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SOME TECHNICAL PROBLEMS OF RF ACCELERATION FOR
PARTICLE ACCELERATORS ABOVE 10 GeV *)

by

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Study Group at Frascati December 1962.

1. Introduction







RF-acceleration is still, to this date, the most practical method for generating high energy beams of charged particles. The proposed super high energy accelerator projects being discussed at present (1, 2, 3, 4, 5) considerably increase the demand for rf power, and for accelerator rf systems of optimum efficiency. A few technical problems in connection with the design of proton- and electron-synchrotrons for energies up to 1000 GeV and 50 GeV respectively, will be considered below.

It is possible to distinguish between the following three types of rf problems:

- a) Generation of extremely high rf power in the uhf range,
- b) Design and fabrication of high Q accelerating units,
- c) Problems related to beam control techniques, especially beam loading.

Table I is a brief summary of some basic parameters and data of rf systems of a few recently proposed super-high-energy synchrotrons. The first column relates to the 300 GeV proton syn-

Rf-parameters of proposed super-high-energy accelerators

	Caltec ¹⁾	Brookhaven Nat. Lab. ²⁾ ³⁾					CEA ²⁾ ⁴⁾	
a) basic parameters	protons	protons					electrons	
maximum energy	300	400	1000	50	[Gev]			
diameter of main ring	≈ 2600	≈ 2350	≈ 5800	≈ 2350	[m]			
injection energy	10	1.9	5.8	1	[Gev]			
1- initial frequency	.005	.05	.01	10 ⁻⁷				
final frequency	27.2	24.8	60.8	24.8	[μs]			
orbital period at c	5	8	8	5-75	[cm]			
radial useful aperture	1	1	1/15	1	.5	76 × 10 ⁻³	[sec]	
acceleration time								
form of energy rise	1/3	1/3	5	1/3	1	60	[cps]	
repetition rate								
b) rf-parameters								
peak acceleration voltage/turn	12	14	200	88	550	924	[Mev]	
proposed acceleration structure	120	19m	300m	115m	730m	2180m		
Princeton-Cavities		disc loaded waveguide (linac sections)						
proposed frequency	≈ 30	≈ 500	≈ 500	≈ 1000	≈ 1000	≈ 1000	[Mc/s]	
harmonic number		≈ 12500	≈ 30000	≈ 25000	≈ 25000	≈ 25000		
number of accelerated particles per pulse		3.6 × 10 ¹²	8.7 × 10 ¹²	2.5 × 10 ¹²	2.5 × 10 ¹²	2.5 × 10 ¹²		
circulating current (average)		23	23	16	16	16	[ma]	
peak rf power:								
cavity losses	1.2	.6	10	3.8	24	14	[Mw]	
beam power load		.2	1.4	.6	3.8	12.8	[Mw]	
average rf power	.4	.3	4	1.5	7	4	[Mw]	
equivalent energies for use as electron accelerator ³⁾		17.5	35	36	57		[Gev]	

Ref. ¹⁾ M. Sands, SL-10 (Sept. 60)

²⁾ J.P. Blewett et al. Design Study for a 300-1000 Bev Accelerator (Aug. 61)

³⁾ K.W. Robinson, CEA-87 (July 61)

⁴⁾ T.L. Collins, M.S. Livingston, J.R. Rees, H.S. Snyder, CEA-88 (July 61)

Table I

chrotron proposed by Sands (1). In the following a selection has been made of several versions of super high energy synchrotrons studied by the Brookhaven Accelerator Study Group under the direction of Blewett (2). The last column gives the parameters of a 50 GeV electron synchrotron proposed by the scientific staff of the Cambridge Electron Accelerator under the direction of Livingston (3). There are four examples of proton machines with a linear form of energy rise, and one proton machine and the electron machine with sinusoidal rise and a repetition rate of 1 and 60 cps respectively.

The accelerating time, energy and equilibrium phase of the particle beam determines the peak rf accelerating voltage per turn for the proton accelerators; for the electron synchrotron on the other hand, the additional voltage needed to compensate for the extremely high radiation losses per turn is dominant, and in this example is 800 MeV per turn at 50 GeV.

There is, as can be seen, a great variety of values for the rf accelerating voltage which must be generated by the accelerating cavities. It is obvious that only high Q cavities are efficient enough to produce these voltages with reasonably limited rf power consumption, with the exception perhaps of the slow 300 GeV and 400 GeV proton synchrotrons where a greater sacrifice of rf power may still be tolerable.

Long disc loaded waveguides forming linac sections operated at a frequency as high as possible seem to be the optimum solution to achieve best efficiency of rf power, because the shunt resistance per unit length increases with the square root of frequency. Upper limits of frequency are set by the aperture needed for passage of the beam; the aperture should not exceed $1/3$ of the wavelength; furthermore, the increased cost per kilowatt of rf power with increased frequency should be considered. High harmonic numbers are also desired for keeping radial amplitudes of phase oscillations small.

