The Higgs couplings and self-coupling in the EFT framework

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DESY & IHEP

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and current work, S. Di Vita, G. Durieux, C. Grojean, JG, Z. Liu, G. Panico,
M. Riembau, T. Vantalon
Introduction

- Higgs and nothing else? What next?
- An $e^+ e^-$ collider is an obvious direction to go.
- Higgs factory ($e^+ e^- \rightarrow hZ$ at 240-250 GeV, $e^+ e^- \rightarrow \nu\bar{\nu} h$ at higher energies), and many more other measurements.
- The scale of new physics $\Lambda$ is large $\Rightarrow$ effective field theory (EFT) is a good description at low energy.
- A global analysis of the Higgs coupling constraints, in the EFT framework. See also e.g.,
  - [arXiv:1510.04561, 1701.04804] Ellis et al.,

- **Robust constraints** on the **triple Higgs coupling** at both circular and linear colliders. (current work)
Higgs measurements

- $e^+ e^- \rightarrow hZ$, cross section maximized at around 250 GeV.
- $e^+ e^- \rightarrow \nu \bar{\nu} h$, cross section increases with energy.
- $e^+ e^- \rightarrow t\bar{t}h$, can be measured with $\sqrt{s} \gtrsim 500$ GeV.
- $e^+ e^- \rightarrow Zhh$ and $e^+ e^- \rightarrow \nu \bar{\nu} hh$ (triple Higgs coupling).
Conventionally, the constraints on Higgs couplings are obtained from global fits in the so-called “κ” framework.

\[ g^\text{SM}_h \rightarrow \kappa g^\text{SM}_h. \]

Anomalous couplings such as \( hZ^{\mu\nu}Z_{\mu\nu} \) or \( hZ_\mu \partial^\nu Z^{\mu\nu} \) are assumed to be zero.

EFT framework

- Assuming \( \nu \ll \Lambda \), leading contribution from BSM physics are well-parameterized by D6 operators.
- Gauge invariance is built in the parameterization.

Lots of parameters! (Is it practical to perform a global fit?)
The “12-parameter” framework in EFT

- Assume the new physics
  - is CP-even,
  - does not generate dipole interaction of fermions,
  - only modifies the diagonal entries of the Yukawa matrix,
  - has no corrections to $Z$-pole observables and $W$ mass (more justified if the machine will run at $Z$-pole).

- Additional measurements
  - Triple gauge couplings from $e^+ e^- \rightarrow WW$. (The LEP constraints will be improved at future colliders.)
  - Angular observables in $e^+ e^- \rightarrow hZ$.
  - $h \rightarrow Z \gamma$ is also important.

- Only 12 combinations of operators are relevant for the measurements considered (with the inclusion of the Yukawa couplings of $t, c, b, \tau, \mu$).

- All 12 EFT parameters can be constrained reasonably well in the global fit!
EFT basis

- We work in the Higgs basis (LHCHXSWG-INT-2015-001, A. Falkowski) with the following 12 parameters,
  \[ \delta c_Z, \ c_{ZZ}, \ c_{Z\Box}, \ c_{\gamma\gamma}, \ c_{Z\gamma}, \ c_{gg}, \ \delta y_t, \ \delta y_c, \ \delta y_b, \ \delta y_\tau, \ \delta y_\mu, \ \lambda_Z. \]

- The Higgs basis is defined in the broken electroweak phase.
  - \[ \delta c_Z \leftrightarrow hZ^\mu Z_\mu, \ c_{ZZ} \leftrightarrow hZ^{\mu\nu} Z_{\mu\nu}, \ c_{Z\Box} \leftrightarrow hZ_\mu \partial_\nu Z^{\mu\nu}. \]

- Couplings of h to W are written in terms of couplings of h to Z and \( \gamma \).

- 3 aTGC parameters (\( \delta g_{1,Z}, \ \delta \kappa_\gamma, \ \lambda_Z \)), 2 written in terms of Higgs parameters.

