The Higgs couplings and self-coupling in the EFT framework

Jiayin Gu

DESY & IHEP

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and current work, S. Di Vita, G. Durieux, C. Grojean, JG, Z. Liu, G. Panico,
M. Riembau, T. Vantalon
Introduction

▷ Higgs and nothing else? What next?
▷ An $e^+ e^-$ collider is an obvious direction to go.
▷ Higgs factory ($e^+ e^- \rightarrow hZ$ at 240-250 GeV, $e^+ e^- \rightarrow \nu \bar{\nu} h$ at higher energies), and many more other measurements.
▷ The scale of new physics $\Lambda$ is large $\Rightarrow$ effective field theory (EFT) is a good description at low energy.
▷ A global analysis of the Higgs coupling constraints, in the EFT framework. See also e.g.,
   ▷ [arXiv:1510.04561, 1701.04804] Ellis et al.,

▷ Robust constraints on the triple Higgs coupling at both circular and linear colliders. (current work)
Higgs measurements

- $e^+ e^- \rightarrow hZ$, cross section maximized at around 250 GeV.
- $e^+ e^- \rightarrow \nu\bar{\nu}h$, cross section increases with energy.
- $e^+ e^- \rightarrow t\bar{t}h$, can be measured with $\sqrt{s} \gtrsim 500$ GeV.
- $e^+ e^- \rightarrow Zh$ and $e^+ e^- \rightarrow \nu\bar{\nu}hh$ (triple Higgs coupling).
\( \kappa \) framework vs. EFT

- Conventionally, the constraints on Higgs couplings are obtained from global fits in the so-called \( \kappa \) framework.

\[
g_h^{\text{SM}} \rightarrow \kappa g_h^{\text{SM}}.
\]

- Anomalous couplings such as \( hZ_{\mu\nu} Z_{\mu\nu} \) or \( hZ_{\mu} \partial_{\nu} Z^{\mu\nu} \) are assumed to be zero.

- EFT framework
  - Assuming \( \nu \ll \Lambda \), leading contribution from BSM physics are well-parameterized by D6 operators.
  - Gauge invariance is built in the parameterization.

- Lots of parameters! (Is it practical to perform a global fit?)

From the CEPC preCDR and "Physics Case for the ILC" ([arXiv:1506.05992])
The “12-parameter” framework in EFT

- Assume the new physics
  - is CP-even,
  - does not generate dipole interaction of fermions,
  - only modifies the diagonal entries of the Yukawa matrix,
  - has no corrections to $Z$-pole observables and $W$ mass (more justified if the machine will run at $Z$-pole).

- Additional measurements
  - Triple gauge couplings from $e^+e^- \rightarrow WW$. (The LEP constraints will be improved at future colliders.)
  - Angular observables in $e^+e^- \rightarrow hZ$.
  - $h \rightarrow Z\gamma$ is also important.

- Only 12 combinations of operators are relevant for the measurements considered (with the inclusion of the Yukawa couplings of $t$, $c$, $b$, $\tau$, $\mu$).

- All 12 EFT parameters can be constrained reasonably well in the global fit!
EFT basis

- We work in the Higgs basis (LHCHXSWG-INT-2015-001, A. Falkowski) with the following 12 parameters,

\[ \delta c_Z, \ c_{ZZ}, \ c_{Z\square}, \ c_{\gamma\gamma}, \ c_{Z\gamma}, \ c_{gg}, \ \delta y_t, \ \delta y_c, \ \delta y_b, \ \delta y_\tau, \ \delta y_\mu, \ \lambda_Z. \]

- The Higgs basis is defined in the broken electroweak phase.

- Couplings of h to W are written in terms of couplings of h to Z and \( \gamma \).

- 3 aTGC parameters (\( \delta g_{1,Z}, \delta \kappa_{\gamma}, \lambda_Z \)), 2 written in terms of Higgs parameters.

- It can be easily mapped to the following basis with D6 operators.

