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We study the recurrences $\rho(1^-)$, $g(3^-)$, $i(5^-)$, etc. in the 3π decay of the J/ψ and show that the Virasoro dual amplitude for this decay gives the following ratios

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We study the recurrences $\rho(1^-)$, $g(3^-)$, $i(5^-)$, etc. in the 3π decay of the J/ψ and show that the Virasoro dual amplitude for this decay gives the following ratios

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In a recent Letter, Cohen-Tannoudji et al.[1] have pointed out that the hadronic decays of the J/ψ may provide us with the possibility of studying heavy resonances such as recurrences of the ρ, ω, B etc. It is obvious that since the J/ψ is heavy, enough energy is available in a given subchannel of the J/ψ decay in order to reach the higher recurrences of a Regge trajectory. The more important part of the argument is that since the c quark in the J/ψ cannot end up in the final state hadrons (Fig. 1), the exchanges between the final state hadrons have no twist and therefore the subchannels are maximally resonating. The experimental data on the 3π decay of the J/ψ, showing that 70% of the 3π decay is the $\rho\pi$ mode [2], gives credence to this argument. (Another support to this argument comes also from the decay $\phi \to 3\pi$. Very recently the Orsay group [3] has reached the conclusion that greater than 80% of the 3π decay of the ϕ proceed through the $\rho\pi$ mode).

In this short note we want to study the problem quantitatively for the recurrences $\rho(1^-)$, $g(3^-)$, $i(5^-)$, ... appearing in the 3π decay of the J/ψ. The preliminary analysis of an experiment at DESY [4] gives an upper limit, i.e.,

$$\frac{\Gamma(J/\psi \to g\pi)}{\Gamma(J/\psi \to \rho\pi)} < 0.5 \tag{1}$$

Using the width formula [5] for the decay of a spin-J meson into a spin-1 and a spin-0 meson, we can write

$$\frac{\Gamma(J/\psi \to g\pi)}{\Gamma(J/\psi \to \rho\pi)} = \frac{G^2_{\psi g\pi}}{G^2_{\psi\rho\pi}} \cdot \frac{4}{15} \cdot \frac{p_g^7}{p_\rho^3} \cdot \left(\frac{m_\psi}{m_g}\right)^4 , \tag{2}$$

where p_g (p_ρ) is the magnitude of the 3-momentum of the g (ρ) in the rest frame of the J/ψ. It seems that the factor $(m_\psi/m_g)^4$ leads to a large enhancement. However, we have to know also the factor $G^2_{\psi g\pi}/G^2_{\psi\rho\pi}$. It is known that there is some arbitrariness in the definition of coupling constants because of their dimensions; namely one can define them involving arbitrary masses. Therefore, it would be nice if one extracts these coupling constants from a dynamical theory. This can be achieved in a dual model, since a dual model relates dynamically the particles with different spins. As pointed out in Ref. (1), the Virasoro amplitude, and not the Veneziano amplitude, should be used for the $J/\psi \to 3\pi$. The reason for this is that the process $J/\psi \to 3\pi$ has very different duality properties in comparison to the process $\omega \to 3\pi$. For example, in the $\omega \to 3\pi$ decay one needs three diagrams in the lowest order in order to have poles in all three channels which correspond to the (s,t), (s,u) and (u,t) terms of the Veneziano amplitude, but in the $J/\psi \to 3\pi$ case one diagram has poles in all channels. Furthermore, since $\alpha(s)+\alpha(t)+\alpha(u) \neq 2$ for $J/\psi \to 3\pi$, it is not possible to impose the necessary condition to cancel all the undesired poles at even integer values of α in the case of the Veneziano amplitude. The Virasoro amplitude however, does not suffer from the defect of having poles at even integers.

