DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 82-025
May 1982

APPLICATION OF GEGENBAUER INTEGRATION METHOD TO e^+e^- ANNIHILATION PROCESS

by

B. Lampe and G. Kramer

II. Institut für Theoretische Physik der Universität Hamburg

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 82-025
May 1982

Application of Gegenbauer Integration Method to e^+e^- Annihilation Process

B. Lampe and G. Kramer

II. Institut für Theoretische Physik der Universität Hamburg

Abstract: We apply the Gegenbauer expansion method to the evaluation of integrals encountered in virtual higher order QCD corrections to e^+e^- annihilation 3-jet cross sections. With these integrals the cross sections $\sigma = \sigma_U + \sigma_L$ and σ_L are calculated.

1. Introduction

The usefulness of evaluating momentum-space integrals associated with Feynman diagrams by expanding propagators in terms of Gegenbauer polynomials is well known [1]. We apply this technique to the calculation of higher order virtual QCD corrections to e^+e^- annihilation into three jets. These higher order corrections have been computed previously by two groups [2] independently. In these papers the integrals encountered in the evaluation of Feynman integrals were obtained by employing Feynman parametrization to combine propagator denominators [3]. In this paper we work out these integrals employing Gegenbauer expansions. This way we obtain an independent check on these earlier calculations. We shall see that the Gegenbauer method is quite straightforward and leads us to the final result rather easily. After doing the integrals the necessary trace calculations for obtaining the three-jet cross sections were also repeated. This provides a further check of the results already reported. In addition we calculated the longitudinal part to the three-jet cross section.

The outline of the paper is as follows. In section 2 we collect all the contributions to the $O(\alpha_s^2)$ corrections to the cross section $e^+e^- \to q\bar{q}g$ from which the necessary momentum space integrals can be read off. Then in section 3 the evaluation of the loop integrals with the Gegenbauer method is described. Section 4 contains the final formula for the cross section $\sigma = \sigma_U + \sigma_L$ and for σ_L separately.

2. Virtual Corrections to $q\bar{q}g$ Final State.

In this section we give the explicit expressions for the α_s^2 ($\alpha_s = g^2/4\pi$, g = quark-gluon coupling) contributions to the cross section of

$$e^+(p_+) + e^-(p_-) \to q(p_1) + \bar{q}(p_2) + g(p_3) \qquad (2.1)$$

The symbols in brackets denote the momenta of the particles in the initial and final state respectively. The quarks are assumed to be massless.

The diagrams contributing to (2.1) in order α_s^2 are shown in Fig. 2. The cross section summed over all polarizations is given by the product of the first order diagrams in Fig. 1 and the diagrams in Fig. 2. For our calculations we use the Feynman gauge.

As is well known the higher order diagrams in Fig. 2 have ultraviolet as well as infrared and collinear divergences. We regulate these divergences by continuing the dimensionality of spacetime to arbitrary $n = 4 - 2\varepsilon$ [4]. For general n the cross section for the process (2.1) is obtained from ($q = p_+ + p_-$)

$$d\sigma = \frac{e^4}{2q^6} L^{\mu\nu} H_{\mu\nu} \cdot \text{(phase space)} \qquad (2.2)$$

where (phase space) is the 3-particle phase space in n dimensions. $L_{\mu\nu}$ is the lepton tensor and $H_{\mu\nu}$ the hadron tensor including summations over final spin, colour and flavour together with the quark charge factors.

$H_{\mu\nu}$ must be calculated for arbitrary dimension n before it is multiplied with the lepton tensor $L^{\mu\nu}$. The formula (2.2) is quite general and contains also the various angular correlations between the final state particles and the direction of the incoming beam [5].

These various contributions can easily be disentangled by expanding $L_{\mu\nu}$ into products of polarization vectors of the incoming virtual photon: $\varepsilon_\mu^{(\pm)} = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$ for the transverse polarization and $\varepsilon_\mu^{(0)} = (0, 0, 0, 1)$ for the longitudinal polarization in the rest system of the photon $\vec{q} = 0$. This expansion is:

$$L_{\mu\nu} = p_{+\mu} p_{-\nu} + p_{-\mu} p_{+\nu} - \frac{q^2}{2} g_{\mu\nu}$$

$$= \mathcal{U}\left(\epsilon_\mu^{(+)}\epsilon_\nu^{(+)*} + \epsilon_\mu^{(-)}\epsilon_\nu^{(-)*}\right) + \mathcal{L}\,\epsilon_\mu^{(0)}\epsilon_\nu^{(0)}$$

$$+ \mathcal{T}\left(\epsilon_\mu^{(+)}\epsilon_\nu^{(-)*} + \epsilon_\mu^{(+)*}\epsilon_\nu^{(-)}\right) + \mathcal{I}\left(\epsilon_\mu^{(+)}\epsilon_\nu^{(0)} + \epsilon_\mu^{(+)*}\epsilon_\nu^{(0)}\right) \qquad (2.3)$$

The coordinate system x,y,z is determined by the momenta of the final state $q\bar{q}g$. With respect to this system the positron momentum as $\hat{p}_+ = (-\sin\theta\cos\chi, \sin\theta\sin\chi, \cos\theta)$. Then

$$\mathcal{U} = \frac{q^2}{4}(1 + \cos^2\theta)$$

$$\mathcal{L} = \frac{q^2}{2}\sin^2\theta \qquad (2.4)$$

$$\mathcal{T} = \frac{q^2}{4}\cos 2\chi \sin^2\theta$$

$$\mathcal{I} = \frac{q^2}{4\sqrt{2}}\cos\chi \sin 2\theta$$

With (2.3) we can write for the product of lepton and hadron tensor

$$L_{\mu\nu}H^{\mu\nu} = \mathcal{U}\,\mathcal{H}_U + \mathcal{L}\,\mathcal{H}_L + \mathcal{T}\,\mathcal{H}_T + \mathcal{I}\,\mathcal{H}_I \qquad (2.5)$$

where

$$\mathcal{H}_U = \epsilon_\mu^{(+)} H^{\mu\nu} \epsilon_\nu^{(+)*} + \epsilon_\mu^{(-)} H^{\mu\nu} \epsilon_\nu^{(-)*} \qquad (2.6)$$

is the unpolarized transverse part of H and

$$\mathcal{H}_L = \epsilon_\mu^{(0)} H^{\mu\nu} \epsilon_\nu^{(0)} \qquad (2.7)$$

is the longitudinal part of H and similar for H_T and H_I. If (2.5) is integrated over angles θ and χ only H_U and H_L remain:

$$\int \frac{d\cos\theta\, d\chi}{4\pi} L_{\mu\nu} H^{\mu\nu} = \frac{q^2}{3}\left(\mathcal{H}_U + \mathcal{H}_L\right) \qquad (2.8)$$

Therefore the integrated cross section with all correlation angles integrated out is obtained by replacing $L^{\mu\nu}$ in (2.2) by

$$\frac{q^2}{3}\left(\epsilon_\mu^{(+)}\epsilon_\nu^{(+)*} + \epsilon_\mu^{(-)}\epsilon_\nu^{(-)*} + \epsilon_\mu^{(0)}\epsilon_\nu^{(0)}\right) = \frac{q^2}{3}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \qquad (2.9)$$

Since $q^\mu H_{\mu\nu} = 0$ the sum of the unpolarized transverse and the longitudinal cross section $\sigma = \sigma_U + \sigma_L$ follows from replacing $L_{\mu\nu}$ in (2.2) by $(-\frac{q^2}{3} g_{\mu\nu})$. The longitudinal cross section σ_L is obtained by substituting $\frac{q^2}{3}\varepsilon_\mu^{(0)}\varepsilon_\nu^{(0)}$ instead of $L_{\mu\nu}$ in (2.2). Of course σ_L depends on the choice of coordinate system. This will be specified later.

