, but the problem of the pre-asymptotic corrections is a very complicated one. The situation with the inclusive semileptonic decays seems to be considerably more clear. Finally, one more interesting aspect is the decays of the type $D(B) \to l\nu_l$ (where $l = e, \mu$ or τ). The QCD sum rules enable one to determine coupling constants for such decays, $f_D(f_B)$, to a rather good accuracy.

All these subjects will be discussed briefly below. It is worth noting that there is another field of research, that has appeared recently, has been developing rapidly and become a sort of industry, both in theory and experiment. We mean exclusive nonleptonic decays of D mesons (see Section 1.1 of Chapter 1, Subsection C). In principle, they give a rich information on quark and gluon dynamics. Unfortunately, adequate exposition of the issue would lead us far aside and we have to limit ourselves by a few fragmentary comments.

3.1 SPECTRUM

Masses of mesons with open charm and beauty were discussed within the sum rule method in Refs. 134, 135. Unlike the cases of charmonium and bottonium, the answer depends drastically on the quark condensate in vacuum

$$\langle vac | \bar{\psi}\psi \; | vac \rangle \neq 0 \; , \qquad (3.2)$$

where $\psi = u, d$, or s. The existence of the quark condensate was established in the sixtees[136], but until now it revealed itself in the pion physics only.

The graphs dominating in the current correlation function are presented in Figure 19, where as an example we have chosen the current with the quark contents $(c\bar{u})$. As usually, the cross on a quark or gluon line means that the corresponding field is vacuum, i.e. it forms the vacuum condensate.

In the limit $m_Q \to \infty$, the states differing only by the quark spin direction are degenerate. In other words, $m(J^{PC} = 0^{-+}) = m(J^{PC} = 1^{--})$ and $m(J^{PC} = 0^{++}) = m(J^{PC} = 1^{++})$. A spin dependence appears with m_Q decreasing. To give an idea on the magnitude of the effect, we quote the result of Reinders et al. referring to the $b\bar{q}$ system. According to Ref. 134, the splitting between the pseudoscalar and vector states amounts to about -50 MeV. A close number was obtained for the splitting between the scalar and the axial-vector*). Both predictions are still to be confirmed experimentally.

*) Analogous result, about 50 MeV, stems also from the potential model[29].

As for the $c\bar{q}$ system, here we have reliable data on the D and D* mesons[31]. The experimental value of the D*-D mass difference is about 140 MeV. Unfortunately, the complete analysis of the QCD sum rules is not yet performed for this case and this number, 140 MeV, can be compared only with the nonrelativistic model[137].

There is a complete unanimity in the QCD theory on the orbital excitations. In both works[134, 135] the authors have noted a strikingly large effect of the quark condensate which induces a large splitting between the different parity states, i.e. $\Delta m_0 = m(J^{PC} = 0^{-+}) - m(J^{PC} = 0^{++})$ and $\Delta m_1 = m(J^{PC} = 1^{--}) - m(J^{PC} = 1^{++})$. The results of Refs. 134, 135 are compiled in Table 14 (the numbers are somewhat rounded off). Technically, the methods are different - the nonrelativistic Borel summation in one case and the moment technique in the other case - but the predictions agree well within the theoretical ambiguity of order of 100 MeV. The mass difference between the positive and negative parity states is 0.8 GeV. Still larger splittings Δm_0 and Δm_1 should be expected for the $c\bar{q}$ system. Recall for comparison that the potential model yields only 0.5 GeV in this case. Therefore the experimental searches for a scalar partner to the D meson seem to be a very important task.

It should be emphasized once more that the analysis based on the QCD sum rules does not need model assumptions: the fundamental vacuum parameters are translated in the language of observable quantities.

3.2 COUPLINGS OF PURELY LEPTONIC DECAYS

Decays into an $l\nu$ pair are possible for mesons with the quark content $c\bar{s}$ (the F meson), $c\bar{d}$ (the D^+ meson), and $b\bar{u}$ (the B^- meson). In the second case, the decay is suppressed as compared to the first case because the Cabibbo angle is small, the ratio of the probabilities is proportional to $tg^2\,\theta_C$ (see below). As well known the transition of a pseudoscalar particle into a charged lepton and the left-handed neutrino is proportional to the lepton mass, by kinematical reasons, therefore most probable are the decays into a $\tau\nu$ pair. For example,

$$\frac{\Gamma(D^+ \to \tau^+\nu)}{\Gamma(D^+ \to \mu^+\nu)} = \left(\frac{m_\tau}{m_\mu}\right)^2 \left[\frac{1 - m_\tau^2/m_D^2}{1 - m_\mu^2/m_D^2}\right]^2 \approx 2.5 \qquad (3.3)$$

where the second factor reflects the difference of the phase spaces. In the F^+ and B^- cases the dominance of the $\tau\nu$ channel is absolute.

As to the branching ratio BR(D(B) $\to\tau\nu$), it is determined by the width of the $\tau\nu$ transition as well as by the total width of the decaying meson. At first, we shall consider the absolute value of the width Γ (D(B)$\to\tau\nu$).

It is convenient to introduce the coupling constants f_D, f_B, parametrizing the corresponding amplitudes by analogy to the familiar coupling f_π,

$$< D(B)|\bar{Q}\gamma_\mu\gamma_5 q|o> = -if_{D(B)}\, p_\mu \qquad (3.4)$$

Then we get for the branching ratios

$$BR(D^+ \to \tau^+\nu) = 2.5 \cdot 10^{-4}(f_D/f_\pi)^2 \quad ,$$
$$BR(F^+ \to \tau^+\nu) = 3.3 \cdot 10^{-2}(f_F/f_\pi)^2 \quad ; \qquad (3.5)$$

here we have used the value Γ_{tot} $(D^+) = 1.25 \cdot 10^{12}$ sec^{-1}, which is close to the experimental total width of the D^+ meson, see Section 1.1 of Chapter 1 (Subsection C), and have assumed that Γ_{tot} $(F^+) \sim \Gamma_{tot}$ (D^+) (see below). Let us also parenthetically note that investigating the secondary particle distribution in the cascade

$$F^+ \to \tau^+ + \nu_\tau$$
$$\qquad \hookrightarrow \pi^+ \;(\rho^+) + \bar{\nu}_\tau \qquad (3.6)$$

provides, in principle, with a limit on the ν_τ mass, see Ref.138 .

The key question is: "how far are the couplings f_D and f_B from f_π ?" ($f_\pi \sim 133$ MeV; the couplings f_D and f_F are equal to each other up to O(10%)). One can answer this question by analysing the two-point functions induced by the current $\bar{Q}\gamma_\mu\gamma_5 q$ in the framework of the sum rule method. We will not dwell on the details - the procedure is quite standard - but quote directly the result of Ref. 135:

$$f_D \sim 230 \text{ MeV} \quad , \quad f_B \sim 140 \text{ MeV} \qquad (3.7)$$

Similar sum rules were considered in Ref. 134, where a somewhat larger value of f_B was obtained. This fact is, perhaps, due to an underestimation of the continuum whose threshold was pushed too high in that work[134]. In any case, these numbers are much less than those proposed some time ago in the course of theoretical speculations aimed to explain the "D^+/D^0 anomaly". As we have

already mentioned in Chapter 1 there is no need now for such speculations.

The result given in (3.7) is in good agreement with the phenomenological trend. Indeed, $f_D/f_\pi \sim 1.7$. On the other hand, it is known from experiment that making one of the quarks heavier, increasing its mass from 0 up to 150 MeV (we mean the s quark), we increase the coupling by a quarter,

$$(f_K/f_\pi)_{exp} \sim 1.27 \qquad (3.8)$$

It is instructive to compare (3.7) with the results based on other principles. Say, predictions[139] of the naive bag model are several times larger. We believe, of course, that future experiments will select the adequate scheme.

Assuming that $f_D \sim f_F \sim 230$ MeV, one gets

$$BR(D^+ \to \tau\nu) \sim 7.5 \cdot 10^{-4}, \quad BR(F^+ \to \tau^+\nu) \sim 0.1 \; .$$

3.3 TOTAL WIDTHS

If $m_Q \to \infty$, and we sum over all decay modes, the parton model is quite adequate. The inclusive decay rate of the $Q\bar{q}$ state is determined just by a probability of the weak transition of the Q quark into three light fermions, while the light antiquark $\bar{q}$ plays the role of a passive spectator. The process is depicted in Figure 5, where $c \to s u\bar{d}$, $s\mu^+\nu$, $se^+\nu$, and the Cabibbo suppressed channels are neglected.

Virtual hard gluons (Fig. 20) enhance nonleptonic modes slightly, but the effect is small even for $m_c = 1.5$ GeV, about 10% in the probability, and it falls further with the quark mass increasing. What is most important, the logarithmic corrections due to hard gluons are investigated well and do not bear any surprise (see e.g. the book[7]).

If the logarithmic renormalization effects are neglected at all, the c-quark life-time is obtained from the muon life-time by a simple rescaling

$$\tau_c = \frac{1}{5}\left(\frac{m_\mu}{m_c}\right)^5 \tau_\mu \qquad (3.9)$$

where the factor 1/5 is due to the fact that the number of the allowed channels is 5 ($se\nu$, $s\mu\nu$, $su_i\bar{d}_i$; i = 1, 2, 3 is the colour subscript). Note that the corresponding value of the branching ratio is BR($D \to e\nu X$) ~ 0.2 in good agreement with the data on D^+.

What quark mass is to be substituted in eq. (3.9): the current quark mass or that referring to constituent quarks ? Or, perhaps, one must take the D-meson mass ? In order to give a correct answer to this question one should know the pre-asymptotic non-perturbative corrections. In the asymptotics, $m_c \to \infty$, the difference between the current and constituent c-quark masses, and the D-meson, would not be essential. In the real world the numbers are: 1.35, 1.55, and 1.86 GeV, respectively. The scattering of the estimates for τ_c in the extremal cases amounts to a factor $(1.86/1.35)^5 \sim 5$. Even a larger ambiguity would emerge if one tried to account for the mass of the s quark in the final state. Say, taking the constituent quark with $m_s \sim 500$ MeV, one would reduce the available phase space by one half.

All the above arguments are presented only in order to demonstrate an acute need for an understanding of the pre-asymptotic non-perturbative effects, even a very crude one. It is just these effects that convert the current mass figuring at short distances into the constituent mass, and result in substantial deviations from the parton model. Unfortunately, consistent analysis of such effects, based on QCD, is still absent. Below we shall consider in brief one of the corrections (the interference one), which is formally suppressed by powers of $1/m_c$, but actually amounts to ~ 0.5 in the D-meson decays, because of large numerical factors. Certain corrections are known surely to exist and, perhaps, are not small. But they are absolutely beyond the reach of theorists at present.

The situation in the theory, unsatisfactory as it is in this aspect, inspired some theorists to renounce the asymptotical formulae at all, at least in the case of D-mesons, and to calculate Γ_{tot} summing up all the available two-particle and quasi-two-particle decay modes[140]. The theoretical result is in reasonable agreement with the experimental life-time; and, what is more important, $\Gamma_{tot}(D^+)$ and $\Gamma_{tot}(D^0)$ turned out to be approximately equal. Further details can be found in the original paper[140].

Such a radical step as the renouncement of the quark-parton formulae for D-mesons seems to be unnecessary, though. Without a consistent theoretical apparatus at hand, one can nevertheless rely upon one's intuition, which prompts that it is the constituent c-quark mass, $m_c = 1.55$ GeV, that figures in Eq. (3.9), while the strange quark mass may be neglected in the final state. Then

$$\Gamma^{-1}_{tot}(D^+) \sim \Gamma^{-1}_{tot}(D^0) \sim 6.5 \cdot 10^{-13} \text{ sec.} , \qquad (3.10)$$

in excellent agreement with the latest data, cf. Section 1.1, and Ref. 26. Encouraged by this success, we expect that the F-meson life-time is also close to this value, i.e. $6.5 \cdot 10^{-13}$ sec.*) It seems that the non-perturbative corrections, having different phases, are subject to a "destructive interference", and their total effect reduces, mainly, to a renormalization of m_c.

In the B-meson case the question which mass is to be substituted into the expression (3.9) is unimportant at all, since the ambiguity is relatively small. On the other hand, the information on the quark mixing angles is yet far from being complete (see Section 4.1). As it was discussed in Section 1.2, Subsection B, experiment indicates a dominance of the b-c, not b-u, transition. The branching ratio of the semileptonic decay is $BR(B \to e\nu X) \sim (15\text{-}17)$ %, depending on the assumptions on masses m_c and m_b[67, 141].

3.4 PRE-ASYMPTOTICAL EFFECTS

As was mentioned above, the life-time of the $Q\bar{q}$ meson coincides with the Q-quark life-time in the limit $m_Q \to \infty$ (Figures 5, 11, 20). There is a lot of corrections of various nature vanishing as powers of $1/m_Q$. Examples are shown in Figure 21. Some of the graphs depend on the spectator flavour (a, b). It is this contribution that is responsible for difference between the life-times of D^+, D^0, F, etc. For other graphs (c, d, e) the flavour of the spectator is inessential. As usual, the cross on the lines means the interaction with the vacuum fields. The graphs d and e describe the effect of the quark mass growth due to the gluon and quark condensates - in the intuitive language, a conversion of the current mass into the constituent one.

In this section we will concentrate on a very simple effect due to the Pauli principle. Turn back to the graph of Figure 5(a) and assume, first, that we deal with the D^+ meson. Then integrating over the phase space of the newly born $\bar{d}$ quark we find ourselves, inevitably, in the domain occupied by the spectator $\bar{d}$-quark. The Pauli principle forbids two identical quarks to occupy the same position in the phase space. Nothing of the kind takes

*) Since we believe that the F-meson non-leptonic decays are determined by diagram 5a, the final state in these decays necessarily contains a pair of $s\bar{s}$, and, hence, is enriched by η, $K\bar{K}$, ...

place in the D^0 case, where the spectator quark is $\bar{u}$. Of course, the Pauli principle is effective only in a limited region of the phase space, where the $\bar{d}$-quark momenta are approximately identical, while the total phase space increases rapidly with m_Q. However, for the real D-meson decays this "limited" region occupies a considerable part of the phase-space volume.

The interference contribution, in terms of the Feynman graphs, is depicted in Figure 22*).

Omitting the calculational details, we give here the answer, obtained by M. Voloshin and one of the authors (M.A.S.) a few years ago:

$$\Gamma_{hadr}(D^+) = \frac{1}{M_D} < D^+ | \frac{G_F^2 m_c^5}{64\pi^3} \, {}^1\!/_2 \, \bar{c}\,(1+\gamma_5)c$$
$$- \frac{G_F^2}{\pi} m_c^2 \,(C_+^2 + C_-^2)\,(\bar{c}_i\gamma_\mu(1+\gamma_5)d_j)\,(\bar{d}_j\gamma_\mu(1+\gamma_5)c_i) \qquad (3.11)$$
$$+ \frac{1}{2}\,(C_+^2 - C_-^2)\,(\bar{c}\gamma_\mu(1+\gamma_5)d)\,(\bar{d}\gamma_\mu(1+\gamma_5)c) \quad | D^+ >$$

where $C_\pm$ are known coefficients, describing renormalization of the operators $(\bar{c}s)_L(\bar{d}u)_L \pm (\bar{c}u)_L(\bar{d}s)_L$ due to hard gluons (see e.g. the book[7]).

Let us suppose for a moment that the flavour degrees of freedom of quarks are absent, and $C_+ = C_- = 1$. Taking into account that

$$<D^+|\bar{c}c|D^+> \sim 2\, m_D$$

$$<D^+|\bar{c}\gamma_\mu\gamma_5 d|o> = -\, i f_D p_\mu , \quad <o|\bar{d}\gamma_\mu\gamma_5 c|D^+> = i f_D p_\mu$$

we would conclude that the interference term decreases the decay probability, in complete agreement with the Pauli principle,

$$\{\,\Gamma_{hadr}\,(D^+)\}_{no\ color} \sim \frac{G_F^2 m_c^5}{64\pi^3} - \frac{G_F^2}{4\pi}\, m_c^2\, \frac{1}{m_D} \qquad (3.12)$$
$$\times \;\; <D^+|\bar{c}\gamma_\mu\gamma_5 d)\,(\bar{c}\gamma_\mu\gamma_5 d)^+ \,|\, D^+ > \approx$$

*) A consideration of the interference effects close in spirit was given in Ref. 142, but we do not agree with the final results obtained in these works.

$$\approx \frac{G_F^2 m_c^5}{64\pi^3} - \frac{G_F^2}{4\pi}\, m_c^2\, m_D\, f_D^2 \quad .$$

It turns out that, with the colour indices switched on, the answer is no longer unambiguous. The reason is that now two $\bar{d}$ quarks having identical momenta can be in different colour states, so that the interference may be constructive, as well as destructive. One should have in mind also the fact that the coefficients $C_\pm$ are not unity, namely, $C_+^2 \approx 1/\sqrt{2}$, $C_-^2 \approx 2$ (see Ref. 7). As to the magnitude of the effect, it depends on the value of the matrix elements of the four-fermion operators over the D meson state.

A naive factorization (i.e. saturation by the vacuum in the intermediate state) is, probably, not too good, quantitatively, since, according to Ref. 143, the non-factorizable contribution (Figure 23) has the same order of magnitude as the factorizable one.

If, for the purpose of orientation, one makes use of factorization, the correction to $\Gamma_{hadr}\,(D^+)$ turns out to be positive (the interference is constructive !),

$$\frac{\Delta\Gamma}{\Gamma_{parton}} = 16\pi^2\, \frac{f_D^2}{m_c^2}\, \{\frac{1}{2}\,(C_-^2 - C_+^2) - \frac{1}{6}\,(C_-^2 + C_+^2)\} \approx \frac{16\pi^2}{3}\, \frac{f_D^2}{m_c^2} \qquad (3.13)$$

In other words, this effect tends to make $\tau(D^+) < \tau\,(D^-)$, which seems to be welcome by the latest SLAC data, Table 5. The factor f_D^2/m_c^2 vanishes for $m_c \to \infty$. In the real world it amounts to $\sim 10^{-2}$, but this suppression is compensated by a large numerical factor, $(16\pi^2/3) \sim 50$. Where do such large numbers come from ? The reason is quite transparent: the parton mechanism corresponds to the three-particle decay of the quark, the interference mechanism corresponds to the two-particle decay. The ratio of the (dimensionless) phase space volumes is of order of $4\pi^2$. Due to the same reason the annihilation graph (Figure 21a) is enhanced, generally speaking. (Though it is forbidden by the chirality in the case of pseudoscalar mesons.)

To summarize, the natural scale of the pre-asymptotical power corrections, at least some of them, is

$$4\pi^2\, f_Q^2/m_Q^2 \sim \begin{cases} 0.5 \text{ for F, D mesons,} \\ 0.05 \text{ for B mesons} \end{cases} .$$

In decays of $(c\bar{q})$ states one may, in principle, expect considerable deviations from the parton-model predictions ($\lesssim$ 100%). In the decays of $(b\bar{q})$ and especially of $(t\bar{q})$ states, the parton picture must work with a good accuracy.

3.5 EXCLUSIVE WEAK DECAYS

Both theorists and experimentalists are concerned, with a rare exception, by the D-meson decays. This subject is in a very intricate situation. On one hand, a large number of particular modes has been observed experimentally, but statistical and systematical errors of measurements are still substantial (see Ref. 31). On the other hand, theoretical understanding, unfortunately, is not up to the mark either. A considerable number of models has been proposed[144, 145], but all the models contain ambiguities and the connection with the basic principles is not always clear. A review of the models was given in Ref. 46.

In order to show a nontrivial character of the problems, which present, in essence, puzzles of the quark-gluon dynamics, we shall discuss here a single example - the ratio of the widths

$$\frac{\Gamma(D^0 \to \bar{K}^0 \pi^0)}{\Gamma(D^0 \to K^- \pi^+)} \quad . \tag{3.14}$$

If we accept the naive quark model, the decay amplitudes are described by the graphs of Figure 24. Then we get for the ratio of the widths

$$\frac{\Gamma \bar{K}^0 \pi^0}{\Gamma K^- \pi^+} = \frac{1}{2} \cdot \frac{1}{9} \frac{(2C_+ - C_-)^2}{(\frac{2}{3}C_+ + C_-)^2} \tag{3.15}$$

where the factor 1/2 is due to the isotopic symmetry, and 1/9 is an effect of the colour symmetry (the $u\bar{d}$ pair, converting into π^+, is in the colourless state from the very beginning, while to select this state in the $\bar{K}^0\pi^0$ decay one has to loose a factor of 3 in the amplitude). As to the coefficients C_+, C_-, they are related to hard gluons and are given in Ref. 7; the result is $(2C_+ - C_-)^2/(\frac{2}{3}C_+ + C_-)^2 \approx 2 \cdot 10^{-2}$. Thus, according to the naive quark model, the ratio $\Gamma(\bar{K}^0\pi^0)/\Gamma(K^-\pi^+) \sim 10^{-3}$. The experimental data show that $BR(D^0 \to \bar{K}^0\pi^0)$ and $BR(D^0 \to K^-\pi^+)$ and $BR(D^0 \to K^-\pi^+)$ are of the same order of magnitude. The contrast with the theoretical prediction is striking. Probably one has to take into account the graphs of Figure 25, which are enhanced numerically and do improve the situation, according to the estimates presented in Ref. 144.