- It can be easily mapped to the following basis with D6 operators.

\[
\begin{align*}
\mathcal{O}_H &= \frac{1}{2}(\partial_\mu |H|^2)^2 \\
\mathcal{O}_{WW} &= g^2 |H|^2 W_{\mu\nu}^a W^{a,\mu\nu} \\
\mathcal{O}_{BB} &= g^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \\
\mathcal{O}_{HW} &= ig (D^\mu H) \dagger \sigma^a (D^\nu H) W_{\mu\nu}^a \\
\mathcal{O}_{HB} &= ig' (D^\mu H) \dagger (D^\nu H) B_{\mu\nu} \\
\mathcal{O}_{GG} &= g_s^2 |H|^2 G^A_\mu \nu G^{A,\mu\nu} \\
\mathcal{O}_{y_u} &= y_u |H|^2 \bar{Q}_L \tilde{H} u_R \\
\mathcal{O}_{y_d} &= y_d |H|^2 \bar{Q}_L \tilde{H} d_R \\
\mathcal{O}_{y_e} &= y_e |H|^2 \bar{L}_L \tilde{H} e_R \\
\mathcal{O}_{3W} &= \frac{1}{3!} g \epsilon_{abc} W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c \end{align*}
\]
Results of the “12-parameter” fit

Assuming the following run plans (no official plan for CEPC 350 GeV run)

- CEPC 240 GeV (5/ab) + 350 GeV (200/fb)
- FCC-ee 240 GeV (10/ab) + 350 GeV (2.6/ab)
- ILC 250 GeV (2/ab) + 350 GeV (200/fb) + 500 GeV (4/ab)
- CLIC 350 GeV (500/fb) + 1.4 TeV (1.5/ab) + 3 TeV (2/ab)
Beam polarization helps discriminate different parameters.

Two polarization configurations are considered, $P(e^-, e^+) = (-0.8, +0.3)$ and $(+0.8, -0.3)$.

$F(-+)$ in the range of 0.6-0.8 gives an optimal overall results.

Runs with different polarizations probe different combinations of EFT parameters in Higgs production.
Introduction

Global fit in the EFT framework

Results

Conclusion

Triple Higgs coupling in the EFT framework
(global fit with 12+1 parameters)

One loop corrections to all Higgs couplings (production and decay).

250 GeV: $hZ$ near threshold (more sensitive to $\delta \kappa_\lambda$)

at 350 GeV:

- WW fusion
- $hZ$ at a different energy

$h \rightarrow WW^*/ZZ^*$ also have some discriminating power (but turned out to be not enough).
Triple Higgs coupling at low energies (250 & 350 GeV)

Run at both 250 GeV and 350 GeV are needed to obtain good constraints on $\delta \kappa_\lambda$!

Bounds are further improved if combined with HL-LHC measurements.

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Double-Higgs measurements ($e^+e^- \rightarrow Zhh$ & $e^+e^- \rightarrow \nu\bar{\nu}hh$)

$P(e^-, e^+) = (-0.8, +0.3)$

$e^+e^- \rightarrow Zhh$

$e^+e^- \rightarrow \nu\bar{\nu}hh$ (W-fusion only)

$e^+e^- \rightarrow \nu\bar{\nu}hh$ (W-fusion only)

$\sigma/\sigma_{SM}$ vs $\sqrt{s}$ [GeV]

Destructive interference in $e^+e^- \rightarrow \nu\bar{\nu}hh$! The square term is important.

$hh$ invariant mass distribution helps discriminate the “2nd solution.”
\( \chi^2 \text{ vs. } \delta \kappa_\lambda, \text{ ILC} \)

**Inputs:**
- 500 GeV (4 ab\(^{-1}\)): \( \sigma(Zhh) \) measured to 16.8% [C. F. Dürig, PhD thesis, Hamburg U. (2016)]
- 1 TeV (2 ab\(^{-1}\)): \( \sigma(\nu\bar{\nu}hh) \) measured to 2.7\( \sigma \) significance \( \Rightarrow \sim 37\% \) [talk by Dürig at ALCW15]

**Complementarity between the 500 GeV run and the 1 TeV run.**

**Single Higgs measurements provide non-negligible improvement.**
- up to 500 GeV: \([-0.31, +0.28]\) \( \rightarrow \) \([-0.26, +0.25]\),
- up to 1 TeV: \([-0.20, +0.23]\) \( \rightarrow \) \([-0.18, +0.20]\),

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The measurement of $Zhh$ or the $M_{hh}$ distribution of $\nu\bar{\nu}hh$ can help resolve the “2nd solution.”