Then the three-jet distribution up to order α_s has the following simple form [2]

$$\frac{d^2\sigma}{dy_{13}\,dy_{23}} = \sigma^{(0)} \frac{\alpha_s(\mu^2)}{2\pi} C_F \left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} \frac{1}{\Gamma(1-\varepsilon)}$$

$$\Theta(1 - y_{13} - y_{23})\left(y_{13}\, y_{23}\,(1 - y_{13} - y_{23})\right)^{-\varepsilon} B(y_{13}, y_{23}) \qquad (2.10)$$

where

$$\mathcal{B}(y_{13}, y_{23}) = \mathcal{B}^V(y_{13}, y_{13}) - \varepsilon\, \mathcal{B}^S(y_{13}, y_{23})$$

$$\mathcal{B}^V(y_{13}, y_{23}) = \frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} + \frac{2y_{12}}{y_{13}\, y_{23}}$$

$$\mathcal{B}^S(y_{13}, y_{23}) = \frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} + 2 \qquad (2.11)$$

We introduced the variables $y_{ij} = \frac{s_{ij}}{q^2} = 2p_i p_j/q^2$ so that $y_{12} + y_{13} + y_{23} = 1$. Furthermore $C_F = \frac{4}{3}$ and μ is an arbitrary parameter to define the coupling constant g in such a way that g is dimensionless for arbitrary n: $g \to g\mu^{\varepsilon}$. $B(y_{13}, y_{23})$ is the full three-jet matrix element squared for arbitrary dimension n. $\sigma^{(2)}$ is the cross section for $q\bar{q}$ production for arbitrary n:

$$\sigma^{(2)} = \sigma_0 \left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} \frac{\Gamma(2-\varepsilon)}{\Gamma(2-2\varepsilon)} \qquad (2.12)$$

with

$$\sigma_0 = \frac{4\pi\alpha^2}{3q^2} N_c \sum_{i=1}^{N_f} Q_i^2 \qquad (2.13)$$

being the $q\bar{q}$ cross section for $n = 4$. N_c is the number of colours, N_f the number of flavours and Q_i the quark charge with flavour i.

In the following we write down the contributions of the various diagrams to $H_{\mu\nu}$ in order to exhibit the momentum space integrals which will be calculated with the Gegenbauer technique in the next section.

We start with the first two diagrams in Fig. 2. They consist of a quark self-energy insertion. They are multiplied with the two lowest order diagrams in Fig. 1. Then the first diagram in Fig. 2 produces the term

$$F_{\mu\nu}^{(1)} = (i t^a t^b t^a t^b + f^{abc} t^b t^c t^a) \int d^n k \, \frac{k_\varrho}{k^2 (k - p_1 - p_2)^2}$$

$$Sp\Big\{ \not{p}_1 \gamma_\beta (\not{p}_1 + \not{p}_3) \gamma_\alpha \gamma^\varrho \gamma^\alpha (\not{p}_1 + \not{p}_3) \gamma_\mu \not{p}_2 \gamma^\beta (\not{p}_2 + \not{p}_3) \gamma_\nu \frac{1}{s_{13}^2 s_{23}}$$

$$- \not{p}_1 \gamma_\beta (\not{p}_1 + \not{p}_3) \gamma_\alpha \gamma^\varrho \gamma^\alpha (\not{p}_1 + \not{p}_3) \gamma_\mu \not{p}_2 \gamma^\nu (\not{p}_1 + \not{p}_3) \gamma^\beta \frac{1}{s_{13}^3} \Big\} \qquad (2.14)$$

t^a denote the colour matrices and repeated indices have to be summed.

In (2.14) we left out a factor which is common to all diagrams. The contribution of the second diagram in Fig. 2, called $F_{\mu\nu}^{(2)}$, is of similar structure and will not be written down here.

The third and the fourth diagram have a vertex insertion for the quark-photon vertex. The contribution from the third diagram is

$$F_{\mu\nu}^{(3)} = (i t^a t^b t^a t^b + f^{abc} t^b t^c t^a) \int d^n k \, \frac{(q-k)_\sigma k_\varrho}{k^2 (k-p_2)^2 (k-q)^2}$$

$$Sp\Big\{ \not{p}_1 \gamma_\beta (\not{p}_1 + \not{p}_3) \gamma_\alpha \gamma^\sigma \gamma_\mu \gamma^\varrho \gamma^\alpha \not{p}_2 \gamma_\nu (\not{p}_1 + \not{p}_3) \gamma^\beta \frac{1}{s_{13}^2}$$

$$- \not{p}_1 \gamma_\beta (\not{p}_1 + \not{p}_3) \gamma_\alpha \gamma_\sigma \gamma_\mu \gamma^\varrho \gamma^\alpha \not{p}_2 \gamma^\beta (\not{p}_2 + \not{p}_3) \gamma_\nu \frac{1}{s_{13}\, s_{23}} \Big\}$$

(2.15)

$F_{\mu\nu}^{(4)}$, the contribution of the fourth diagram, looks very similar to $F_{\mu\nu}^{(3)}$. $F_{\mu\nu}^{(5)}$ and $F_{\mu\nu}^{(6)}$ are the expressions coming from the fifth and sixth diagram in Fig. 2 which have a vertex insertion at the quark-gluon vertex. $F_{\mu\nu}^{(5)}$ is:

$$F_{\mu\nu}^{(5)} = i\, t^a t^b t^a t^b \int d^n k \, \frac{(k-p_3)_\sigma k_\rho}{k^2 (k-p_3)^2 (k-p_1-p_3)^2}$$

$$Sp\Big\{ \not{p}_1 \gamma_\alpha \gamma^\sigma \gamma_\beta \gamma^\rho \gamma^\alpha (\not{p}_1+\not{p}_3) \gamma_\mu \not{p}_2 \gamma^\beta (\not{p}_2+\not{p}_3) \gamma_\nu \frac{1}{S_{13} S_{23}}$$

$$- \not{p}_1 \gamma_\alpha \gamma^\sigma \gamma_\beta \gamma^\rho \gamma^\alpha (\not{p}_2+\not{p}_3) \gamma_\mu \not{p}_2 \gamma_\nu (\not{p}_1+\not{p}_3) \gamma^\beta \frac{1}{S_{13}^2} \Big\} \tag{2.16}$$

