Simulation study on optimization of cavity design for axion search experiments using COMSOL multiphysics

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A conventional axion search experiment utilizes microwave resonant cavities, where axions are converted into photons under a strong magnetic field. Optimized cavity dimension is essential to enhance signal power from the axion-to-photon coupling, to broaden the frequency range, to minimize mode crossings, etc. An extensive study has been performed to optimize the dimension of a cavity and frequency tuning system using the COMSOL multiphysics simulation software. We introduce a figure of merit for this purpose, and present the results from the simulation study.

1 Introduction

Axions can be detected by the conversion to photons in a strong magnetic field[1]. When we search axion using microwave cavity experiments, the axion's conversion to photon power has proportionality as below.

$$P_{a\to\gamma\gamma} \sim B_0^2 V f C Q \tag{1}$$

where B_0 means external magnetic field, V is volume of cavity, f is resonant frequency of cavity, C is form factor of cavity mode, and Q means quality factor of cavity. Form factor C means how resonant mode of cavity is aligned with external magnetic field[2].

$$C_E = \frac{|\int \vec{E}\cdot\vec{B}dV|^2}{B_0^2\int \epsilon |E|^2 dV} \tag{2}$$

When one use solenoid magnet and cylindrical cavity to search axions, TM010 mode has the largest value of form factor, C.
Because no one knows exact mass of the axion, we have to sweep the resonant frequency of a microwave cavity. To change the resonant frequency of a cavity, cavity geometry or material inside cavity should be changed. In this simulation, I introduced the long dielectric rod and by changing position of the rod, changed the resonant .
For a given SNR, and $g_{a\gamma\gamma}$, which means axion-photon coupling constant, frequency scanning rate has below proportionality[2].

$$\frac{df}{dt} \sim B_0^4 V^2 f^2 C^2 Q \tag{3}$$

For a cavity with radius = 20mm, and height = 80mm, and a rod with radius = 2mm, and dielectric constant = 9, the resonant frequency dependency of rod position is given as below graph.

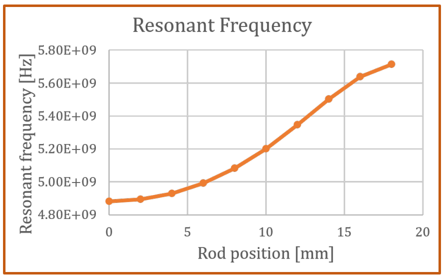

Figure 1: Resonant frequency of TM_{010} vs. rod position where R=20mm, L=80mm, r=2mm, and ϵ_r=9.

Then one can define the totla scan range and total scan time.

$$\Delta F \equiv f_f - f_i \tag{4}$$

$$\Delta T \equiv \int_{t_i}^{t_f} dt = \int_{f_i}^{f_f} \frac{df}{df/dt} \sim \sum_i \frac{\delta f_i}{V^2 C_i^2 f_i^2 Q_i} \tag{5}$$

where f_f and f_i refers that final resonant frequency and initial resonant frequency, respectively. δf_i means the frequency difference between f_i and f_{i+1}, and index i means rod positions. From this, averaged scan rate can be defined as below.

$$\left.\frac{df}{dt}\right|_{averaged} = \frac{\Delta F}{\Delta T} \tag{6}$$

The below graph shows a plot of scan rate of each rod position vs. resonant frequency.

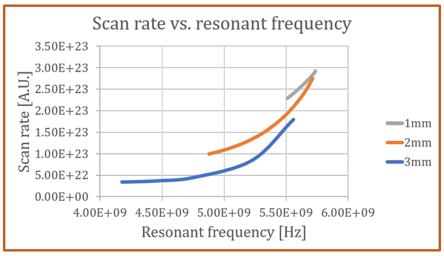

Figure 2: Scan rate vs. resonant frequency for various rod radius.