If the reliable theory for the exclusive weak decays of the charmed particles is still absent, one may wonder what the chances are to succeed in this question in the future. Indeed, we know a number of inveterate problems in the field of the traditional strong interactions which are not solved up to now. Nevertheless, we think that a progress in understanding the exclusive D-decays is quite probable. Our optimism is based on the fact that we have a large parameter at hand, the c-quark mass; exploiting this parameter skillfully one will be able to improve the theoretical description considerably.

4. HEAVY QUARKS AND WEAK INTERACTIONS

Heavy quark decays are a beautiful testing ground for investigation of the weak interaction structure. Not only they enable one to study the standard six-quark model and get information on the quark mixing angles, but they also shed light on properties of such exotic objects as the Higgs bosons and the axion.

In this Chapter we shall discuss in brief the quark mixing parameters. Notice that there exists a recent review[5] devoted to the issue, where the reader will find a more detailed discussion and an exhaustive list of references. We shall consider also the decays of heavy hadrons with production of the Higgs bosons and axions in the final state. Useful information on the subject is contained in Refs. 10, 146, 147. Finally, a special subsection is devoted to weak neutral currents of heavy quarks. Practically, we shall leave aside such interesting questions as the CP-invariance violation and mixing of heavy neutral mesons, refering the reader to other sources[7, 5].

4.1 PHENOMENOLOGY OF THE QUARK MIXING

Discovery of the b quark (and the third charged lepton τ) has led to a natural replacement of the Glashow - Iliopoulos - Maiani four-quark model[148] by the Kobayashi - Maskawa (KM) six-quark model[149] with three left-handed quark doublets,

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L , \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \; ; \quad q_L = \frac{1}{2}(1+\gamma_5)q \quad , \tag{4.1}$$

and the right-handed singlets,

$$u_R, \; d_R, \; c_R, \; s_R, \; t_R, \; b_R \; ; \quad q_R = \frac{1}{2}(1-\gamma_5)q \quad . \tag{4.2}$$

Here t is the sixth, not yet discovered quark; the present lower bound for its mass is $m_t > 18.3$ GeV [20, 21].

The general form of the weak charged current of the quarks is

$$j_\mu = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad (4.3)$$

where U is a unitary 3 x 3 matrix ($UU^+ = 1$). The Kobayashi-Maskawa parametrization[149] is often used for the matrix U, in terms of three Euler angles θ_1, θ_2, θ_3, describing the rotations in the flavour space, and a phase δ, related to the CP-parity violation*),

U =	d	s	b
u	c_1	s_1c_3	s_1s_3
c	$-s_1c_2$	$c_1c_2c_3-e^{i\delta}\ s_2s_3$	$c_1c_2c_3 + e^{i\delta}\ s_2c_3$
t	s_1s_2	$-c_1s_2c_3-e^{i\delta}\ c_2s_3$	$-c_1s_2s_3 + e^{i\delta}\ c_2c_3$

(4.4)

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$.

According to Eqs. (4.3), (4.4), the experimental data on various weak transitions allow one to determine the matrix elements U_{ik}.

1) $\underline{U_{ud}}$. The only angle known completely from experiment is θ_1. Actually, since $\overline{U_{ud}} = c_1$, the angle θ_1 is just the Cabibbo angle θ_C; its value is measured in the strangeness conserving β decays,

$$c_1 \approx 0.9737 \pm 0.0025;\ s_1 = 0.2270 {}^{+0.0104}_{-0.0110} \quad . \qquad (4.5)$$

2) $\underline{U_{us}}$. It is not difficult to see that in the limit $\theta_2, \theta_3 \to 0$, the matrix U connects the (d,s) quarks with (u,c), while the b quark is connected only with the

*) The choice of the phase sign here coincides with that in the book[7]. Recall also that the CP violation cannot be introduced in a natural way within the four-quark scheme in the minimal SU(2) x U(1) model with a single Higgs doublet.

t quark. Therefore the b-quark decays, discussed in Chapters 1,3 proceed only if $s_3 \neq 0$ and/or $s_2 \neq 0$. Bounds on the angle θ_3 emerge from the data on semi-leptonic kaon and hyperon decays, because $U_{us} = s_1c_3$: $|s_1c_3| \sim 0.219 \pm 0.002$. With the known value of θ_1, one gets, hence, a value of c_3 close to 1. Accounting for all uncertainties[46, 150], one has

$$|\sin\theta_3| = 0.28^{+0.21}_{-0.28} \quad . \qquad (4.6)$$

3) $\underline{U_{ub}}$. The $b \to uW^-$ transition is determined by the matrix element $U_{ub} = s_1s_3$. From the unitarity condition

$$|U_{ud}|^2 + |U_{us}|^2 + |U_{ub}|^2 = 1 \qquad (4.7)$$

and the measured values of U_{ud},$|U_{us}|$ it follows that the matrix element U_{ub} is small,

$$|U_{ub}| = 0.06 {}^{+0.03}_{-0.06} \quad , \qquad (4.7a)$$

in complete agreement with the set of data on the B-meson decays (see Section 1.2.). As mentioned in Chapter 1, the present-day experiment does not yet provide with a more accurate information on the fraction of the $b \to uW^-$ decays. If $\theta_3 = 0$, the Cabibbo universality would be exact, and the phase factor $\exp(i\delta)$ might be eliminated by a redefinition of the b-quark phase. It is noteworthy that obtaining stricter bounds on θ_3 is of great importance, from the viewpoint of the CP-violation problem. In particular, if the experimental data indicated $s_3 \ll s_1 \sim s_2$, the phase δ might be of order 1. If, however, $s_3 \sim s_2 \sim s_1$, then $\delta \sim 10^{-2}$.

4) $\underline{U_{cd}}$. The matrix element $U_{cd} = s_1c_2$ can be extracted more or less accurately from analysis of the K_L-K_S system. (The virtual c quark contributes to the K^0-$\bar{K}^0$ transition.) In Ref. 5 it was found that

$$0.19 \leq |U_{cd}| \leq 0.23 \qquad (4.8)$$

Practically the same limit on $|U_{cd}|$ stems from the data on charm production in neutrino reactions. To be more exact, we mean the cross sections' difference

$$\sigma(\nu_\mu(d+s) \to \mu^- cX) - \sigma(\bar{\nu}_\mu(\bar{d} + \bar{s}) \to \mu^+ \bar{c}X) \quad .$$

Comparing eqs. (4.5) and (4.8) we see that c_2 is close to unity and the corresponding angle is not large. Quantitatively,

$$|s_2| < 0.5 \quad .$$

5) $\underline{U_{cs}}$. The most straightforward way of measuring the matrix element $U_{cs} = (c_1c_2c_3 - e^{i\delta} s_2s_3)$ is the decays $D \to K^* e\nu_e$, $D \to Ke\nu_e$. One should keep in mind that until the t-quark discovery these decays are the best source of the experimental information on the angle θ_2. Unfortunately, the experimental situation with the semileptonic exclusive D meson decays is not yet established entirely. Still one can use the D^+ data which were not revised essentially. The analysis of the electron spectrum in the $D^+ \to e^+X$ decay shows that roughly speaking half of the events in $D^+ \to e^+X$ refer to the channel $D^+ \to \bar{K}^0 e^+\nu_e$. This implies that

$$\Gamma(D^+ \to \bar{K}^0 e^+\nu_e) \sim \frac{1}{2} \Gamma(D^+ \to e^+X) = (1 \pm 0.5)\ 10^{11}\ \text{sec}^{-1} \quad .$$

On the other hand, theoretically

$$\Gamma(D^+ \to \bar{K}^0 e^+\nu_e) \sim 1.5 \cdot 10^{11}\ \text{sec}^{-1}\ |f_+^{D \to K}(o)|^2 |U_{cs}|^2 \quad ,$$

and, if the form factor $f_+^{D \to K}(o)$, describing the $D \to K$ transition induced by the vector current, is close to unity, then

$$|U_{cs}| = 0.8 \pm 0.2$$

Actually, one may think that $f_+^{D \to K}(o) \sim 0.5$, which automatically entails

$$|U_{cs}| \sim 1 \quad .$$

The analysis of the $K_L - K_S$ system yields[5] a close constraint, $0.8 < |U_{cs}| < 1.0$.

6) $\underline{U_{cb}}$. As may be inferred from the discussion in Section 1.2 (Subsection B) all available data on the B-meson decays confirm the hypothesis

$$|U_{cb}/U_{ub}| >> 1 \quad ,$$

see also Eq. (1.13). Depending on assumptions on the phase δ, the available information on the matrix elements U_{cb}, U_{cs}, U_{cd} can be summarized in terms of combined bounds on the angles θ_2 and θ_3 (see e.g. Ref. 5).

As we have mentioned already in Section 1.2 (Subsection B) the existing bounds for the B-meson life-time (Eq. (1.16)) lead to a combined lower bound for the quark mixing angles. Indeed, for instance under the assumption of the free b-quark decay (Figure 11), and accounting for the phase space effects, we have

$$\tau_B = \frac{3.7 \cdot 10^{-15}\ \text{sec}}{3|U_{cb}|^2 + 2.5\ |U_{ub}|^2}$$

(see e.g. Refs.67,141). According to (1.13),(1.16) and unitarity the following inequality should be valid: $0.57 > |U_{cb}| > 0.05$. The dominance of the $b \to c$ transition, $b \to c + W^-_{\hookrightarrow \bar{u}d}$ must result in a high multiplicity of particles in the final state in the B-meson decay - the prediction nicely confirmed by experiment (cf. Section 1.2). This is just why it is difficult to study traditional exclusive channels in the B-meson decay. Note that about 1/5 of all hadronic events corresponds to the transition $b \to c + W^- \to c + \bar{c}s$, with the $c\bar{c}$-system mass, predominantly, less than $2M_D$, so that this state converts into charmonium (see Figure 26). Therefore one should expect a rather high branching ratio for the transition $B \to J/\Psi + X$[151] : BR $(B \to J/\Psi + X) \cong (3\text{-}1)\%$. The higher estimate, 3%, assumes actually that the formation of the colourless $c\bar{c}$ bound state proceeds with the unit probability. More realistic assumptions lead to $BR(B \to J/\Psi + X) \gtrsim 1\%$.

Experimental observation of this decay with the 1% branching ratio would be an additional argument in favor of the b-c transition dominance. Of great interest here is investigation of exclusive channels, since according to Ref. 151, the typical final state is $J/\Psi\ K\pi$ and the number of particles is not large.

The beautiful baryon Λ_b = bud has, probably, also an appreciable (of order of a few percent) decay channel of the type $\Lambda_b \to J/\Psi + pK(\Lambda\pi)$.

7) $\underline{U_{td(s,\ b)}}$. It goes without saying that a direct experimental information on these matrix elements will be available only after the discovery of the t quark. The $t \to b$ transition must be dominant. The matrix element $U_{td} = s_1s_2$ is of particular interest, as it shows how small the angle θ_2 is. To find U_{td} one should investigate experimentally the transition $t \to e^+\nu_e +$ pions.

An important bound on the angle θ_2 stems from a theoretical analysis of the t-quark contribution to the $K^0 \to \bar{K}^0$ transitions, see Figure 27. To avoid a too large t-quark contribution (which is proportional to $m_t^2 s_2^4$) to the $K_L^0 - K_S^0$ mass difference, as compared with the c-quark contribution, one must suppose that $tg\ \theta_2 \lesssim (m_c/m_t)^{1/2} \lesssim 0.3$. On the other hand, if $m_c = 1.35$ GeV the t-quark contribution is needed in order to ensure the correct value of $\Delta M(K_L K_S)$; this means that the angle θ_2 cannot be too small. The conclusion is confirmed also by analysis of the CP-symmetry violation in the K^0-$\bar{K}^0$ system (see e.g. Ref. 7). It follows from unitarity: $|U_{td}| < 0.13$, $|U_{ts}| < 0.56$; $0.99 > |U_{tb}| > 0.82$.

A parametrization of the mixing matrix, different from (4.4), is also often used. Introducing new three angles θ, β, γ ($0 \le \theta,\ \beta \le \frac{1}{2}\pi,\ -\frac{1}{2}\pi \le \gamma$) and a new phase $\delta' \le \frac{1}{2}\pi$ we have[153]:

$$U = \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -c_\gamma c_\theta s_\beta e^{i\delta'} & c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{-i\delta} & -c_\gamma s_\beta s_\theta - s_\gamma c_\theta e^{-i\delta'} & c_\gamma c_\beta \end{pmatrix} \qquad (4.9)$$

Here $c_{\theta,\beta,\gamma} = \cos(\theta, \beta, \gamma)$, $s_{\theta,\beta,\gamma} = \sin(\theta, \beta, \gamma)$.

The angle θ coincides with the Cabibbo angle θ_C, but for $\beta \neq 0$ the Cabibbo universality is broken:

$$tg\theta = U_{us}/U_{ud};\ |U_{us}|^2 + |U_{ud}|^2 = \cos^2\beta < 1 .$$

This parametrization is especially appropriate for description of weak b-quark transitions, since the angles β and γ are related directly to its decays,

$$\sin\beta = U_{ub}\ ;\ |\sin\gamma\ \cos\beta| = |U_{cb}|\ ;\ tg\gamma = |U_{cb}/U_{tb}| . \qquad (4.10)$$

Because the violation of the Cabibbo universality is small (cf. (4.7), (4.7a)) the angle β must also be small: $\beta \le 6 \cdot 10^{-2} \sim \frac{1}{4}\theta$. Bounds on the angle γ arise, in particular, from the data on the K^0-$\bar{K}^0$ transitions. To illustrate this point, we present in Figure 28 a compilation of the available bounds[68, 20, 21] for the quantities $\sin\beta$ and $|\sin\gamma|$, resulting from the universality of the weak coupling, data on the $K_L^0 - K_S^0$ mass difference, and the upper bound for τ_B (see Eqs. (1.16) and (4.10)). The dashed areas are ruled out by the present experimental data, the lower left angle corresponds to $\tau_B < 1.4 \cdot 10^{-12}$ sec.[68].

4.2 WEAK NEUTRAL CURRENTS OF HEAVY QUARKS

In the standard Glashow - Weinberg - Salam model[9, 154], the weak neutral quark current

$$j_\mu^0 = \frac{1}{2} \sum_q (v_q \bar{q}\gamma_\mu q + a_q \bar{q}\gamma_\mu\gamma_5 q) \qquad (4.11)$$

has the following values of the vector and axial-vector couplings:

$$a_{u,c,t} = 1\ ,\quad v_{u,c,t} = 1 - \frac{8}{3}\sin^2\theta_W\ ;$$
$$a_{d,s,b} = -1 = a_e\ ,\quad v_{d,s,b} = -1 + \frac{4}{3}\sin^2\theta_W \qquad (4.12)$$

where θ_W is the Weinberg angle ($(\sin^2\theta_W)_{exp} = 0.229 \pm 0.009$).

The available experimental data on the J/Ψ meson yields in the neutrino-induced reactions (see Figure 29), as well as on the hadron production cross sections in the e^+e^- annihilation (see Figure 30), are in agreement with the assumption of the standard model according to which the $\bar{c}c$ and $\bar{u}u$ currents have identical structure. Besides, the data on the e^+e^- annihilation rule out anomalously large coupling constants in the $\bar{b}b$-current.

A detailed information on the heavy quark neutral currents can be extracted from e^+e^- collisions, if one investigates the distributions in the polar δ and azimuthal ϕ angles describing the jets generated by these quarks (the ϕ distribution is useful if the initial particles are polarized, see e.g. Refs. 67, 155). In contrast to the case of light quarks, these jets can be identified quite unambiguously, for instance, by detecting hard definite sign muons produced in decays of the corresponding leading heavy hadrons (the so called-method of tagged quarks[67, 155], see Section 5.1 of Chapter 5). The contributions from semileptonic decays of B and D mesons can be reliably separated by means of a selection in the muon transverse momentum.

In particular, measurement of the forward-backward asymmetry for charm and beauty is of great interest,

$$A_C^q = \frac{d\sigma_q(\theta) - d\sigma_q(\pi-\theta)}{d\sigma_q(\theta) + d\sigma_q(\pi-\theta)} \qquad (4.13)$$

From this quantity one can immediately determine the parameter a_q. For $W^2 \ll M_Z^2$ the quantity A_c^q is determined by the interference of the graphs of Figures 30a, b, and is equal to

$$A_c^q = \rho_c^q \frac{2\cos\theta}{1+\cos^2\theta} \qquad (4.14)$$

$$\rho_c^q = 0.11 \frac{a_e a_q}{Q_q} \left(\frac{W}{35\ \text{GeV}}\right)^2 \left(1-\frac{W^2}{M_Z^2}\right)^{-1} \qquad (4.15)$$

cf. Eqs. (6.2), (6.3).

According to Eq. (4.15), the asymmetry in $e^+e^- \to q\bar{q}$ is larger than in $e^+e^- \to \mu^+\mu^-$ by a factor of 1.5 (c quarks) or by a factor of 3 (b quarks). The asymmetry increases rapidly with energy and, as seen from calculations exploiting more accurate formulae, for W = 60 GeV the result is $\rho_c^b \approx -0.8$; the effect is decreasing for higher energies[67]. Note the possibility to determine the Z^0 mass via propagator effects in A_c^q.

The coupling constants v_q can be found in experiments with longitudinally polarized e^-, e^+ beams. For example, with the electron helicity λ, a P-odd correlation appears in the total cross section for the process $e^+e^- \to \bar{q}q$,

$$A_p^q = \frac{1}{\lambda} \frac{\sigma_q(\lambda) - \sigma_q}{\sigma_q} \approx - \frac{v_q}{a_q} \rho_c^q \qquad (4.16)$$

$(W^2 \ll M_Z^2)$. This correlation is especially substantial in the case of the b quark: for $W \approx 40$ GeV one has $A_p^b \approx 0.4$, while $A_p^c \approx 0.1$.

An elegant possibility for investigation of the interaction of the neutral axial electron current with the vector c and b quark current directly at the J/Ψ and Υ peaks was considered in Ref. 156. The dependence of the total hadronproduction cross section on the sign of the longitudinal polarization should result in a relative effect $A_p^b \approx 1.6 \cdot 10^{-2}$ in the Υ resonance, and $A_p^c \approx 4 \cdot 10^{-4}$ in the J/Ψ resonance. This effect is especially striking in the case of the t quark where A_p^t becomes of order unity (below the Z^0 mass).

Another possibility is to measure the longitudinal polarization of the final $\mu^+\mu^-$ $(\tau^+\tau^-)$ at the narrow resonance peak.

4.3 ELUSIVE SCALAR PARTICLE FACTORY

Decays of hadrons containing heavy quarks are one of the best sources of information on various exotic objects appearing in the theory. Such objects are, in particular, the Higgs bosons (see e.g. Refs. 10, 146) and the axion (e.g. Ref. 147) - their couplings to quarks are proportional to m_q.

In the minimal standard model of electroweak interaction (with a single doublet of Higgs particles)[154, 9] there is a single physical neutral boson H^0; in models containing several scalar multiplets there are also physical charged particles $H^{\pm}$. The experimental search for the scalar bosons H is one of the foremost problems in modern physics. However, it is no chance that these particles were called elusive. On one hand, the theory is not able yet to predict their masses and the detailed structure of the Higgs sector. On the other hand, the expected production cross sections for the Higgs bosons are, as a rule, quite small, and, moreover, it is very difficult to identify the final state.

Since the Higgs boson-quark coupling constant is of the order of $(G_F\sqrt{2})^{\frac{1}{2}} m_q$ in the standard model, the heavy quarkonia decay is an intensive source of the H^0 bosons with masses $m_H < 2m_Q$. In particular, for the vector quarkonium V_Q the $V_Q \to H^0 + \gamma$ decay probability is given by the following relation (see Fig. 31)[157]:

$$\frac{\Gamma(V_Q \to H^0\gamma)}{\Gamma(V_Q \to \mu^+\mu^-)} = \frac{G_F m_Q^2}{\sqrt{2}\pi\alpha} \left(1 - \frac{m_H^2}{M_V^2}\right) \qquad (4.17)$$

For example, in the Υ meson case this relation yields $(m_H^2/M_V^2 \ll 1)$ $BR(\Upsilon \to H^0\gamma) \approx 3 \cdot 10^{-4}$, so one may hope to find the $\Upsilon \to H^0\gamma$ decay by detecting monochromatic photons. If $m_H \gtrsim 4$ GeV, the main channels of the H boson decay are $H^0 \to \tau^+\tau^-$, $c\bar{c}$. Note also that in the standard model with a single Higgs doublet two requirements - the vacuum stability and applicability of the perturbation theory - imply that[158]

$$1\ \text{TeV} \gtrsim m_H \gtrsim 7\ \text{GeV} \quad .$$

In the models with several doublets lighter scalar particles may well exist. At present, the CB group is analysing the data on the $J/\psi \to \gamma\mu^+\mu^-$ decay[41] in order to obtain bounds on the cascade transition $J/\psi \to \gamma H^0 \to \gamma\mu^+\mu^+$. They assume that 400 MeV $\lesssim m_H \lesssim$ 3 GeV.

In the case of the t quark, the ratio (4.17) is essentially enhanced, and accounting for the existing bound $m_t > 18.3$ GeV it is not difficult to get $\Gamma(V_t \to H^0\gamma)/\Gamma(V_t \to \mu^+\mu^-) > 0.12$ (unless m_t is considerably higher than $\frac{1}{2}M_Z$).