$F_{\mu\nu}^{(7)}$ and $F_{\mu\nu}^{(8)}$ stand for the contribution of the box diagrams in Fig. 2 which have quark-gluon couplings only (QED box). The former one has the following form

$$F_{\mu\nu}^{(7)} = i\, t^a t^b t^a t^b \int d^n k \, \frac{(p_1-k)_\sigma (p_1+p_3-k)_\rho (p_2+k)_\lambda}{k^2 (k+p_2)^2 (k-p_1)^2 (k-p_1-p_2)^2}$$

$$Sp\Big\{ \not{p}_1 \gamma_\alpha \gamma^\sigma \gamma_\beta \gamma^\rho \gamma_\mu \gamma^\lambda \gamma^\alpha \not{p}_2 \gamma_\nu (\not{p}_1+\not{p}_3) \gamma^\beta \frac{1}{S_{13}}$$

$$- \not{p}_1 \gamma_\alpha \gamma^\sigma \gamma_\beta \gamma^\rho \gamma_\mu \gamma^\lambda \gamma^\alpha \not{p}_2 \gamma^\beta (\not{p}_2+\not{p}_3) \gamma_\nu \frac{1}{S_{23}} \Big\} \tag{2.17}$$

$F_{\mu\nu}^{(8)}$ is of similar form. The next three diagrams in Fig. 2, which yield $F_{\mu\nu}^{(i)}$ (i = 9, 10, 11) contain the three-gluon vertex. $F_{\mu\nu}^{(9)}$ and $F_{\mu\nu}^{(10)}$ come from the diagrams with vertex insertion at the quark-gluon vertex. $F_{\mu\nu}^{(9)}$ is

$$F_{\mu\nu}^{(9)} = f^{abc} t^b t^c t^a \int d^n k \, \frac{(p_1+p_3-k)_\sigma \left[(2p_3-k)_\lambda g_{\alpha\beta} + (2k-p_3)_\beta g_{\alpha\lambda} - (p_3+k)_\alpha g_{\beta\lambda} \right]}{k^2 (k-p_3)^2 (k-p_1-p_3)^2}$$

$$Sp\Big\{ \not{p}_1 \gamma^\alpha \gamma^\sigma \gamma^\lambda (\not{p}_1+\not{p}_3) \gamma_\mu \not{p}_2 \gamma^\beta (\not{p}_2+\not{p}_3) \gamma_\nu \frac{1}{S_{13} S_{23}}$$

$$- \not{p}_1 \gamma^\alpha \gamma^\sigma \gamma^\lambda (\not{p}_1+\not{p}_3) \gamma_\mu \not{p}_2 \gamma_\nu (\not{p}_1+\not{p}_3) \gamma^\beta \frac{1}{S_{13}^2} \Big\} \tag{2.18}$$

and a similar expression for $F_{\mu\nu}^{(10)}$. $F_{\mu\nu}^{(11)}$ comes from the box diagram with the three-gluon coupling (QCD box). It is

$$F_{\mu\nu}^{(11)} = f^{abc} t^b t^c t^a \int d^n k \, (p_1-k)_\sigma (p_2+p_3+k)_\tau$$

$$\frac{\left[(p_3-k)_\rho g_{\beta\lambda} + (2k+p_3)_\beta g_{\lambda\rho} - (2p_3+k)_\lambda g_{\rho\beta} \right]}{k^2 (k-p_1)^2 (k+p_2+p_3)^2 (k+p_3)^2}$$

$$Sp\Big\{ \not{p}_1 \gamma^\lambda \gamma^\sigma \gamma_\mu \gamma^\tau \gamma^\rho \not{p}_2 \gamma_\nu (\not{p}_1+\not{p}_3) \gamma^\beta \frac{1}{S_{13}}$$

$$- \not{p}_1 \gamma^\lambda \gamma^\sigma \gamma_\mu \gamma^\tau \gamma^\rho \not{p}_2 \gamma^\beta (\not{p}_2+\not{p}_3) \gamma_\nu \frac{1}{S_{23}} \Big\} \tag{2.19}$$

The remaining diagrams in Fig. 2 are absorbed in the counter term which renormalizes the quark-gluon coupling g. It will not be derived here and is taken from earlier work [2].

Let us denote the total contribution of the first eleven diagrams by $F_{\mu\nu}$. We split $F_{\mu\nu}$ into two terms according to whether they are proportional to the colour factor

$$t^a t^b t^a t^b = C_F \left(C_F - \frac{N_c}{2} \right) \tag{2.20}$$

or

$$f^{abc} t^b t^c t^a = i\, C_F \frac{N_c}{2} \tag{2.21}$$

The decomposition is:

$$F_{\mu\nu} = C_F \left(C_F - \frac{N_c}{2} \right) \hat{F}_{\mu\nu} + \frac{1}{2} C_F N_c \tilde{F}_{\mu\nu} \tag{2.22}$$

The contributions to $\hat{F}_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are easily derived from the formulas for the $F_{\mu\nu}^{(i)}$ given above. They yield

$$\hat{F}_{\mu\nu} = \sum_{i=1}^{8} F_{\mu\nu}^{(i)}$$

$$\tilde{F}_{\mu\nu} = \sum_{i=1}^{4} F_{\mu\nu}^{(i)} + \sum_{i=8}^{11} F_{\mu\nu}^{(i)} \tag{2.23}$$

The expressions of the Feynman diagrams contain five types of integrals which are evaluated by the Gegenbauer expansion technique in the next section.

3. Evaluation of Loop Integrals.

For the computation of diagrams written down in the last section we need the following integrals

$$A_\rho = \int d^n k \, \frac{k_\rho}{k^2 (k-p)^2} \tag{3.1}$$

$$B_\rho = \int d^n k \, \frac{k_\rho}{k^2 (k-p_1)^2 (k-p_1-p_3)^2} \tag{3.2}$$

$$B_{\rho\sigma} = \int d^n k \, \frac{k_\rho k_\sigma}{k^2 (k-p_1)^2 (k-p_1-p_3)^2} \tag{3.3}$$

$$C = \int d^n k \, \frac{1}{k^2 (k+p_2)^2 (k-p_1)^2 (k-p_1-p_3)^2} \tag{3.4}$$

We shall demonstrate the method with the simplest diagram of the type (3.1) which is

$$A = \int d^n k \, \frac{1}{k^2 (k-p_1)^2} \tag{3.5}$$

For using the Gegenbauer expansion technique the integrals must be analytically continued into the Euclidean region. This is easily acomplished by letting the zero component of each external and internal line aquire the same phase factor i. Then the external lines are all spacelike, so that it will be necessary to analytically continue the external momenta back to the timelike region. This will be done after the evaluation of the integrals. In the Euclidean region A is

$$A = i \int d^n k \, \frac{1}{k^2 (k-p)^2} \tag{3.6}$$

Having obtained the Euclidean integrand, we introduce four-dimensional spherical coordinates for the loop momentum and expand in the integrand each propagator in a series of Gegenbauer polynomials [6]:

$$\frac{1}{[(p-k)^2]^\rho} = \frac{\Gamma(1-\varepsilon)}{(pk)^\rho \, \Gamma(\rho)} \sum_{j=0}^{\infty} \frac{\Gamma(j+\rho)}{\Gamma(j+1-\varepsilon)} \, T^{j+\rho}(p,k) \, {}_2F_1(\rho-1+\varepsilon, \rho+j, j+2-\varepsilon, T^2(p,k)) \, C_j^{1-\varepsilon}(\hat{p}\hat{k}) \tag{3.7}$$

$C_j^{1-\varepsilon}$ are the $n = 4-2\varepsilon$ dimensional hyperspherical harmonics (Gegenbauer polynomials), $T(p,k) = \min(\frac{p}{k}, \frac{k}{p})$, $\hat{p}$, $\hat{k}$ are unit vectors along p and k and p and k denote the lenght of p and k on the right side of (3.7). The properties of the $C_j^{1-\varepsilon}$ needed in this paper are summarized in appendix A.