The bounds on $\delta K_{\lambda}$ can be further improved by having a $hZ$ threshold run (e.g., by combining with CEPC 240 GeV or ILC 250 GeV).
A summary of the (future) bounds on $\delta\kappa_\lambda$

**Results**

- **CLIC**
  - $240\text{GeV}(5/ab)$ only (CEPC)
  - $240\text{GeV}(5/ab)+350\text{GeV}(200/fb)$
  - $240\text{GeV}(5/ab)+350\text{GeV}(1.5/ab)$ (FCC–ee)
  - FCC–ee with zero aTGCs

- **ILC**
  - $250\text{GeV}(2/ab)$ only
  - $250\text{GeV}(2/ab)+350\text{GeV}(200/fb)$
  - above + $500\text{GeV}(4/ab)$
  - above + $1\text{TeV}(2/ab)$

- **CLIC**
  - $350\text{GeV}(500/fb)+1.4\text{TeV}(1.5/ab)+3\text{TeV}(2/ab)$
  - + $\text{Zhh at 1.4 TeV}$
  - binned $M_{hh}$ in $\nu\nu hh$ (4 bins)

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(HL-LHC results from arXiv:1704.01953)

14TeV(3/ab), rates & distributions

68\%, 95\% CL bounds (combined with HL-LHC)

 bounds on $\delta\kappa_\lambda$ from EFT global fit

-2 -1 0 1 2 3

68\%, 95\% CL bounds, lepton collider only

68\%, 95\% CL bounds, combined with HL-LHC

-2 -1 0 1 2 3

binned $M_{hh}$ in $\nu\nu hh$ (4 bins)
After the discovery of Higgs at the LHC, a plausible “next step” is to build an $e^+e^-$ collider to perform Higgs precision measurements.

It makes sense to go beyond the “$\kappa$” frame and study Higgs physics in the EFT framework.

We can obtain strong and robust constraints on the coefficients of the relevant dimension-6 operators!

We can obtain robust constraints on the triple Higgs coupling!
## backup slides
Global Determinant Parameter ($GDP \equiv \frac{2^n}{\sqrt{\det \sigma^2}}$).

- Ratios of GDPs are basis-independent.
- Smaller GDP $\rightarrow$ better precision!
Impact of the single Higgs measurements

What if the single Higgs measurements are much better or much worse?

- Much better: can further improve the bounds on $\delta_{K,\lambda}$ from double-Higgs measurements.
- Much worse: can significantly worsen the bounds on $\delta_{K,\lambda}$ from double-Higgs measurements.
Angular observables in $e^+ e^- \rightarrow hZ$

- Angular distributions in $e^+ e^- \rightarrow hZ$ can provide information in addition to the rate measurement alone.

- Previous studies

- 6 independent asymmetry observables from 3 angles

  \[ A_{\theta_1}, \ A^{(1)}_{\phi}, \ A^{(2)}_{\phi}, \ A^{(3)}_{\phi}, \ A^{(4)}_{\phi}, \ A_{\theta_1, c\theta_2}. \]

- Focusing on leptonic decays of $Z$ (good resolution, small background, statistical uncertainty dominates).
3 aTGC parameters, 2 ($\delta g_{1,Z}$ & $\delta \kappa_\gamma$) related to Higgs observables.

Linear colliders (large $\sqrt{s}$, beam polarizations) could potentially constrain the aTGCs very well.

At the moment we simply use ILC numbers

I. Marchesini, PhD thesis, Hamburg U. (2011), assuming 500 fb$^{-1}$ data at 500 GeV with $P(e^-, e^+) = (\pm 0.8, \pm 0.3)$.

CLIC can potentially do much better!

An updated experimental study that performs a global fit among all three aTGC parameters is desired.

Will $e^+e^- \rightarrow WW$ be much better measured than the $Z$-pole measurements? The “TGC dominance” assumption may not be valid anymore.
If you don’t like the Higgs basis...