There are, however, a few facts which complicate the situation when high Q cavities are used.

One is that on proton machines the orbital frequency still changes in the order of one or several percent depending on the energy of the injector, the other is that high impedance rf systems suffer from beam loading effects; this last effect, however, is the sign of good rf power efficiency.

The table contains a few characteristic data of rf systems using disc loaded waveguides as accelerating structures at frequencies of 500 Mc/s and 1000 Mc/s respectively, proposed by Robinson (6, 7) and Rees (8); the total length of waveguide for the 50 GeV electron synchrotron is more than 2 kilometers, hence becomes a technical project similar to the Stanford 2 Mile Linear Accelerator. In spite of the lower energy, an electron synchrotron of this type is comparable in magnitude with the 1000 GeV-proton synchrotron with regard to its output power, i.e. energy x average beam current for an average beam power of 1,2 Megawatts. This is of the same magnitude as for the faster 1 cps proton synchrotron and would result in an average efficiency of about 25% for the rf power.

With an additional injector the rf system of the 4 versions of proton machines could also be used for electron acceleration; the maximum energies which can be achieved are indicated in the last line of Table I.

The basic parameters of super high energy synchrotrons which are given in Table I would lead to different rf peak power in the range between about 1 and 30 MW.

As to the effects of beam loading in the proposed rf structures, the most critical of the proton machines would be the 1000 GeV-synchrotron with a sinusoidal rise of energy. At the beginning of the acceleration the cavity voltage would be relatively small and perhaps 1/10 of the maximum rf voltage; the voltage induced by the beam, however, would be constant and would amount to 250 Mill.Volts, which is about half the maximum rf acceleration voltage. Still more critical are the conditions of the electron synchrotron where the voltage induced by the beam is nearly the same as the maximum rf accelerating voltage. These values are based on a shunt resistance*) of 15 Megohms

*) The shunt resistance of an accelerating cavity is defined as
$$\frac{(\text{effective acceleration voltage})^2}{\text{power loss}}$$

for the proton synchrotron and 28 Megohms for the electron synchrotron per meter of waveguide.

With regard to the question of frequency change of the injected beam with proton synchrotrons, several methods of solving this problem have been studied:

- a) One general method uses a corresponding frequency modulation of the rf accelerating voltage. The technical problem is then to tune the cavities fast and precisely enough without too much additional rf power loss (9, 10)
- b) A second method which has been proposed and studied by Robinson (6, 7), would be to use a constant rf frequency and to modulate the phase on the individual accelerating units. This would be possible without much additional rf power, because the filling and decay time for the stored energy of a high-Q-uhf cavity is about the same as the revolution period of the particle beam. The technical problem then is mainly to find and transmit the correct phase program for the individual accelerating units.

With both methods the practical problems are considerably facilitated if the energy of the injector is increased as much as possible. The amount of frequency change decreases with the 2nd power of the energy at injection. A certain compromise will be necessary for economic reasons but perhaps on this point one should remember the opinion expressed by some experts in the accelerator field that every machine is as good as its injector. Hence, it seems to be wise to decide in favour of a safe injection technique.

2. Generation of extremely high power in the uhf range

For many reasons, the rf systems of super high energy accelerators would be divided into several units, so that the rf peak power per unit would be less than 1 Megawatt, and would not be a very serious technical problem in the uhf range. Much more critical would be the demand for average rf power as determined by the length of the duty period and by the repetition rate for circular accelerators. Hence, power amplifier tubes must be suited for high duty cycle operation and high average anode dissipation. A typical range for

the average rf power per tube would be 50 - 200 kW.

There are already two institutes where experience is being collected about the operation of such high power tubes, namely the 6 GeV Cambridge Electron Accelerator and the German Electron Synchrotron at Hamburg. These machines need about 400 kW peak and 100 kW average rf power at 475 and 500 Mc/s respectively. A possible extension of the energy up to 7,5 GeV with good beam intensity demands about 1,6 - 2 MW peak and perhaps 300 kW average rf power. At CEA one super power triode from RCA is being operated; at DESY 2 Eimac klystrons are used for an experimental 400 kW power amplifier (see Fig.1, the synchrotron itself is not yet in operation).