Substituting (3.7) with $\rho = 1$ into (3.5) the angular integral becomes trivial and

$$A = \frac{2i\pi^{2-\varepsilon}}{\Gamma(2-\varepsilon)\, p} \int_0^\infty dk \, k^{-2\varepsilon} \, T(p,k) \, {}_2F_1(\varepsilon, 1, 2-\varepsilon, T^2(p,k)) \tag{3.8}$$

The next step is to expand the hypergeometric function into the well known power series (see appendix B). Then the radial integral is easily done and

$$A = \frac{i\pi^{2-\varepsilon}}{\Gamma(\varepsilon)} \, p^{-2\varepsilon} \sum_{j=0}^{\infty} \left(\frac{1}{j+1-\varepsilon} + \frac{1}{j+\varepsilon} \right) \frac{\Gamma(j+\varepsilon)}{\Gamma(j+2-\varepsilon)} \tag{3.9}$$

The infinite sum over j in (3.9) can be performed with the summation formula in appendix B. If applied to (3.9) this is

$$\sum_{j=0}^{\infty} \frac{\Gamma(j+\varepsilon)}{\Gamma(j+2-\varepsilon)} \left(\frac{1}{j+1-\varepsilon} + \frac{1}{j+\varepsilon} \right) = \frac{(\Gamma(\varepsilon))^2 (\Gamma(1-\varepsilon))^2}{\Gamma(2-2\varepsilon)} \tag{3.10}$$

With this we get for A:

$$A = \frac{i\pi^{2-\varepsilon}}{p^{2\varepsilon}} \; \frac{\Gamma(\varepsilon)\,(\Gamma(1-\varepsilon))^2}{\Gamma(2-2\varepsilon)} \tag{3.11}$$

which can easily be continued into Minkowski space

$$A = \frac{i\pi^{2-\varepsilon}}{(-p^2)^\varepsilon} \; \frac{\Gamma(1+\varepsilon)\,(\Gamma(1-\varepsilon))^2}{\varepsilon \; \Gamma(2-2\varepsilon)} \tag{3.12}$$

We see that A has a pole in ε. It agrees with earlier evaluations utilysing different methods [2,6].

The computation of A_ρ proceeds along the same line. We quote just the final result:

$$A_\rho = \frac{i\pi^{2-\varepsilon}}{(-p^2)^{\varepsilon}} \frac{\Gamma(1+\varepsilon)\,\Gamma(2-\varepsilon)\,\Gamma(1-\varepsilon)}{\varepsilon\,\Gamma(3-2\varepsilon)}\, p_\rho \tag{3.13}$$

Next we calculate B_ρ and $B_{\rho\sigma}$. The method for doing these integrals can be demonstrated more easily with the scalar integral

$$B = \int d^n k \frac{1}{k^2\,(k-p_1)^2\,(k-p_1-p_3)^2} \tag{3.14}$$

In (3.14) we encounter two propagators which can be expanded in hyperspherical harmonics. But this leads to complicated double sums. It is more economical to combine two propagators with the standard Feynman parameter method. Then B is, now in Euclidean space

$$B = -i\int_0^1 dx \int d^n k \frac{1}{k^2\left[(k-p_1-xp_3)^2 + p_3^2\,x(1-x)\right]^2} \tag{3.15}$$

For the special case $p_3^2 = 0$ - this is actually needed only - the integral (3.15) can be treated by the Gegenbauer expansion method in the same way as we did it for the integral A. The result, which we obtained after integration over angles, the lenght k and x, is:

$$B = \frac{i\pi^{2-\varepsilon}}{\varepsilon\,\Gamma(1+\varepsilon)} \frac{1}{2p_1p_3} \left\{ (p_1^2+2p_1p_3)^{-\varepsilon} + (p_1^2)^{-\varepsilon} \right\}$$

$$\sum_{j=0}^{\infty} \frac{\Gamma(j+2)\,\Gamma(j+1+\varepsilon)}{j!\,\Gamma(j+2-\varepsilon)} \left[\frac{1}{j+1-\varepsilon} + \frac{1}{j+1+\varepsilon}\right] \tag{3.16}$$

The sum over j can be done with the summation formula in appendix B. The final result, continued to Minkowski-space, is:

$$B = \frac{i\pi^{2-\varepsilon}\,\Gamma(1+\varepsilon)\,(\Gamma(1-\varepsilon))^2}{\varepsilon^2\,\Gamma(1-2\varepsilon)} \frac{1}{2p_1p_3} \left\{(-p_1^2-2p_1p_3)^{-\varepsilon} - (-p_1^2)^{-\varepsilon}\right\} \tag{3.17}$$

We remind the reader that (3.17) is valid only for $p_3^2 = 0$.

The calculation of the more complicated integrals B_ρ and $B_{\rho\sigma}$ is done with the same technique. The needed angular integrals are given in appendix A. The final summation over j which comes from the expansion of the hypergeometric function can again be performed with the general summation formula in appendix B. The result, continued to Minkowski-space is:

$$B_\rho = \frac{i\pi^{2-\varepsilon}\,\Gamma(1+\varepsilon)\Gamma(1-\varepsilon)\Gamma(2-\varepsilon)}{\varepsilon\,\Gamma(2-2\varepsilon)} \frac{1}{2p_1p_3}$$

$$\left\{ \frac{1}{\varepsilon}\left(p_{1\rho} - \frac{p_1^2}{2p_1p_3}\,p_{3\rho}\right) f(p_1^2, p_1p_3) + \frac{1}{1-\varepsilon}\, p_{3\rho}\, g(p_1^2, p_1p_3)\right\} \tag{3.18}$$

and

$$B_{\varrho\sigma} = \frac{i\pi^{2-\varepsilon}\,\Gamma(1+\varepsilon)\,\Gamma(1-\varepsilon)\,\Gamma(3-\varepsilon)}{\varepsilon\;\Gamma(3-2\varepsilon)}\;\frac{1}{2p_1p_3}$$

$$\Big\{-\frac{1}{(1-\varepsilon)(2-\varepsilon)}\;g_{\varrho\sigma}\;p_1p_3\;g(p_1^2,p_1p_3) + \frac{1}{\varepsilon}\;p_{1\varrho}p_{1\sigma}\,f(p_1^2,p_1p_3)$$

$$+\,(p_{1\varrho}p_{3\sigma}+p_{3\varrho}p_{1\sigma})\Big(\frac{1}{1-\varepsilon}\;g(p_1^2,p_1p_3) - \frac{1}{\varepsilon}\,\frac{p_1^2}{2p_1p_3}\;f(p_1^2,p_1p_3)\Big)$$

$$-\,p_{3\sigma}p_{3\varrho}\Big(\frac{1}{2-\varepsilon}\;h(p_1^2,p_1p_3) + \frac{1}{1-\varepsilon}\,\frac{p_1^2}{p_1p_3}\;g(p_1^2,p_1p_3) - \frac{1}{\varepsilon}\Big(\frac{p_1^2}{2p_1p_3}\Big)^2 f(p_1^2,p_1p_3)\Big)\Big\} \qquad (3.19)$$

where we introduced the functions f, g and h:

$$f(p_1^2,p_1p_3) = (-p_1^2-2p_1p_3)^{-\varepsilon} - (-p_1^2)^{-\varepsilon}$$

$$g(p_1^2,p_1p_3) = \frac{1}{2p_1p_3}\Big[(-p_1^2-2p_1p_3)^{1-\varepsilon} - (-p_1^2)^{1-\varepsilon}\Big]$$

$$h(p_1^2,p_1p_3) = \frac{1}{(2p_1p_3)^2}\Big[(-p_1^2-2p_1p_3)^{2-\varepsilon} - (-p_1^2)^{2-\varepsilon}\Big] \qquad (3.20)$$

The integrals B, B_ρ and $B_{\rho\sigma}$ are evaluated for general p_1^2. We need them for $p_1^2 = 0$ only.