Results in the SILH’(-like) basis ($O_{W,B} \rightarrow O_{WW, WB}$)

$$
\mathcal{L}_{D6} = \frac{c_H}{v^2} O_H + \frac{\kappa_{WW}}{m_W^2} O_{WW} + \frac{\kappa_{BB}}{m_W^2} O_{BB} + \frac{\kappa_{HW}}{m_W^2} O_{HW} + \frac{\kappa_{HB}}{m_W^2} O_{HB} \\
+ \frac{\kappa_{GG}}{m_W^2} O_{GG} + \frac{\kappa_{3W}}{m_W^2} O_{3W} + \sum_{f=t,c,b,\tau,\mu} \frac{c_y_f}{v^2} O_{y_f}.
$$
Impact of running at different energies (using CEPC as an example)

- **Advantages of the runs at higher energies**
  - Much better measurement of the $WW$ fusion process ($e^+ e^- \rightarrow \nu \bar{\nu} h$).
  - Probing $e^+ e^- \rightarrow hZ$ at different energies.
  - Improving constraints on aTGCs ($e^+ e^- \rightarrow WW$).

- Very helpful in resolving the degeneracies among parameters!
The importance of combining all measurements

- The results are much worse if we only include the rates of Higgs measurements alone!
- There is some overlap in the information from different measurements.
- Measurements at different energies can be very helpful.
The precision reach of $\delta K_\lambda$ at circular colliders

- The precision reach of $\delta K_\lambda$ in the luminosity plane (luminosities at 240 GeV and 350 GeV).

- $e^+ e^- \rightarrow WW$ measured very well $\Rightarrow$ setting aTGCs to zero is a good approximation.
### Impact of $\delta\kappa_\lambda$ on the other parameters

- Adding one more parameter could worsen the bounds on others.
- The effect is under control if the degeneracies are well-resolved.
- The HL-LHC bounds on $\delta\kappa_\lambda$ can also help.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta\kappa_\lambda/10^2$</th>
<th>$\delta c_z$</th>
<th>$c_{zz}$</th>
<th>$c_{c\gamma}/10$</th>
<th>$c_{Z\gamma}/10$</th>
<th>$c_{Z\gamma}/10$</th>
<th>$\delta y_c$</th>
<th>$\delta y_b$</th>
<th>$\delta y_t$</th>
<th>$\delta y_\mu/10$</th>
<th>$\lambda_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEPC 240GeV (5/ab) only</td>
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<td>CEPC 240GeV (5/ab) + 350GeV (200/fb)</td>
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<td>CEPC 240GeV (5/ab) + 350GeV (1/ab)</td>
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<td>CEPC 240GeV (5/ab) + 350GeV (2/ab)</td>
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Precision reach at CEPC with and without $\delta\kappa_\lambda$ at CEPC:

- Light shade: 12-parameter fit including $\delta\kappa_\lambda$.
- Solid shade: 11-parameter fit with $\delta\kappa_\lambda=0$.

(4.6/10^2)

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**Impact of the Higher energy runs**

### Precision reach at CEPC with different luminosities at 350 GeV

- CEPC 240GeV (5ab) only
- CEPC 240GeV (5ab) + 350GeV (200/fb)
- CEPC 240GeV (5ab) + 350GeV (500/fb)
- CEPC 240GeV (5ab) + 350GeV (1/ab)
- CEPC 240GeV (5ab) + 350GeV (2/ab)

### Precision reach at FCC-ee with different luminosities at 350 GeV

- Only e + e - → νν h and e + e - → tth are included

### Precision reach at ILC with different run scenarios at 1 TeV

- Light shades for columns 2&3: e + e - → WW measurements at 350 GeV not included

### Conclusion

**Global fit in the EFT framework**

- Results
\[ e^+ e^- \rightarrow \nu \bar{\nu} h \]

- It is hard to separate the \( WW \) fusion process from \( e^+ e^- \rightarrow hZ, Z \rightarrow \nu \bar{\nu} \) at 240 GeV.
- It is not consistent to focus on one process and treat the other one as SM-like!
- For CEPC/FCC-ee 240 GeV, we analyze the combined \( e^+ e^- \rightarrow \nu \bar{\nu} h \) process, assuming new physics can contribute to both processes.
\( e^+ e^- \rightarrow WW \)