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_H )</td>
<td>( \frac{1}{2} (\partial_\mu</td>
</tr>
<tr>
<td>( O_{WW} )</td>
<td>( g^2</td>
</tr>
<tr>
<td>( O_{BB} )</td>
<td>( g' \ )^2</td>
</tr>
<tr>
<td>( O_{HW} )</td>
<td>( ig(D_{\mu} H)^\dagger \sigma^a (D_{\nu} H) W_{\mu\nu}^a )</td>
</tr>
<tr>
<td>( O_{HB} )</td>
<td>( ig'(D_{\mu} H)^\dagger (D_{\nu} H) B_{\mu\nu} )</td>
</tr>
<tr>
<td>( O_{GG} )</td>
<td>( g_s^2</td>
</tr>
<tr>
<td>( O_{yy} )</td>
<td>( y_u</td>
</tr>
<tr>
<td>( O_{yd} )</td>
<td>( y_d</td>
</tr>
<tr>
<td>( O_{ye} )</td>
<td>( y_e</td>
</tr>
<tr>
<td>( O_{3W} )</td>
<td>( \frac{1}{3!} g_{\epsilon abc} W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c )</td>
</tr>
</tbody>
</table>
Results of the “12-parameter” fit

precision reach of the 12-parameter fit in Higgs basis

Assuming the following run plans (no official plan for CEPC 350 GeV run)

- CEPC 240 GeV(5/ab) + 350 GeV(200/fb)
- FCC-ee 240 GeV(10/ab) + 350 GeV(2.6/ab)
- ILC 250 GeV(2/ab) + 350 GeV(200/fb) + 500 GeV(4/ab)
- CLIC 350 GeV(500/fb) + 1.4 TeV(1.5/ab) + 3 TeV(2/ab)
Impact of beam polarization

- Beam polarization helps discriminate different parameters.
  - Two polarization configurations are considered, $P(e^-, e^+) = (-0.8, +0.3)$ and $(+0.8, -0.3)$.
  - $F(-+)$ in the range of 0.6-0.8 gives an optimal overall results.
- Runs with different polarizations probe different combinations of EFT parameters in Higgs production.
Triple Higgs coupling in the EFT framework
(global fit with 12+1 parameters)

Triple Higgs coupling at low energies (250 & 350 GeV)

- $K_{\lambda} \equiv \frac{\lambda_{hhh}}{\lambda_{SM}^{hhh}}$, 
  $\delta K_{\lambda} \equiv K_{\lambda} - 1 = c_6 - \frac{3}{2} c_H$, 
  with $\mathcal{L} \supset -\frac{c_6}{\nu^2} (H^\dagger H)^3$.

- One loop corrections to all Higgs couplings (production and decay).

- 250 GeV: $hZ$ near threshold (more sensitive to $\delta K_{\lambda}$)

- at 350 GeV:
  - $WW$ fusion
  - $hZ$ at a different energy

- $h \rightarrow WW^*/ZZ^*$ also have some discriminating power (but turned out to be not enough).
Triplet Higgs coupling at low energies (250 & 350 GeV)

- Runs at both 250 GeV and 350 GeV are needed to obtain good constraints on $\delta K_{\lambda}$!
- Bounds are further improved if combined with HL-LHC measurements.

<table>
<thead>
<tr>
<th></th>
<th>ILC alone</th>
<th>ILC + HL-LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-zero aTGCs</td>
<td>zero aTGCs</td>
</tr>
<tr>
<td>250 GeV(2 ab)</td>
<td>[-5.72, +5.87]</td>
<td>[-5.39, +5.62]</td>
</tr>
<tr>
<td>250 GeV(2 ab)+350 GeV(200 fb)</td>
<td>[-1.26, +1.26]</td>
<td>[-1.18, +1.18]</td>
</tr>
<tr>
<td>250 GeV(2 ab)+350 GeV(1.5 ab)</td>
<td>[-0.84, +0.84]</td>
<td>[-0.56, +0.56]</td>
</tr>
</tbody>
</table>

Double-Higgs measurements \((e^+e^- \rightarrow Zhh \& e^+e^- \rightarrow \nu\bar{\nu}hh)\)

- Destructive interference in \(e^+e^- \rightarrow \nu\bar{\nu}hh!\) The square term is important.
- \(hh\) invariant mass distribution helps discriminate the “2nd solution.”
\( \chi^2 \) vs. \( \delta \kappa_\lambda \), ILC