During the last years both, the triode and the klystron techniques have made considerable progress; the following may be mentioned here: RCA has combined the active portion of a super power triode with the rf input and output circuits as a completely evacuated amplifier system. This amplifier is called "Coaxitron" (11, 12). At the A.I.E.E. conference of January 1962 a detailed description of the Coaxitron was given. A model has been made for the 400 - 450 Mc/s frequency range which can be operated without any frequency tuning for rf peak powers up to 5 Megawatts. This model has been tested with a pulse length of 26 microseconds and a duty cycle of about 1%, values typical for linear accelerators. It is clear that for circular accelerators much longer pulses and greater duty factors are necessary; the peak pulse power will then be reduced and limited by thermal effects. With increasing frequencies these problems become more critical because of the smaller size of anode, grids, etc. Furthermore, the peak anode voltage is limited by decreasing distances inside the tube.

The development of klystrons is today proceeding along the following lines (13):

- a) Increased perveance of electron guns. Typical values are $2 \cdot 10^{-6} \text{ A/V}^{3/2}$ instead of $1 \cdot 10^{-6} \text{ A/V}^{3/2}$. Research is being done on so called "magnetron injection type guns" which can be constructed with a perveance of one order of magnitude higher.
- b) Technological and constructive improvements concerning ceramic

windows.

- c) Better reliability by using internal cavities for smaller frequency ranges.

Optimum performance can be achieved with internal cavities for fixed frequency operation.

The separation of gun, rf circuits and collector on velocity modulated tubes is advantageous for higher frequencies. Probably it would not be too difficult to build a klystron with, for instance, 400 kW average collector dissipation at 1000 Mc/s. About 30 of these tubes probably would be able to produce the peak and average rf power for a 1000 GeV proton synchrotron.

3. Design and fabrication of acceleration units

The problem of how to build high-Q-cavities as acceleration units in the uhf range already existed for the above mentioned electron accelerators. CEA is using 16 completely evacuated cavities at 475 Mc/s, each with two resonators excited in the π -mode (14). DESY had originally provided for cavities of similar structure with 5 resonators per accelerating unit, but is now constructing 16 accelerating units, each containing 3 resonators excited in the π -mode. Each accelerating unit will produce an rf peak voltage of about 1 Million Volts for the eventual operation of the synchrotron at 7.5 GeV. In principle, the accelerating units of DESY are disc loaded waveguides which in a similar form, but of greater length, would also be needed for super high energy proton synchrotrons. Therefore, the fabrication technique and the performance of the DESY cavities are briefly described in the following and illustrated by a few pictures*).

Fig. 2 shows laboratory measurements on a non-evacuated 5-section model which is 5 foot long. For the fabrication of vacuum-tight cavities the electroforming method has been successfully applied; the principle was developed at Stanford University for S-Band linacs several years ago (15). Furthermore, many technical problems had to

*) A more detailed report is in preparation.

be solved in order to make vacuumtight joints between the cavity and additional components as, for instance, tuning plungers, rf input couplers, monitoring loops, etc. These components are mounted on stainless steel flanges which enable the use of vacuum-tight metal seal for high bake out temperatures.

Fig. 3 shows a complete 3-section model with a molecular pump on one side and a Vacion pump on the other side.

The shunt resistance of this cavity structure is between 10 and 12 Megohms per meter, and the Q-factor is about 40,000. The resonant frequency can be controlled by plungers and servomechanisms on each section over the range of $\pm 5 \cdot 10^{-4}$. A very efficient water cooling was desired and actually realized with slots on the internal discs and longitudinal channels on the outer surface of the cavity wall.

High power tests at 500 Mc/s were carried out on these accelerating units with 200 - 300 kW pulsed power and up to 30 kW average power, corresponding to an effective accelerating voltage of about 1.5 Million Volts. It was expected that the vacuum window at the rf input would limit the maximum permissible rf power.

Some very impressive studies were made on windows of different material, such as 99.5% alumina and quartz. These windows were tested up to their limit of rf load. The events taking place on the vacuum side could be observed and photographed through observation windows. Fig. 4 and 5 show typical examples, a quartz window which was brought to glow and fluorescence, and a disc of alumina which gave off intense fluorescent light. These tests have contributed considerably to an understanding of the mechanism of how and why a ceramic window is destroyed in high power electron tubes. The mechanism of window failure has already been the subject of many investigations by others but was not completely understood, especially not the process of puncturing. The results of window tests on the DESY-cavity and the observed effects may be explained as follows:

At sufficiently high rf field strength small glowing points can be observed on the vacuum side of the ceramic window (presumably caused by very tiny local impurities). The number of such glowing

points and the intensity of the emitted light increase or decrease with changes of peak and average rf power (on Fig. 5 two such points are visible). These glowing points, which probably start to glow by local dielectric losses, emit electrons the number of which increases very fast with higher rf power. The emitted electrons are accelerated and decelerated by the rf field and a number of them produce secondary electrons by bombardment of the surface. Furthermore, and this seems to be important, the cloud of electrons attracts the ions inside the vacuum chamber, hence causes recombinations with molecules which then contaminate the surface of the window and its surroundings more and more. As an example, a very fast contamination was observed on a ceramic window within a few minutes after an oil diffusion pump was used without adequate baffle. The attractive power for ions is maximum, of course, on the glowing spots themselves, with the result that a very heavy ion bombardment occurs there, and leads to additional local contamination. This may be seen on a window after it has been used.