- \( e^+ e^- \rightarrow WW \) offers a great way to probe the anomalous triple gauge couplings (aTGCs, parameterized by \( \delta g_{1,Z} \), \( \delta \kappa_{\gamma} \), \( \lambda_Z \)).
- \( \delta g_{1,Z} \) and \( \delta \kappa_{\gamma} \) are related to Higgs observables.
- CEPC with 5 \( ab^{-1} \) data at 240 GeV can produce \( \sim 9 \times 10^7 \) \( e^+ e^- \rightarrow WW \) events.
- With such large statistics, the aTGCs can be very well constrained ([1507.02238] Bian, Shu, Zhang), but with two potential issues:
  - Systematic uncertainties can be important!
  - If \( e^+ e^- \rightarrow WW \) is measured more precisely than the \( Z \)-pole measurements, is it still ok to assume the fermion gauge couplings are SM-like?
The interplay between Higgs and TGC

precision reach of aTGCs at CEPC 240GeV (5/ab)

precision reach at CEPC 240GeV (5/ab) assuming different systematics for $e^+e^-\rightarrow WW$

- $\delta g_{1,Z}$, $\delta \kappa_{\gamma}$ ↔ $c_{ZZ}$, $c_{Z\Box}$, $c_{\gamma\gamma}$, $c_{Z\gamma}$

- We try different assumptions on the systematic uncertainties (in each bin with the differential distribution divided into 20 bins).

- Detailed study of $e^+e^-\rightarrow WW$ required to estimate the systematic uncertainties!
Asymmetry observables

\[ A_{\theta_1} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \text{sgn}(\cos(2\theta_1)) \frac{d\sigma}{d\cos \theta_1}, \]

\[ A^{(1)}_\phi = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \text{sgn}(\sin \phi) \frac{d\sigma}{d\phi}, \]

\[ A^{(2)}_\phi = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \text{sgn}(\sin(2\phi)) \frac{d\sigma}{d\phi}, \]

\[ A^{(3)}_\phi = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \text{sgn}(\cos \phi) \frac{d\sigma}{d\phi}, \]

\[ A^{(4)}_\phi = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \text{sgn}(\cos(2\phi)) \frac{d\sigma}{d\phi}, \]

(1)

\[ A_{c\theta_1, c\theta_2} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \text{sgn}(\cos \theta_1) \int_{-1}^{1} d\cos \theta_2 \text{sgn}(\cos \theta_2) \frac{d^2\sigma}{d\cos \theta_1 d\cos \theta_2}, \]

(2)
The “12-parameter” framework in the Higgs basis

- The relevant terms in the EFT Lagrangian are

\[ \mathcal{L} \supset \mathcal{L}_{hVV} + \mathcal{L}_{hff} + \mathcal{L}_{tgc}, \]  

(3)

- The Higgs couplings with a pair of gauge bosons

\[ \mathcal{L}_{hVV} = \frac{h}{v} \left[ (1 + \delta c_W) \frac{g^2 v^2}{2} W^+_{\mu} W^-_{\mu} + (1 + \delta c_Z) \frac{(g^2 + g'^2) v^2}{4} Z_\mu Z^\mu \right. \]

\[ + c_{WW} \frac{g^2}{2} W^+_{\mu\nu} W^-_{\mu\nu} + c_W \Box g^2 (W^-_{\mu} \partial_\nu W^+_{\mu\nu} + \text{h.c.}) \]

\[ + c_{gg} \frac{g_s^2}{4} G^a_{\mu\nu} G_{\mu\nu} + c_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} A_{\mu\nu} + c_{Z\gamma} \frac{e \sqrt{g^2 + g'^2}}{2} Z_{\mu\nu} A_{\mu\nu} \]

\[ + c_{ZZ} \frac{g^2 + g'^2}{4} Z_{\mu\nu} Z_{\mu\nu} + c_{Z\Box} g^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g' g' Z_\mu \partial_\nu A_{\mu\nu} \right]. \]  