The amplitude for the reaction $J/\psi(p) + \bar{\pi}(-p_1) \longrightarrow \pi(p_2) + \pi(p_3)$ is given by

$$A(s,t,u) = \varepsilon_{\mu\nu\sigma\rho}\, \varepsilon^\mu p_1^\nu p_2^\sigma p_3^\rho\, V(s,t,u) \tag{3}$$

where $s = (p_1+p_2)^2$, $t = (p_2+p_3)^2$, $u = (p_1+p_3)^2$, and the scalar amplitude $V(s,t,u)$ has the Virasoro form:

$$V(s,t,u) = \beta\, \frac{\Gamma\left(\frac{1}{2}-\frac{1}{2}\alpha(s)\right)\Gamma\left(\frac{1}{2}-\frac{1}{2}\alpha(t)\right)\Gamma\left(\frac{1}{2}-\frac{1}{2}\alpha(u)\right)}{\Gamma\left(1-\frac{1}{2}\alpha(s)-\frac{1}{2}\alpha(t)\right)\Gamma\left(1-\frac{1}{2}\alpha(s)-\frac{1}{2}\alpha(u)\right)\Gamma\left(1-\frac{1}{2}\alpha(t)-\frac{1}{2}\alpha(u)\right)} \tag{4}$$

Here we have only the ρ-trajectory in all three channels.

We now expand the amplitude (4) as a sum of the pole terms:

$$V(s,t,u) = \beta \sum_{n=0}^{\infty} \frac{2(-1)^n}{(2n+1)-\alpha(s)} \cdot \frac{\prod_{r=1}^{n} \frac{1}{2}\left[\alpha(t)+2r-1\right] \frac{1}{2}\left[\alpha(u)+2r-1\right]}{\Gamma\left(1-\frac{1}{2}\left[\alpha(t)+\alpha(u)\right]\right)} \tag{5}$$

One should note that because of the gamma function in the denominator the Virasoro amplitude is regular if $\alpha(s) + \alpha(t) + \alpha(u) \equiv N =$ odd integer number. Since $\alpha_\rho(s) = 0.47 + 0.88\, s$, N is not at all an odd integer (actually $N \approx 10$) for $J/\psi \to 3\pi$, so that poles are really present.

In the center of mass frame of the s-channel, Eq. (3) can be written as

$$A = \sqrt{s}\; q\, q' \sin\theta\, \left(V_1 + V_3 + V_5 + \cdots\right) \tag{6}$$

where q and q' are the magnitudes of the center of mass three - momenta before and after the collision and θ is the center of mass scattering angle; and $V_1, V_3, V_5, \ldots$ stand for the pole terms at $\alpha(s) = 1,3,5,\ldots$ (in Eq. 5) with residues evaluated also in the center of mass frame. Now if we expand our helicity amplitude A into partial waves as:

$$A = \sum_{n,\ell} \frac{\Gamma_{n,\ell}}{n-\alpha(s)}\, d^\ell_{10}(\theta) \tag{7}$$

we can easily identify the so-called relative strengths of the resonances (or partial wave residues), $\Gamma_{n,\ell}$:

$$\Gamma_{11} = \beta \, m_\rho \, q_\rho \, q'_\rho \, \frac{2\sqrt{2}}{\Gamma\left(\frac{3-N}{2}\right)} \quad ,$$

$$\Gamma_{33} = \beta \, m_g \, q_g^3 \, q_g'^3 \, \frac{8}{5\sqrt{3}} \cdot \frac{\alpha'^2}{\Gamma\left(\frac{5-N}{2}\right)} \quad , \qquad (8)$$

$$\Gamma_{55} = \beta \, m_i \, q_i^5 \, q_i'^5 \, \frac{16}{21}\sqrt{\frac{2}{15}} \, \frac{\alpha'^4}{\Gamma\left(\frac{7-N}{2}\right)}$$

where α' is the slope of the ρ-trajectory. At this point we can say that roughly $\Gamma_{11}^2 : \Gamma_{33}^2 : \Gamma_{55}^2$ are the ratios for the densities of points around $s = m_\rho^2$, m_g^2 and m_i^2 in the Dalitz plot of the $J/\psi \to 3\pi$ decay. Numerically we have

$$\frac{\Gamma_{11}^2}{\Gamma_{33}^2} \approx 9 \quad \text{and} \quad \frac{\Gamma_{33}^2}{\Gamma_{55}^2} \approx 400 \, . \qquad (9)$$

These imply that there is no hope to see the higher recurrences beyond the $g(3^-)$ in the Dalitz plot.