We remark that the results obtained so far are valid for all ε for which the original integrals exist. For later applications we need only the terms proportional to ε^{-2}, ε^{-1} and ε^{0}. They are deduced from the well-known expansion of the Γ function and the other functions which appear.

The last integrals to be calculated is C which is needed for the various box diagrams. C has four denominators. For applying the Gegenbauer expansion method in the same way as for the simpler integral A we combine just two denominators by introducing one Feynman parameter respectively. This gives

$$C = i\int d^nk \int_0^1 dx \int_0^1 dy\;\frac{1}{k^4(k-p_1-xp_2-yp_3)^4} \qquad (3.21)$$

In (3.21) we assumed $p_1^2 = p_2^2 = p_3^2 = 0$. Now we can use the expansion (3.7). Then, after integration over angles and k:

$$C = -\,\frac{2i\pi^{2-\varepsilon}\,(\Gamma(1-\varepsilon))^2\,\Gamma(2+\varepsilon)}{\varepsilon\;\Gamma(1-2\varepsilon)}\int_0^1 dx\int_0^1 dy\;p^{-4-2\varepsilon} \qquad (3.22)$$

where $p = p_1 + xp_2 + yp_3$. To derive (3.22) we used again the summation formula (B.3). The integration over x and y can be done easily with the help of standard formulae [7]. The final result for C, continued to Minkowski-space, is:

$$C = -\,2i\pi^{2-\varepsilon}\,\frac{(\Gamma(1-\varepsilon))^2\,\Gamma(1+\varepsilon)}{\varepsilon^2\;\Gamma(1-2\varepsilon)}\;\frac{1}{S_{12}\,S_{13}}$$

$$\Big\{(-S_{12}-S_{13}-S_{23})^{-\varepsilon}\,{}_2F_1\Big(1,-\varepsilon,1-\varepsilon,-\frac{S_{23}(S_{12}+S_{13}+S_{23})}{S_{12}\,S_{13}}\Big)$$

$$-\,(-S_{13})^{-\varepsilon}\,{}_2F_1\Big(1,-\varepsilon,1-\varepsilon,-\frac{S_{23}}{S_{12}}\Big) - (-S_{12})^{-\varepsilon}\,{}_2F_1\Big(1,-\varepsilon,1-\varepsilon,-\frac{S_{23}}{S_{13}}\Big)\Big\}$$

$$(3.23)$$

The hypergeometric functions in (3.23) have the power series expansion

$${}_2F_1(1,-\varepsilon,1-\varepsilon,z) = 1 - \varepsilon\sum_{j=1}^{\infty}\frac{1}{j-\varepsilon}\,z^j \qquad (3.24)$$

This series is the expansion of a logarithm generalized to $n = 4-2\varepsilon$ dimensions. If expanded in a power series in ε we obtain

$$\ell\ell^{n}(\varepsilon, z) \equiv \sum_{j=1}^{\infty} \frac{1}{j-\varepsilon} z^{j} = -\ln(1-z) + \sum_{j=1}^{\infty} \varepsilon^{j} \mathcal{L}_{1+j}(z) \qquad (3.25)$$

where $\mathcal{L}_n(z)$ is the generalized dilogarithm

$$\mathcal{L}_n(z) = \sum_{j=1}^{\infty} \frac{z^j}{j^n} \qquad (3.26)$$

Therefore C can be expressed by logarithms and generalized dilogarithms.

With (3.25) and (3.26) the terms $\sim \varepsilon^{-2}$, $\sim \varepsilon^{-1}$ and $\sim \varepsilon^{0}$ in (3.23) can be obtained. The result agrees with the formula in [3] where a completely different method had been used. We remark that our result (3.23) is valid for time and space-like q^2 whereas the method in [3] was limited to time-like q^2.

This completes the evaluation of the integrals needed to evaluate the various terms in section 2.

4. Cross Sections.

The loop integrals derived in the last section are substituted into the various $F_{\mu\nu}^{(i)}$ terms written down explicitly in section 2. Then the traces have been processed with the help of Schoonship [8]. In intermediate stages of the calculation expressions become quite lengthly. The final result has, however, a relatively compact form. First we report the result for $\sigma = \sigma_U + \sigma_L$, the sum of the unpolarized transverse and the longitudinal cross section which is obtained by contracting (2.22) with $(-\frac{q^2}{3} g_{\mu\nu})$ in n-dimensional space. The result is written in the following form:

$$\frac{d^2\sigma}{d y_{13}\, d y_{23}} = \sigma^{(1)} \frac{\alpha_s(\mu^2)}{2\pi} C_F \left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} \frac{1}{\Gamma(1-\varepsilon)}$$

$$\Theta(1-y_{13}-y_{23}) \left(y_{13}\, y_{23}\, (1-y_{13}-y_{23})\right)^{-\varepsilon} T(y_{13}, y_{23}) \qquad (4.1)$$

where T is decomposed as

$$T = C_F \left(C_F - \frac{N_C}{2}\right) \hat{T} + \frac{1}{2} C_F N_C \tilde{T} \qquad (4.2)$$

in complete analogy to (2.22), so that the contributions to the two colour factors are given separately. For $\hat{T}$ we obtain:

$$\hat{T} = \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} \frac{\alpha_s(\mu^2)}{2\pi} \Big\{ B(y_{13}, y_{23}) \Big[-\frac{2}{\varepsilon^2}$$

$$+ \frac{1}{\varepsilon}\left(2\ln y_{12} - 3\right) - 8 + \frac{2\pi^2}{3} - \ln^2 y_{12} \Big]$$

$$+ 4 \ln y_{12} \left[\frac{2 y_{12}}{y_{13}+y_{23}} + \frac{y_{12}^2}{(y_{13}+y_{23})^2} \right]$$

$$+\ln y_{13}\left[\frac{4y_{12}}{y_{12}+y_{23}}+\frac{2y_{13}}{y_{12}+y_{23}}-\frac{y_{13}y_{23}}{(y_{12}+y_{23})^2}\right]$$

$$+\ln y_{23}\left[\frac{4y_{12}}{y_{12}+y_{13}}+\frac{2y_{23}}{y_{12}+y_{13}}-\frac{y_{13}y_{23}}{(y_{12}+y_{23})^2}\right]$$

$$-2r(y_{12},y_{13})\frac{y_{12}^2+(y_{12}+y_{23})^2}{y_{13}y_{23}}-2r(y_{12},y_{23})\frac{y_{12}^2+(y_{12}+y_{13})^2}{y_{13}y_{23}}$$

$$+\frac{y_{12}}{y_{12}+y_{13}}+\frac{y_{12}}{y_{12}+y_{23}}+\frac{4y_{12}}{y_{13}+y_{23}}-\frac{y_{12}}{y_{13}}-\frac{y_{12}}{y_{23}}-\frac{y_{13}}{y_{23}}-\frac{y_{23}}{y_{13}}\Big\} \tag{4.3}$$