If the process is too "hot", i.e. if the material is already partly exploding, the ceramic is blown up along the hottest spots, usually in the area of maximum field density. These experimental results are in full conformity with the experience which has been gained over many years at Stanford University. ~~Fig. 6 shows a few examples of punctured and broken windows of S-Band klystrons.~~

In an accelerator with energies from 300 to 1000 GeV the components inside the beam tunnel will not be accessible for repairs for a long time after shut down of the machine, because of the intense activation by radiation. For this reason the problem of reliability of the accelerating equipment merits special attention.

4. Beam control and beam loading

Beam control seems to be one of the most serious rf problems in super high energy accelerators.

The method of phase modulation on the individual acceleration units would demand, for instance, a precise program of the phase of a 500 Mc/s signal which has to be transmitted by cables of considerable length or by other transmitting systems. On a 1000 GeV proton

synchrotron the diameter of 5800 m would result in a total phase shift of $3,5 \times 10^6$ degrees if a signal at this frequency must be transmitted from one point of the circumference to the opposite side. If the error in phase is to be kept within a few degrees the length of the cable must be kept constant within 10^{-6} , at least during the interval between readjustment of the electrical length. Experience in this respect may be expected at Stanford where a 2 mile linear accelerator must be properly phased at 2856 Mc/s.

A second problem arises at low injection energies, if the period of coherent beam phase oscillation is of the same magnitude as one revolution period or even only a fraction of it, and if an automatic beam control system demands a fast correction of the individual cavity amplitudes or phases. In this case a correcting signal originating from a fast beam monitor and going to the point of correction will offer only a very limited rise time for correction; different conditions are illustrated by Fig. 6.

Finally, the problem of beam loading, which has already been mentioned briefly, must be considered. It is important, first of all, on electron synchrotrons, where it exists already at lower energies, because the radiation losses of these machines require a high impedance accelerating system. At present CEA as well as DESY will have to solve the problems related to beam loading effects. In the following a few ideas are mentioned about different methods which are under discussion at present. In general they may be classified into:

- a) Reduction of the cavity-impedance at low energies for a given cavity voltage program.
- b) Compensation methods.

The first general method can be realized either by detuning the cavities with frequency modulation or by damping them with ferrite (as proposed by Rees), so that in a cavity the current coming from the rf generator is of the same magnitude or greater than the circulating beam current, with the result that the beam current will have less influence on the cavity voltage and phase.

The second method is based on the principle of cancelling the

effect of the circulating beam by an equivalent current from the rf power amplifier. Some control systems which could be used to realize this principle are illustrated by the block diagrams of Fig. 7 and Fig. 8.

Fig. 7 shows a so called "beam wave feedback system". A low Q pickup cavity measures the intensity and phase of the beam current I_B , and generates a wave which is fed back to the high Q-cavity system as $-I_B$ after correct phasing and amplification. In this manner, only errors in the reproduction of I_B will cause transients on the cavity voltage program, which is determined by $I_C = I_B - I_G$ (I_G being the total current of the rf amplifier). A cavity voltage monitor and a differential amplifier may possibly be used to reduce the effect of errors. Errors, for instance, must be expected by nonlinearities of the rf power amplifier. An upper frequency limit of stable beam control for coherent beam oscillations is given by the length of cables in the total control loop, i.e. by the distance between rf power amplifier and beam, and also by the rise time of the rf amplifier. The compensation of the voltage induced by the beam at higher frequencies will depend on the total group velocity $d\omega/d\kappa$ of the system and will be delayed. Therefore, at higher beam intensity an improvement of this system will be necessary to take into account the very critical situation at injection.

It should be possible to overcome the difficulties at injection by using a system which triggers an independent anti-beam-wave at the right instant before injection. Fig. 8 should illustrate one of several possible versions of such a system.

In this case, a prebuncher cavity between gun and waveguide of the injector is fed by a signal coming from the master oscillator of the rf system and bunches the injected beam at the same frequency. Shortly before the gun is fired, a coherent wave as an anti-beam-wave is started and amplified by the rf power amplifier. By means of an adjustable timing in the triggering scheme the coincidence of beam wave and anti-beam-wave can be achieved. By adjusting the initial conditions, i.e. amplitude and phase of the coherent anti-beam-wave, an optimum cancellation of the beam induced voltage on the high Q cavities can thus be

achieved; i.e. the cavity will continue to give the desired voltage and phase program without transient.

