Vladimir Chekelian (MPI for Physics, Munich) on behalf of the H1 Collaboration

**New measurements of jets in DIS and extraction of $\alpha_s$ at NNLO**

H1: 0.5 fb$^{-1}$ of the $ep$ collision data with $E_e=27.5$ GeV and $E_p=920/820/575/460$ GeV, $\sqrt{s} = 319/300/252/225$ GeV

Completion of the jet measurements by the H1 collaboration at HERA:

- new multi-jets cross sections measurements in DIS at low $Q^2$
- $\alpha_s$ determination at NNLO using jet measurements in DIS by H1
  H1prelim-17-031

**ep collider**

**HERA I:** 1992-2000

**HERA II:** 2003-2007
Jets in deep-inelastic $ep$ scattering at HERA

**DIS kinematics:**
- $Q^2 = -q^2 = -(e-e')^2$ virtuality
- $x = Q^2 / 2(pq)$ Bjorken $x$
- $y = (pq) / (pe)$ inelasticity

**Breit frame:**

**Jet production in DIS:**
- Defined in the Breit frame
  - (e.g. $k_T$ algorithm with $R=1$)
- Sensitive to $\alpha_s$ already at LO
- Dominated by boson-gluon fusion and directly sensitive to gluon
- Leading order for trijets is $O(\alpha_s^2)$

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New jet measurements in DIS by H1

Inclusive jet, dijet and trijet production cross sections in ep NC DIS at low $Q^2$ ($Q^2 < 100 \text{ GeV}^2$) with scattered electron in Spacal ($E > 10.5 \text{ GeV}$) at high $Q^2$ ($Q^2 > 150 \text{ GeV}^2$) - an extension to the low $P_T$ bin: $5 < P_T^{jet} < 7 \text{ GeV}$

- HERA II data (290 pb$^{-1}$, $\sqrt{s}=319 \text{ GeV}$):
  - in the Breit frame using $k_T$ algorithm with $R=1$
  - as a function of $Q^2$ and $P_T$ at the hadron level
→ also jet cross sections normalised to inclusive NC DIS

<table>
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<th>Application</th>
<th>Low-$Q^2$ extended phase space</th>
<th>Low-$Q^2$ measurement phase space</th>
<th>High-$Q^2$ measurement phase space extension</th>
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<tr>
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<td>Used for event selection and unfolding</td>
<td>Phase space of jet cross sections</td>
<td>Phase space of jet cross sections</td>
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<tr>
<td>NC DIS phase space</td>
<td>$3 &lt; Q^2 &lt; 120 \text{ GeV}^2$</td>
<td>$5.5 &lt; Q^2 &lt; 80 \text{ GeV}^2$</td>
<td>$150 &lt; Q^2 &lt; 15 000 \text{ GeV}^2$</td>
</tr>
<tr>
<td></td>
<td>$0.08 &lt; y &lt; 0.7$</td>
<td>$0.2 &lt; y &lt; 0.6$</td>
<td>$0.2 &lt; y &lt; 0.7$</td>
</tr>
<tr>
<td>Phase space common for all jets</td>
<td>$-1.5 &lt; \eta_{lab}^{jet} &lt; 2.75$</td>
<td>$-1.0 &lt; \eta_{lab}^{jet} &lt; 2.5$</td>
<td>$-1.0 &lt; \eta_{lab}^{jet} &lt; 2.5$</td>
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<tr>
<td></td>
<td>$P_T^{jet} &gt; 3 \text{ GeV}$</td>
<td>$P_T^{jet} &gt; 4 \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>Inclusive jet</td>
<td>$P_T^{jet} &gt; 3 \text{ GeV}$</td>
<td>$4.5 &lt; P_T^{jet} &lt; 50 \text{ GeV}$</td>
<td>$5 &lt; P_T^{jet} &lt; 7 \text{ GeV}$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$(7 &lt; P_T^{jet} &lt; 50 \text{ GeV published in [26])}$</td>
</tr>
<tr>
<td>Dijet</td>
<td>$N_{jet} \geq 2$</td>
<td>$N_{jet} \geq 2$</td>
<td></td>
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<tr>
<td></td>
<td>$\langle P_T^{jet} \rangle_2 &gt; 3 \text{ GeV}$</td>
<td>$5 &lt; \langle P_T^{jet} \rangle_2 &lt; 50 \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>Trijet</td>
<td>$N_{jet} \geq 3$</td>
<td>$N_{jet} \geq 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\langle P_T^{jet} \rangle_3 &gt; 3 \text{ GeV}$</td>
<td>$5.5 &lt; \langle P_T^{jet} \rangle_3 &lt; 40 \text{ GeV}$</td>
<td></td>
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</tbody>
</table>