**Inputs:**
- 500 GeV (4 ab\(^{-1}\)) : \( \sigma(Zhh) \) measured to 16.8\% [C. F. Düorig, PhD thesis, Hamburg U. (2016)]
- 1 TeV (2 ab\(^{-1}\)) : \( \sigma(\nu\bar{\nu}hh) \) measured to 2.7\( \sigma \) significance \( \Rightarrow \sim 37\% \) [talk by Düorig at ALCW15]

**Complementarity between the 500 GeV run and the 1 TeV run.**

**Single Higgs measurements provide non-negligible improvement.**
- up to 500 GeV: \([-0.31, +0.28] \rightarrow [-0.26, +0.25]\),
- up to 1 TeV: \([-0.20, +0.23] \rightarrow [-0.18, +0.20]\),

The Higgs couplings and self-coupling in the EFT framework
\( \chi^2 \) vs. \( \delta K_\lambda \), CLIC

Input:

- \( \sigma(\nu \bar{\nu} hh) \) measured to 44% at 1.4 TeV and 20% at 3 TeV (Higgs Physics at the CLIC Electron-Positron Linear Collider [arXiv:1608.07538], Assuming unpolarized beam.)

- \( \sigma(Zhh) \) measured to \( \sim 50\% \) at 1.4 TeV (our own naive estimation).

- The measurement of \( Zhh \) or the \( M_{hh} \) distribution of \( \nu \bar{\nu} hh \) can help resolve the “2nd solution.”

- The bounds on \( \delta K_\lambda \) can be further improved by having a \( hZ \) threshold run (e.g., by combining with CEPC 240 GeV or ILC 250 GeV).
A summary of the (future) bounds on $\delta \kappa_\lambda$

**bounds on $\delta \kappa_\lambda$ from EFT global fit**

- **HL-LHC**
  - 68%, 95% CL bounds, lepton collider only
  - 68%, 95% CL bounds, combined with HL-LHC

- **CEPC & FCC-ee**
  - HL-LHC results from arXiv:1704.01953
  - 14 TeV (3/ab), rates & distributions

- **ILC**
  - 240 GeV (5/ab) only (CEPC)
  - 240 GeV (5/ab) + 350 GeV (200/ab)
  - 240 GeV (5/ab) + 350 GeV (1.5/ab) (FCC-ee)
  - FCC-ee with zero αTGCs

- **CLIC**
  - 250 GeV (2/ab) only
  - 250 GeV (2/ab) + 350 GeV (200/ab)
  - above + 500 GeV (4/ab)
  - above + 1 TeV (2/ab)

**$\delta \kappa_\lambda = \frac{\lambda_{hh}}{\lambda_{hh}^{SM}} - 1$**

Jiayin Gu
DESY & IHEP

The Higgs couplings and self-coupling in the EFT framework
After the discovery of Higgs at the LHC, a plausible “next step” is to build an $e^+e^-$ collider to perform Higgs precision measurements.

It makes sense to go beyond the “$\kappa$” frame and study Higgs physics in the EFT framework.

We can obtain strong and robust constraints on the coefficients of the relevant dimension-6 operators!

We can obtain robust constraints on the triple Higgs coupling!
backup slides
Global Determinant Parameter ($GDP \equiv \sqrt[n]{\text{det} \sigma^2}$).

Ratios of GDPs are basis-independent.

Smaller GDP $\rightarrow$ better precision!
Impact of the single Higgs measurements

- What if the single Higgs measurements are much better or much worse?

- Much better: can further improve the bounds on \( \delta K_\lambda \) from double-Higgs measurements.

- Much worse: can significantly worsen the bounds on \( \delta K_\lambda \) from double-Higgs measurements.
Angular observables in $e^+ e^- \rightarrow hZ$

- Angular distributions in $e^+ e^- \rightarrow hZ$ can provide information in addition to the rate measurement alone.