The system may then be switched over to an automatic beam control; for instance, fast beam-energy and phase monitors could be used to determine corrections of the amplitude and phase of the rf power amplifier to keep the system stable. These corrections have to be performed within a fraction of the period of coherent phase oscillations. Hence, the beam monitor also must announce fast enough the behavior of the beam.

How a fast beam energy monitor could be built, is shown in Fig.9. Two low Q-pickup cavities are placed at an angle Θ on the circumference of the machine. They deliver signals of the same amplitude but of different phase to a diplexer which produces the sum and difference of the voltages V_1 and V_2 . The amplitudes of $V_1 - V_2$ and $V_1 + V_2$ are then compared. If V_1 and V_2 arrive with a phase shift of 90° on the diplexer, $|V_1 - V_2|$ and $|V_1 + V_2|$ will be equal and will cancel on a bridge. The phase relation for any desired position of the beam orbit can be adjusted by means of a variable phase shifter. If then a radial orbit displacement Δr occurs due to errors in the energy of the beam, the phase relation between V_1 and V_2 is shifted by

$$\Delta \varphi = \frac{\Delta r}{R} \cdot \Theta \cdot h \quad (h = \text{harmonic number}).$$

A positive or negative signal results from comparing $|V_1 - V_2|$ and $|V_1 + V_2|$, which is proportional to the orbit displacement Δr and the beam intensity I_B . This monitor has the advantage of delivering relatively high signal levels within very short time and of being rather unaffected by noise. The sensitivity is proportional to the angle Θ which, however, also determines the time needed for monitoring.

A similar method may also be applied for a beam phase monitor using the signal from the rf master oscillator as reference.

All these considerations about beam loading effects have been related to electron synchrotrons; they are not directly applicable to proton synchrotrons because of the velocity modulation of the proton beam. But, if beam loading effects are studied and solved on existing electron machines, it may be expected that some good ideas will also be born and developed for application to bigger proton synchrotrons.

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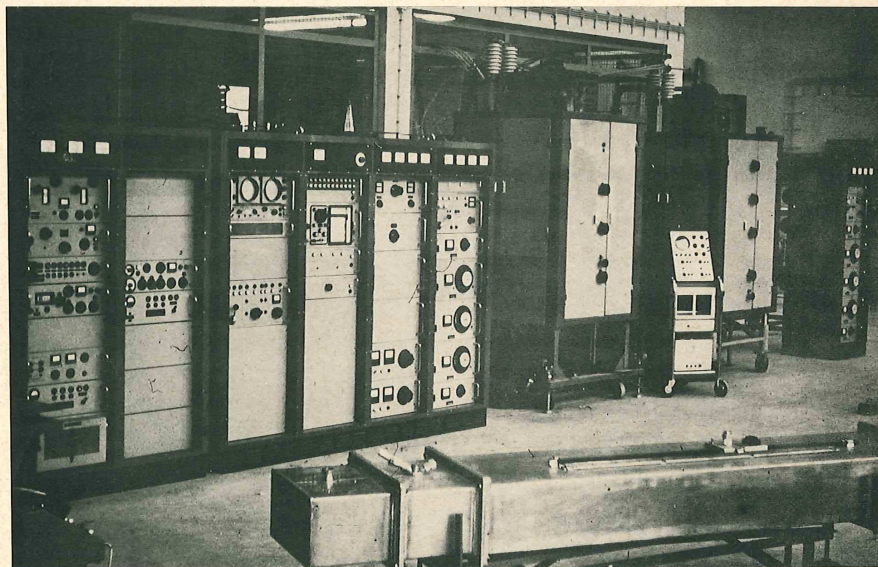


