ELECTROPRODUCTION OF PIONS NEAR THE $\Lambda(1236)$ ISOBAR
AND THE FORM FACTOR $G_M(q^2)$ OF THE ($\gamma\Lambda$)-VERTEX


Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany
and
II. Institut für Experimentalphysik der Universität, Hamburg, Germany

# Volkswagen Foundation Fellow, now at Laboratoire de l'Accélérateur Linéaire, Orsay, France
+ Now at Sektion Physik, Universität München, Germany
\* Member of Institut für Kernphysik der Universität Mainz, Germany
ABSTRACT

The cross section for inelastic electron-proton scattering was measured at incident electron energies of 1.5 to 6 GeV by magnetic analysis of the scattered electrons at angles between 10° and 35°. For invariant masses of the hadronic final state $M \leq 1.4$ GeV, the measured spectra are compared with theoretical predictions for electroproduction of the $\Delta(1236)$ isobar. The magnetic dipole transition form factor $f_M(q^2)$ of the $(\gamma N \Delta)$-vertex is derived for momentum transfers $q^2 = 0.2 - 2.34$ (GeV/c)$^2$ and found to decrease more rapidly with $q^2$ than the proton form factors.
The results of an experiment on inelastic electron-proton scattering are reported, in which the spectra of the scattered electrons were measured for incident electron energies of 1.5 to 6 GeV and electron scattering angles of 10° to 35°. The experimental set-up and the method of data evaluation have been described in a previous article). 

Fig. 1 shows the spectrum of the scattered electrons for an incident energy of 4.379 GeV and a scattering angle of 10°. This spectrum is uncorrected for radiative effects. On the far right one has the elastic peak, demagnified by a factor 15, then the peaks indicating isobar excitation near \( W = 1.236 \), \( W = 1.52 \) and \( W = 1.68 \) GeV, \( W \) being the invariant mass of the hadronic final state. The momentum transfer at the peak of the \( \Delta(1236) \) isobar was \( q^2 = 0.63 \) (GeV/c)^2. Within the level of accuracy of this experiment, no other resonances with \( W \leq 2.0 \) GeV are observed.

In the previous communication), we have presented part of the results of this experiment in terms of \( \sigma_T(q^2, W) \) and \( \sigma_L(q^2, W) \) (the cross sections for the absorption by the proton of transverse and longitudinal photons) for \( W = 1.220 \) GeV and \( W = 1.350 \) GeV and for momentum transfers of \( q^2 = 0.2 - 2.4 \) (GeV/c)^2. \( \sigma_T \) and \( \sigma_L \) are defined by
the following formula for the inelastic (e-p) scattering cross section:

\[
\frac{1}{P^2} \frac{d^2\sigma}{d\Omega dE} = \sigma_T(q^2, W) + \epsilon \sigma_L(q^2, W),
\]

(1)

where

\[
\begin{align*}
\Gamma_T &= \frac{a}{4\pi} \frac{E}{E'} \frac{(W^2 - m^2)^2}{2M q^2} \left\{ 2 + \cot^2 \frac{q/2}{q_0/2} \right\} \\
\epsilon &= \left\{ 1 + 2(1 + q_0^2/q^2) \tan^2 \frac{q/2}{q_0/2} \right\}^{-1}
\end{align*}
\]

are kinematic parameters whose physical meanings have been discussed in ref. 1. In this letter we present the spectra for \( W \leq 1.4 \) GeV and different values of the incident energy \( E \) and the scattering angle \( \theta \) without separating them into longitudinal and transverse parts. The cross section is split instead into a resonant part \( \sigma_R \) and a background contribution \( \sigma_B \), i.e.

\[
\frac{1}{P^2} \frac{d^2\sigma}{d\Omega dE} = \sigma_R(q^2, W) + \sigma_B(q^2, W).
\]

(2)

\( \sigma_R \) corresponds to the excitation of the \( \Delta(1236) \) isobar and the background \( \sigma_B \) includes both non-resonant production and the contribution of the second resonance. According to current theories\(^2\), \( \sigma_R \) is expected to be mainly transverse and dominated by the magnetic dipole part. We therefore
approximate it by $^{3/4}$)

$$\sigma_R = \frac{g_{\pi N}^2}{w(w^2-M^2)(w-M)^2} \frac{\Gamma(w)}{1 + \frac{q^2}{M^2}} \sigma_M(q^2).$$  \hspace{1cm} (3)

$M$ and $\Gamma(w)$ are the resonant mass and the width function of the $\Delta(1236)$ isobar, $q$ is the three-momentum of the virtual photon in the laboratory system and the other quantities have been defined in ref. 1. $\sigma_M(q^2)$ is the magnetic dipole transition form factor for the excitation of the $\Delta(1236)$ isobar. The definition of $\sigma_M(q^2)$ as given by eq. (3) is the same as that of Ash et al. 5).