Now we turn to the issue of the charged Higgs bosons $H^\pm$. If this particle existed and had a moderate mass, $m_H < (m_b - m_c)$, then the b quark would decay entirely by means of the semiweak transition $b \to H^- c$: $\Gamma(b \to H^- c)/\Gamma(b \to c\bar{u}d) \approx 6\pi^2/Gm_b^2 \sim 10^5$. If $m_{H^-} > 2$ GeV, the dominating decay channels would be $H^- \to \tau^- \tilde{\nu}_\tau$, $\bar{c}s$, and their experimental signatures are bright. As discussed in Section 1.2 (Subsection B), the data on the B-meson decays reject such a possibility. Moreover, analysis of data on the e^+e^- annihilation at energies $W \lesssim 30$ GeV rule out the existence of $H^\pm$ within the mass interval 5 - 15 GeV (provided H decays in the following way: $H^- \to \tau^- \tilde{\nu}_\tau$, $\bar{c}s$)[20, 21]. Therefore, if there is any hope to discover the $H^\pm$ bosons in heavy quark decays, it refers to t quarks, $t \to H^+ + b$.

Heavy quarkonia decays provide with very important information on such a theoretical invention as the axion (a), a new very light pseudoscalar particle ensuring the natural CP invariance of strong interactions (see e.g. Ref. 147). During the past few years, this object has been intensively searched for in experiments on proton and electron accelerators, reactors, as well as in decays of excited nuclei.

The standard axion theory[159,160] predicts that its coupling to u, c, t quarks is of order $(G\sqrt{2})^{1/2} m_q x$, while that to d, s, b quarks is of order $(G\sqrt{2})^{1/2} m_q x^{-1}$, where x is a standard parameter in the axion theory, which is equal to the ratio of the vacuum expectation values for two Higgs fields. Just as the decay $V_Q \to H^0 + \gamma$ the ratio of the $V_Q \to a + \gamma$ width to the leptonic width, say for the J/ψ meson, is (cf. (4.17)),

$$\Gamma\frac{(J/\psi \to a + \gamma)}{(J/\psi \to \mu^+\mu^-} = \frac{G_F m_c^2 x^2}{\sqrt{2}\,\pi\alpha} \qquad (4.18)$$

(the corresponding formula for the b quark is obtained by replacing the factor x^2 by x^{-2} and $m_c \to m_b$).

The CB group in a recent experiment[161] has searched for the J/ψ decay into a photon (with energy $E_\gamma \cong \frac{1}{2}m_{J/\psi}$) and a long-living non-interacting object having a low mass $m_a < 1$ GeV. (It may be the axion or another exotic object with similar properties.) Working with the statistics of about $2 \cdot 10^6$ J/ψ events, the authors have obtained an upper bound for the branching ratio of the decay

$$BR(J/\psi \to a + \gamma) < 1.4 \cdot 10^{-5} \text{ (90\% C.L.)} \qquad (4.19)$$

Comparing (4.18) and (4.19) ($BR(J/\psi \to \mu^+\mu^-) = (7\pm1)$ %), it is not difficult to get a bound on the parameter x, $x < 0.6$. Thus, we rule out immediately the value x=3, which seemed to be most preferable in the Feissner experiments[162], where the discovery of the axion was claimed.

Since $BR(\Upsilon \to a\gamma) \sim M_\Upsilon^2$ and the product $BR(\Upsilon \to a\gamma)BR(J/\Psi \to a\gamma)$ is independent of x and is unambiguously predicted within the standard Weinberg-Wilczek scheme it may be instructive to perform combined analysis of $J/\psi \to \gamma a$ and $\Upsilon \to \gamma a$ reactions. The data on monochromatic photons obtained by the LENA group ($\Upsilon \to a\gamma$) and the CUSB group ($\Upsilon'' \to a\gamma$) decisively rule out the existence of the standard axion[41]. Let us emphasize that in the axion searches the heavy quark decays have demonstrated an essentially higher effectivity than other methods of investigation. Moreover, the heavy quark decays, as the best testing ground for new axion models, will hardly lose its role in future.

5. HEAVY QUARKS AND THE PERTURBATION THEORY

Since the heavy quark physics is connected with short distances where asymptotic freedom is operative, many relevant problems look much simpler here than for light quarks. The large quark mass enables one to use the standard perturbation theory (PT); say, the leading log summation.(e.g. 11,163,164), gives a description of final states in various hard processes, or, in other words, the heavy quark fragmentation function. Combining the PT results with the parton model approach one can obtain a consistent picture of production and hadronization of heavy quarks in which the bremsstrahlung and confinement effects are accounted for simultaneously.

Besides, decays of heavy quarkonia are, in principle, the clearest source of the gluonic jets. The vector quarkonia can decay into three gluons, while the pseudoscalar and P-wave quarkonia with $J^{PC} = 2^{++}, 0^{++}$ may decay also into two gluons (see Figure 32). Investigation of heavy quarkonia decays provides, thus, with an important information on the gluon jet properties in particular, on specific features of the gluon hadronization. Determination of the gluon quantum numbers (spin, parity etc.) is also related to investigation of the gluon jets. One should keep in mind, however, that PT is not a self-consistent approximation, so that one should always remember about effects lying beyond the PT control. Experiments on heavy quark fragmentation reveal conditions under which confinement does not preclude coloured partons from appearing as individual objects with quite definite properties. In this way one learns whether the hadronization mechanism is sufficiently "soft" or not.

Let us finally note that since in QCD the intensity of soft gluon emission by a gluon is higher, by a factor 9/4, than that by a quark there appears a possibility to compare yields of various particles in decays of quarkonia and in e^+e^- annihilation (see e.g. Refs. 11, 164). This is a way of testing consequences of the conventional picture for the cascade multiple fission of gluons and the subsequent hadronization.

We will discuss here in brief the problem of the heavy hadron spectrum in e^+e^- annihilation, the tests of the gluon spin and parity, as well as the QCD-based ideas on yields of various hadrons in quarkonia decays.

5.1 DISTRIBUTION OF HEAVY HADRONS IN JETS

First of all, we shall discuss the peculiarities of the gluonic bremsstrahlung and hadronization due to a large quark mass[67, 164]. Suppose that $M_Q R \gg 1$, where R is the confinement radius, or, to be more exact, a quantity determined by the non-perturbative interaction of bremsstrahlung gluons with $k_\perp \sim R^{-1}$ ($\alpha_s(k^2 \sim R^{-2}) \sim 1$) with the vacuum light quark condensate: $R^{-1} \sim$ (250-300) MeV. Because the quark Q is very massive, the formation time for the bremsstrahlung with energy ω and transverse momentum $k_\perp$,

$$t_{rad} \cong E_Q z(1-z)/(k_\perp^2 + M_Q^2 z^2) \quad ,$$

is always parametrically shorter (for $z \sim 1$) than the time of the gluon hadronization, $t_{hadr} \cong \omega R^2$, corresponding to distances at which the interaction becomes strong. For energetic gluons the characteristic transverse momenta are $k_\perp \gtrsim M_Q$, and only parametrically soft gluons with $z \lesssim (M_Q R)^{-1} \ll 1$, $k_\perp \sim R^{-1}$ emitted intensively. Hence, we draw an important conclusion that the radiative energy loss by a heavy quark is small and is controlled by PT[155, 164]. Along this line we get an explanation of the spectacular effect of leading heavy quarks (and hadrons)[165] which is reminiscent (kinematically) of the leading baryon effect in the pp-interaction.

If the parameter $M_Q R$ is large enough, the spectra of hadrons containing heavy Q quarks - call them H_Q - should approximately coincide with the spectrum of the quark Q: $|x_Q - x_{H_Q}| \lesssim (M_Q R)^{-1}$ for $1 - x_{Q,H_Q} \gg (M_Q R)^{-1}$ ($x_{Q,H_Q} = 2E_{Q,H_Q}/W$). The coincidence should become even better after summation over all types of hadrons with the heavy quark Q when $|1 - x_{QH}| < (M_Q R)^{-1}$ the spectra should decrease as powers of $(1 - X_{HQ})$.

Since the distances essential for the hard gluon radiation by Q are determined by the quantity $M_Q^{-1} \ll R$, the Q spectrum for $(1-x_Q) > (M_Q R)^{-1}$ is described completely by PT and is infrared-stable.

The Q-quark inclusive distribution $\bar{D}_Q(x)$ in the process $e^+e^- \to Q(x)+\ldots$ is given in the leading log approximation (LLA) by the following expression:

$$\bar{D}_Q^{LLA}(x,W) = \int \frac{dj}{2\pi i}\, x^{-i} \exp\left\{ \Delta\xi \neq C_2 \left[3 + \frac{2}{j(j+1)} - 4\psi(j+1) - 4\gamma_E \right] \right\} \quad (5.1)$$

where $\psi = \Gamma'/\Gamma$, $\gamma_E = 0.577$ is the Euler constant,

$$C_2 = \frac{N^2-1}{2N} \quad ; \quad \Delta\xi = \xi(W^2) - \xi(M_Q^2) = \frac{1}{b} \ln \frac{\alpha_s(M_Q^2)}{\alpha_s(W^2)} \quad ; \quad (5.2)$$

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f = 9 \text{ (u, d, s, quarks)} \quad .$$

Here N_c is the number of colors (N = 3), N_f is the number of "unfrozen" quarks. The integration in Eq. (5.1) goes over a contour, parallel to the imaginary axis, to the right of all singularities, i.e. Re j > 0. The distribution (5.1) grows with x increasing. The magnitude of $\bar{D}_Q^{LLA}(x)$ can be found numerically for arbitrary $\Delta\xi$ and x. However, for real energies the value of $\Delta\xi$, say, for the b quark, is not large, and one can use simple interpolations, for instance

$$\bar{D}_Q(x) = N(\Delta\xi)\, \frac{1+x^2}{2}\, (1-x)^{-1+4C_2\Delta\xi} \quad (5.3)$$

where $N(\Delta\xi)$ is determined by normalization conditions. More accurate calculations enable one, in principle, to get logarithmic corrections to the LLA formula (5.1), as a series in powers of α_s.

The mean fraction of the energy carried away by the heavy quark is given in LLA by the following expression[164,166]:

$$\langle x_Q \rangle^{LLA} = \exp(-8C_2\Delta\xi/3) = \left(\frac{\alpha_s(M_Q^2)}{\alpha_s(W^2)}\right)^{-\frac{32}{81}} \quad (5.4)$$

which corresponds to the well-known results for deep inelastic scattering (the valence quark distributions)[157]. Applied to the c quark, this estimate means that its radiation loss, for energies available at present ($W \sim 30$-40 GeV), amounts only to a quarter of its momentum: $\langle x_c \rangle = 0.75$; the b quark spends its energy even more stingily: $\langle x_b \rangle = 0.85$. Experimental tests of this PT predictions would be very interesting. Note that the first data from MARK II (and CDHS, EMC groups at SPS) really indicate that the charm fragmentation function is fairly hard.

Properties of jets generated by heavy quarks can be investigated experimentally via the specific decay modes of the corresponding leading heavy hadrons, e.g. by detecting hard leptons from the semileptonic decay[155,67]. Analysis of the lepton spectra allows one not only to separate events with heavy quarks, but also to discriminate between contributions of different heavy quarks. Besides that, it is possible to discern the jets from a quark and a gluon, from a quark and antiquark. This fact opens the way to investigate apart from the strong interaction problems, say, the gluon properties[155], various manifestations of weak interactions of heavy quarks (see Section 4.2). A discrimination between the b and c quark events may be performed by using a selection in the transverse lepton momentum $p_\perp^l$ (with respect to the hadronic jet axis): under the condition $p_\perp^l > 1.2 - 1.5$ GeV one gets, mainly, the b-quark events[67]. It is possible to indicate other specific features of events with heavy quarks in e^+e^- annihilation. In particular, apart from a lepton one should observe in such events an appreciable energy loss (the energy is carried away by neutrinos) and an apparent disbalance of the transverse momenta.

5.2 TESTING THE GLUON SPIN AND PARITY

At present we have a number of PT methods for determination of the gluon spin and parity, and they were tried in analysis of experimental data (see e.g. Refs. 16, 11, 168). The data favor the hypothesis $J_g^{PC} = 1^{--}$, although with different degree of reliability. However, an interpretation of any experiment on investigation of quark and gluon properties requires some additional assumptions, concerning, in particular, the mechanism of parton hadronization. Therefore it is especially important to measure the parton quantum numbers in various experiments, in order to test various stages in the hierarchy of the hadronization hypotheses[11]. In this case confronting the results means testing the perturbative QCD predictions, as well as the concepts of the parton-hadron relation, i.e. the character of confinement.

In order to get an idea of how strongly this or that QCD prediction depends on particular features of the theory, it is commonly accepted to compare the prediction with those of other models, in particular assuming different quantum numbers for the gluon. One should keep in mind, of course, that QCD has no rivals, that such models are just auxiliary, since by no means they present versions of a consistent theory.

A) Decays of heavy quarkonia

The most reliable confirmation of the conventional gluon spin and parity is, evidently, the comparison of the heavy quarkonia decay widths, since it is based only on a minimal assumption of the completeness of parton states[168, 169]. In particular, the ratio of the direct hadronic decay widths (Γ^{dir}) for pseudoscalar ($^1S_0$) and vector ($^3S_1$) states,

$$r^S = \Gamma^{dir}(^1S_0)/\,\Gamma^{dir}(^3S_1) \quad , \qquad (5.5)$$

predicted by QCD is $r_{QCD} \sim 6\,(\alpha_s/\pi)^{-1}\,(1+O(\alpha_s))$ (see Figure 32). For instance, for the J/Ψ and η_c states we have $r^S_{QCD} \sim 10^2$, cf. Chapter 2. A close result is expected also for Υ and η_b.

If the gluons were pseudoscalar ($J_g^P = 0^-$), then the $^1S_0$ state could not decay into gg, and both states, $^1S_0$ and $^3S_1$, go into three gluons, in spite of different C parities. Respectively, the predicted difference in the decay widths is substantially less than that for $J_g^P = 1^{--}$. The estimate for J/Ψ and η_c would be[168] $r_P^S \sim 10$. For $J_g^P = 1^+$ the estimated width $\Gamma^{dir}(^1S_0)$ is considerably less than in the standard case $J_g^P = 1^-$, perhaps, it is even less than $\Gamma^{dir}(^3S_1)$. Finally, if gluons are scalars, $J_g^P = 0^+$, the $^3S_1$ state can decay, as before, into three gluons, while the number of gluons generated in the direct decay of $^1S_0$ is no less than four. In this case one should expect $\Gamma^{tot}_{\eta_c} < \Gamma^{tot}_{J/\Psi}$[11].

Thus the η_c/Ψ ratio observed experimentally, $r^S_{exp} = (3\pm1)\cdot 10^2$ (Table 13) is in perfect agreement with the standard QCD gluon, and all other possibilities are ruled out. The measurements of the widths for the S-wave $\bar{b}b$ states will give even more reliable test of the J_g^P assignment.

The comparison of the decay widths for the $^3P_0$ and $^3P_2$ states, proposed in Ref. 169, also requires only the completeness of the parton states.

B) $\Upsilon \to 3g$

In order to test the gluon spin in three-jet processes $e^+e^- \to q\bar{q}g$ and $\Upsilon \to 3g$[170-172, 63, 17], one needs more serious assumptions; first of all, it is necessary to assume that the hadrons "remember" the direction and properties of the parent partons (the so-called soft decolorization hypothesis).

In experiments at Υ, Υ' peaks the distribution of the so-called oblateness axis*) T with respect to the direction of the initial e^+e^- beams has been measured; it is proportional to $1+\alpha_T\cos^2\theta$, $(\alpha_T)_{exp} = 0.35 \pm 0.11$[62]. Let us denote by β the angle between a vector normal to the gluon emission plane and the oblateness axis. The β distribution has also been measured. It has the form $1+\alpha_N\cos^2\beta$, where $(\alpha_N)^{\Upsilon}_{exp} = -0.29 \pm 0.06$[62]. The results are in agreement with the QCD predictions[171]: $(\alpha_T)_{QCD} = 0.39$, $(\alpha_N)^{\Upsilon}_{QCD} = -0.33$, and rule out completely the assumptions $J^P_g = 0^-, 0^+$.

C) $J/\psi \to \gamma f \to \gamma\pi\pi$[173], $2^3S_1\ \pi^+\pi^- + (1^3S_1)$[116, 118]

Graphs describing these decays in the two-gluon approximation are given in Figure 33(a, b). These decays naturally occupy a lower position in the "hierarchy of hypotheses".

In particular, the masslessness of the intermediate gluons is usually assumed in calculations of the $J/\psi \to \gamma f$ decay characteristics. It is no surprise, therefore, that we observe no good agreement of the oversimplified QCD prediction[173] with the measurements of the f meson polarization state in the radiative decay[42] (see Table 2, footnote, $X_{th} = 0.76$, $y_{th} = 0.54$).

As it was shown in Refs. 116, 118 with the help of the multipole expansion for the gluon fields, the ratio of the cascade decay widths for ψ' and Υ',

$$k = \Gamma(\Upsilon' \to \Upsilon\pi\pi)/\ \Gamma(\psi' \to J/\psi\pi\pi) \quad , \qquad (5.7)$$

must depend essentially on the gluon spin: $k_{QCD} \sim 0.1$, for $J^P_g = 0^+$ $k \sim 1$. The point is that, unlike the scalar gluon variant, in QCD the process amplitude is proportional to $\langle r^2 \rangle$ (see Section 2.3 and Eq. (2.52)).

*) The axis T is close to the direction of the most energetic gluon jet, for details see Ref. 11.

The experimental result $k_{exp} = (8.5 \pm 6) \cdot 10^{-2}$ clearly favors the QCD prediction.

The fact that various methods mentioned above do agree between each other is very important, as it confirms our ideas of the confinement properties. In particular, the "soft decolorization" hypothesis gets an additional experimental support.

5.3 HADRON MULTIPLICITIES IN QUARKONIUM DECAYS

Experimental data on the multiple hadron production in e^+e^- annihilation at high energies[23, 174] provide with a good qualitative confirmation of spectacular predictions, specific for perturbative QCD, of the gluon cascade fission (e.g. (11, 164)): a sharp rise of the mean multiplicity of charged particles $\langle n_{ch} \rangle$ with increasing energy, a characteristic structure of the plateau, etc. The picture of the gluon fission is also in agreement with the observed increase of the baryon yields (p, $\bar{p}$ and Λ, $\bar{\Lambda}$) in hadronic jets[164]. The latter fact indicates, among other things, that the light glueball formation does not dominate in the fragmentation of gluons into hadrons (the gluonium proliferation in the final state would suppress the relative baryon yield if $m_{glueball} < 2.5$ GeV).

Within the double-log approximation of perturbative QCD, there is a general functional technique of calculating asymptotics of various characteristics referring to hadron production in jets, multiplicities, spectra correlation functions etc.[175]. Using the known gluon distributions in quarkonia decays, this method can be applied also to calculation of some characteristics of the multiple particle production in these decays if the relative emission angles of gluons are sufficiently large. However, if the gluon configurations in the $V_Q \to 3g$ decay are more or less symmetric, one can obtain simple formulae relating directly experimental characteristics for the direct quarkonium decay to the corresponding characteristics of e^+e^- annihilation at $W = \frac{2}{3}M_{V_Q}$; see Ref. 11.

In this way we supress theoretical uncertainties, say, those due to corrections to the double-log approximation. Of course, one should keep in mind that the resulting estimates (given below) may be rather crude, in particular, because the symmetrical three-jet configurations are actually not absolutely dominant in the V_Q decay.

Here we shall consider yields of various type hadrons h in the direct decays of the quarkonium V_Q. The mean multiplicities $\langle n_h(V_Q)\rangle$ can be related to the bremsstrahlung gluon contribution to the corresponding multiplicities in e^+e^- annihilation, $\langle n_h(e^+e^-)\rangle$, by means of the equality[11, 164],

$$\langle n_h(V_Q)\rangle = \frac{2}{3}\cdot\frac{9}{4}\,\Delta\langle n_h(e^+e^-, W=\tfrac{2}{3}M_{V_Q})\rangle + \langle n_h(J/\psi)\rangle \qquad (5.8)$$

where $\Delta\langle n_h\rangle$ is the growth of the multiplicity for e^+e^- annihilation with energy increasing from ~ 2 GeV up to W. The factor of 3/2 accounts for the replacement of two jets by three jets, and the factor of 9/4 is due to a higher probability of soft gluon emission by a gluon, as compared with the quark case (e.g. Ref. 176).

For the Υ meson Eq. (5.8) provides with a reasonable estimate of the charged particle multiplicity, $\langle n_{ch}(\Upsilon)\rangle = 8 - 10$. The estimate for the $t\bar{t}$ quarkonium (toponium) is $\langle n_{ch}(T)\rangle = 35 - 40$ for the mass range 40 - 50 GeV.

Since for moderate e^+e^- annihilation energies, $W \lesssim 3$ GeV, the baryon yields are low ($n_p(e^+e^-) = (2\pm1)\cdot 10^{-2}$ for $W \sim 3$ GeV), practically, all the observed multiplicity of the non-leading baryons can be attributed to the cascade fission of the PT gluons. Hence, replace $\Delta\langle n_p(e^+e^-)\rangle$ in Eq. (5.8) by $\langle n_{\bar{p}}(e^+e^-)\rangle$. One immediately predicts that the baryon multiplicity in the direct quarkonium decay must be considerably higher than that in the background[164]. It is remarkable that the situation is realized already for the J/ψ meson. For instance, $n_p(J/\psi) = (6.3 \pm 1.8)\cdot 10^{-2}$ [177], by a factor of 2 - 3 higher than off the resonance. This result can be considered as an argument in favour of the three-gluon nature of the direct J/ψ decays. Using the data on $p,\bar{p}$ and Λ, $\bar{\Lambda}$ yields in e^+e^- annihilation (e.g.[23]), one gets from Eq. (5.8) for the Υ resonance $\langle n_{p\bar{p}}(\Upsilon)\rangle = 0.74 \pm 0.08$ and $\langle n_{\Lambda\bar{\Lambda}}(\Upsilon)\rangle = 0.22 \pm 0.04$, in good agreement with experiment (see Table 6, and, especially, the DASP-2 data).