Asymmetric cuts $\langle P_T^{jet} \rangle_{2,3} \gg P_T^{jet}$ to avoid IR sensitive regions in the theory calculations.
Simultaneous regularised unfolding of inclusive jets, dijets, trijets and NC DIS

Detector effects like migrations, acceptance, efficiency are corrected for in regularised unfolding by minimising

\[ \chi^2(x, \tau) = (y - Ax)^T V^{-1}_y (y - Ax) + \tau L^T L \]

**Migration Matrix**

<table>
<thead>
<tr>
<th>( \vec{e} )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( e_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstructed Trijet events which are not generated as Trijet event</td>
<td>Trijet</td>
<td>( Q^2, &lt;p_T&gt;^2, y, \text{Trijet-cuts} )</td>
<td></td>
</tr>
<tr>
<td>Reconstructed Dijet events which are not generated as Dijet event</td>
<td>Dijet</td>
<td>( Q^2, &lt;p_T&gt;^2, y, \text{Dijet-cuts} )</td>
<td></td>
</tr>
<tr>
<td>Reconstructed jets without match to generator level</td>
<td>Incl. Jet</td>
<td>( p_T, Q^2, y, \eta )</td>
<td></td>
</tr>
<tr>
<td>NC DIS ( Q^2, y )</td>
<td>Hadron level</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \rightarrow \) two times more bins in \( p_T \) - combined later

**Statistical correlations**

- all stat. correlations are provided
- systematics: total & eight correlated unc.
- normalisation/lumi uncertainty - 2.5%
- hadronisation corr. to compare to theory
Control distributions

Inclusive NC DIS

Extended phase space \(\rightarrow\) grey areas

Two NC DIS generators:
- Django (blue) / Rapgap (red) reweighed to describe data well
  - half a difference is assigned to syst.

Background (green): Pythia normalised to bkg enriched sample
  \(\rightarrow\) good overall description of data

Inclusive jets

Dijets

Trijets

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V. Chekelian, Jets in DIS and alpha_s at NNLO
Doble differential dijets cross sections

\[ \sigma(\text{bin}) / \Delta Q^2 \Delta \langle P_T \rangle_2 \]
- as a function of \( Q^2 \) and \( \langle P_T \rangle_2 = (P_{T\text{jet}}^1 + P_{T\text{jet}}^2)/2 \) with \( P_{T\text{jet}}^{1,2} > 4 \text{ GeV} \)

\[ 5.5 < Q^2 < 80 \text{ GeV}^2 \]
\[ 5 < \langle P_T \rangle_2 < 50 \text{ GeV} \]

- compared to calculations at NLO, aNNLO, NNLO (NNPDF3.0, \( \alpha_s(m_Z) = 0.118 \)) multiplied by hadronic corr.