Fig.1

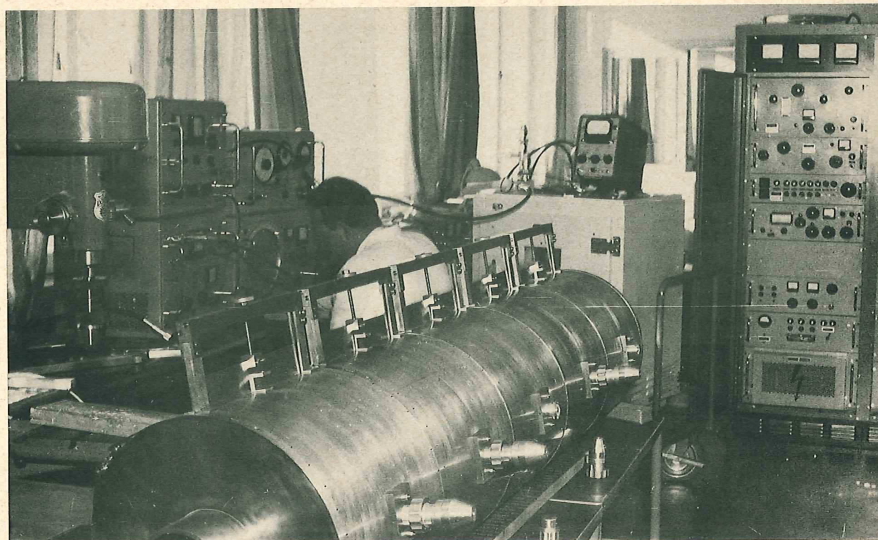


Fig.2

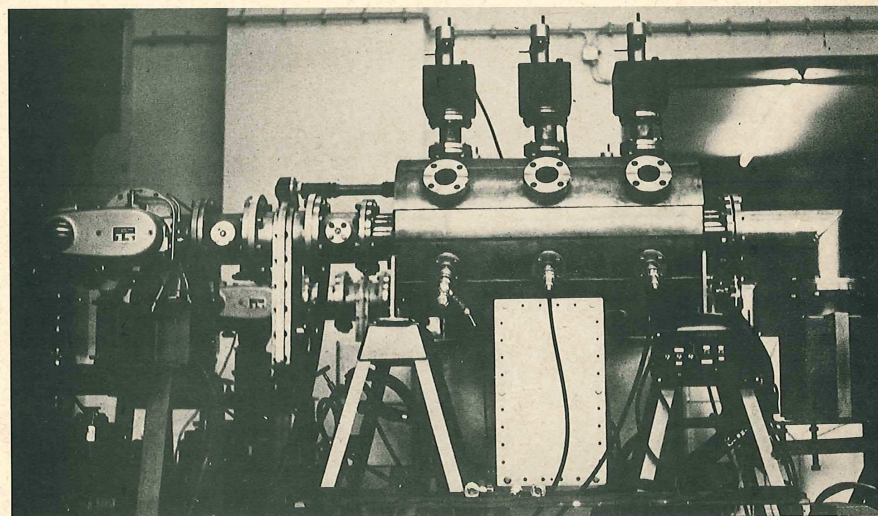


Fig.3

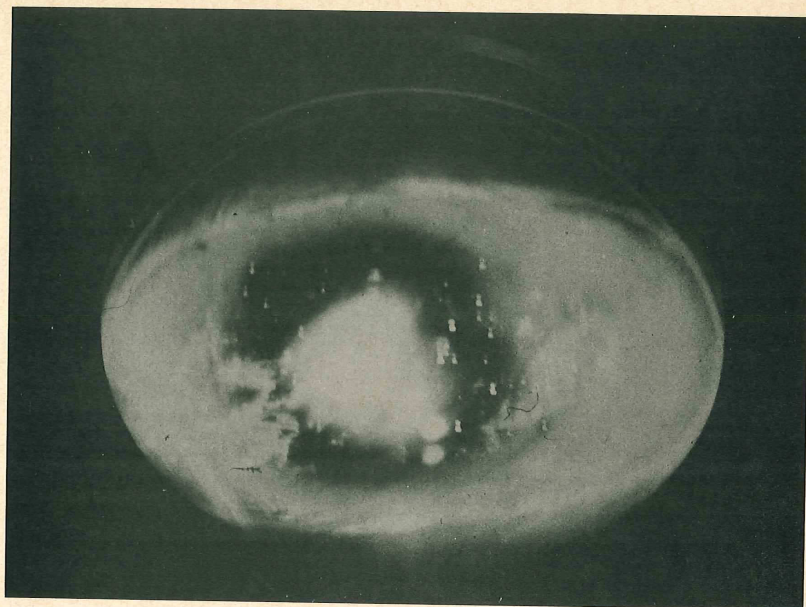


Fig.4

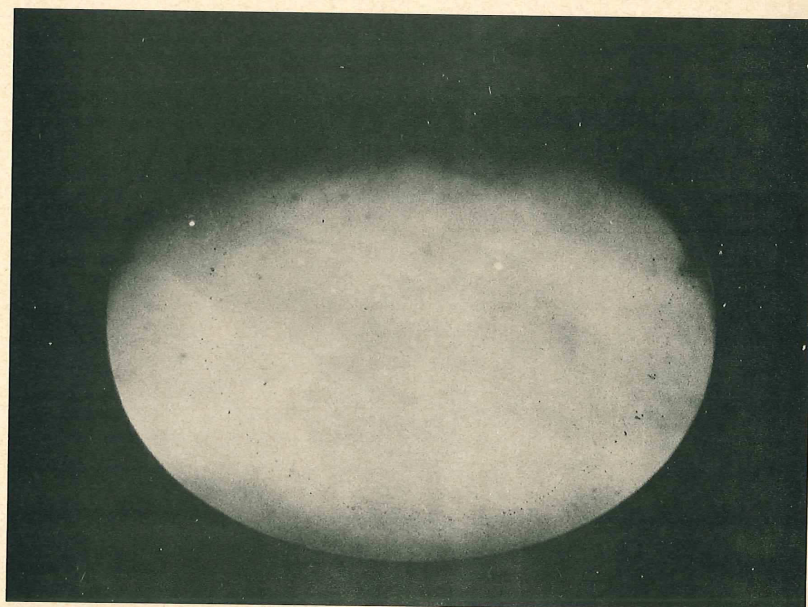
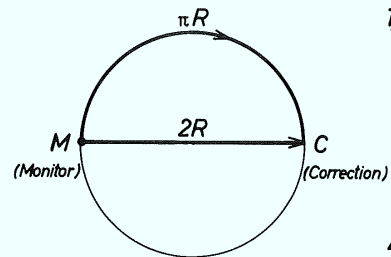


Fig.5

Fig.6

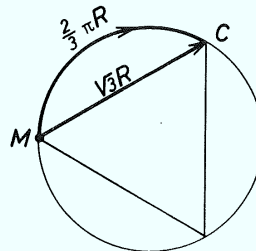
Available rise time for fast beam control



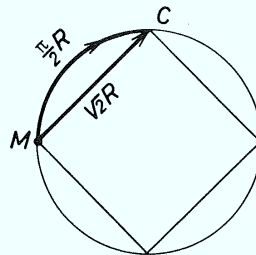
Typical values for a 1000 GeV-Synchrotron