We would like to emphasize that the increase of multiplicities for different type hadrons is approximately one and the same since it is connected with the gluon fission. In the case of the Υ quarkonium, where the pre-asymptotic effects are small, one should expect an approximately equal baryon-to-meson yield ratio on and off the resonance.

6. TOPONIUM AS IT MIGHT BE

The superheavy quarkonium is, probably, a fascinating world where the roles of the weak, electromagnetic and strong interactions are interchanged as compared with those we have got used to. The weak interaction effects, negligible in charmonium and bottonium, may be considerable, or even dominating, in the toponium. From the theoretical point of view, the situation in $t\bar{t}$ is quite clear: the Coulombian description is valid within the limits determined by the theory; but the decay properties are peculiar, indeed. We will consider here in brief the expected properties of the T toponium, the 1^3S_1 state of the $t\bar{t}$ system; a more detailed exposition of the issue can be found, say, in Refs.86,87,141. As it was mentioned above, the toponium is expected to be a "gluon factory", where it will be easy to investigate exhaustively the gluon properties and the mechanism of their hadronization. Possibilities of studying weak interactions in the T particle decays are also unique (see Chapter 4).

The expected toponium properties depend essentially on its mass, M_T. Unfortunately, the existing theoretical predictions for this quantity are scattered in a very broad interval beginning slightly above the experimental lower bound (37.6 GeV) up to $M_T = 150$ GeV (see e.g. Ref. 178).

The main decays of the superheavy vector $Q\bar{Q}$ quarkonium can be described by the diagrams of Figure 34. Transitions involving the Higgs bosons like $Q\bar{Q} \to Z^0H$ may also exist, in principle (see Figure 31).

One should pay special attention to the transitions, described by the graphs of Figures 34h and 34i, corresponding to weak decays of one of the quarks constituting heavy quarkonium. In the case of the t quark, the graph, the graph of Figure 34h represents the transition

$$t\bar{t} \to t + \bar{b} + \text{leptons or hadrons} \quad . \qquad (6.1)$$

The produced quarks t and $\bar{b}$ may then form a new hadron ($t\bar{q}$) plus the B meson, or bound themselves in a superheavy hadron $t\bar{b}$. Thus there appears a real possibility to study new elements of the quark mixing matrix U (in particular, U_{tb}, see Eq. (4.4) and Section 4.1).

The contribution (6.1) to the total T-meson width becomes noticeable ($\gtrsim 5\%$) for $m_t \gtrsim 25$ GeV and grows further with the t-quark mass; if $M_Z \lesssim M_T \lesssim 2M_W$ this transition becomes dominant (see below). The decays can be reliably detected experimentally, say, by hard leptons from the $W^- \to e^- \bar{\nu}_e$ transition or from a semileptonic decay of the t quark. For $m_t > M_H$ the graph of Figure 34(a) becomes operative.

Let us dwell now on the graphs of Figures 34a-g. They all correspond to the $Q\bar{Q}$ annihilation at short distances (so they are called annihilation graphs) and are proportional to the quark wave function at the origin squared. Accounting for the γ and Z^0 exchanges (Figures 34a, b), the leptonic width $\Gamma(T \to e^+e^-)$ is determined by the following relation*) (cf. Eq. (2.17), Section 4.2 and Eq. (4.11))

$$\Gamma(T \to e^+e^-) = \Gamma_T^e \left[1 - \frac{2v_e v_t R_Z}{Q_t} + \frac{(v_e^2 + a_e^2) v_t^2 R_Z^2}{Q_t^2}\right] \qquad (6.2)$$

where

$$R_Z = \frac{G_F}{8\pi\alpha\sqrt{2}} \frac{M_T^2 M_Z^2}{M_T^2 - M_Z^2} \approx 5.5 \cdot 10^{-2} \left(\frac{M_T}{35\ \text{GeV}}\right)^2 \frac{1}{\frac{M_T^2}{M_Z^2} - 1} \qquad (6.3)$$

and $\Gamma_T^e \equiv (T \to e^+e^-)_{QED}$ is the purely electromagnetic contribution to the leptonic T width. The axial-vector (a_t) and the vector (v_t) coupling constants for the t quark are determined within the standard model by Eqs. (4.12), and for electron $a_e = -1$, $v_e = 4\sin^2\theta_W - 1$, so that $v_e \ll 1$, since $\sin^2\theta_W \approx 0.23$. Recall (see Section 2.1 Subsection C) that the quantity Γ_T^e has a relatively slow dependence on the quark mass, and the energy dependence of the leptonic width is determined, almost completely, by the last term in the square brackets of Eq. (6.2). In particular, $\Gamma(T \to e^+e^-)$ sharply increases if M_T is close to M_Z.

*) Near the Z^0 peak Eq. (6.2) must be modified, in particular, because R_Z is changed by radiative effects and by effects due to Γ_Z (details can be found in Refs. 179, 180).

Since the neutrino couplings are equal to unity, $a_\nu = v_\nu = 1$, the width of the $T \to \nu\bar{\nu}$ transition (Figure 34(b)) is given by the following expression

$$\Gamma(T \to \nu\bar{\nu}) = 2\,\Gamma_T \frac{v_t^2}{Q_t^2} R_Z^2 . \qquad (6.4)$$

For $M_T \sim M_Z$, the transition $T \to \sum_\nu \nu\bar{\nu}$ amounts to a considerable (10 - 20%) fraction of the total T width, and its observation, say, by the radiation tail $e^+e^- \to \gamma + T$ "nothing"[179] is of considerable interest, in particular, from the point of view of counting the number of the lepton generations. On the other hand, the $T \to \nu\bar{\nu}$ decay might be observed in the cascade*)

$$2^3S_1 \to 1^3S_1 + \pi^+ + \pi^- . \qquad (6.5)$$
$$\quad\ \ \ \llcorner\!\!\to \nu\bar{\nu}$$

The $T \to q\bar{q}$ decay width due to the graphs of Figures 34(c,d) is (cf. Eq. (6.2))

$$\Gamma(T \to q\bar{q}) = 3\,\Gamma_T^e \left[Q_q^2 - \frac{2v_q v_t Q_q}{Q_t} R_Z + \frac{(v_q^2 + a_q^2) R_Z^2}{Q_t^2} \right] \qquad (6.6)$$

Note that in the case of the $T \to b\bar{b}$ transition one should add a contribution from the t-channel W boson exchange (Figure 34(g)) to the amplitude represented by the graph 34(d).

The W-boson exchange contribution to the total width is, for $U_{cb} \approx 1$[181])

$$\Gamma_W^T = \frac{1}{2} \frac{M_T^4}{(M_W^2 + \frac{1}{4} M_T^2)^2} \left(\frac{\Gamma_t}{Q_t^2}\right)^2 \qquad (6.7)$$

The ratio of the $T \to f\bar{f}$ width to the electromagnetic width Γ_T^e

$$\Gamma_f^T = \Gamma(T \to f\bar{f})/\Gamma_T^e \quad , \ f = e, \nu, q \ , \qquad (6.8)$$

*) Bounds on the $1^3S_1 \to \nu\bar{\nu}$ decay from the cascade (6.5) were intensively discussed immediately after the J/Ψ and Ψ' discoveries (see e.g. Ref. 97).

as a function of the T-meson mass is shown on Figure 35. The ratio $r^T_{3g} = \Gamma\,(T \to 3g)/\,\Gamma^e_T$, calculated with the help of formulae like (2.41), is also presented for comparison. As one sees from the Figure (see also below), the relative weight of the three-gluon annihilation decreases with increasing M_T, and it does not exceed 30% at $M_T \gtrsim 60$ GeV.

The difference in the $d\bar{d}$ and $b\bar{b}$ contributions is mainly due to the constructive interference of the graphs 34(c,d) with that of Figure 34(g). Near the Z^0-boson pole (M_I = 80 - 90 GeV) the behaviour of the $T \to f\bar{f}$ channels is determined completely by the pole term, modified according to Refs. 179, 180 (see the footnote to Eq. (6.2)). A further growth of the quark mass results in a spectacular interplay of different forces. For instance, in the vicinity of the Z-boson pole the leptonic widths of the excited states lying nearer to Z, are larger than the leptonic width of the ground level[182].

Note that the $T \to q\bar{q}$ transitions would produce two-jet configurations experimentally, and the $T \to b\bar{b}$ channel can be separated rather easily because of specific properties of the b-quark decays (see Section 1.2 and Section 4.1).

For $M_T \gtrsim 100$ GeV the decay channels $T \to H^0\gamma$ (see Eq. (4.17)), $T \to Z^0\gamma$, $T \to H^0_T Z^0$ are comparable with the annihilation channels discussed above. However, as we have already mentioned, the free t-quark decays are dominant at such energies (see Figures 34 h, i)).The point is that the corresponding contribution to the total T-meson width is proportional to m_t^5, in contrast to the annihilation graphs (where the M_T-dependence is rather weak, except the region near the Z^0-pole).

For $m_t < M_W$, one can write an expression for $\Gamma^T_{t\to W+\ldots}$, neglecting all fermion masses, except m_t and m_b, and putting $|U_{tb}| = 1$,

$$\Gamma^T_{t\to W+\ldots} = 2\cdot 9\,\frac{G_F^2 m_t^5}{192\pi^3}\; f\left(\frac{m_t^2}{M_W^2}\,,\;\frac{m_b^2}{M_W^2}\right) . \qquad (6.9)$$

The factor 9 here is due to the existence of 9 allowed decay channels. The function f accounts for the effects due to the phase space and the W-boson propagator (see Figure 36).

Figure 37 (taken from Ref. 87) compares the quantities $\Gamma\,(T \to 3g)$, $\Gamma^I_f = \sum_{f=l,\nu,q} \Gamma(T \to f\bar{f})$ and $18\,\Gamma\,(t \to be^+\nu_e)$ for different magnitudes of the mass M_{I_T}. As it is seen from the Figure, the free quark decay becomes noticeable ($\Gamma^T_{t\to W+\ldots}/\,\Gamma_T \gtrsim 6\%$) only for $M_T \gtrsim 50$GeV; for $M_T \gtrsim 70$GeV its probability is, approximately, twice higher than $\Gamma(T \to 3g)$ and amounts to about 1/9 of the total width Γ_T.

For $M_T > 2M_W$ the dominant channel is the t-quark decay with production of the real W boson (Figure 34i). The corresponding width of the $T \to W$ decay is (we put $U_{tb} = 1$)

$$\Gamma\,(T \to W+\ldots) = 2\frac{G_F}{8\pi\sqrt{2}}\; m_t^3\left(1+2\frac{M_W^2}{m_t^2}\right)\left(1-\frac{M_W^2}{m_t^2}\right)^2 . \qquad (6.10)$$

We emphasize that different hadronic channels of the T-boson decay can be seperated in experiments, if one pays attention to specific differences in the corresponding events.For example, the $T \to q\bar{q}$ transitions produce, mainly, two-jet events; $T \to 3g$ events have a planar topology, while the free t-quark decays result, in the average, in isotropic events. Besides that, as we have already mentioned, one should observe in the latter case direct hard leptons correlated with kaons(from the b quark decays).

At the T-resonance peak, the e^+e^- annihilation cross section, corresponding to the free t-quark decay ,is[179]:

$$\sigma(e^+e^- \to t\bar{t})_{\text{free quark decay}} = R_{\text{free quark decay}}\;\sigma(e^+e^- \to \mu^+\mu^-) \qquad (6.11)$$

$$R_{\text{free quark decay}} = \frac{9}{2\alpha^2}\sqrt{\frac{\pi}{2}}\left(\frac{\Gamma(T\to e^+e^-)}{\Gamma_T}\right)\left(\frac{\Gamma^T_{t\to W+\ldots}}{\sigma}\right)\left(\frac{2\sqrt{2}\sigma}{M_T}\right)^\beta$$

where

$$\beta = \frac{4\alpha}{\pi}\left(\ln\left(\frac{M_T}{m_e}\right) - \frac{1}{2}\right) \qquad (6.12)$$

and σ is the energy spread in the e^+e^- beams ($\sigma \sim W^2$). Note that for $M_T \lesssim 70$ GeV the branching ratio $\Gamma(T \to e^+e^-)/\Gamma_T$, and, consequently, the ratio $R_{\text{free quark decay}}$, have a weak dependence on the quark wave function.

The cross sections corresponding to other T-meson decay channels, $T \to f'$, can be obtained from (6.11) by substituting $\Gamma^T_{t\to W+\ldots} \to \Gamma(T\to f')$.

As we see, the possibility of experimental observation and investigation of the T meson depends substantially on the magnitude of σ(e.g.[59]). Say for the toponium mass in the interval $M_T \sim 40$ - 50 GeV and with σ (MeV) = $22 \cdot 10^{-3}\, M_T^2$ (GeV^2) (the numbers are relevant to PETRA) we get that the maximum of the cross section exceeds the background in the hadron channel by a factor of 2 - 3 and that in the muon channel by a factor of 1.5 - 2.5 . Thus the leptonic width measurement and reconstruction of the total width does not seem to be a difficult task. Note that if $M_T > M_Z$ the conditions for experimental investigations in toponium become worse because of a strong radiative Z^0 tail, which should be eliminated in this or that way[155, 179].

7. PERSPECTIVES

Rather rich information on the heavy quarks is available at present, still the field does not seem to be exhausted. One may expect here many striking and important discoveries. In particular, the t quark observation and measurement of its mass may produce a considerable effect upon the further development of the theory of quarks and leptons. Moreover, the t quarks might be a unique factory for production of exotic objects, like the Higgs particles, etc. There are some chances to encounter the fourth generation of quarks (if it exists in nature).

There is still a lot of unsolved problems relevant to the charm and beauty sectors. The information which can be obtained here concerns, mainly, the strong interaction properties, in particular, special features of gluodynamics. As always, the most attractive are the frontiers that are not yet achieved by the present-day theory. From this point of view strikingly rich are the $c\bar{q}$ and $b\bar{q}$ systems - charmed and beautiful particles. First of all, one should gain an understanding of exclusive weak decays, work out reliable methods for calculating transition form factors like $f_+^{D\to K}$, completely solve the problem of charmed particles' life-time. Analysis of $D^* \to D\gamma$ and $D^* \to D\pi$ type transitions is also of interest. It goes without saying that many unanswered questions are still waiting for their solution in charmonium and, especially, in bottonium. Possible surprises are hidden in radially excited Ψ levels above the open flavor threshold. Unique and important data might be obtained in hadronic transitions between the charmonium (bottonium) levels and in radiative J/Ψ (Υ) decays.

Accumulating data on various decays of heavy hadrons, we learn elements of the quark mixing matrix. the data on the b-quark transitions into c and u quarks are especially urgent.

The progress in the heavy quark physics has been amazingly rapid. A list of main theoretical and experimental findings of last year contains at least 10-15 items; and, what is more important, new efforts in this field will be surely highly productive.

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FIGURES

Figure 1. Level structure in charmonium and radiative transitions between the levels. Solid lines represent E1 transitions, dashed dotted lines represent M1 transitions. States and transitions not yet observed are shown by dashed lines.

Figure 2. Inclusive photon spectrum in the ψ' radiative decay measured by the CB group[16]. The observed peaks are identified with the radiative transitions between the charmonium levels. In the upper part we give the distributions in the η_c and η_c' resonance regions (the background is subtracted).

Figure 3. Graphs representing the quarkonium transitions: (a) $Q\bar{Q} \to 3g$, (b) $Q\bar{Q} \to \gamma\, gg$.

Figure 4a. Normalized inclusive photon spectrum from the J/ψ decay; $r = (dN/dx)_{exp}/(dN/dx)_{QCD,\text{lower order}}$. The Breit-Wigner curve shows a fit which includes a broad tensor resonance with mass M = 2 GeV and width Γ = 0.6 GeV. The experimental points are taken from Ref. 39.

Figure 4b. The photon spectrum from the decay $J/\psi \to \gamma$ + light hadrons $0.8 < E_\gamma < 1.6$ GeV[25].

Figure 5. Graphs describing charmed meson decays in the free quark decay approximation ($\Delta C = \Delta S$).

Figure 6. Mass spectrum of $\mu^+\mu^-$ pairs measured in the pN collisions at p = 400 GeV/c[13].

Figure 7a. The observed cross section for the process $e^+e^- \to$ hadrons in the Υ resonance region, measurements of the CLEO group[17].

Figure 7b. Energy behaviour of R in the region of Υ'''-resonance. The darker curve shows R below Υ'''.

Figure 8. Level structure in the $b\bar{b}$ quarkonium. Solid lines correspond to observed states (transitions). Unobserved states (transitions) are denoted by dashed lines. The dashed-dotted line refers to the transition $\Upsilon'' \to 1^3P_J$, whose observation is not absolutely trustworthy (see the text).

Figure 9. The pion invariant mass distribution in the $\Upsilon'' \to \Upsilon\pi^+\pi^-$ decay[65]. Curve 1 - the analogous distribution for $\psi' \to J/\psi\ \pi\pi$. Curve 2 - the phase space volume. Curve 3 - the best fit.

Figure 10. The photon spectrum in the cascade $\Upsilon'' \to \gamma + 2^3P_J \to \gamma\gamma + \Upsilon$ (the background is subtracted; taken from Ref. 25). Curves 1, 2, 3 correspond to transitions $\Upsilon'' \to \gamma + 2^3P_J$ where J = 0, 1, 2. Curve 4 - the total contribution of all transitions.

Figure 11. Quark graphs for the B-meson decays in the free quark decay approximation. The ratios of the partial decay widths to $\Gamma(b \to e^- \bar{\nu}_e c(u))$, including colour factors and the phase space effects, are presented in parenthesis.

Figure 12. Theoretical prediction for the reduced leptonic width (2.17) versus the quark mass. For comparison, the J/ψ and Υ experimental points are also presented. Numbers in the upper part of the graph indicate the relative magnitude of the $\langle G^2 \rangle$ correction.

Figure 13a. The lowest order graph for the correlation function (2.21). Solid lines denote heavy quarks, wavy lines denote currents.

Figure 13b. Coupling of quarks to vacuum fields. Dashed lines are gluons.

Figure 14. Ratio of moments versus n (definitions are given in Eq. (2.22)). Arrow A indicates the 20 % level of the $\langle G^2 \rangle$ correction. Arrow B separates the regions of small and large experimental uncertainties: to the right of the arrow the uncertainty is less than 1%. Arrow D indicates the asymptotical value of r_n.

Figure 15. Masses of che charmonium P levels derived from the QCD sum rules (taken from Ref. 74).

Figure 16. Graphical representation of the QCD duality relations.

Figure 17. Effects due to the gluon condensate in correlation functions included by quark and gluon currents. In the gluon case (b) we deal with the Born graphs. In the quark case (a) any graph necessarily contains a loop. Every additional loop leads to a suppression $\sim 1/16\pi^2$.

Figure 18. Exclusive radiative annihilation of J/Ψ (Υ). Additional factors $(\alpha_s)^{1/2}$ (cf. Figure 3b)are compensated a large logarithmic factor ln $(M^2 R^2_{conf})$ due to the loop integration.

Figure 19. Graphs describing the correlation function $i\int e^{iqx} d^4x < 0 \mid T\{\bar{c}\Gamma u(x), \bar{u}\,\Gamma c(0)\}\mid 0>$, where $\Gamma = 1,\ \gamma_5$, or γ_μ.

Figure 20. Graph for the $c \to sud$ decay accounting for hard virtual gluons. Black point stands for the weak Hamiltonian.

Figure 21. Examples of graphs corresponding to corrections $\sim m_Q^{-2}$ to heavy meson decays.

Figure 22. Interference contribution to the hadronic width of the D^+-meson decay.

Figure 23. Non-factorizable contribution to the matrix element of four-fermion operators over the D-meson states.

Figure 24. Graphs describing $D^0 \to K^0\pi^0$ and $D^0 \to K^-\pi^+$ decays (in the free c-quark decay approximation).

Figure 25. Pre-asymptotical contribution to the $D^0 \to \pi^0 \bar{K}^0$ decay.

Figure 26. Graph for the $B \to J/\Psi + X$ decay.

Figure 27. Graphs determining the $K_L^0 - K_S^0$ mass difference.

Figure 28. The existing bounds for the quark mixing angles, depicted in the $\{\sin\beta,\ |\sin\gamma|\}$plane. The domain in the left lower angle corresponds to $\tau_B < 1.4 \cdot 10^{-12}$ sec.[68]. The domains which are ruled out are dashed. Values to the left of the dashed line are ruled out by the CLEO data on kaon yields in the $B\bar{B}$ events[17].

Figure 29. Graph describing the $c\bar{c}$ state production in neutrino induced reactions.

Figure 30. Lowest-order graph for the process $e^+e^- \to q\bar{q}$.

Figure 31. Graph describing the decay of the vector quarkonium, $V_Q \to H^0 + \gamma$.

Figure 32. Graphs describing the decays of vector (a) and C-even (b) quarkonia in QCD.

Figure 33. Graphic representation for the decays (a) $J/\Psi \to \gamma f$, $f \to \pi\pi$ and (b) $(2^3S_1) \to (1^3S_1)\ \pi\pi$.