As you can see in graph, both total scan range and averaged scan rate become important parameters for a given experiment. Scan range maximizes as rod radius becomes larger, but averaged scan rate maximizes as rod radius becomes smaller. By giving equal weight to each parameter, figure of merit can be used as below.

$$F.O.M. = \Delta F \times \frac{\Delta F}{\Delta T} \tag{7}$$

2 Optimization of cavity and rod dimension

Parameters of the tuning system are cavity radius(R), cavity height(L), relative rod radius(r_{rr}), and rod dielectric constant(ϵ_r). We defined relative rod radius as rod radius divide by cavity radius, so it means how rod size is large compared to cavity size. We set rod dielectric constant as 9 for this section.

Simulation procedure was as follows. Firstly, we choose dimension of the tuning system(R, L, r_{rr}). Secondly, we shift the rod positions from cavity center to wall of cavity. And finally, we calculate $F.O.M.$.

The field distribution when rod exists inside cavity is as below.

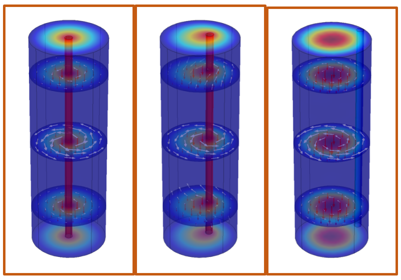

Figure 3: Field distributions of the TM_{010} mode for different dielectric rod positions.

Where the color refers the relative norm of resonant electric field, red arrow means resonant electric field, and white arrow means resonant magnetic field.

The simulation result is as below.

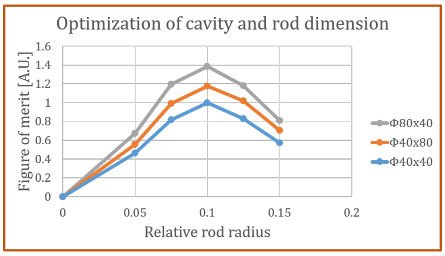

Figure 4: Figure of merit vs. relative rod radius at ϵ_r=9 for various cavity dimensions.

Cavity dimension does not influence the relative optimal size of the tuning rod. So the optimal r_{rr} is independent of cavity dimension. At $\epsilon_r = 9$, the optimal r_{rr} is 0.1

3 Dependence of optimal rod dimension on dielectric constant

We set cavity dimension as R=20mm, and L=80mm, because the cavity dimension was independent with optimal relative rod size. From this, we changed rod dielectric constant(ϵ_r)

from 6 to 11. The simulation procedure was same with previous simulation but from this, we changed not cavity dimension but dielectric constant.

The simulation result is as below.

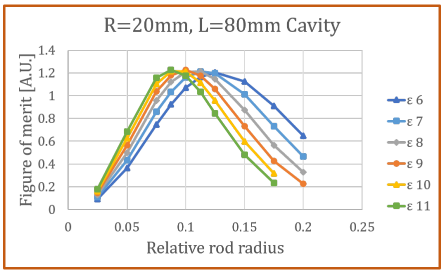

Figure 5: Figure of merit vs. relative rod radius for various rod dielectric constants.

So the optimal relative rod radius is as below.

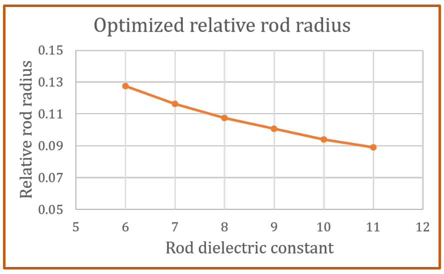

Figure 6: Optimized relative rod radius vs. rod dielectric constant.

As you can see from graph, the optimal r_{rr} is inversely proportional to ϵ_r, and the maximum value of F.O.M. almost independent of ϵ_r.

4 Conclusion

In short, from this simulation study, we can get two important results. First, for a given dielectric constant, the optimal relative size of a tuning system, especially when we use dielectric rod, is constant regardless of cavity dimension. Second, the optimal rod size has a dependence on (inversely proportional to) dielectric constant.

5 Acknowledgments

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