 $(R \approx 2800 \text{ m}; T \approx 60 \mu\text{s}; \beta = \frac{v}{c} \approx 1)$



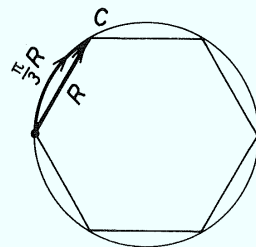
$$\Delta t = \left(1 - \frac{2}{\pi}\right) \frac{T_0}{2} \approx .36 \frac{T_0}{6} \quad (11 \mu\text{s})$$



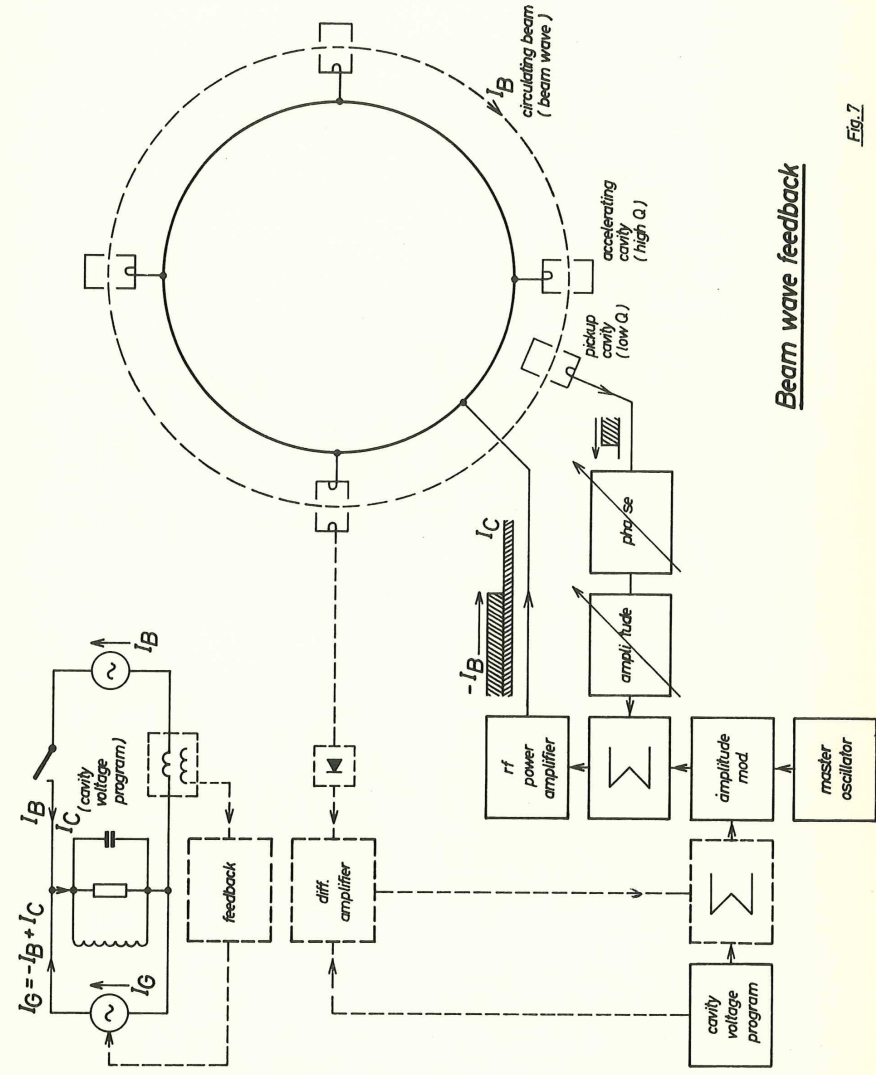
$$\Delta t = \left(1 - \frac{3\sqrt{3}}{2\pi}\right) \frac{T_0}{3} \approx .17 \frac{T_0}{3} \quad (3.3 \mu\text{s})$$