Figure 34. Main graphs corresponding to the decays of superheavy vector $Q\bar{Q}$ quarkonium

Figure 35. Ratios $r_f^T = \Gamma(T \to f\bar{f})/\Gamma(T \to e^+e^-)_{QED}$ and $r_{ggg}^T = \Gamma(T \to ggg)/\Gamma(T \to e^+e^-)_{QED}$ versus the toponium mass[87].

Figure 36. The function $f(m_Q^2/M_W^2,\ m_b^2/m_Q^2)$ for m_b=5GeV, M_W=80GeV.

Figure 37. Comparison of the decay widths for $T \to 3g$, $T \to \sum_{f=e,\nu,q} f\bar{f}$, and $t \to be^+\nu_e$ [87].

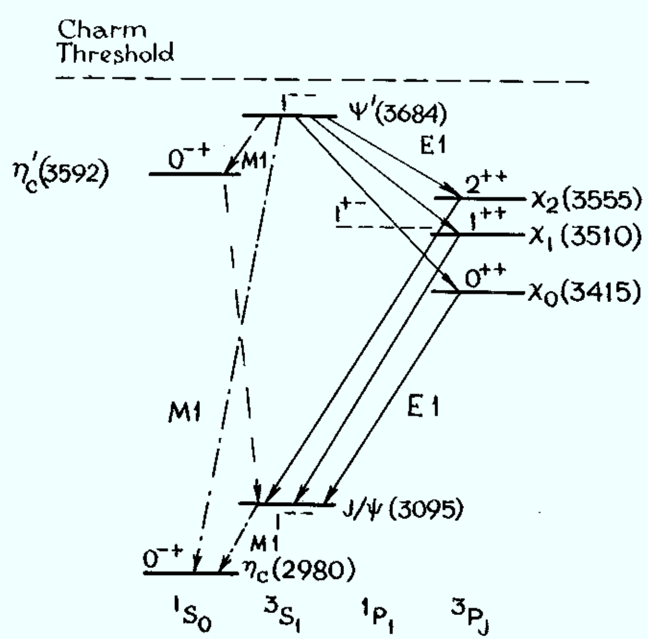


Fig. 1

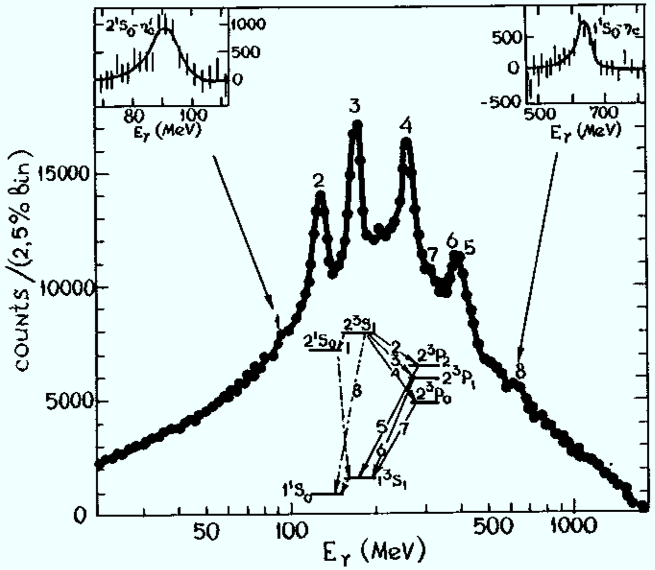


Fig. 2

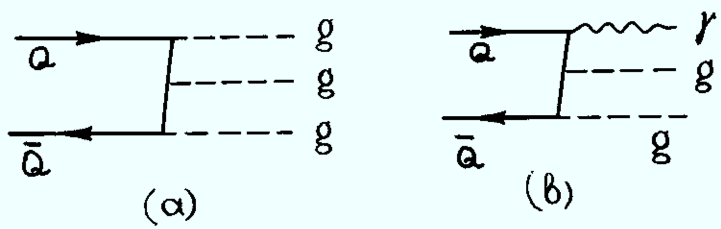


Fig 3

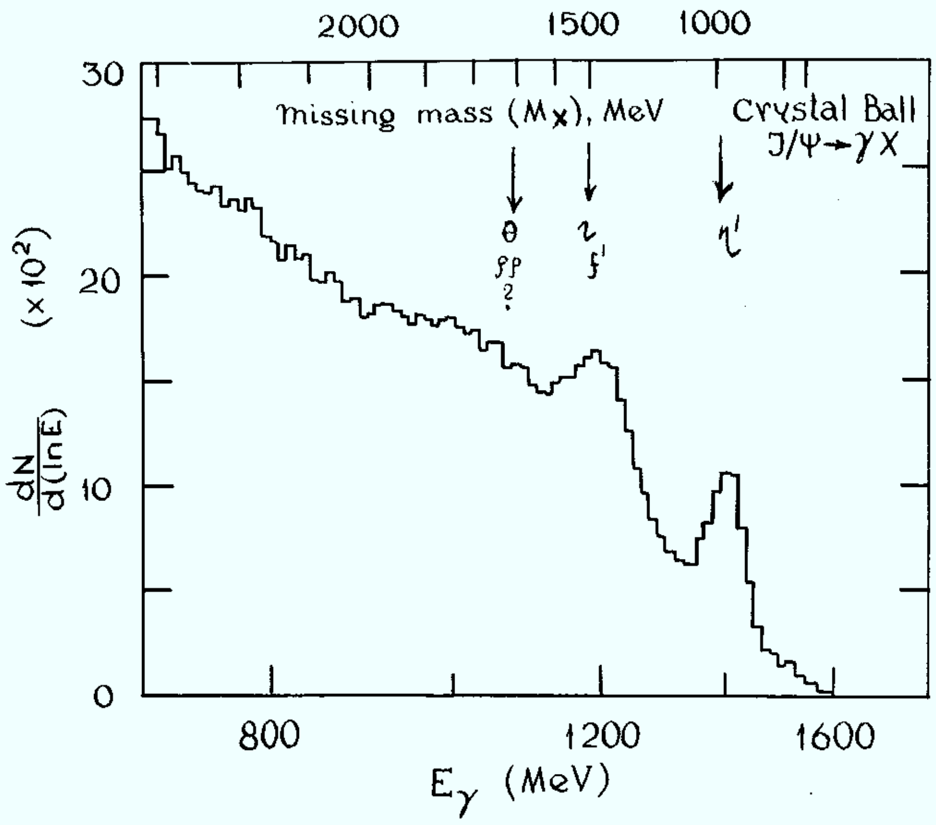


Fig. 4 (b)

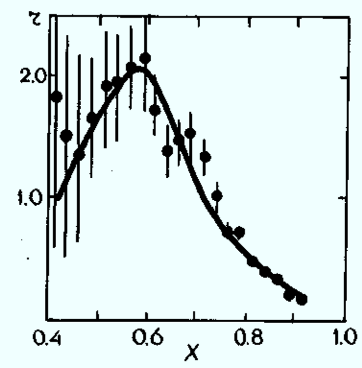


Fig. 4 (a)

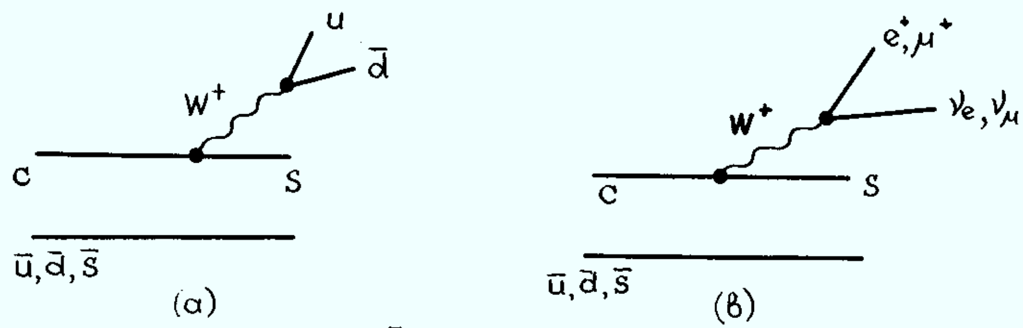


Fig. 5

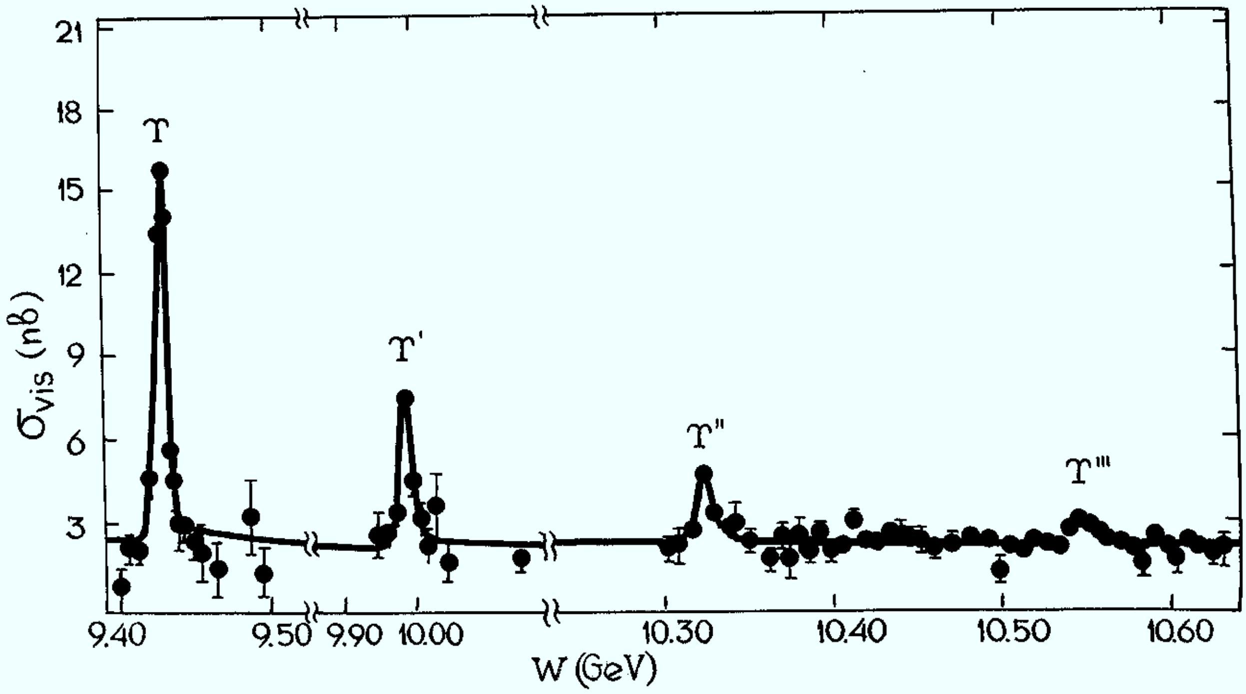

Υ
Υ'
Υ''
Υ'''
σvis (nb)
21
18
15
12
9
6
3
9.40
9.50
9.90
10.00
10.30
10.40
10.50
10.60
W (GeV)

Fig. 7 (a)

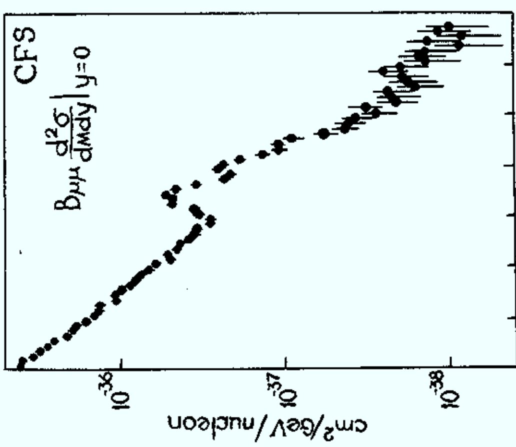

CFS
$B_{\mu\mu}\frac{d^2\sigma}{dMdy}\Big|_{y=0}$
cm²/GeV/nucleon
10⁻³⁶
10⁻³⁷
10⁻³⁸
6
7
8
9
10
11
12
13
M, GeV

Fig. 6

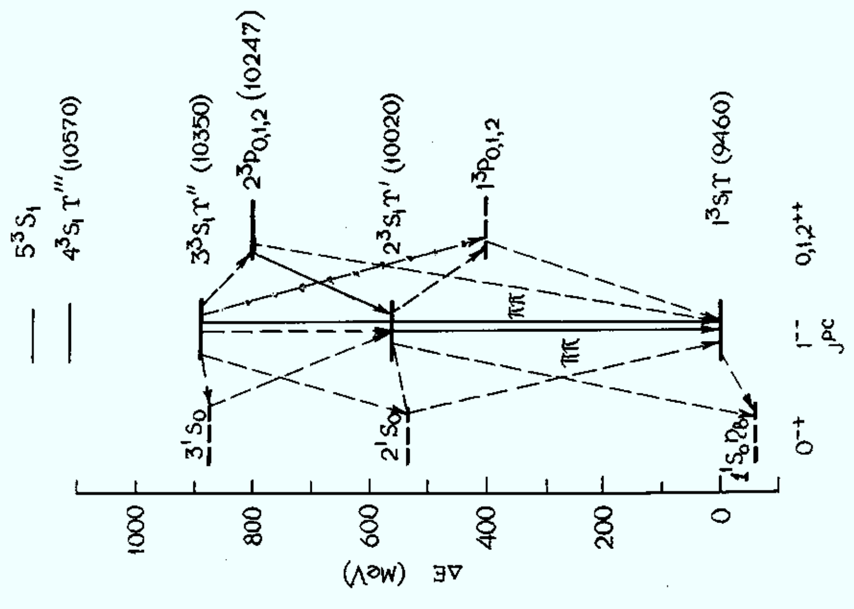

5^3S_1
$4^3S_1\ \Upsilon'''$ (10570)
$3^3S_1\ \Upsilon''$ (10350)
$2^3P_{0,1,2}$ (10247)
$2^3S_1\ \Upsilon'$ (10020)
$1^3P_{0,1,2}$
$1^3S_1\ \Upsilon$ (9460)
3^1S_0
2^1S_0
1^1S_0
ππ
ππ
0^{-+}
1^{--}
$0,1,2^{++}$
J^{PC}
1000
800
600
400
200
0
ΔE (MeV)

Fig. 8

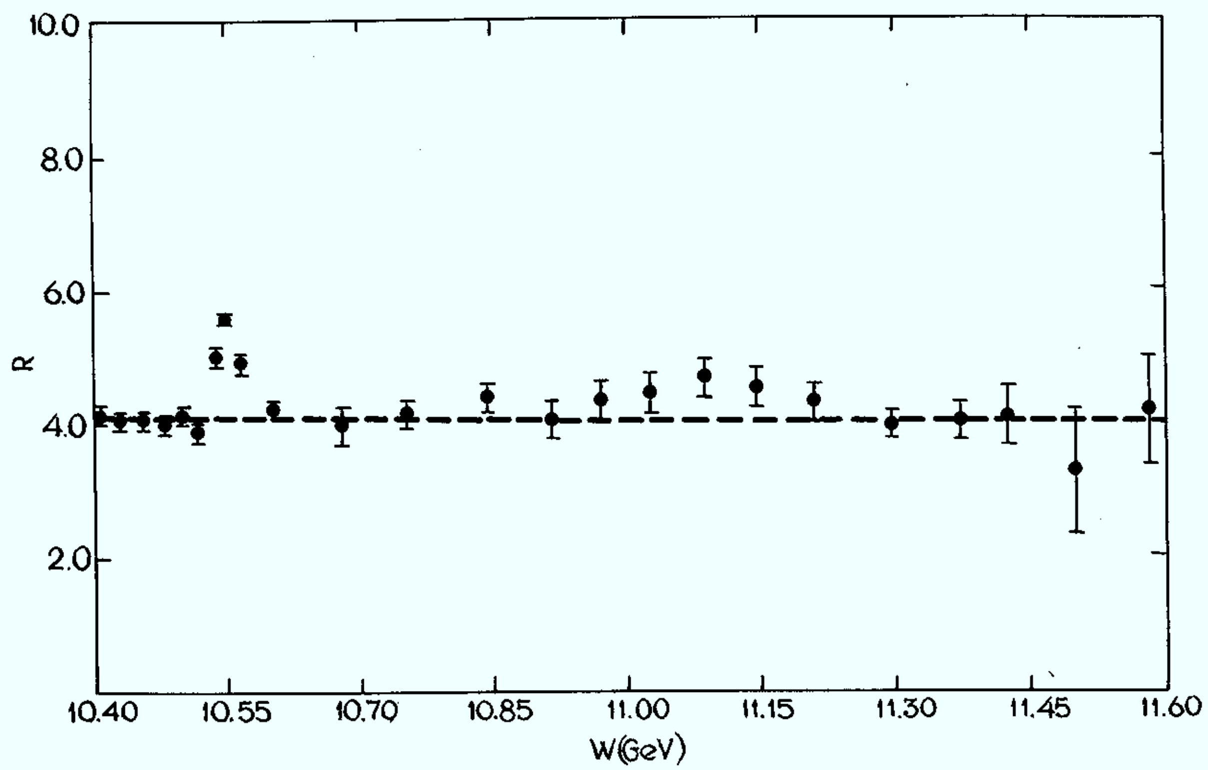

10.0
8.0
6.0
R
4.0
2.0
10.40
10.55
10.70
10.85
11.00
11.15
11.30
11.45
11.60
W(GeV)

Fig.7 (b)

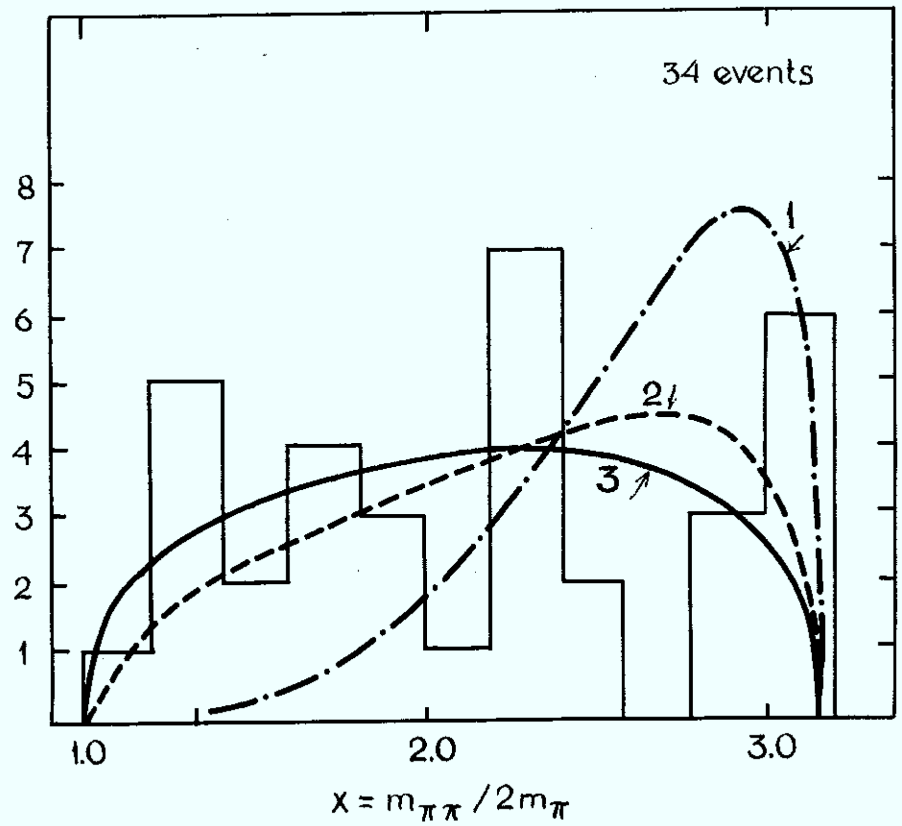

34 events
1
2
3
1.0
2.0
3.0
x = mππ / 2mπ
1
2
3
4
5
6
7
8

Fig. 9

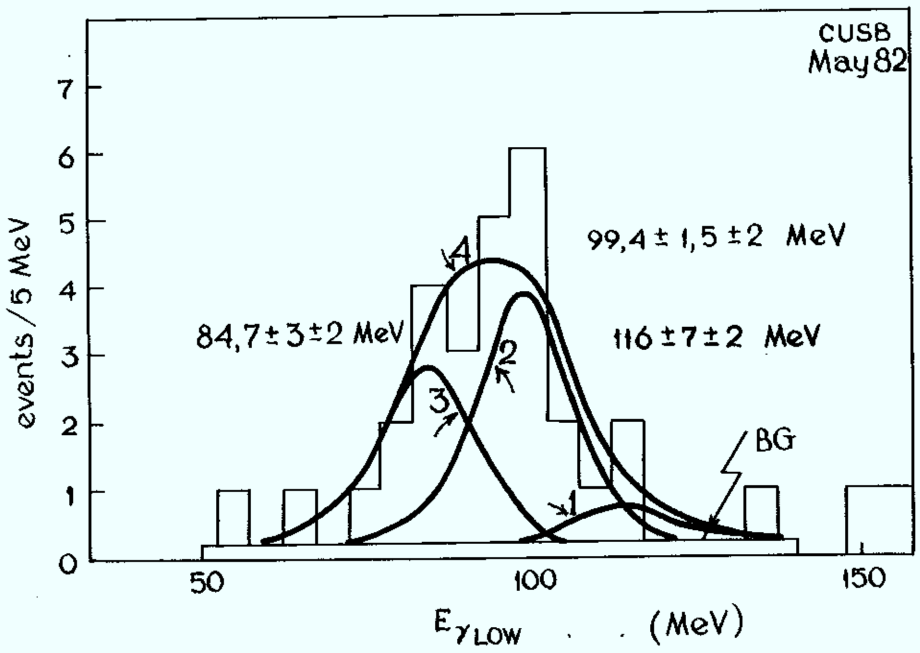

CUSB
May 82
99,4 ± 1,5 ± 2 MeV
84,7 ± 3 ± 2 MeV
116 ± 7 ± 2 MeV
4
3
2
1
BG
events / 5 MeV
0
1
2
3
4
5
6
7
50
100
150
Eγ LOW (MeV)