To be more precise, we evaluate the Born terms for the ρ , g and i poles, and equate the Γ_{11}^{Born}, Γ_{33}^{Born} and Γ_{55}^{Born} so found to the corresponding quantities in Eq. (8). We arrive in this way at the following relations:

$$G_{\psi\rho\pi} \, G_{\rho\pi\pi} = \beta \, \frac{1}{\Gamma\left(\frac{3-N}{2}\right)}$$

$$G_{\psi g\pi} \, G_{g\pi\pi} = \beta \, \frac{\alpha'^2}{4\,\Gamma\left(\frac{5-N}{2}\right)} \qquad (10)$$

$$G_{\psi i\pi} \, G_{i\pi\pi} = \beta \, \frac{\alpha'^4}{16\,\Gamma\left(\frac{7-N}{2}\right)}$$

Thus we obtain the ratio

$$\frac{G^2_{\psi g\pi}}{G^2_{\psi\rho\pi}} = \frac{\alpha'^4}{16}\left[\frac{\Gamma\left(\frac{3-N}{2}\right)}{\Gamma\left(\frac{5-N}{2}\right)}\right]^2 \frac{G^2_{\rho\pi\pi}}{G^2_{g\pi\pi}} , \tag{11}$$

where $G^2_{\rho\pi\pi}/G^2_{g\pi\pi}$ can be found either using the known experimental decay widths

$$\frac{G^2_{\rho\pi\pi}}{G^2_{g\pi\pi}} = \frac{96}{35}\cdot\frac{m_\rho^2}{m_g^2}\cdot\frac{q_g'^7}{q_\rho'^3}\cdot\frac{\Gamma(\rho\to\pi\pi)}{\Gamma(g\to\pi\pi)} \approx 11.5\ \mathrm{GeV}^4 \tag{12}$$

or, using the elastic widths [6], obtained from the Veneziano amplitude for $\pi\pi \to \pi\pi$. In the latter case

$$\frac{G^2_{\rho\pi\pi}}{G^2_{g\pi\pi}} = \frac{8}{\alpha'^2} \tag{13}$$

For N = 10, Eq. (11) gives, with the value of Eq. (12) or Eq. (13),

$$\frac{G^2_{\psi g\pi}}{G^2_{\psi\rho\pi}} \approx 0.035 \tag{14}$$

Finally , if we use this value in Eq. (2) we obtain

$$\frac{\Gamma(J/\psi \to g\pi)}{\Gamma(J/\psi \to \rho\pi)} = 0.06 \tag{15}$$

which is one order of magnitude smaller than the experimental upper limit (1).

Similarly we find

$$\frac{\Gamma(J/\psi \to i\pi)}{\Gamma(J/\psi \to \rho\pi)} = 1 \times 10^{-4} \tag{16}$$

In order to get the last result we used the appropriate formula corresponding to Eq. (2), Eq. (10) and the ratio

$$\frac{G^2_{\rho\pi\pi}}{G^2_{i\pi\pi}} = \frac{384}{\alpha'^4} \tag{17}$$

In conclusion, we emphasize that the Virasoro model for the 3π decay of the J/ψ suggests that this decay can provide us with the opportunity of studying only the $\rho(1^-)$ and $g(3^-)$, but not the $i(5^-)$ and the higher recurrences of $\rho(1^-)$.

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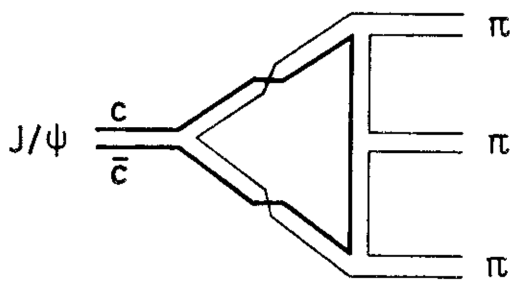

Fig. 1