- Previous studies

- 6 independent asymmetry observables from 3 angles

  \[ A_{\theta_1}, \ A_{\phi}^{(1)}, \ A_{\phi}^{(2)}, \ A_{\phi}^{(3)}, \ A_{\phi}^{(4)}, \ A_{c \theta_1, c \theta_2}. \]

- Focusing on leptonic decays of $Z$ (good resolution, small background, statistical uncertainty dominates).
TGC measurements (using ILC 500 GeV results)

- 3 aTGC parameters, 2 ($\delta g_{1,Z}$ & $\delta \kappa_{\gamma}$) related to Higgs observables.
- Linear colliders (large $\sqrt{s}$, beam polarizations) could potentially constrain the aTGCs very well.
- At the moment we simply use ILC numbers
  - I. Marchesini, PhD thesis, Hamburg U. (2011), assuming 500 fb$^{-1}$ data at 500 GeV with $P(e^-, e^+) = (\pm 0.8, \pm 0.3)$.
- CLIC can potentially do much better!
  - An updated experimental study that performs a global fit among all three aTGC parameters is desired.
- Will $e^+ e^- \rightarrow WW$ be much better measured than the $Z$-pole measurements? The “TGC dominance” assumption may not be valid anymore.

<table>
<thead>
<tr>
<th>ILC</th>
<th>uncertainty</th>
<th>$\delta g_{1,Z}$</th>
<th>$\delta \kappa_{\gamma}$</th>
<th>$\lambda_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta g_{1,Z}$</td>
<td>$6.1 \times 10^{-4}$</td>
<td>1</td>
<td>0.634</td>
<td>0.477</td>
</tr>
<tr>
<td>$\delta \kappa_{\gamma}$</td>
<td>$6.4 \times 10^{-4}$</td>
<td>1</td>
<td>1</td>
<td>0.354</td>
</tr>
<tr>
<td>$\lambda_Z$</td>
<td>$7.2 \times 10^{-4}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
If you don’t like the Higgs basis...

Results in the SILH'(-like) basis ($\mathcal{O}_{W, B} \rightarrow \mathcal{O}_{WW, WB}$)

$$\mathcal{L}_{D6} = \frac{c_H}{v^2} \mathcal{O}_H + \frac{\kappa_{WW}}{m_W^2} \mathcal{O}_{WW} + \frac{\kappa_{BB}}{m_W^2} \mathcal{O}_{BB} + \frac{\kappa_{HW}}{m_W^2} \mathcal{O}_{HW} + \frac{\kappa_{HB}}{m_W^2} \mathcal{O}_{HB}$$

$$+ \frac{\kappa_{GG}}{m_W^2} \mathcal{O}_{GG} + \frac{\kappa_{3W}}{m_W^2} \mathcal{O}_{3W} + \sum_{f=t,c,b,\tau,\mu} \frac{c_{y_f}}{v^2} \mathcal{O}_{y_f}.$$
Advantages of the runs at higher energies

- Much better measurement of the $WW$ fusion process ($e^+ e^- \rightarrow \nu \bar{\nu} h$).
- Probing $e^+ e^- \rightarrow hZ$ at different energies.
- Improving constraints on aTGCs ($e^+ e^- \rightarrow WW$).

Very helpful in resolving the degeneracies among parameters!
The importance of combining all measurements

- The results are much worse if we only include the rates of Higgs measurements alone!
- There is some overlap in the information from different measurements.
- Measurements at different energies can be very helpful.
The precision reach of $\delta K_\lambda$ at circular colliders

- The precision reach of $\delta K_\lambda$ in the luminosity plane (luminosities at 240 GeV and 350 GeV).

- $e^+ e^- \rightarrow WW$ measured very well $\Rightarrow$ setting aTGCs to zero is a good approximation.
Impact of $\delta K_\lambda$ on the other parameters

- Adding one more parameter could worsen the bounds on others.
- The effect is under control if the degeneracies are well-resolved.
- The HL-LHC bounds on $\delta K_\lambda$ can also help.
Impact of the Higher energy runs

precision reach at CEPC with different luminosities at 350 GeV

precision reach at FCC-ee with different luminosities at 350 GeV

precision reach at LHC with different run scenarios

precision reach at LHC with different run scenarios at 1 TeV

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The Higgs couplings and self-coupling in the EFT framework
\[ e^+ e^- \rightarrow \nu \bar{\nu} h \]