Here $B(y_{13},y_{23})$ was defined in (2.11) and the function $r(x,y)$ is defined as

$$r(x,y)=\ln x\ln y-\ln x\ln(1-x)-\ln y\ln(1-y)$$
$$-\mathcal{L}_2(x)-\mathcal{L}_2(y)+\frac{\pi^2}{6} \tag{4.4}$$

Similarly $\tilde{T}$ is:

$$\tilde{T}=\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{\alpha_s(\mu^2)}{2\pi}\Big\{B(y_{13},y_{23})\Big[-\frac{4}{\varepsilon^2}$$

$$+\frac{1}{\varepsilon}\left(2\ln y_{13}+2\ln y_{23}-3\right)-8+\frac{4\pi^2}{3}-\ln^2 y_{13}-\ln^2 y_{23}-2r(y_{13},y_{23})\Big]$$

$$+\ln y_{13}\left[4\,\frac{y_{12}+y_{13}}{y_{12}+y_{23}}-\frac{y_{13}y_{23}}{(y_{12}+y_{23})^2}\right]$$

$$+\ln y_{23}\left[4\,\frac{y_{12}+y_{23}}{y_{12}+y_{13}}-\frac{y_{13}y_{23}}{(y_{12}+y_{13})^2}\right]$$

$$+\frac{y_{12}}{y_{12}+y_{13}}+\frac{y_{12}}{y_{12}+y_{23}}+\frac{y_{12}}{y_{13}}+\frac{y_{12}}{y_{23}}+\frac{y_{13}}{y_{23}}+\frac{y_{23}}{y_{13}}\Big\} \tag{4.5}$$

This is our final result for the virtual correction to the differential cross section $d^2\sigma/dy_{12}dy_{23}$. It agrees with the result in [2] if we add the counter term in the MS [9] renormalization scheme which contributes to T the term

$$T^{(ct)}=\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}B(y_{13},y_{23})\left(\tfrac{1}{3}N_f-\tfrac{11}{6}N_c\right)$$
$$\left(\frac{1}{\varepsilon}+\gamma-\ln\frac{4\pi\mu^2}{q^2}\right) \tag{4.6}$$

This term takes into account the last six diagrams in Fig. 8 through the renormalization of the quark-gluon coupling constant as was explained in detail in [2].

We remark that the flavour dependence, the terms proportional to N_f, come in through the counter term (4.6) only, i.e. the renormalization diagrams in Fig. 2. This contribution is also responsible for the terms which are absorbed in the running coupling constant $\alpha_s(q^2)$.

In order to calculate the longitudinal part of the cross section we must specify the z direction in the system $\vec{q}=0$. First we identify the z direction with the direction of the quark momentum $\vec{p}_1$. Then $\varepsilon_\mu^{(o)}=p_{1\mu}/p_{1o}-q_\mu/q_o$ for $\vec{q}=o$, so that we can write $p_{1\mu}/p_{1o}$ for $\varepsilon_\mu^{(o)}$ in (2.7). In lowest order the longitudinal cross section is in this case:

$$\frac{d^2\sigma^{(1)}}{dy_{13}dy_{23}}=\sigma^{(a)}\frac{\alpha_s(\mu^2)}{2\pi}C_F\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{1}{\Gamma(2-\varepsilon)}$$
$$\Theta(1-y_{13}-y_{23})\left(y_{13}y_{23}(1-y_{13}-y_{23})\right)^{-\varepsilon}B_L^{(1)}(y_{13},y_{23}) \tag{4.7}$$

where

$$B_L^{(1)}(y_{13},y_{23})=\frac{2y_{12}}{(1-y_{23})^2}(1-\varepsilon) \tag{4.8}$$

The virtual $O(\alpha_s^2)$ contribution can be written again in the form (4.7) with the function $B_L^{(1)}$ replaced by $T_L^{(1)}$ which we decompose as (4.2):

$$T_L^{(1)} = C_F\left(C_F - \frac{N_C}{2}\right)\hat{T}_L^{(1)} + \frac{1}{2}C_F N_C \tilde{T}_L^{(1)} \tag{4.9}$$

Then we obtained:

$$\begin{aligned}\hat{T}_L^{(1)} = & \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{\alpha_s(\mu^2)}{2\pi} B_L^{(1)}(y_{13}, y_{23})\Big[-\frac{2}{\varepsilon^2}\\ & + \frac{1}{\varepsilon}\left(2\ln y_{12} - 3\right) - 9 + \frac{2\pi^2}{3} - \ln^2 y_{12}\\ & + 2\ln y_{23} + \frac{2y_{13}}{y_{13}+y_{23}}\\ & + 2\ln y_{12}\left(\frac{y_{13}}{y_{13}+y_{23}} + \frac{y_{12}y_{13}}{(y_{13}+y_{23})^2}\right)\\ & - 2r(y_{12}, y_{13}) + 2\frac{y_{12}}{y_{13}} r(y_{12}, y_{23})\Big]\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}\tilde{T}_L^{(1)} = & \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{\alpha_s(\mu^2)}{2\pi} B_L^{(1)}(y_{13}, y_{23})\Big[-\frac{4}{\varepsilon^2}\\ & + \frac{1}{\varepsilon}\left(2\ln y_{13} + 2\ln y_{23} - 3\right) - 7 + \frac{4\pi^2}{3} - \ln^2 y_{13} - \ln^2 y_{23}\\ & + 2\ln y_{23} - 2r(y_{13}, y_{23})\Big]\end{aligned} \tag{4.11}$$

For the calculation of jet variable distributions, as for example thrust distributions, one needs also the longituinal cross section

with the z axis taken along the antiquark $(z\,||\,\vec{p}_2)$ or the gluon momentum $(z\,||\,\vec{p}_3)$ [5]. σ_L with $z\,||\,\vec{p}_2$ follows from (4.10) and (4.11) by the interchange of $y_{13} \leftrightarrow y_{23}$. $d^2\sigma_L/dy_{13}dy_{23}$ with $z\,||\,\vec{p}_3$ must be calculated separately. The result, in lowest order, is [5]:

$$\begin{aligned}\frac{d^2\sigma^{(3)}}{dy_{13}\,dy_{23}} = & \sigma^{(2)}\frac{\alpha_s(\mu^2)}{2\pi} C_F\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{1}{\Gamma(2-\varepsilon)}\\ & \Theta(1-y_{13}-y_{23})\left(y_{13}y_{23}(1-y_{13}-y_{23})\right)^{-\varepsilon} B_L^{(3)}(y_{13}, y_{23})\end{aligned} \tag{4.12}$$

where

$$B_L^{(3)}(y_{13}, y_{23}) = \frac{4y_{12}}{(1-y_{12})^2} \tag{4.13}$$

We notice that there is no extra term proportional to ε as in (4.8). The virtual $O(\alpha_s^2)$ contribution is again decomposed in the form (4.9). Then $\hat{T}_L$ and $\tilde{T}_L$ for $z\,||\,\vec{p}_3$ have the following form:

$$\begin{aligned}\hat{T}_L^{(3)} = & \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{\alpha_s(\mu^2)}{2\pi} B_L^{(3)}(y_{13}, y_{23})\Big[-\frac{2}{\varepsilon^2}\\ & + \frac{1}{\varepsilon}\left(2\ln y_{12} - 3\right) + \frac{\pi^2}{2} - 8 - \ln^2 y_{12} + 2\ln y_{12}\\ & + \frac{y_{13}+y_{23}}{2y_{12}} - \frac{y_{13}^2}{2y_{12}(y_{12}+y_{13})} - \frac{y_{23}^2}{2y_{12}(y_{12}+y_{23})}\\ & + \frac{1}{2}\ln y_{13}\left(\frac{y_{13}y_{23}}{(y_{12}+y_{23})^2} + \frac{2y_{23}}{y_{12}+y_{23}}\right)\\ & + \frac{1}{2}\ln y_{23}\left(\frac{y_{13}y_{23}}{(y_{12}+y_{13})^2} + \frac{2y_{13}}{y_{12}+y_{13}}\right)\\ & - r(y_{12}, y_{13}) - r(y_{12}, y_{23})\Big]\end{aligned} \tag{4.14}$$

$$\tilde{T}_L^{(3)} = \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} \frac{\alpha_s(\mu^2)}{2\pi}\, B_L^{(3)}(y_{13},y_{23})\Big[-\frac{4}{\varepsilon^2}$$

$$+\frac{1}{\varepsilon}\left(2\ln y_{13}+2\ln y_{23}-3\right) - 8 + \frac{4\pi^2}{3} - \ln^2 y_{13} - \ln^2 y_{23}$$

$$+\frac{y_{13}+y_{23}}{2y_{12}} - \frac{y_{13}^2}{2y_{12}(y_{12}+y_{13})} - \frac{y_{23}^2}{2y_{12}(y_{12}+y_{23})}$$

$$+\frac{1}{2}\ln y_{13}\left(\frac{y_{13}y_{23}}{(y_{12}+y_{23})^2} + 2\,\frac{y_{23}-y_{13}}{y_{12}+y_{23}}\right)$$

$$+\frac{1}{2}\ln y_{23}\left(\frac{y_{13}y_{23}}{(y_{12}+y_{13})^2} + 2\,\frac{y_{13}-y_{23}}{y_{12}+y_{23}}\right)$$

$$-\,2\,r(y_{13},y_{23})\Big] \qquad (4.15)$$

We remark that $\hat{T}_L^{(3)}$ and $\tilde{T}_L^{(3)}$ are symmetric in y_{13} and y_{23} as it should be.

The counter terms which come from the renormalization of the quark-gluon coupling constant have the same structure as (4.6) if the renormalization is done in the MS scheme. The only change is that $B(y_{12},y_{23})$ in (4.6) is replaced by $B_L^{(1)}(y_{13},y_{23})$ or $B_L^{(3)}(y_{13},y_{23})$ depending whether $d^{(1)}\sigma_L$ or $d^{(3)}\sigma_L$ has to be renormalized.

This completes our calculation of the higher order virtual corrections to three-jet cross sections with the help of the Gegenbauer expansion method. In order to produce measurable cross sections one must add the infrared and collinear divergent part of the four-jet cross section as was done in the second paper of [3] for σ but not for σ_L, which still has to be done.

Acknowledgement:

We thank the DESY-Rechenzentrum, in particular K. Dahlmann, for their efforts to install the Schoonship routine.

References:

[1] J.L. Rosner, Ann. Phys. (N.Y.) 44, 11 (1967)
M. Levine, R. Roskies, Phys. Rev. D9, 421 (1974)
M. Levine, R. Perisko, R. Roskies, Phys. Rev. D13, 997 (1976)
V.K. Cung, A. Devetom T. Fulton, W. Repko,
Phys. Rev. D18, 3893 (1978)
K.G. Chetyrkin, A.L. Kataev, F.V. Tkachov,
Phys. Lett. 86B, 277 (1977)
W. Celmaster, R. Gonsalves, Phys. Rev. D21, 3112 (1980)
A.E. Terrano, Phys. Lett. 93B, 424 (1980)

[2] R.K. Ellis, D.A. Ross, A.E. Terrano, Nucl. Phys. B178, 421 (1981)
K. Fabricius, G. Kramer, G. Schierholz, I. Schmidt,
Z. Physik C11, 315 (1982)

[3] K. Fabricius, I. Schmidt, Z. Physik C3, 51 (1979)

[4] G. 't Hooft, M. Veltman, Nucl. Phys. B44, 189 (1972)
C.G. Bollini, J.J. Giambiagi, Nuovo Cim. 12B, 20 (1972)
W.J. Marciano, Phys. Rev. D12, 3861 (1975)

[5] G. Kramer, G. Schierholz, J. Willrodt,
Phys. Lett. 78B, 249 (1978), erratum ibid. 80B, 433 (1979)

[6] A.E. Terrano, Phys. Lett. 93B, 424 (1980)

[7] I.S. Gradshteyn, I.M. Ryzhik, Table of Integrals,
Series and Products, Academic Press, 1980

[8] H. Strubbe, Com. Phys. Commun. 8, 1 (1974)

[9] G. 't Hooft, M. Veltman, Nucl. Phys. B61, 455 (1973)

[10] A. Erdelyi, Higher Transcendental Functions Vol. I
New York (1953)

Appendix A

Orthogonality relation:

$$\int d\Omega_b \, C^{\lambda}_{j_1}(\hat{a}\hat{b}) \, C^{\lambda}_{j_2}(\hat{b}\hat{c}) = \delta_{j_1 j_2} \frac{2\pi^{\lambda+1}}{\Gamma(1+\lambda)} \, \frac{\lambda}{j_1+\lambda} \, C^{\lambda}_{j_1}(\hat{a}\hat{c}) \tag{A.1}$$

Special cases:

$$C_0^{\lambda}(x) = 1 \, , \quad C_1^{\lambda}(x) = 2\lambda x \, , \quad C_j^{\lambda}(1) = \frac{\Gamma(j+2\lambda)}{j! \, \Gamma(2\lambda)} \tag{A.2}$$

Angular integrals:

$$\int d\Omega_k \, \hat{k}_\rho \hat{k}_\sigma = \frac{\pi^{2-\varepsilon}}{\Gamma(3-\varepsilon)} \delta_{\rho\sigma} \tag{A.3}$$

$$\int d\Omega_k \, \hat{k}_\rho \hat{k}_\sigma \, C_j^{1-\varepsilon}(\hat{k}\hat{q}) = \frac{\pi^{2-\varepsilon}}{\Gamma(1-\varepsilon)} \left(2 \hat{q}_\rho \hat{q}_\sigma - \frac{1}{(2-\varepsilon)(3-\varepsilon)} \delta_{\rho\sigma} \right) \tag{A.4}$$