\( \rightarrow \) reasonable description of the dijet data over 4-5 orders of magnitude
Dijets: aNNLO & NNLO calculations

\[ \frac{\sigma}{\sigma_{\text{NLO}}} \]

\begin{align*}
5.5 < Q^2 < 8 \text{ GeV}^2 & \quad 8 < Q^2 < 11 \text{ GeV}^2 & \quad 11 < Q^2 < 16 \text{ GeV}^2 \\
16 < Q^2 < 22 \text{ GeV}^2 & \quad 22 < Q^2 < 30 \text{ GeV}^2 & \quad 30 < Q^2 < 42 \text{ GeV}^2 \\
42 < Q^2 < 60 \text{ GeV}^2 & \quad 60 < Q^2 < 80 \text{ GeV}^2 \end{align*}

**aNNLO** (approximate NNLO)

**NNLO**
Rev.Lett.117(2016)042001

- scale unc. from variation of \( \mu_r \) and \( \mu_f \) by factors 0.5/2, excluding (0.5,2) and (2,0.5)

→ aNNLO and NNLO improve \( P_T \) shape dependence
→ NNLO reduced scale unc. at high \( P_T \) compared to NLO

H1 Dijets

\[ \frac{\sigma}{\sigma_{\text{NLO}}} \]

\[ P_T \] [GeV]

**H1 HERA-II**

Systematic uncertainty

NLO \( \otimes \) hadr. corr.

aNNLO \( \otimes \) hadr. corr.

NNLO \( \otimes \) hadr. corr.
Normalised dijet cross sections divided by $\sigma_{\text{NLO}}$

\[ \sigma_i^{\text{norm}} = \frac{\sigma_i}{\sigma_{\text{NC}}} \]

- jet cross sections
- incl. NC DIS in $Q^2$ bin

- some reduction of exp. unc.
- NNLO overshoots dijet data a bit
- best suited for possible "PDF+$\alpha_s$" fits together with inclusive NC & CC DIS data

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V. Chekelian, Jets in DIS and $\alpha_s$ at NNLO
Double diff. inclusive jet cross sections divided by $\sigma_{NLO}$

$$\sigma(\text{bin}) / \Delta Q^2 \Delta P_T^{\text{jet}}$$

New measurements:
- low $Q^2$: 5.5 - 80 GeV$^2$
  \hspace{1em} 4.5 < P_T < 50 GeV
- high $Q^2$: 150 - 15000 GeV$^2$
  \hspace{1em} 5 < P_T < 7 GeV

Similar to dijets:
- scale unc. from variation
  of $\mu_T$ and $\mu_F$ by factors 0.5/2,
  excluding (0.5,2) and (2,0.5)

→ aNNLO and NNLO
improve $P_T$ shape dependence
→ NNLO
reduced scale unc. at high $P_T$
compared to NLO

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alpha_s at NNLO
Normalised inclusive jet cross sections
divided by $\sigma_{\text{NLO}}$

$$\sigma/\sigma_{\text{NC}} / \Delta P_{T,jet}$$

New measurements:
- low $Q^2$: 5.5 - 80 GeV$^2$
  4.5 < $P_T < 50$ GeV
- high $Q^2$: 150 - 15000 GeV$^2$
  5 < $P_T < 7$ GeV

Similar to dijets:
- scale unc. from variation of $\mu_T$ and $\mu_F$ by factors 0.5/2, excluding (0.5,2) and (2,0.5)

$\rightarrow$ aNNLO and NNLO
improve $P_T$ shape dependence
$\rightarrow$ NNLO
reduced scale unc. at high $P_T$
compared to NLO
Trijet cross sections  \( \langle P_T \rangle_3 = (P_{T\text{jet1}} + P_{T\text{jet2}} + P_{T\text{jet3}})/3 \)

absolute (divided by \( \sigma_{\text{NLO}} \)) normalised

\[
\begin{align*}
\text{H1 Trijets} & \quad \uparrow \text{H1 HERA-II} \\
\text{Systematic uncertainty} & \quad \text{NLO } \otimes \text{ hadr. corr.}
\end{align*}
\]

→ good description of the data by calculations at NLO
→ NNLO is not available yet

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V. Chekelian, Jets in DIS and \( \alpha_s \) at NNLO
Extraction of $\alpha_s$ at NNLO from jet data in DIS


Input jet data in DIS: 5 inclusive jet sets and 4 dijet sets published by H1

Jet cross section & $\alpha_s$-dependence:

$$\sigma_i = \sum_{n=1}^{\infty} \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}^{(n)}_{i,k}(x, \mu_R, \mu_F) \cdot c_{\text{had},i}$$