$$\Delta t = \left(1 - \frac{2\sqrt{2}}{\pi}\right) \frac{T_0}{4} \approx .1 \frac{T_0}{4} \quad (1.5 \mu\text{s})$$



$$\Delta t = \left(1 - \frac{3}{\pi}\right) \frac{T_0}{6} \approx .046 \frac{T_0}{6} \quad (.5 \mu\text{s})$$



Beam wave feedback

Fig. 7

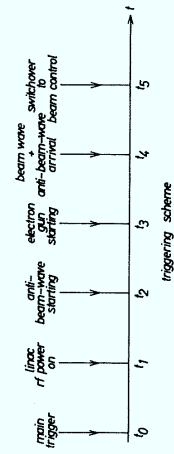
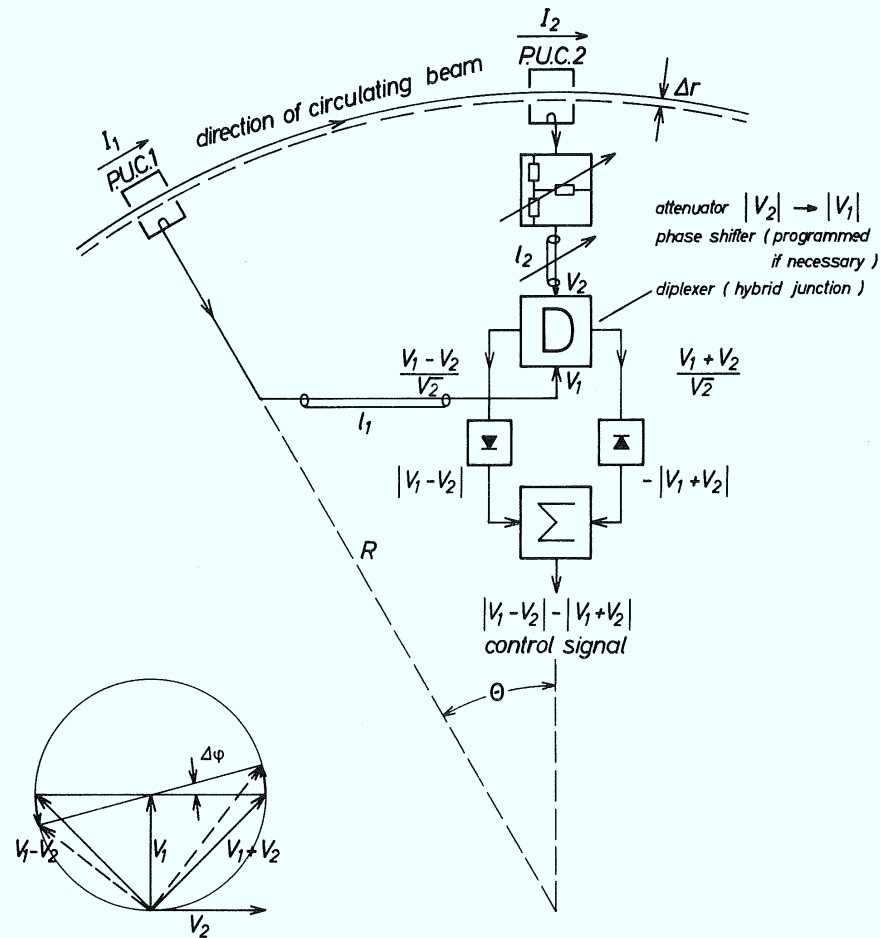


Fig. 8

Fig.9

Beam energy monitor



Line adjustment for equilibrium orbit

$$l_1 - l_2 = \Theta R; \angle V_1 = \angle V_2 + \frac{\pi}{2}$$

Phase shift for radial displacement Δr

$$\Delta\phi = \frac{\Delta r}{R} \cdot \Theta \cdot h \quad (h = \text{harmonic number})$$