(4)
Not all the couplings are independent, for instance one could write the following couplings as

\[
\delta c_W = \delta c_Z + 4 \delta m, \\
c_{WW} = c_{ZZ} + 2 s_{\theta_W}^2 c_{Z\gamma} + s_{\theta_W}^4 c_{\gamma\gamma}, \\
c_{W\Box} = \frac{1}{g^2 - g'^2} \left[ g^2 c_{Z\Box} + g'^2 c_{ZZ} - e^2 s_{\theta_W}^2 c_{\gamma\gamma} - (g^2 - g'^2) s_{\theta_W}^2 c_{Z\gamma} \right], \\
c_{\gamma\Box} = \frac{1}{g^2 - g'^2} \left[ 2 g^2 c_{Z\Box} + (g^2 + g'^2) c_{ZZ} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{Z\gamma} \right],
\]

we only consider the diagonal elements in the Yukawa matrices relevant for the measurements considered,

\[
\mathcal{L}_{hff} = -\frac{h}{\sqrt{2}} \sum_{f=t,c,b,\tau,\mu} m_f (1 + \delta y_f) \bar{f}_R f_L + \text{h.c.}.
\]
\[ \mathcal{L}_{\text{tgc}} = ig s_{\theta_W} A^\mu (W^- \nu W^+_{\mu \nu} - W^+ \nu W^-_{\mu \nu}) \\
+ ig (1 + \delta g_1^Z) c_{\theta_W} Z^\mu (W^- \nu W^+_{\mu \nu} - W^+ \nu W^-_{\mu \nu}) \\
+ ig \left[ (1 + \delta \kappa Z) c_{\theta_W} Z^{\mu \nu} + (1 + \delta \kappa \gamma) s_{\theta_W} A^{\mu \nu} \right] W^-_{\mu} W^+_{\nu} \\
+ \frac{ig}{m_W^2} (\lambda_Z c_{\theta_W} Z^{\mu \nu} + \lambda_{\gamma} s_{\theta_W} A^{\mu \nu}) W^-_{\nu} W^+_{\rho} W^+_{\rho \mu} , \quad (7) \]

- \[ V_{\mu \nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu \text{ for } V = W^{\pm}, Z, A. \] Imposing Gauge invariance one obtains \( \delta \kappa Z = \delta g_{1, Z} - t_{\theta_W}^2 \delta \kappa \gamma \) and \( \lambda_Z = \lambda_{\gamma} . \)

- 3 aTGCs parameters \( \delta g_{1, Z}, \delta \kappa \gamma \) and \( \lambda_Z \), 2 of them related to Higgs observables by

\[ \delta g_{1, Z} = \frac{1}{2 (g^2 - g'^2)} \left[ -g^2 (g^2 + g'^2) c_{Z\Box} - g'^2 (g^2 + g'^2) c_{ZZ} + e^2 g'^2 c_{\gamma \gamma} + g'^2 (g^2 - g'^2) c_{Z\gamma} \right] \]

\[ \delta \kappa \gamma = - \frac{g^2}{2} \left( c_{\gamma \gamma} \frac{e^2}{g^2 + g'^2} + c_{Z \gamma} \frac{g^2 - g'^2}{g^2 + g'^2} - c_{ZZ} \right) . \quad (8) \]
## CEPC/FCC-ee Higgs rate measurements

<table>
<thead>
<tr>
<th>CEPC</th>
<th>FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>production</td>
<td>[240 GeV, 5 ab$^{-1}$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Zh $\nu\bar{\nu}h$</td>
</tr>
<tr>
<td>$\sigma \times BR$</td>
<td>0.20% 0.28%</td>
</tr>
</tbody>
</table>

| $h \rightarrow b\bar{b}$ | 0.21% | 0.39% | 2.0% | 2.6% | 0.20% | 0.28% |
| $h \rightarrow c\bar{c}$ | 2.5% | - | 15% | 26% | 1.2% | - |
| $h \rightarrow gg$ | 1.2% | - | 11% | 17% | 1.4% | - |
| $h \rightarrow \tau^+\tau^-$ | 1.0% | - | 5.3% | 37% | 0.7% | - |
| $h \rightarrow WW^*$ | 1.0% | - | 10% | 9.8% | 0.9% | - |
| $h \rightarrow ZZ^*$ | 4.3% | - | 33% | 33% | 3.1% | - |
| $h \rightarrow \gamma\gamma$ | 9.0% | - | 51% | 77% | 3.0% | - |
| $h \rightarrow \mu\mu$ | 12% | - | 115% | 275% | 13% | - |
| $h \rightarrow Z\gamma$ | 25% | - | 144% | - | 18% | - |