PDFs ME

NNLO calculations for ep DIS jet production (2016):

Double-real

Real-virtual

Double-virtual

using antenna subtraction technique

Input H1 jet data compared to $\alpha_s$ NNLO fit

5 inclusive jet cross section sets
- data period: 300 GeV / HERA-I / HERA-II
- $Q^2$ range:
  - low-$Q^2$ (5/5.5-100 GeV$^2$)
  - high-$Q^2$ (150-15000 GeV$^2$)
- $P_T$ ranges:
  - 4.5/5/7 < $P_T$ < 50 GeV
- common for all sets:
  - $-1 < \eta_{\text{jet, lab}} < 2.5$, 0.2 < $y$ < 0.7

4 dijet cross section sets
- $\langle P_T \rangle_2$ ranges
  - 5/7 < $\langle P_T \rangle_2$ < 50 GeV
    (m$_{12}$ > 16/18 GeV)
- open points with $\mu = \sqrt{Q^2 + P_T^2} < 2m_b$
  are excluded from $\alpha_s$ NNLO fit
Scale dependence of jet cross sections at NNLO

Scales (renormalisation and factorisation) are chosen to be

\[ \mu_R^2 = \mu_F^2 = Q^2 + P_T^2 \]

- scale dependence by varying multiplicative factors to \( \mu_R, \mu_F \)
in four phase space domains (low & high \( \mu \), incl.jets & dijets)

\( \rightarrow \) reduction of scale dependency at NNLO compared to NLO

\( \rightarrow \) still relevant scale dependence at NNLO at low scales

- \( \mu_F \) dependence small (green band)
Methodology of the $\alpha_s(m_Z)$ determination

The strong coupling constant is determined in a fit of theory to jet data with free parameter $\alpha_s(m_Z)$ by minimizing $\chi^2$ based on log-normal probabilities

$$\chi^2 = \sum_{i,j} \log \frac{S_i}{\sigma_i} (V_{\text{exp}} + V_{\text{had}} + V_{\text{PDF}})_{ij}^{-1} \log \frac{S_j}{\sigma_j}$$

$\zeta=$Data, $\sigma_i=$NNLO, $V=$covariance matrices

- experimental uncertainties (stat. & syst.)
- scale uncertainty (varying multiplicative factors to $\mu_{R,F}$ by 0.5, 2)
- PDF uncertainties (repeating fits without $V_{\text{PDF}}$ in $\chi^2$)
- hadronisation unc. (repeating fits without $V_{\text{had}}$ in $\chi^2$)

**Theory:** $\alpha_s$ dependences of the jet cross sections (factorisation theorem)

$$\sigma_i = \sum_{n=1}^{\infty} \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \tilde{\sigma}^{(n)}_{i,k}(x, \mu_R, \mu_F) \cdot c_{\text{had},i}$$

explicit dependence in hard ME:

$$\tilde{\sigma}^{(n)}_{i,k} = \alpha_s^n(\mu_R) \tilde{\sigma}^{(n)}_{i,k}(x, \mu_R, \mu_F)$$

perturbative expansion in orders of $\alpha_s$

implicit dependence in PDFs:

$$\frac{\partial f}{\partial \alpha_s} = \frac{\mathcal{P} \otimes f}{\beta}$$

splitting kernels $\mathcal{P}$

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s)$$
ME & PDF dependencies on $\alpha_s^\sigma(m_Z)$, $\alpha_s^f(m_Z)$

$$\sigma_i = f(\alpha_s^f(m_Z)) \otimes \hat{\sigma}_i(\alpha_s^\sigma(m_Z)) \cdot c_{\text{had},i}$$