Fig. 10

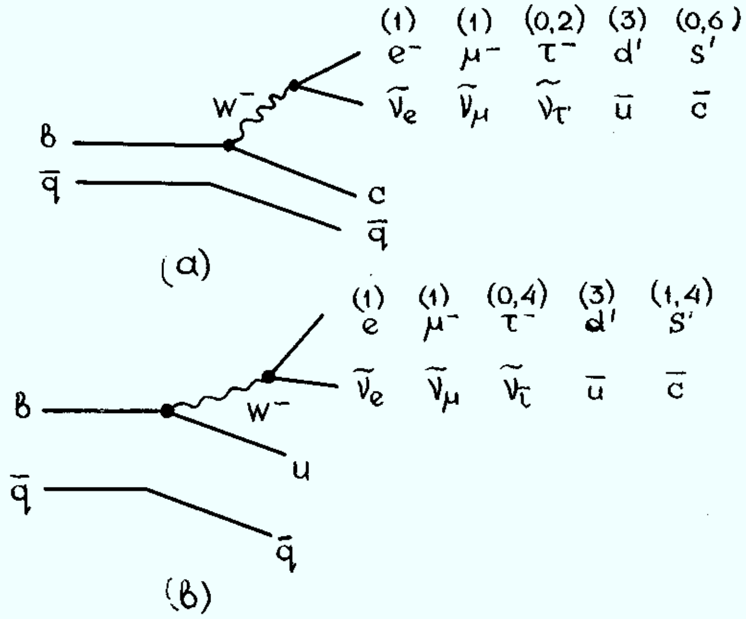


Fig. 11

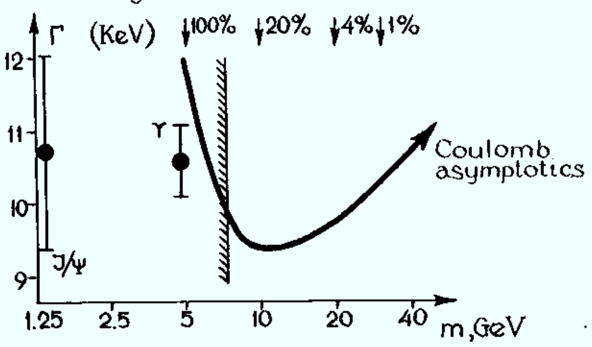


Fig. 12

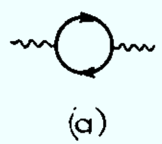


Fig 13

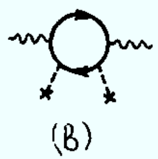


Fig. 13

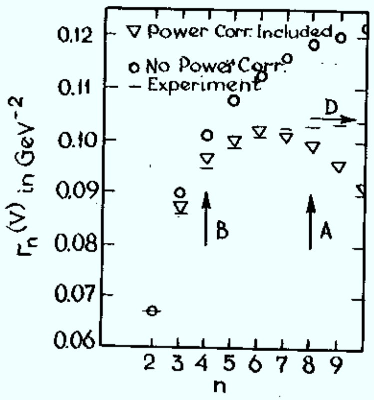


Fig. 14

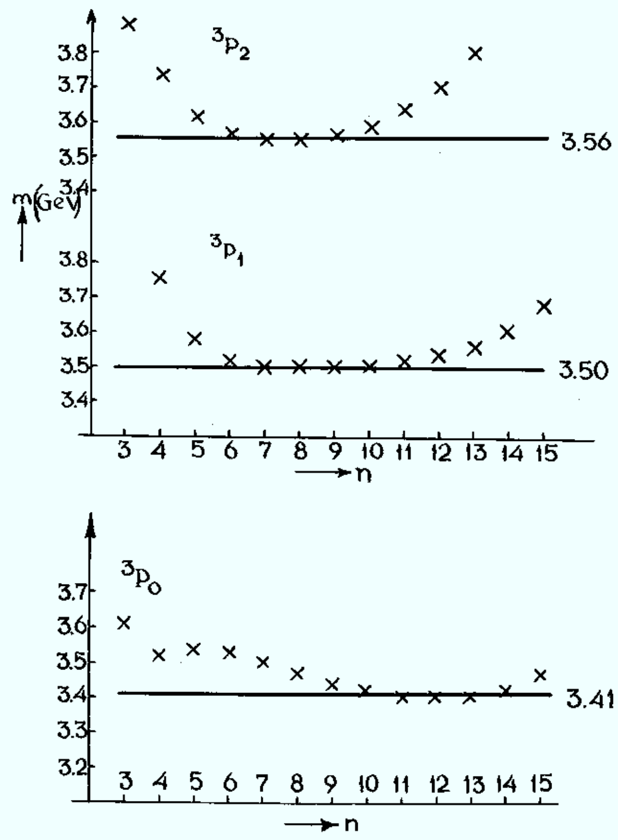


Fig. 15

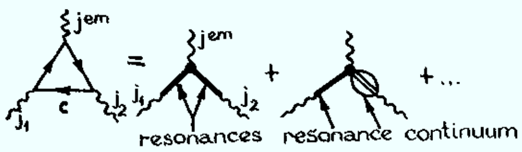


Fig. 16

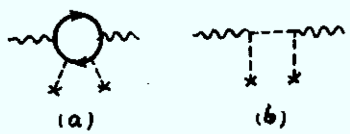


Fig. 17

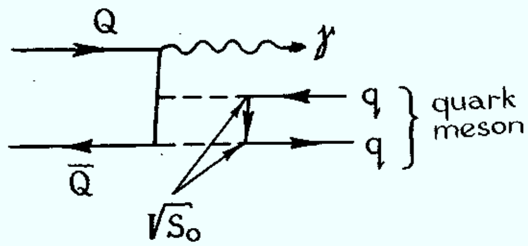


Fig. 18

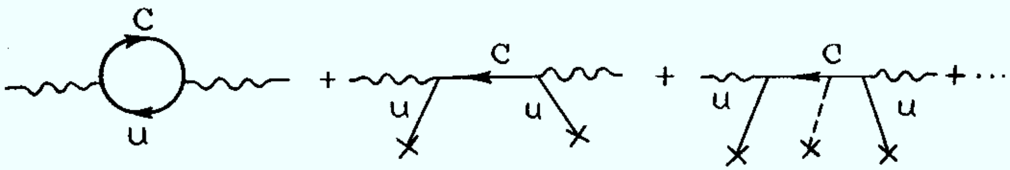


Fig. 19

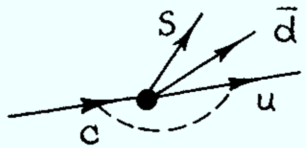


Fig. 20

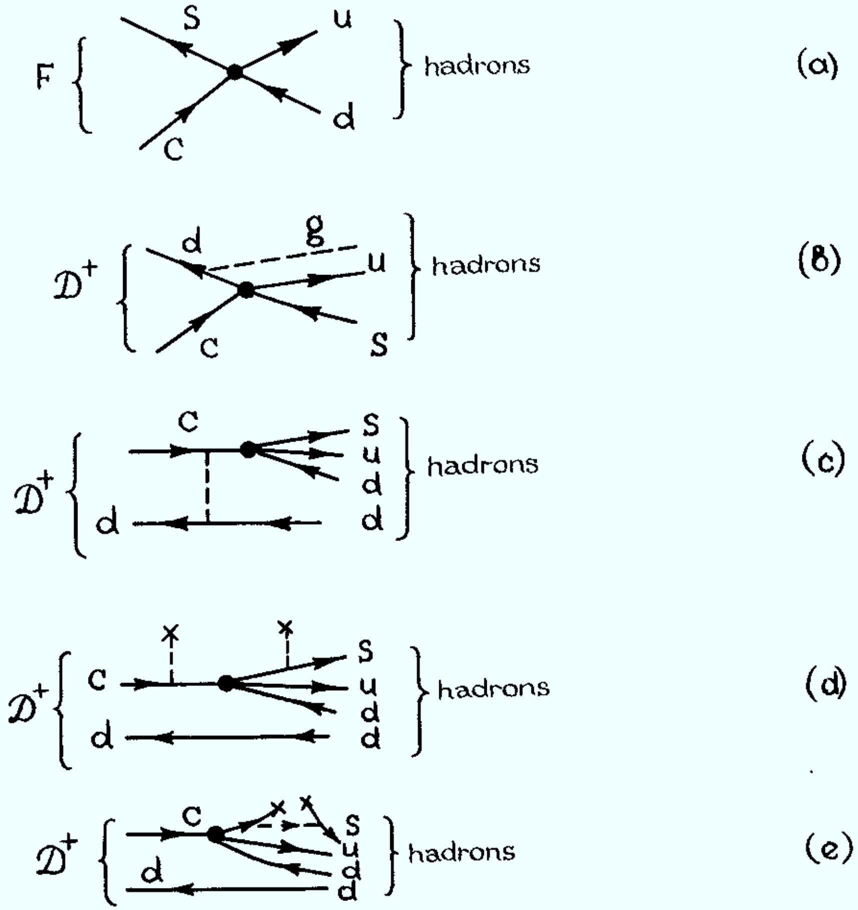


Fig. 21

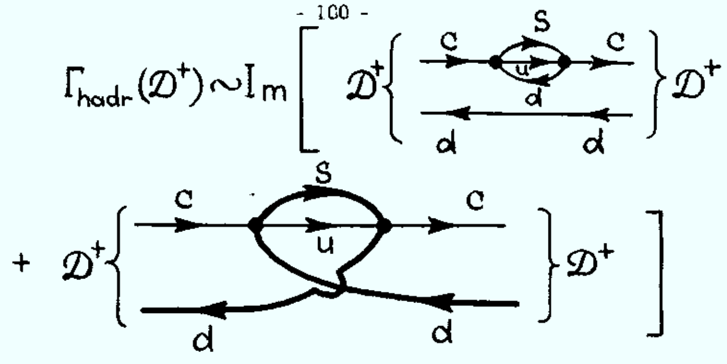
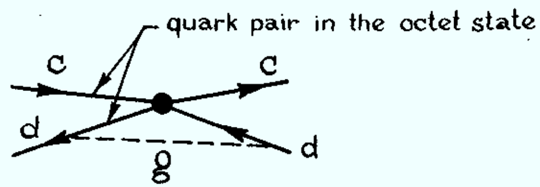


Fig. 22



$\bullet = (\bar{c}_i \gamma_\mu (1+\gamma_5) d_j)(\bar{d}_j \gamma_\mu (1+\gamma_5) c_i)$

Fig 23

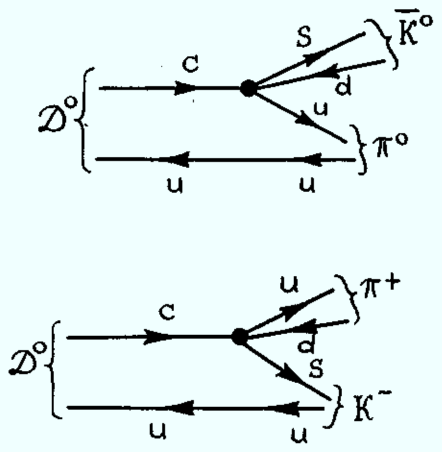


Fig. 24

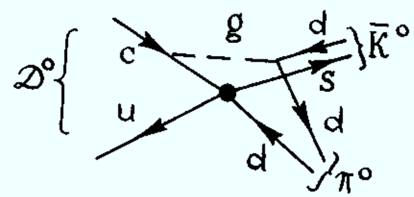


Fig. 25

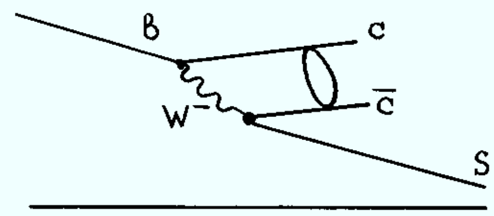


Fig. 26

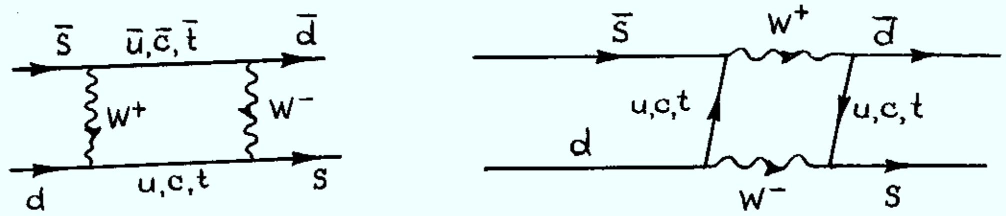


Fig. 27

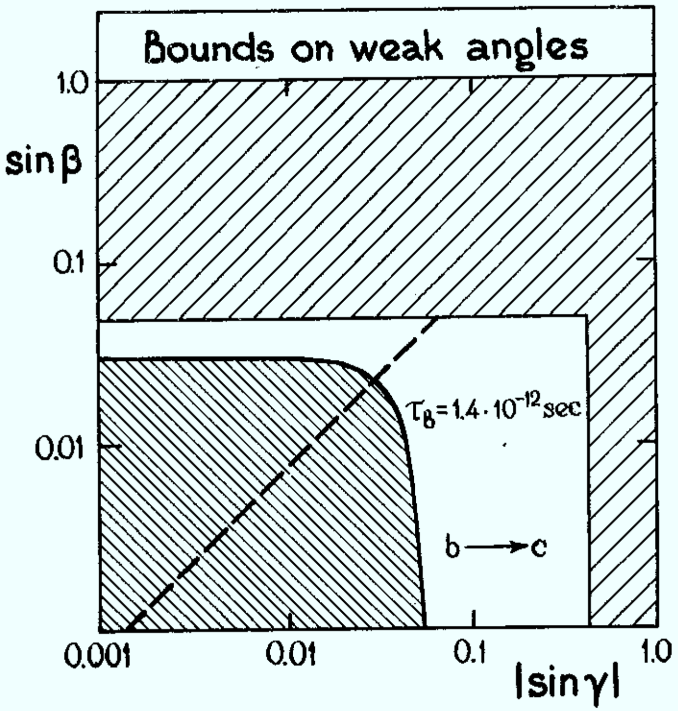


Fig. 28

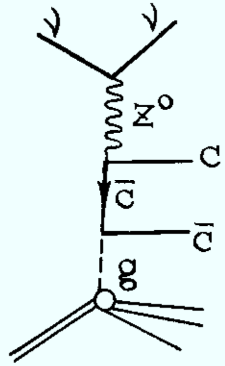


Fig. 29

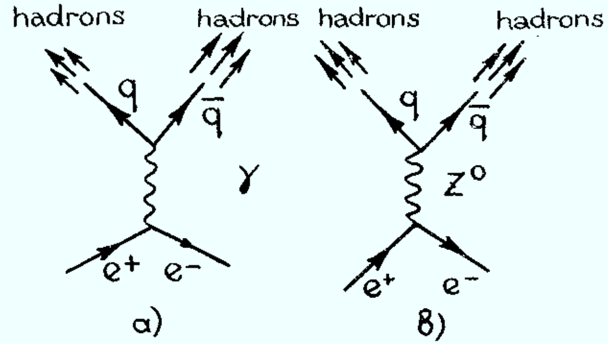


Fig. 30

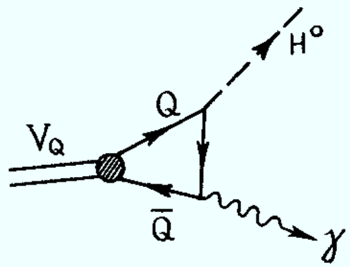


Fig. 31

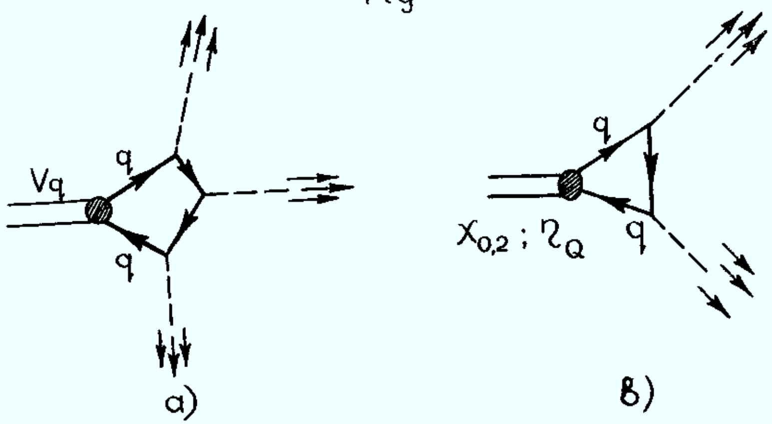


Fig. 32

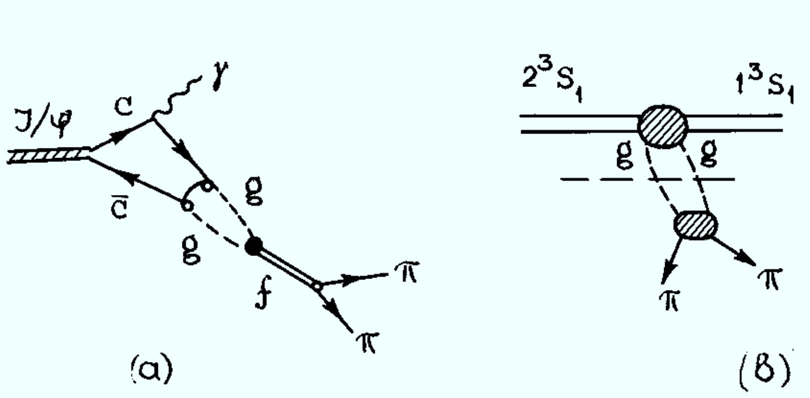


Fig. 33

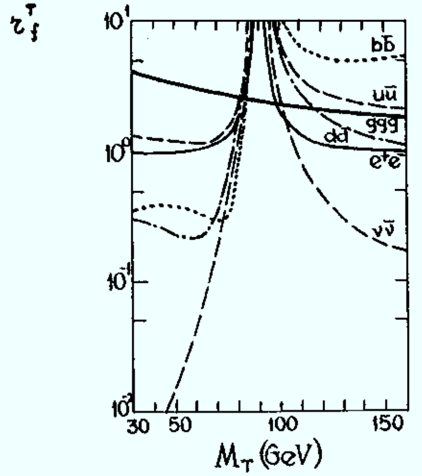


Fig. 35

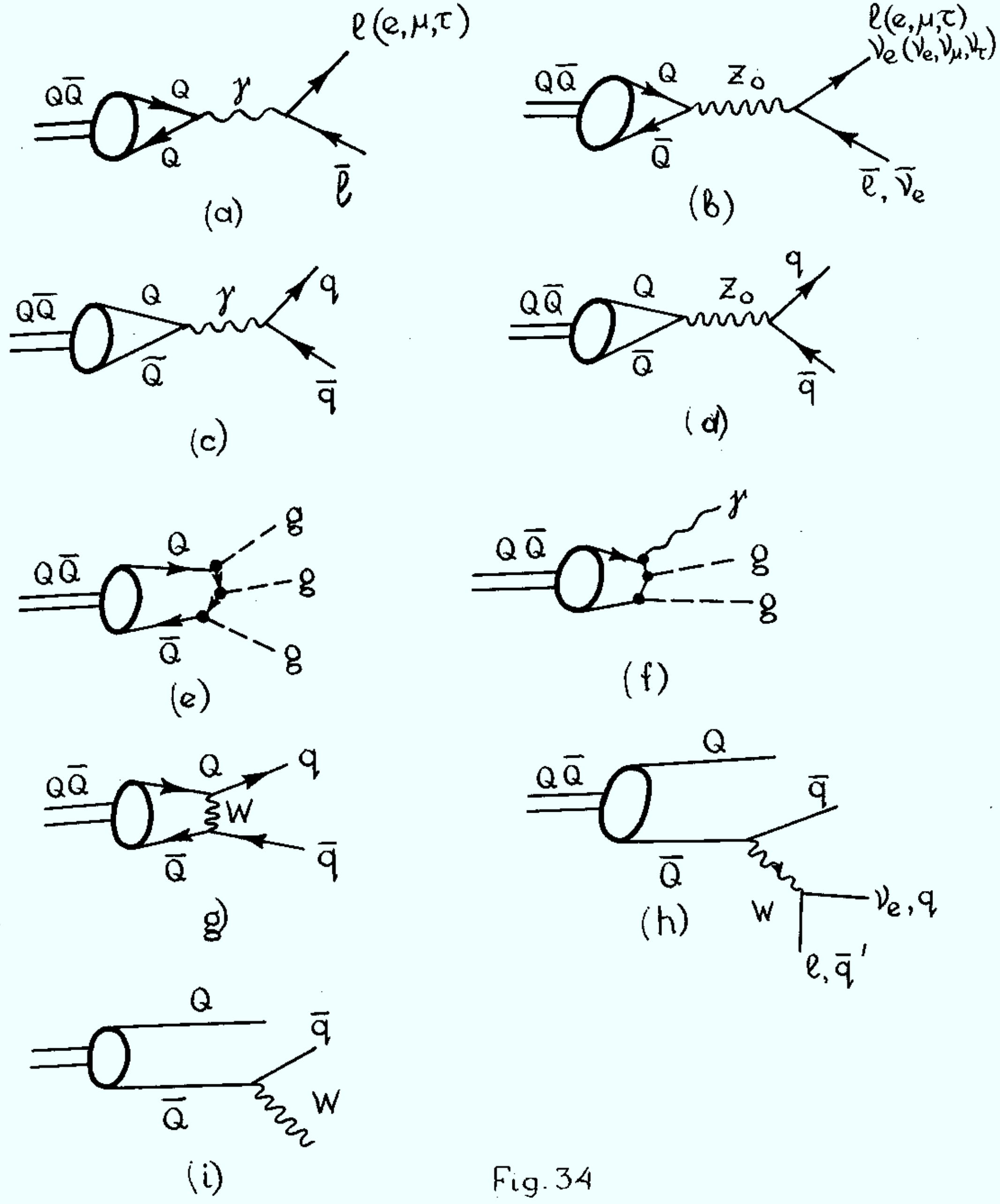


Fig. 34

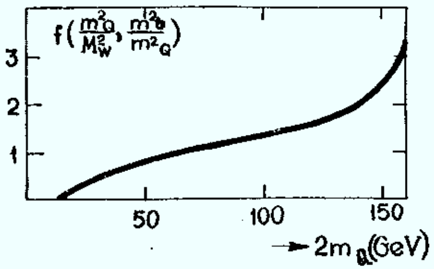


Fig. 36

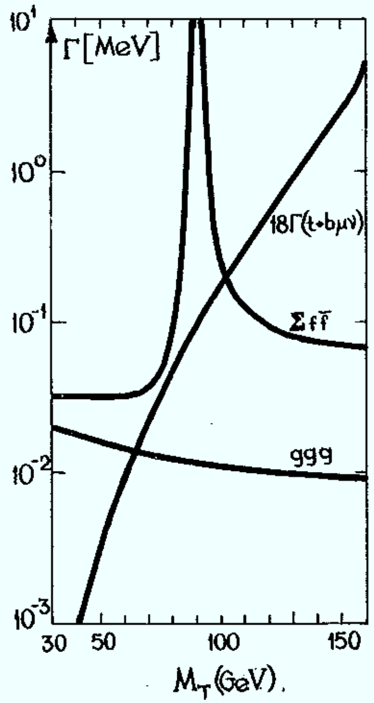


Fig. 37

Table 1:

Parameters of the J/ψ , ψ' and higher vector states of $c\bar{c}$.