**Table:** For $e^+ e^- \rightarrow \nu \bar{\nu} h$, the precisions marked with a diamond $\diamond$ are normalized to the cross section of the inclusive channel which includes both the $WW$ fusion and $e^+ e^- \rightarrow hZ, Z \rightarrow \nu \bar{\nu}$, while the unmarked ones include $WW$ fusion only.
ILC Higgs rate measurements

<table>
<thead>
<tr>
<th>Production</th>
<th>ILC [250 GeV, 2 ab⁻¹]</th>
<th>[350 GeV, 200 fb⁻¹]</th>
<th>[500 GeV, 4 ab⁻¹]</th>
<th>[1 TeV, 1 ab⁻¹]</th>
<th>[1 TeV, 2.5 ab⁻¹]</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( Zh ) ( \nu \bar{\nu} h )</td>
<td>( Zh ) ( \nu \bar{\nu} h )</td>
<td>( Zh ) ( \nu \bar{\nu} h ) ( tth )</td>
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<td>( \nu \bar{\nu} h ) ( tth )</td>
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<tr>
<td>( h \to bb )</td>
<td>0.42%</td>
<td>3.7%</td>
<td>1.7%</td>
<td>1.7%</td>
<td>0.64%</td>
</tr>
<tr>
<td>( h \to c\bar{c} )</td>
<td>2.9%</td>
<td>-</td>
<td>13%</td>
<td>17%</td>
<td>4.6%</td>
</tr>
<tr>
<td>( h \to gg )</td>
<td>2.5%</td>
<td>-</td>
<td>9.4%</td>
<td>11%</td>
<td>3.9%</td>
</tr>
<tr>
<td>( h \to \tau \tau )</td>
<td>1.1%</td>
<td>-</td>
<td>4.5%</td>
<td>24%</td>
<td>1.9%</td>
</tr>
<tr>
<td>( h \to WW^* )</td>
<td>2.3%</td>
<td>-</td>
<td>8.7%</td>
<td>6.4%</td>
<td>3.3%</td>
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<tr>
<td>( h \to ZZ^* )</td>
<td>6.7%</td>
<td>-</td>
<td>28%</td>
<td>22%</td>
<td>8.8%</td>
</tr>
<tr>
<td>( h \to \gamma \gamma )</td>
<td>12%</td>
<td>-</td>
<td>44%</td>
<td>50%</td>
<td>12%</td>
</tr>
<tr>
<td>( h \to \mu \mu )</td>
<td>25%</td>
<td>-</td>
<td>98%</td>
<td>180%</td>
<td>31%</td>
</tr>
<tr>
<td>( h \to Z \gamma )</td>
<td>34%</td>
<td>-</td>
<td>145%</td>
<td>-</td>
<td>49%</td>
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CLIC Higgs rate measurements

<table>
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<tr>
<td></td>
<td>[350 GeV, 500 fb$^{-1}$]</td>
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<tr>
<td>production</td>
<td>$Zh$</td>
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<tr>
<td>$\sigma$</td>
<td>1.6%</td>
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<table>
<thead>
<tr>
<th>$\sigma \times \text{BR}$</th>
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<tr>
<td>$h \to bb$</td>
<td>0.84% 1.9%</td>
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<tr>
<td>$h \to c\bar{c}$</td>
<td>10.3% 14.3%</td>
</tr>
<tr>
<td>$h \to gg$</td>
<td>4.5% 5.7%</td>
</tr>
<tr>
<td>$h \to \tau\tau$</td>
<td>6.2% -</td>
</tr>
<tr>
<td>$h \to WW^*$</td>
<td>5.1% -</td>
</tr>
<tr>
<td>$h \to ZZ^*$</td>
<td>- -</td>
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<tr>
<td>$h \to \gamma\gamma$</td>
<td>- -</td>
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<tr>
<td>$h \to \mu\mu$</td>
<td>- -</td>
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<tr>
<td>$h \to Z\gamma$</td>
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</tr>
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</table>

Table: We also include the estimations for $\sigma(hZ) \times \text{BR}(h \to b\bar{b})$ at high energies in [arXiv:1701.04804] (Ellis et al.), which are 3.3% (6.8%) at 1.4 TeV (3 TeV). For simplicity, the measurements of $ZZ$ fusion ($e^+ e^- \to e^+ e^- h$) are not included in our analysis.