ME: orders of $\alpha_s^{(n)}$

PDF: by integration of

$$\frac{\partial f}{\partial \alpha_s} = \frac{\mathcal{P} \otimes f}{\beta}$$

or

$$\tilde{f}(\mu) = f(\sqrt{K}\mu), \exp\left[\frac{1}{2} \int_{\alpha_s^{(\text{ref})}}^{\alpha_s'} \frac{d\alpha_s'}{\beta(\alpha_s')}\right] = \sqrt{K}$$

- cross sections are sensitive to both $\alpha_s^\sigma(m_Z)$
- cross sections are sensitive to both $\alpha_s^\sigma(m_Z)$

Jet cross sections at NNLO

Simultaneous fit to $\alpha_s^\sigma(m_Z)$ & $\alpha_s^f(m_Z)$

- both $\alpha_s^\sigma(m_Z)$ from fits are consistent within unc. using NNPDF3.0_nnlo_as_0.118
Variations of the scale and the scale choices

- our choice of scales: \( \mu_R^2 = \mu_F^2 = Q^2 + P_T^2 \)
- \( \mu_R \) variation has more impact than \( \mu_F \)
- theory uncertainty related to scale from variation of \( \mu_R, \mu_F \) by 0.5 & 2.0

- Q² as scale is disfavored (larger \( \chi^2 \))
- other choices are within scale unc.
- NNLO has smaller scale uncertainty compared to NLO

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Variation of the input PDF sets

\[ \alpha_s(m_Z) \equiv \alpha_s^\sigma(m_Z) \equiv \alpha_s^f(m_Z) \] and \( \chi^2/\text{ndf} \) from repetitive fits with different input PDF sets as a function of "\( \alpha_s(m_Z) \) of PDF"

→ different PDF sets obtained for our default \( \alpha_s(m_Z) = 0.118 \) deliver very stable fit results:

\[ \text{additional PDF unc. "PDFset": } \frac{1}{2} \max[\Delta(\text{all PDFs at 0.118})] \]

→ the \( \alpha_s(m_Z) \) results are sensitive to input "\( \alpha_s(m_Z) \) of PDF"
- minimum of \( \chi^2/\text{ndf} \) is obtained around our default value 0.118

\[ \text{additional PDF unc. "PDF} \alpha_s": \frac{1}{2} [\Delta \alpha_s(m_Z) = 0.004] \]
(2nd largest unc. after scale unc.)
Strong coupling from jets in DIS at NNLO

Results for $\alpha_s(m_Z)$ at NNLO using
- 9 individual H1 data sets separately
- all H1 inclusive jets data
- all H1 dijets data
- all H1 jets (excluding dijets HERA-I since no correlations to incl. jets)

all H1 jet data sets are consistent:
- $\chi^2$/ndf is around unity for all fits

all $\alpha_s(m_Z)$ results are consistent

H1 jets (203 data points, $\chi^2$/ndf=1.03)

$\alpha_s(m_Z) = 0.1157 (6)_{\text{exp}} (3)_{\text{had}} (6)_{\text{PDF}}$

$ (12)_{\text{PDF}} (2)_{\text{PDF set}} (\pm 27)_{\text{scale}}$

- excellent experimental precision
- still scale uncertainty is the largest
- in agreement with the world average
Fits are performed for groups of jet data points at similar scales and resulting $\alpha_s(m_Z)$ are transported to the average $\mu_R$ of the group.

- running of $\alpha_s$ in one experiment from 7 to 90 GeV is demonstrated

- in the full range $\alpha_s$ is in agreement with other $\alpha_s$ results at NNLO and the world average with a tendency to be a bit lower

- scale uncertainty is about the same at all $\mu_R$ values
Conclusions

The last missing piece in the jet measurements by H1 is on place:

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<th>HERA-II</th>
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<td></td>
<td>Trijets</td>
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</table>

The first determination of the strong coupling constant $\alpha_s(m_Z)$ at NNLO using $ep$ DIS jet data from H1

$$\alpha_s(m_Z) = 0.1157(6)_{\text{exp}}(^{+31}_{-26})_{\text{theo}}$$

→ very close and nice cooperation of theoreticians and experimentalists

Jets in DIS: precision QCD phenomenology with NNLO accuracy