- It is hard to separate the \( WW \) fusion process from \( e^+ e^- \rightarrow hZ, \ Z \rightarrow \nu \bar{\nu} \) at 240 GeV.
- It is not consistent to focus on one process and treat the other one as SM-like!
- For CEPC/FCC-ee 240 GeV, we analyze the combined \( e^+ e^- \rightarrow \nu \bar{\nu} h \) process, assuming new physics can contribute to both processes.
\[ e^+ e^- \to WW \]

- \( e^+ e^- \to WW \) offers a great way to probe the anomalous triple gauge couplings (aTGCs, parameterized by \( \delta g_{1,Z} \), \( \delta \kappa_{\gamma} \), \( \lambda_Z \)).
- \( \delta g_{1,Z} \) and \( \delta \kappa_{\gamma} \) are related to Higgs observables.
- CEPC with 5 \( ab^{-1} \) data at 240 GeV can produce \( \sim 9 \times 10^7 \) \( e^+ e^- \to WW \) events.
- With such large statistics, the aTGCs can be very well constrained ([1507.02238] Bian, Shu, Zhang), but with two potential issues:
  - Systematic uncertainties can be important!
  - If \( e^+ e^- \to WW \) is measured more precisely than the \( Z \)-pole measurements, is it still ok to assume the fermion gauge couplings are SM-like?
The interplay between Higgs and TGC

- $\delta_{g_{1,2}}, \delta_{\kappa_\gamma}$ ↔ $c_{ZZ}, c_{Z\square}, c_{\gamma\gamma}, c_{Z\gamma}$

- We try different assumptions on the systematic uncertainties (in each bin with the differential distribution divided into 20 bins).

- Detailed study of $e^+e^- \rightarrow WW$ required to estimate the systematic uncertainties!
Asymmetry observables

\[ A_{\theta_1} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \ \text{sgn}(\cos(2\theta_1)) \ \frac{d\sigma}{d\cos \theta_1}, \]
\[ A^{(1)}_{\phi} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ \text{sgn}(\sin \phi) \ \frac{d\sigma}{d\phi}, \]
\[ A^{(2)}_{\phi} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ \text{sgn}(\sin(2\phi)) \ \frac{d\sigma}{d\phi}, \]
\[ A^{(3)}_{\phi} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ \text{sgn}(\cos \phi) \ \frac{d\sigma}{d\phi}, \]
\[ A^{(4)}_{\phi} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ \text{sgn}(\cos(2\phi)) \ \frac{d\sigma}{d\phi}, \]

\[ A_{c\theta_1,c\theta_2} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \ \text{sgn}(\cos \theta_1) \ \int_{-1}^{1} d\cos \theta_2 \ \text{sgn}(\cos \theta_2) \ \frac{d^2\sigma}{d\cos \theta_1 d\cos \theta_2}, \]
The “12-parameter” framework in the Higgs basis

- The relevant terms in the EFT Lagrangian are

\[ \mathcal{L} \supset \mathcal{L}_{hVV} + \mathcal{L}_{hff} + \mathcal{L}_{tgc}, \]

- the Higgs couplings with a pair of gauge bosons

\[
\mathcal{L}_{hVV} = \frac{h}{v} \left[ (1 + \delta c_W) \frac{g^2 v^2}{2} W^{+}_\mu W^{-}_\mu + (1 + \delta c_Z) \frac{(g^2 + g'^2)v^2}{4} Z_{\mu} Z_{\mu} \right. \\
+ c_{WW} \frac{g^2}{2} W^{+}_{\mu \nu} W^{-}_{\mu \nu} + c_{W\square} g^2 (W^{-}_\mu \partial_\nu W^{+}_{\mu \nu} + \text{h.c.}) \\
+ c_{gg} \frac{g_s^2}{4} G^{a}_{\mu \nu} G^{2}_{\mu \nu} + c_{\gamma\gamma} \frac{e^2}{4} A_{\mu \nu} A_{\mu \nu} + c_{Z\gamma} \frac{e \sqrt{g^2 + g'^2}}{2} Z_{\mu \nu} A_{\mu \nu} \\
+ c_{ZZ} \frac{g^2 + g'^2}{4} Z_{\mu \nu} Z_{\mu \nu} + c_{Z\square} g^2 Z_{\mu} \partial_\nu Z_{\mu \nu} + c_{\gamma\square} gg' Z_{\mu} \partial_\nu A_{\mu \nu} \right].
\]