	Mass (MeV)	$n^{2S+1}L_y$ *	Γ(MeV)	$\Gamma(\psi \to e^+e^-)$ (keV)
J/ψ	3096.93 ± 0.09 [a) 32)]	1^3S_1	0.063 ± 0.009	4.6 ± 0.4
ψ'	3686.0 ± 0.10 [32)]	2^3S_1	0.215 ± 0.040	2.05 ± 0.2
ψ''	3770.0 ± 3	1^3D_1	25 ± 3	0.26 ± 0.05
ψ(4030)	4030.0 ± 5	3^3S_1	52 ± 10	0.75 ± 0.10
ψ(4160)	4159.0 ± 20	2^3D_1	78 ± 20	0.78 ± 0.31
ψ(4415)	4415.0 ± 6	4^3S_1	43 ± 20	0.43 ± 0.13

Notes:

* Here and below (see Table 7) the spectroscopic assignments are those favored by the potential models [29,30)].

a) Here and below we point out the experimental group only when its result is the most precise. In other cases the world average results are presented.

Table 2:

Exclusive radiative decays of J/ψ.

Decay Channel	BR($\cdot 10^{-4}$)	Notes
$J/\psi \to \gamma\pi^0$	$(3.6 \pm 1.1 \pm 0.8) \cdot 10^{-1}$	CB[41]
$J/\psi \to \gamma\eta'$	36 ± 5	
$J/\psi \to \gamma\eta$	8.6 ± 0.9	$\frac{BR(J/\psi \to \gamma\eta')}{BR(J/\psi \to \gamma\eta)} = 4.7 \pm 0.6$ CB[41]
$J/\psi \to \gamma f(1270)$	15 ± 4	CB[42], $f \to \pi^0\pi^0$ $x = A_1/A_0 = 0.88 \pm 0.13$ $y = A_2/A_0 = 0.04 \pm 0.19$ (A_λ - amplitude with helicity λ)
$J/\psi \to \gamma f'(1515)$ $f' \to K\bar{K}$ $f' \to \eta\eta$	$(1.6 \pm 0.5 \pm 0.8)$ (0.9 ± 0.9)	MARK II } 25) CB }

Table 3:

Parameters of the new states ι and θ [25].

Parameter	MARK II	Crystal Ball
M_ι (MeV)	1440^{+10}_{-15}	1440^{+20}_{-15}
$\Gamma(\iota \to all)$ (MeV)	50^{+30}_{-20}	55^{+20}_{-30}
J^P	-	0^- (channel $\iota \to \delta\pi$)
C	+	+
$BR(J/\psi \to \gamma\iota)BR(\iota \to K\bar{K}\pi)$	$(4.3 \pm 1.7) \cdot 10^{-3}$	$(4.0 \pm 1.2) \cdot 10^{-3}$
$BR(J/\psi \to \gamma\iota)BR(\iota \to \eta\pi\pi)$	-	$<2 \cdot 10^{-3}$ (90% c.l.)
M_θ (MeV)	1700 ± 20 (channel $\theta \to K^+K^-$, two resonance fit)	1670 ± 50 (channel $\theta \to \eta\eta$, two resonance fit)
$\Gamma(\theta \to all)$ (MeV)	160 ± 80	156 ± 30
J^{PC}	-	2^{++}, 95% c.l.
$BR(J/\psi \to \gamma\theta)BR(\theta \to \eta\eta)$	$(3.8 \pm 1.6)\ 10^{-4}$	-
$BR(J/\psi \to \gamma\theta)BR(\theta \to K\bar{K})$	-	$(12.4 \pm 18 \pm 5.0) \cdot 10^{-4}$
$BR(J/\psi \to \gamma\theta)BR(\theta \to \pi\pi)$	$< 6 \cdot 10^{-4}$	$< 2.4 \cdot 10^{-4}$ (90% c.l.)

Table 4:

Parameters of the D* mesons 44).

	Decay Channel	BR (%)	Note
D^{*0}	$D^0\pi^0$ $D^0\gamma$	47 ± 9 53 ± 9	It was assumed that the angular distribution of D relative to its parent D* in $e^+e^- \rightarrow D^*\bar{D}^* \rightarrow D + \ldots$ was isotropic.
D^{*+}	$D^0\pi^+$ $D^+\pi^0$ $D^+\gamma$	44 ± 7 28 ± 7 28 ± 10	

Table 5:

Charmed particle lifetimes 26

Experiment (a) / Parameter	FNAL (ν) E531	CERN (π,p) NA16	CERN (π) NA18	CERN (γ) NA1	SLAC (e^+e^-) MARK II	SLAC (γ) BC72/73
$\tau(D^0)$ ($\times 10^{-13}$ sec)	$3.2^{+1.0}_{-0.7}$ (14)	$3.9^{+1.4}_{-0.9}$ (14)	$4.1^{+2.6}_{-1.3}$ (9)	-	$3.7^{+2.5}_{-1.5}\pm 1$	$7.7^{+3.0}_{-2.5}$ (13)
$\tau(D^+)$ ($\times 10^{-13}$ sec)	$11.4^{+6.6}_{-4.4}$ (11)	$9.2^{+4.4}_{-2.5}$ (13)	$6.3^{+4.4}_{-2.3}\pm 1.5$ (7)	$9.5^{+3.1}_{-1.5}$ (98)	-	$7.3^{+3.0}_{-2.5}$ (14)
$R = \frac{\tau(D^+)}{\tau(D^0)}$	$3.6^{+2.5}_{-1.5}$	$2.4^{+1.5}_{-0.8}$	1.5 ± 1	-	-	$0.9^{+0.7}_{-0.4}$
$\tau(F^+)$ ($\times 10^{-13}$ sec)	$2.0^{+1.8}_{-0.8}$ (3)	$1.9^{+1.7}_{-0.8}$ (3)	$4.4^{+5.0}_{-1.7}\pm 1.5$ (5)	$5.5^{+5.0}_{-2.5}$ (5)	-	-
$\tau(\Lambda_c^+)$ ($\times 10^{-13}$ sec)	$2.3^{+1.0}_{-0.6}$ (8)	$1.9^{+1.4}_{-0.7}$ (4)	-	-	-	-

(a) In brackets the number of events is indicated.

Table 6:

Υ meson parameters [18,31]

Parameter		Notes
M_Υ (MeV)	9459.7 ± 0.6	Novosibirsk, resonance depolarization method [60]
$BR(\Upsilon\to\mu^+\mu^-)$%	3.3 ± 0.5	assumed: $BR(\Upsilon\to e^+e^-) = BR(\Upsilon\to\mu^+\mu^-)$
$\Gamma(\Upsilon\to e^+e^-)$ (keV)	1.17 ± 0.05	
Γ_Υ (keV)	35.5^{+8}_{-6}	
$\langle n^{ch}_\Upsilon\rangle$	7.9 ± 0.6 [61]	DASP-2. Direct decays. Off resonance $\langle n^{ch}_{off}\rangle$ = = 6.9±0.6
$\langle n^{p+\bar p}_\Upsilon\rangle$	0.64±0.16 (DASP-2) [61] 0.32±0.07 (CLEO)	$\langle n^{p+\bar p}_{off}\rangle$ = 0.22 ± 0.04 (CLEO)
$\langle n^{\Lambda+\bar\Lambda}_\Upsilon\rangle$	0.23 ± 0.05 [17]	$\langle n^{\Lambda+\bar\Lambda}_{off}\rangle$ = 0.14 ± 0.03 (CLEO)

Table 7:

Summary of results on Υ', Υ'' and Υ''' - resonances [18,62]

Parameter	Υ'	Υ''	Υ'''
$M - M_\Upsilon$ (MeV)	559.5 ± 0.3 (CLEO)	890.7 ± 0.5 (CLEO)	1113.0 ± 4.0
$n\,^{2S+1}L_J$	2^3S_1	3^3S_1	4^3S_1
$\Gamma(\Upsilon(nS)\to e^+e^-)$ (keV)	0.54 ± 0.03	0.39 ± 0.02 ± 0.03 (CLEO)	0.275 ± 0.06
$BR(\Upsilon(n,S)\to e^+e^-)$ (%)	1.6 ± 1.0 (CLEO) 2.0 ± 0.4 *)	~ 1.9 (CLEO)	$(1.9 \pm 0.8)\cdot 10^{-3}$
Γ_{tot} ($\Upsilon(nS)$) (keV)	27.3 ± 4.7	17.5 ± 3.5	
$\Gamma(\Upsilon(nS)\to 3g)$ (keV)	13.0 ± 3.4	8.7 ± 2.6	
$BR(\Upsilon(nS)\to\Upsilon\pi^+\pi^-)$ (%)	19.2 ± 2.6	4.9 ± 0.9 ± 0.5	
$\langle n^{ch}\rangle$	8.93 ± 0.26		12.0 ± 0.4 $\langle n^{ch}\rangle_{off}$=8.2±0.4
$\langle n^{\Lambda+\bar\Lambda}\rangle$	0.31 ± 0.03	0.17 ± 0.06	-0.07 ± 0.10
$\langle n^{p+\bar p}\rangle$	0.41 ± 0.08	0.38 ± 0.10	0.29 ± 0.19

Note.

*) $\Gamma_{\Upsilon'}$ is determined by formula $\Gamma_{\Upsilon'} = \left[\frac{\Gamma(\Upsilon'\to e^+e^-)}{BR(\Upsilon\to\mu^+\mu^-)} + \Gamma(\Upsilon'\to\gamma P)\right] \times (1 - BR(\Upsilon'\to\Upsilon\pi\pi))^{-1}$

where $\Gamma(\Upsilon'\to\gamma P)$ is estimated by $\frac{\Gamma(\Upsilon'\to\gamma P)}{\Gamma(\psi'\to\gamma P)} \approx \left(\frac{Q_b}{Q_c}\right)^2 \frac{m_c^2}{m_b^2}$.

Table 8:

Semileptonic branching ratio $B \to \ell \nu_\ell X$ [26]

BR $(B \to e \nu_e X)$	BR $(B \to \mu \nu_\mu X)$
0.127 ± 0.017 ± 0.013 (CLEO)	0.122 ± 0.017 ± 0.031 (CLEO)
0.131 ± 0.012 ± 0.020 (CUSB)	0.15 ± 0.035 ± 0.035 (TASSO)
0.11 ± 0.03 ± 0.02 (MARK II)	0.093 ± 0.029 ± 0.020 (MARK J)
0.136 ± 0.05 ± 0.04 (TASSO)	

Table 9:

Bound $b\bar{b}$ states in the potential model [77].
For each level its position relative to $\Upsilon(1^3S_1)$ is indicated (in MeV).

Levels	nonrelativ. approximation	Corrections $\sim (v/c)^2$ accounted	Experiment [25,62]
3^3D_1	1278	1231	
4^3S_1	1204	1160	1113 ± 4
4^1S_0		1133	
3^3P_2		1091	
3^3P_1	1120	1074	
3^3P_0		1037	
3^1P_1		1081	
2^3D_1	1026	991	
3^3S_1	928	895	890.7 ± 0.5
3^1S_0		864	
2^3P_2		820	804 ± 5
2^3P_1	838	801	790 ± 5
2^3P_0		761	773 ± 8
2^1P_1		809	
1^3D_1	734	710	
2^3S_1	585	560	560 ± 0.3
2^1S_0		520	
1^3P_2		478	
1^3P_1	483	456	
1^3P_0		407	
1^1P_1		465	
1^1S_0	0	-101	

Table 10:

The ratios $\Gamma\,(n^3S_1 \to e^+e^-)/\,\Gamma\,(1^3S_1 \to e^+e^-)$ in the potential models

State	Buchmüller and Tye, ref. 29)	Martin, ref. 103)	Experiment, ref. 18)
2^3S_1, Υ'	0.44	0.51	0.46 ± 0.02
3^3S_1, Υ''	0.32	0.35	0.34 ± 0.02
4^3S_1, Υ'''	0.26	0.27	0.23 ± 0.02

Table 11:

Radiative decay rates in charmonium (Γ in keV)

Decay mode	ref. 107)	Potential model 77)		Experiment 18, 109)
		nonrelat. approx.	with corr. $\sim(v/c)^2$	
$\psi' \to \chi_0\,\gamma$	8	45	19	21 ± 4
$\psi' \to \chi_1\,\gamma$	31	40	31	19 ± 4
$\psi' \to \chi_2\,\gamma$	31	27	27	17 ± 4
$\chi_0 \to J/\psi\,\gamma$	108	121	128	100 ± 4 0
$\chi_1 \to J/\psi\,\gamma$	160	250	270	< 700
$\chi_2 \to J/\psi\,\gamma$	136	362	347	330 ± 16 5

Table 12:

Radiative transitions in bottonium[77)]

Transition	Nonrel. value of < r > (in Fermi)	J	< r > (Fermi) corrections $\sim(v/c)^2$ accounted	$\Gamma_{rad.}$, (keV)	E_γ (keV)	Radiative decay BR, Experiment
$3^3P_J \to 3^3S_1$	0.47	0	0.55	8.0	141	
		1	0.48	12	177	
		2	0.44	14	194	
$3^1P_J \to 3^1S_0$			0.39	15	215	
$1^3P_J \to 1^3S_1$	0.064	0	0.026	0.69	466	
		1	0.055	3.9	501	
		2	0.067	6.1	518	
$3^1P_1 \to 2^1S_0$			0.069	7.7	546	
$3^3S_1 \to 2^3P_J$	-0.52	0	-0.42	1.3 BR	133	$BR(3^3S_1 \to 2^3P_J)=$
		1	-0.49	1.9 (3S→→2P)=	94	=(33±3±3)%
		2	-0.53	1.8 =(29	75	25)
$3^1S_0 \to 2^1P_1$			-0.54	1.3 ±5)%	55	
$2^3P_J \to 2^3S_1$	0.34	0	0.39	12	199	
		1	0.35	16	238	
		2	0.32	17	257	
$2^1P_1 \to 2^1S_0$			0.28	18	285	
$2^3P_J \to 1^3S_1$	0.051	0	0.022	2.1	733	
		1	0.043	9.3	770	
		2	0.053	15	787	
$2^1P_1 \to 1^1S_0$			0.054	21	870	
$2^3S_1 \to 1^3P_J$	-0.33	0	-0.27	0.78 BR	152	$BR(2^3S_1 \to 1^3P_J)=$
		1	-0.31	1.01 (2S→→1P)=	103	=(12±3) %
		2	-0.33	0.96 =(10	82	18)
$2^1S_0 \to 1^1P_1$			-0.34	0.53 ±2)%	55	
$1^3P_J \to 1^3S_1$	0.19	0	0.20	26	399	
		1	0.20	36	446	
		2	0.20	39	467	
$1^1P_1 \to 1^1S_0$			0.16	46	550	

Table 13:

Widths of the C-even charmonium levels [33,41,109]

	η_c	η_c'	χ_0	χ_1	χ_2
Γ (MeV)	12.4 ± 3.4	<8 (95 % C.L.)	$16.1^{+1.5}_{-1.3}$	$1.7^{+0.3}_{-1.7}$ 0.75 ± 0.3 (a)	$2.1^{+1.0}_{-0.7}$ (b)

Notes.

a) Assuming scaling E1 theory and relation

$\Gamma(\chi_1 \to \gamma_1 J/\psi) = \Gamma(\chi_0 \to \gamma_0 J/\psi)\ (K_{\gamma_1}/K_{\gamma_0})^3$.

b) Γ_{χ_J} values in Table 13 correspond to the inclusive decay $\psi' \to \gamma\, \chi_J$; the exclusive cascade $\psi' \to \gamma\, \chi_2 \to \gamma\gamma\, J/\psi \to \gamma\gamma e^+e^-$ leads to $\Gamma_{\chi_2} = 4 \pm 2$ MeV [109].

Table 14:

Masses of mesons with quark content $b\bar{u}$ (or $b\bar{d}$).
Theoretical uncertainty is about 100 MeV.

Quantum numbers	Mass (GeV)	
	ref. [134]	ref. [135]
0^{-+}, 1^{--}	5.3	5.2
0^{++}, 1^{++}	6.1	6.0

REFERENCES

1. Aubert, J.J. et al. (MIT-BNL Collab.) - Phys.Rev.Lett. 33(1974) 1404; Augustin, J.E. et al. (SLAC-LBL Collab.) - Ibid. 1406.
2. Richter, B. - Nobel Lecture, 1976, in: Les Prix Nobel en 1976, Stockholm (1977) 42; Ting, S.C.C. - Ibid. 76.
3. Zakharov, V.I., Ioffe, B.L.,Okun, L.B. - Uspekhi Fiz.Nauk 117(1975) 227.
4. Vainshtein, A.I. et al. - Ibid. 123(1977) 217.
5. Chau, L.L. - Phys.Rep. (1983) (to be published), Preprint BNL-31859.
6. Mess, K.H., Wiik, B.H. - Preprint DESY 82-011 (1982). Wu, S.L., - Physica Scripta 25(1982) 212.
7. Okun, L.B. - Leptons and Quarks, North Holland (1982).
8. Lederman, L.M. - Scientific American 239(1978) 60.
9. Weinberg, S. - Rev.Mod.Phys. 52(1980) 515; Salam, A. - Ibid. 525; Glashow, Sh.L. - Ibid. 539.
10. Vainshtein, A.I., Zakharov, V.I., Shifman, M.A. - Uspekhi Fiz.Nauk. 131(1980) 537.
11. Azimov, Ya.I., Dolshitzer, Yu.L., Khoze, V.A. - Ibid. 132(1981) 443.
12. Trilling, G. - Phys.Rep. 75(1981) 57.
13. Herb, S.W. et al. (CFS Collab.) - Phys.Rev.Lett. 39(1977) 252; Innes, W.R. et al. (CFS Collab) - Ibid. 1240.
14. Appelquist, T. Politzer, H.D. - Phys.Rev.Lett. 34(1975) 43.
15. Eichten E. et al. - Ibid. 34(1975) 369.
16. Scharre, D.L. - in: Proceedings of the 1981 Int. Symp. on Lepton and Photon Interactions at High Energies, Bonn, August 1981. Ed. by W. Pfeil. Univ. Bonn , p. 163.
17. Silverman, A. - Ibid. 138.
18. Schamberger, D. - Ibid. 217.
19. Bienlein, J.K. - Ibid. 190.
20. Bürger, J. - Ibid. 115.
21. Branson, J.G. - Ibid. 279.
22. Felst, H.E. - Ibid. 703.
23. Treille, D. - Ibid. 750.
24. Fisk, H.E. - Ibid. 703; Foà, L. - Ibid. 775.

25. Bloom, E.D. - Talk at the XXI Int.Conf. on High Energy Physics, Paris, 1982.

26. Kalmus, G. - Talk at the same Conference, Preprint RL 82-090, Rutherford(1982).

27. Rubbia, C. - Talk at the same Conference.

28. Heusch, C.A. - Lectures presented at the SLAC Summer Inst. on Particle Physics, July-August 1981; Preprint SLAC-PUB-2876 (Jan. 1982).