Appendix B

Expansion in hyperspherical harmonics ($n = 4-2\varepsilon$) [6]:

$$(a^2 - 2abx + b^2)^{-\varrho} = (ab)^{-\varrho}\frac{\Gamma(1-\varepsilon)}{\Gamma(\varrho)} \sum_{j=0}^{\infty} \frac{(j+1-\varepsilon)\,\Gamma(j+\varrho)}{\Gamma(j+2-\varepsilon)}$$

$$\left(T(a,b)\right)^{\varrho+j} {}_2F_1\left(\varrho-1+\varepsilon,\ \varrho+j,\ j+2-\varepsilon;\ (T(a,b))^2\right) C_j^{1-\varepsilon}(x)$$

(B.1)

Expansion of the hypergeometric function [7]:

$${}_2F_1(a,b,c;z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)} \sum_{j=0}^{\infty} \frac{\Gamma(j+a)\,\Gamma(j+b)}{j!\ \Gamma(j+c)} z^j$$

(B.2)

Summation formula [10]:

$$\sum_{n=0}^{\infty} \frac{\Gamma(a+n)\,\Gamma(b+n)}{n!\ \Gamma(1-b+a+n)} \left(\frac{1}{n+\frac{a}{2}-z} + \frac{1}{n+\frac{a}{2}+z}\right)$$

$$= \frac{\Gamma(\frac{a}{2}-z)\,\Gamma(\frac{a}{2}+z)\,\Gamma(1-b)\,\Gamma(b)}{\Gamma(1-b+\frac{a}{2}-z)\ \Gamma(1-b+\frac{a}{2}+z)}$$

(B.3)

Figure Captions:

Fig. 1: Diagrams with quark, antiquark and gluon in the final state to order α_s.

Fig. 2: Diagrams with $q\bar{q}g$ in the final state to order α_s^2 interfering with the diagrams in Fig. 1.

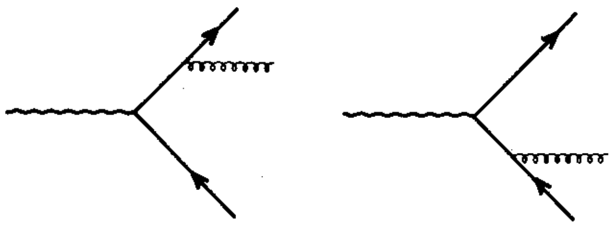

Fig. 1

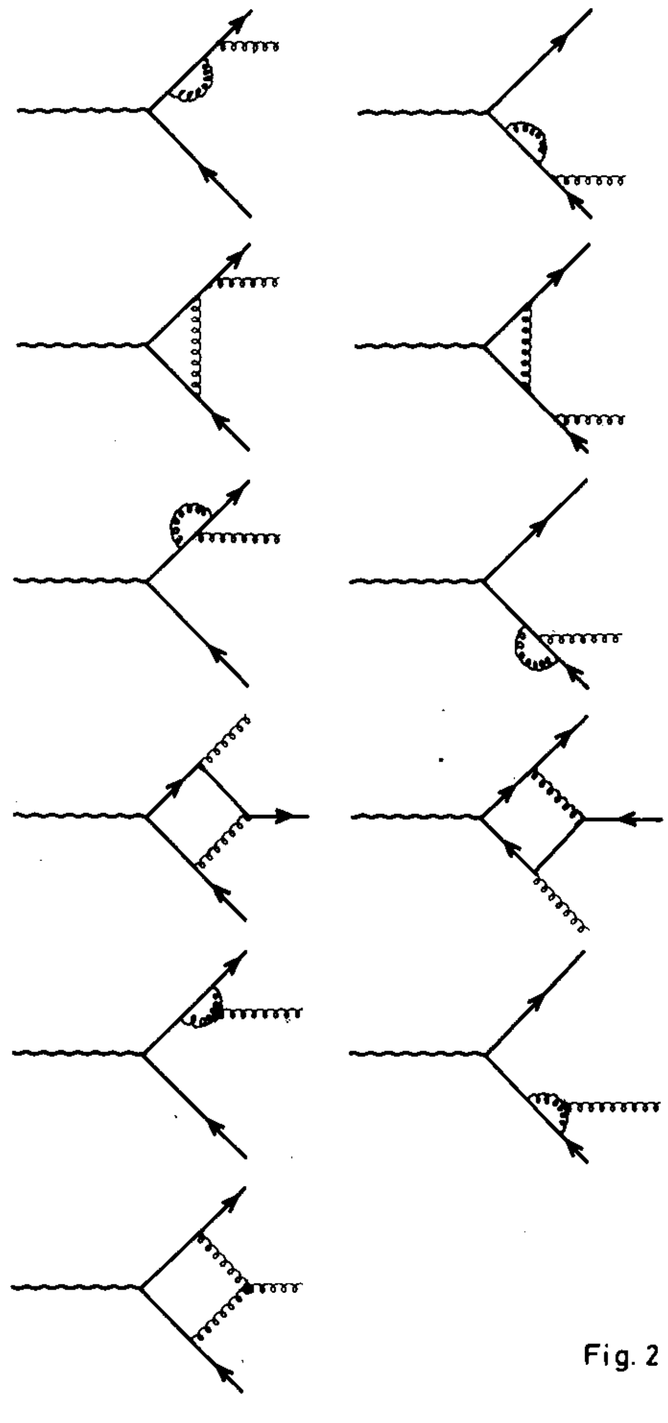

Fig. 2

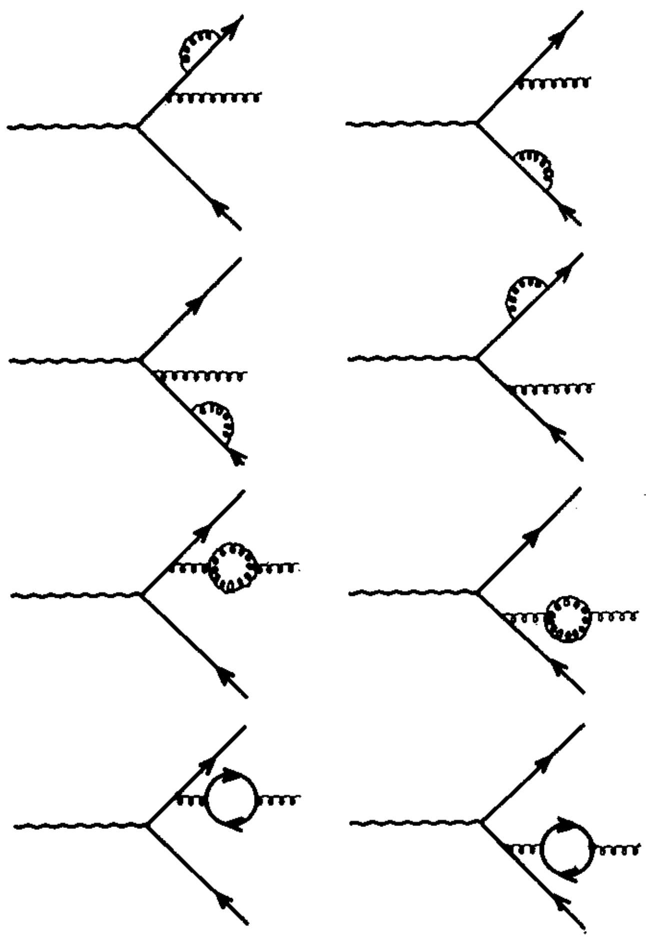

Fig. 2 cont.