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The Higgs couplings and self-coupling in the EFT framework
The “12-parameter” framework in the Higgs basis

- Not all the couplings are independent, for instance one could write the following couplings as

\[
\delta c_W = \delta c_Z + 4\delta m, \\
c_{WW} = c_{ZZ} + 2s_{\theta w}^2 c_{Z\gamma} + s_{\theta w}^4 c_{\gamma\gamma}, \\
c_{WW} = \frac{1}{g^2 - g'^2} \left[ g^2 c_{Z\square} + g'^2 c_{ZZ} - e^2 s_{\theta w}^2 c_{\gamma\gamma} - (g^2 - g'^2) s_{\theta w}^2 c_{Z\gamma} \right], \\
c_{\gamma\square} = \frac{1}{g^2 - g'^2} \left[ 2g^2 c_{Z\square} + (g^2 + g'^2) c_{ZZ} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{Z\gamma} \right],
\]

- we only consider the diagonal elements in the Yukawa matrices relevant for the measurements considered,

\[
\mathcal{L}_{hff} = -\frac{h}{v} \sum_{f=t,c,b,\tau,\mu} m_f (1 + \delta y_f) \bar{f}_R f_L + \text{h.c.}
\]
\[ \mathcal{L}_{\text{tgc}} = i g s_{\theta_W} A_\mu (W^{-\nu} W_{\mu \nu}^+ - W^{+\nu} W_{\mu \nu}^-) \]
\[ + i g (1 + \delta g_1^Z) c_{\theta_W} Z^\mu (W^{-\nu} W_{\mu \nu}^+ - W^{+\nu} W_{\mu \nu}^-) \]
\[ + i g \left[ (1 + \delta \kappa_Z) c_{\theta_W} Z_{\mu \nu} + (1 + \delta \kappa_\gamma) s_{\theta_W} A_{\mu \nu} \right] W_{\mu}^- W_{\nu}^+ \]
\[ + \frac{i g}{m_W^2} (\lambda_Z c_{\theta_W} Z_{\mu \nu} + \lambda_\gamma s_{\theta_W} A_{\mu \nu}) W_{\nu}^- W_{\rho \mu}^+ , \] (7)

\[ V_{\mu \nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu \] for \( V = W^\pm, Z, A, \). Imposing Gauge invariance one obtains \( \delta \kappa_Z = \delta g_1, Z - t_{\theta_W}^2 \delta \kappa_\gamma \) and \( \lambda_Z = \lambda_\gamma. \)

\[ \forall \delta g_1, Z, \delta \kappa_\gamma \text{ and } \lambda_Z, \text{ 2 of them related to Higgs observables by} \]
\[ \delta g_1, Z = \frac{1}{2(g^2 - g'^2)} \left[ -g^2 (g^2 + g'^2) c_{Z \square} - g'^2 (g^2 + g'^2) c_{Z Z} + e^2 g'^2 c_{\gamma \gamma} + g'^2 (g^2 - g'^2) c_{Z \gamma} \right] \]
\[ \delta \kappa_\gamma = - \frac{g^2}{2} \left( c_{\gamma \gamma} \frac{e^2}{g^2 + g'^2} + c_{Z \gamma} \frac{g^2 - g'^2}{g^2 + g'^2} - c_{Z Z} \right) . \] (8)
CEPC/FCC-ee Higgs rate measurements

<table>
<thead>
<tr>
<th>Production</th>
<th>CEPC</th>
<th>FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[240 GeV, 5 ab$^{-1}$]</td>
<td>[350 GeV, 200 fb$^{-1}$]</td>
</tr>
<tr>
<td>Zh $\nu\bar{\nu}h$</td>
<td>0.50% -</td>
<td>2.4% -</td>
</tr>
</tbody>
</table>