29. Gottfried, K. - in: Proc. of the Int. Symp. on Lepton and Photon Interactions at High Energies, Hamburg (1977);
Jackson, J.D. - in: Proc. of the 1977 European Conf. on Particle Physics, Budapest, 1(1977) 603.
Appelquist, T., Barnet, R.M., Lane, K.D. - Ann.Rev.Nucl.Part.Sci. 28(1978) 387;
Jackson, J.D., Quigg, C., Rosner, J.L. - in: Proc. of the 19th Int. Conf. on High Energy Physics, Tokyo (1978) 824;
Krammer, M., Krasemann, H. - Acta Physica Austr., Suppl. XXI(1979) 259;
Quigg, C., Rosner, J.L. - Phys.Rep. 56(1979) 169;
Eichten, E. et al. - Phys.Rev.Ser.D. 21(1980) 203;
Buchmüller, W., Tye, S. - Phys.Rev.Ser.D. 24(1981) 132.

30. Martin, A. - Talk at the 1981 Europ. Conf. on Particle Physics, Lisbon, July 1981; Preprint Ref. CERN-TH.3612 (Sept. 1981);
Preprint CERN-TH.3397 (1982).

31. Particle Data Group - Phys.Lett.Ser.B. 111(1982) 1.

32. Zholentz, A.A. et al. - Phys.Lett.Ser.B. 96(1980) 214.

33. Oreglia, M. et al. (CB Collab.)-Phys.Rev.Ser.D. 25(1982) 2259.

34. Bloom, E.D. (CB Collab.)- Proc. 1979 Int. Symp. on Lepton and Photon interactions at High Energy, Batavia, Ill., Aug. 23-24 (1979), 92;
Partridge, R. et al. (CB Collab.) - Phys.Rev.Lett. 45 (1980) 1150.

35. Himel, T.M. et al. (MARK II Collab.) - Phys.Rev.Lett. 45 (1980) 1146;
Himel, T.M. - Ph. D. thesis SLAC Report-223 (1979).

36. Edwards, C. et al. (CB Collab.) - Phys. Rev. Lett. 48 (1982) 70.

36a Porter, F.C. et al. (CB Collab.)-17th Rencontre de Moriond, Workshop on New Flavors, Les Arcs, France, Jan. 24-30 (1982) 27.

37. Porter, F.C. (CB Collab.) - Preprint SLAC-PUB-2881, (Feb. 1982).

38. Chanowitz, M. - Phys. Rev.Ser.D 12 (1975) 918;
Okün, L.B., Voloshin, M.B. - Preprint ITEP-95, Moscow (1976).

39. Abrams, G.S. et al. (MARK II Collab.) - Phys.Rev.Lett. 44 (1980) 114;
Ronan, M.T. et al. (MARK I, LGW Collab.) - Ibid. 367;
Aschman, D. (CB Collab.)- SLAC preprint, SLAC-PUB-2550 (1980).

40. Scharre, D.L. et al. (MARK II Collab.) - Phys.Lett.Ser.B. 97 (1980) 329.

41. Königsmann, K.C. (CB Collab.) - Preprint SLAC-PUB-2910, (1982).

42. Edwards, C. et al. (CB Collab.) - Phys.Rev.Ser.D. 25 (1982) 3065.

43. Burke, D.L. (MARK II Collab.) - Phys.Rev.Lett. 49 (1982) 632.

44. Coles, M.W. et al. (MARK II Collab.) - Preprint SLAC-PUB-2916 (1982).

45. Bacino, W. et al. (DELCO Collab.) - Phys.Rev.Lett. 45 (1980) 329.

46. Hiltin, D. - Proceed. of SLAC Summer Inst. on Particle Physics, (1980), SLAC Report No.239, ed. by A. Mosher, Stanford (1981) 67;
Harari, H. - Ibid. 141.

47. Kirkby, J. - Proceed. of the IX Int. Symp. on Lepton and Photon Interactions at High Energies, Batavia, Illinois (1979).

48. Brandelik, R. et al. (DASP Collab.) - Phys.Lett.Ser.B. 80(1979) 412;
Z.Phys.Ser.C. 1(1979) 233.

49. Partridge, R. et al. (CB Collab.) - Phys.Rev.Lett. 47 (1981) 760.

50. Abrams, G.S. et al. (MARK II Collab.) - Phys.Rev.Lett. 44(1980) 10.

51. Vella, E. et al. (MARK II Collab.) - Ibid. 48(1982) 1515.

52. Berger, Ch. et al. (PLUTO Collab.) - Phys.Lett.Ser.B. 76(1978) 243.

53. Darden, C.W. et al. (DASP-2 Collab.) - Ibid. 246.

54. Bienlein, J.K. et al. (DESY-Heidelberg Collab.) - Phys.Lett.Ser.B. 78(1978) 360;
Darden, C.W. et al. (DASP-2 Collab.) - Ibid. 364.

55. Böhringer, T. et al. (CUSB Collab.) - Phys.Rev.Lett. 44(1980) 1111.

56. Andrews, D. et al. (CLEO Collab.) - Ibid. 1108; Ibid. 45, p.219.

57. Niczyporuk, B. et al. (LENA Collab.) - Phys.Rev.Lett. 46(1981) 92;
Phys.Lett.Ser.B. 99(1981) 169.

58. Finocchiaro, G. et al. (CUSB Collab.) - Phys.Rev.Lett. 45(1980) 222.

59. Onuchin, A.P. and Khoze, V.A. - Preprint LNPI-408 (1978) (Lecture at the XIII Winter School of LNPI).

60. Artamonov, A.S. et al. - Preprint IYaF-82-94, Novosibirsk (1982); Phys.Lett. Sec.B. 118 (1982) 225.

61. Albrecht, H. et al. (DASP-2 Collab.) - Phys.Lett.Ser.B. 102(1981) 291.

62. Moneti, G. - Preprint HEPSY (Syracuse Univ.)-1-82 (1982).

63. Brandt, S. (PLUTO Collab.) - Proceed. of the Int.Conf. on High Energy Physics, Geneva, June-July 2(1979) 338.

64. Berger, C. et al. (PLUTO Collab.) - Z.Phys.Ser.C. 8(1981) 101.

65. Green, J. et al. - Phys.Rev.Lett. 49(1982) 617.

66. Alam, M.S. et al. - Phys.Rev.Lett. 49(1982) 357.

67. Azimov, Ya.I., Dokshitzer, Yu.L. and Khoze, V.A. - Yad.Fiz. 36(1982) 1510;
Preprint LNPI N806 (1982).

68. Bartel, W. et al. (JADE Collab.) - Phys.Lett.Ser.B. 114(1982) 71.

69. Novikov, V.A. et al. - Phys.Rep. 41(1978) 1.

70. Voloshin, M.B. - Preprint ITEP-21 (1980).

71. Bertlmann, R.A. - Nucl.Phys.Ser.B. (1982) (to be published).

72. Shifman, M., Vainshtein, A., Voloshin, M., Zakharov, V. - Phys.Lett.Ser.B. 77(1978) 80.

73. Shifman, M., Zakharov, V., Vainshtein, A. - Nucl.Phys.Ser.B. 147 (1979) 385; Ibid. 448.

74. Reinders, L.J., Rubinstein, H.R., Yazaki,S. - Ibid. 186 . 109.

75. Callan, C., Dashen R., Gross, D. - Phys.Rev.Ser.D. 19(1979) 1826; Shuryak, E. - Phys.Lett.Ser.B. 79(1978) 135.

76. Voloshin, M.B. - Nucl.Phys.Ser.B. 154(1979) 365.

77. McClary, R., Byers, N. - Preprint UCLA/82/TEP/12, Los Angeles (1982).

78. Crewther, R. - Phys.Rev.Lett. 28(1972) 1421;
Chanowitz, M., Ellis, J. - Phys.Lett.Ser.B. 40(1972) 397;
Collins, J., Duncan A., Joglekar, S. - Phys.Rev.Ser.D. 16(1977) 438.

79. Vainshtein, A.I. et al. -ZhETF Pis'ma 27(1978) 60.

80. Eidelman, S. et al. - Phys.Lett.Ser.B. 82(1979) 278.

81. Bertlmann, R.A. - Acta Phys.Austr. 53(1981) 305.

82. Voloshin, M. - Preprint ITEP-54 (1979).

83. Leutwyler, H. - Phys.Lett.Ser.B. 98(1981) 447.

84. Voloshin, M. - Preprint ITEP-30 (1981).

85. Quigg, C. and Rosner, J.L. - Phys.Lett.Ser.B. 72(1978) 462.

86. Pakvasa, S. et al. - Phys.Rev.Ser.D. 20(1979) 2862;
Goggi G., Penso, G. - Nucl.Phys.Ser.B. 165(1980) 429;
Buchmüller, W. and Tye, S., Ref.29.

87. Kühn, J.H. - Acta Physica Polonica Ser.B. 12(1981) 347.

88. Buchmüller, W., Ng, Y.J., Tye, S. - Phys.Rev.Ser.D. 24(1982) 3003;
Barbieri R., Gatto, R., Remiddi, E. - Phys.Lett.Ser.B. 106(1982) 497.

89. Voloshin, M.B. - Yad.Fiz. 35(1982) 1016.

90. Morozov, A.Yu. - Ibid. 36, p.1302.

91. Appelquist, T., Georgi, H. - Phys.Rev.Ser.D. 8(1973) 4000;
Zee, A. - Ibid. 4038.

92. Poggio, E., Quinn, H., Weinberg, S. - Ibid. 13(1976) 1958.

93. Novikov, V. et al. - Phys.Rev.Lett. 38(1977) 626;
Phys.Lett.Ser.B. 67(1977) 409.

94. Ioffe, B.L. - Nucl.Phys.Ser.B. 188(1981) 817; Ibid. 591;
Chung, Y. et al. - Nucl.Phys.Ser.B. 197(1982) 55;
Belyaev, V.M., Ioffe, B.L. - Preprint ITEP-59 (1982).

95. Ioffe, B.L., Smilga, A.V. - Preprint ITEP-100 (1982);
Nesterenko, V.A., Radyushkin, A.V. - ZhETP Pis'ma 35(1982) 395.

96. Eletsky, V.L., Ioffe, B.L., Kogan, Ya.I. - Preprint ITEP-98 (1982).

97. Frankfurt, L.L. and Khoze, V.A., in: Materials of the X LNPI Winter School, Leningrad II(1975) 196;
Azimov, Ya.I., Frankfurt, L.L., Khoze, V.A., in: Materials of the XII LNPI Winter School, Leningrad I(1977) 72.

98. Voloshin, M.B. - Yad.Fiz. 29(1979) 1368.

99. Voloshin, M., Zakharov, V. - DESY Internal-Report F15-80/03 (1981).

100. Buras, A.J. - Proceed. 1981 Intern. Symp. on Lepton and Photon Interactions at High Energies, Bonn,(August 1981) 636.

101. Bardeen, W.A. et al. - Phys.Rev.Ser.D. 18(1978) 3998.

102. Reinders, L.J., Rubinstein, H.R., Yazaki, S. - Preprint RL-82-017 (1982).

103. Martin, A. - Phys.Lett.Ser.B. 100(1981) 511.

104. Bramon, A., Etim, E., Greco, M. - Ibid. 41(1972) 609;
Sakurai, J.J. - Ibid. 46(1973) 207.

105. Krammer, M., Leal-Perreira, P. - Rev.Bras.Fis. 6(1976) 7;
Quigg, C., Rosner, J. - Phys.Rev.Ser.D. 17(1978) 2364;
Bell, J.S., Pasupathy, A. - Phys.Lett.Ser.B. 83(1979) 389.

106. Shifman, M.A. - Yad.Fiz. 36(1982) 1290.

107. Khodjamirian, A. - Erevan preprint EFI-427 (34)-80 (1980).

108. Jackson, J.D. - Phys.Rev.Lett. 37(1976) 1107.

109. Gaiser, J.E. - Preprint SLAC-PUB-2887 (1982).

110. Shifman, M. - Z.Phys.Ser.C. 4(1980) 345.

111. Shifman, M., Vysotsky, M. - Ibid. 10. 131.

112. Novikov, V. et al. - Nucl.Phys.Ser.B. 191(1981) 301.

113. Mackenzie, P.B., Lepage, G.P. - Phys.Rev.Lett. 47(1981) 1244.

114. Barbieri, R. et al. -Nucl.Phys. Ser.B.154(1979) 535;
Hagiwara, K. et al. - Ibid. 177(1981) 461.

115. Barbieri, R., Caffo, M., Gatto R., Remiddi, E. - Phys.Lett.Ser.B. 95(1981) 93.

116. Gottfried, K. - Phys.Rev.Lett. 40(1978) 538.

117. Voloshin, M. - Nucl.Phys.Ser.B. 154(19) 365;
Peskin, M. - Ibid. 156. 365.
Bhanot G., Peskin M. - Ibid. 391.

118. Yan, T.M. - Phys.Rev.Ser.D. 22(1980) 1652.

119. Shizuya, K. - Ibid. 23. 1180.

120. Voloshin, M., Zakharov, V. - Phys.Rev.Lett. 45(1980) 688.

121. Kuang, Y.-P., Yan, T.M. - Phys.Rev.Ser.D. 24(1981) 2874.

122. Novikov, V., Shifman, M. - Z.Phys.Ser.C. 8(1981) 43.

123. Ioffe, B.L., Shifman, M.A. - Phys.Rev.Lett.Ser.B. 95(1980) 99.

124. Billoire, A. et al. - Ibid. 80. 381.

125. Ishikawa, K. - Phys.Rev.Lett. 46(1981) 978;
Chanowitz, M. - Ibid. 981;
Donoghue, J.F., Johnson, K., Li, B.A. - Phys.Lett.Ser.B. 99(1981) 416.

126. Schnitzer, H.J. - Brandeis preprint (1981).

127. Schechter, J. - Preprint 82-12, Göteborg (1982).

128. Novikov, V. et al. - Nucl.Phys.Ser.B. 165(1980) 55.

129. Witten, E. - Ibid. 156. 269.
Veneziano, G. - Ibid. 159. 213.
Dyakonov, D.I., Eides, M. - Preprint LNPI-639, Leningrad (1981).

130. Novikov, V. et al. - Phys.Lett.Ser.B. 86(1979) 347.

131. Goldberg, H. - Phys.Rev.Lett. 44(1980) 363.

132. Milton, K.A. et al. - Phys.Rev.Ser.D. 22(1980) 1647.

133. Parisi, G., Petronzio, R. - Phys.Lett.Ser.B. 94(1980) 51.

134. Reinders, L.J., Rubinstein, H.R., Yazaki S. - Ibid. 104(1981) 305.

135. Shuryak, E.V. - Nucl.Phys.Ser.B. 198(1981) 83.

136. Gell-Mann, M., Oakes, R., Renner, B. - Phys.Rev. 175(1968) 2195.

137. Eichten, E. et al. - Ref.29.

138. Zhizhin, E.D., Lomonosova, T.A., Nikitin, Yu.P., Khoze, V.A. - Yad.Fiz. 36(1982) 930.

139. Jaffe, R.L., Kiskis, J. - Phys.Rev.Ser.D. 13(1976) 1355;
Donoghue, J.F., Golowich, E. - Ibid. 14, p.1386.

140. Dulyan, L.S., Khodjamirian, A.Y. - Preprint EFI-410(17)-80, (1980).

141. Leville, J.P. - Proc.of CLEO Collab. Workshop, Preprint CLNS 51/505, CLEO 81/05 (1981); UM, HE 81-11 (1981).

142. Peccei, R.D., Rückl, R. - Preprint MPI-PAE/PTh 75-81 (1981);
Altarelli, G., Maiani, L. - Preprint CERN-TH-3367 (1982).

143. Bander, M., Silverman, D., Soni, A. - Phys.Rev.Lett. 44(1980) 7, Erratum 44(1980) 962.

144. Chernyak, V.L., Zhitnitsky, A.R. - Nucl.Phys.Ser.B. 201(1982) 492.

145. Khodjamirian, A.Y. - Preprint EFI-508 (51), Erevan (1981).

146. Barbellini, G. et al. - Preprint DESY 79/27, May (1979);
Ali, A. - Preprint DESY 81-060, September (1981).

147. Anselm, A.A., Uraltsev, N.G. - in: Materials of the XVII LNPI Winter School, Leningrad I (1982) 81.
Ellis, J. - Les Houches Lectures (1981).

148. Glashow, S.L., Illiopoulos, J., Maiani, L. - Phys.Rev.Ser.D. 2(1970) 1285.

149. Kobayashi, M., Maskawa, T. - Progr.Theor.Physics 49(1973) 652.

150. Shrock, R.E., Wang, L.L. - Phys.Rev.Lett. 41(1978) 1692;
Shrock, R.E., Treiman, S.B., Wang, L.L. - Phys.Rev.Lett. 42(1979) 1589.

151. Fritsch, H. - Phys.Lett. Ser.B. 86(1979) 164; Ibid. 343.

152. Ellis, J., Gaillard, M.K., Nanopoulos, D.V., Rudaz, S. - Nucl.Phys.Ser.B. 131(1977) 285.

153. Maiani,L. - Proc. of the 1977 Int.Symp. on Lepton and Photon Interactions at High Energies, Hamburg, ed. by F. Gutbrod (1977) 867.

154. Glashow, S.L. - Nucl.Phys. 22(1961) 579;
Weinberg, S. - Phys. Rev.Lett. 19(1967) 1264;
Salam, A. - Elem. Particle Theory, ed. N. Swartholm (Almquist and Wiksell, Stockholm, 1968) 367.

155. Azimov, Ya.I., Dokshitzer, Yu. L., Khoze, V.A. - in: Materials of the XVI LNPI Winter School, Leningrad I (1981) 2C;
Ioffe, B.L., Khoze, V.A. - Sov.J.Part.Nucl. 9(1978) 50.

156. Skovpen, Yu.I. and Khriplovich, I.B. - Yad.Fiz. 30(1971) 602.

157. Wilczek, F. - Phys.Rev.Lett. 39(1977) 1304.

158. Linde, A.D. - ZhETF Pis'ma 23(1976) 73;
Weinberg, S. - Phys.Rev.Lett. 36 (1976) 294.

159. Weinberg, S. - Phys.Rev.Lett. 40(1978) 223.

160. Wilczek, F. - Phys.Rev.Lett. 40(1978) 279.

161. Edwards, C. et al. (CB Collab.) - Phys.Rev.Lett. 48(1982) 903.

162. Faissner, et al. - Phys.Rev.Lett.Ser.B. 103(1981)234.

163. Dokshitzer, Yu.L., Dyakonov, D.I., Troyan, S.I. - Phys.Rep. 58(1980) 270.

164. Azimov, Ya.I., Dokshitzer, Yu.L., Khoze, V.A.- in: Materials of the XVII LNPI Winter School, Leningrad I (1982) 162.

165. Khoze, V.A., Azimov, Ya.I., Frankfurt, L.L. - Proceed. of the XVII Int. Conf. on High Energy Physics, Tbilisi II(1976) 810, Dubna (1977);
Preprint LNPI-222 (1976);
Suzuki, M. - Phys.Lett.Ser.B (1977) 139;
Bjorken, J.D. - Phys.Rev.Ser.D. 17(1978) 171.

166. Dokshitzer, Yu.L. - ZhETF 71(1977) 1216.

167. Vainshtein, A.I. - ZhETF Pis'ma 24(1976) 376;
Parisi, G., Petronzio, R. - Phys.Lett.Ser.B. 62(1976) 331.

168. Azimov, Ya.I., Dokshitzer, Yu.L., Khoze, V.A. - Yad.Fiz. 34(1981) 1130.

169. Barbieri, R., Gatto, R., Kögerler, R. - Phys.Lett.Ser.B. 60(1974) 183;
Jackson, J.D. - Preprint TH 2730-CERN, August (1979).

170. Ellis, J., Karliner, L. - Nucl.Phys.Ser.B. 148(1979) 141;
Ellis, J., Gaillard, M.K., Ross, G.L. - Ibid. 111. 253.

171. Koller, K., Krasemann, H. - Phys.Lett.Ser.B. 88(1979) 119;
Koller, K., Krasemann, H., Walsh, J.F. - Z.Phys.Ser.C. 1( ) 71.

172. Berger, Ch. et al. (PLUTO Collab.) - Phys.Lett.Ser.B. 82(1979) 449;
Berger, Ch. et al. (PLUTO Collab.) - Phys.Lett.Ser.B. 78(1978) 176.

173. Krammer, M. - Phys.Lett.Ser.B. 74(1978) 361.

174. Brandelik, et al. (TASSO Collab.) - Phys.Lett.Ser.B. 89(1980) 418.

175. Dokshitzer, Yu.L., Fadin, V.S., Khoze, V.A. - Preprint LNPI-745 (1982), No.789 (1982); Phys.Lett.Ser.B. 115(1982) 242.

176. Ioffe, B.L.- in: Materials of the XIII LNPI Winter School, Leningrad II (1978) 84.

177. Braunschweig, W. et al., (DASP Collab.) - Phys.Lett.Ser.B. 63(1976) 115;
Brandelik,R., et al. (DASP Collab.) - Nucl.Phys.Ser.B. 148(1979) 189.

178. Hagiwara, T. - Phys.Lett.Ser.B. 84(1979) 465.

179. Lipatov, L.N., Khoze, V.A.- in: Materials of the X LNPI Winter School, Leningrad II (1978) 84.

180. Baier, V.N., Fadin, V.S., Khoze, V.A., Kuraev, E.A. - Phys.Rep. 78(1981) 293.

181. Goggi, G., Penso, G. - Nucl.Phys.Ser.B. 165(1980) 429.

182. Buchmüller, W., Tye, S. - in Ref.29.