| $h \rightarrow \bar{b}b$ | 0.21%* | 0.39%♦ | 2.0% | 2.6% | 0.20% | 0.28%♦ | 0.54% | 0.71% |
| $h \rightarrow c\bar{c}$ | 2.5% | - | 15% | 26% | 1.2% | - | 4.1% | 7.1% |
| $h \rightarrow gg$ | 1.2% | - | 11% | 17% | 1.4% | - | 3.1% | 4.7% |
| $h \rightarrow \tau\tau$ | 1.0% | - | 5.3% | 37% | 0.7% | - | 1.5% | 10% |
| $h \rightarrow WW^*$ | 1.0% | - | 10% | 9.8% | 0.9% | - | 2.8% | 2.7% |
| $h \rightarrow ZZ^*$ | 4.3% | - | 33% | 33% | 3.1% | - | 9.2% | 9.3% |
| $h \rightarrow \gamma\gamma$ | 9.0% | - | 51% | 77% | 3.0% | - | 14% | 21% |
| $h \rightarrow \mu\mu$ | 12% | - | 115% | 275% | 13% | - | 32% | 76% |
| $h \rightarrow Z\gamma$ | 25% | - | 144% | - | 18% | - | 40% | - |

**Table:** For $e^+e^- \rightarrow \nu\bar{\nu}h$, the precisions marked with a diamond ♦ are normalized to the cross section of the inclusive channel which includes both the $WW$ fusion and $e^+e^- \rightarrow hZ$, $Z \rightarrow \nu\bar{\nu}$, while the unmarked ones include $WW$ fusion only.
## ILC Higgs rate measurements

<table>
<thead>
<tr>
<th>Production</th>
<th>[250 GeV, 2 ab$^{-1}$]</th>
<th>[350 GeV, 200 fb$^{-1}$]</th>
<th>[500 GeV, 4 ab$^{-1}$]</th>
<th>[1 TeV, 1 ab$^{-1}$]</th>
<th>[1 TeV, 2.5 ab$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Zh$</td>
<td>$Zh$</td>
<td>$Zh$</td>
<td>$\nu \bar{\nu} h$</td>
<td>$\nu \bar{\nu} h$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.71%</td>
<td>2.1%</td>
<td>1.1%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h \to bb$</td>
<td>0.42%</td>
<td>1.7%</td>
<td>0.64%</td>
<td>0.5%</td>
<td>0.3%</td>
</tr>
<tr>
<td>$h \to c\bar{c}$</td>
<td>2.9%</td>
<td>13%</td>
<td>4.6%</td>
<td>3.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$h \to gg$</td>
<td>2.5%</td>
<td>9.4%</td>
<td>3.9%</td>
<td>2.3%</td>
<td>-</td>
</tr>
<tr>
<td>$h \to \tau\tau$</td>
<td>1.1%</td>
<td>4.5%</td>
<td>1.9%</td>
<td>1.6%</td>
<td>1.0%</td>
</tr>
<tr>
<td>$h \to WW^*$</td>
<td>2.3%</td>
<td>8.7%</td>
<td>3.3%</td>
<td>3.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$h \to ZZ^*$</td>
<td>6.7%</td>
<td>28%</td>
<td>8.8%</td>
<td>4.1%</td>
<td>2.6%</td>
</tr>
<tr>
<td>$h \to \gamma\gamma$</td>
<td>12%</td>
<td>44%</td>
<td>12%</td>
<td>8.5%</td>
<td>5.4%</td>
</tr>
<tr>
<td>$h \to \mu\mu$</td>
<td>25%</td>
<td>98%</td>
<td>31%</td>
<td>31%</td>
<td>20%</td>
</tr>
<tr>
<td>$h \to Z\gamma$</td>
<td>34%</td>
<td>145%</td>
<td>49%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
CLIC Higgs rate measurements

<table>
<thead>
<tr>
<th>production</th>
<th>[350 GeV, 500 fb$^{-1}$]</th>
<th>[1.4 TeV, 1.5 ab$^{-1}$]</th>
<th>[3 TeV, 2 ab$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Zh$</td>
<td>$\nu\bar{\nu}h$</td>
<td>$\nu\bar{\nu}h$</td>
<td>$tth$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.6%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table:** We also include the estimations for $\sigma(hZ) \times \text{BR}(h \to b\bar{b})$ at high energies in [arXiv:1701.04804] (Ellis et al.), which are 3.3% (6.8%) at 1.4 TeV (3 TeV). For simplicity, the measurements of $ZZ$ fusion ($e^+ e^- \to e^+ e^- h$) are not included in our analysis.