Fig. 2 shows some of the measured spectra, to which all corrections have been applied. The solid curves represent best fits to the data, using eq. (3) for $\sigma_R$ with

$$\Gamma(w) = \frac{0.128(0.85 \ p^2/m^2_N)^3}{1 + (0.85 \ p^2/m^2_N)^2} \text{ (GeV).}$$

This expression for the width function is taken from the work of Dalitz and Sutherland 6) and derives from the measured pion-nucleon phase shifts ($p^2$ is the pion three-momentum in the pion-nucleon CM frame). The following formula for $\sigma_B$ was used:
\[
\sigma_B = \sqrt{W-W_{th}} \left\{ \sum_{i=0}^{N} A_i(q^2) (W-W_{th})^i \right\} \\
+ \frac{B(q^2)r(1.520)}{(W-1.520)^2 + \Pi(1.520)^2/4},
\]

where \( A_i(q^2) \) and \( B(q^2) \) are adjustable parameters. This formula has the required square root behaviour at the pion threshold \( W-W_{th} \). Satisfactory fits were obtained for \( N \leq 3 \) and no significant improvement was achieved by adding higher terms. The background functions obtained by the above procedure are indicated by the dash-dot curves in fig. 2. The dotted curves represent the dispersion theoretical predictions of Gutbrod and Simon\(^7\). The agreement between the model and the data is quite good below and at the peak of the resonance.

The values of \( a_m(q^2) \) obtained are listed in table I and plotted as a function of \( q^2 \) in fig. 3a together with the values for \( a_m(q^2) \) obtained in the coincidence measurements of Ash et al.\(^5\) and Imrie et al.\(^8\). The photoproduction point \( a_m(0) = 3.00 \times 0.08 \) has been derived by Ash et al.\(^5\) from the \( \gamma^0 \)-photoproduction data of Fischer et al.\(^9\). The present analysis was carried out under the assumption that the longitudinal cross sections observed\(^1\) in the region \( 0.2 \leq q^2 \leq 0.6 \text{ (GeV/c)^2} \) were non-resonant. However, the
background cross sections in this region obtained from the fitting procedure are smaller than (although, within the error limits, compatible with) the measured longitudinal cross sections. Since some at least of the background must be transverse, this implies that some fraction of the longitudinal contribution must be resonant. This does not alter the situation as regards the transition form factor $G_M^*(q^2)$. For example, if all of $\sigma_L$ were resonant, the values of $G_M^*(q^2)$ in this region would be reduced by only $6 - 10\%$. The solid curve represents the predictions of the Gubrod-Simon model (normalized to $G_M^*(0) = 3.00$). The data are in good agreement with this model and with the empirical expression of Dufner and Tsai\textsuperscript{4}, derived from the data of Lynch et al.\textsuperscript{10} and Brasse et al.\textsuperscript{11}. The result of Dufner and Tsai\textsuperscript{4} agrees rather well with the Gubrod-Simon model and is therefore not shown in the graph. Our experimental points are consistently below the dipole fit, as is more clearly demonstrated by fig. 3b, where the ratio of $G_M^*(q^2)$ to the dipole fit is plotted against $q^2$. We conclude, therefore, that the transition form factor for the $\Delta(1236)$ isobar as defined by eq. (3) falls off more rapidly with increasing $q^2$ than does the nucleon form factor.
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REFERENCES

1) W. Bartel, B. Dudelzak, H. Krehbiel, J. McElroy,
   U. Meyer-Berkhout, W. Schmidt, V. Walther and G. Weber,

2) S. Fubini, Y. Nambu and V. Wataaghin,
   Phys. Rev. 111 (1958) 329;
   B. Y. Geshkenbein, Physics Letters 16 (1965) 323;
   C. Becchi and G. Morpurgo, Physics Letters 17 (1965) 352;
   A. Salam, R. Delbourgo and J. Strathdee,
   N. Zagury, Phys. Rev. 148 (1966) 1112,
   Phys. Rev. 150 (1966) 1406;
   S. L. Adler, Preprint Princeton (1968), to be published
   in Ann. Phys.;
   N. S. Thornber, Institute of Theoretical Physics,
   Stanford University, Report No. ITP-294 (1968);
   P. L. Pritchett, N. S. Thornber, J. D. Walecka and
   P. A. Zucker, Stanford University, Report No. ITP-297
   (1968) and references quoted therein.

REFERENCES
cont'd

4) A.J. Dufner and Y.S. Tsai, Phys. Rev. 168 (1968) 1801.


We have used for $G_M^a(q^2)$ the expression appearing in eq. (22) of this paper with the spinor normalization term $\sqrt{(E_1-M)/(E_1+M)}$ absorbed into $G_M^a$ and the nucleon and pion form factors approximated by the dipole fit. The non-resonant background in this model has been approximated by Born terms (F. Gutbrod, private communication).


9) G. Fischer, H. Fischer, H.J. Kämpgen, G. Knop, P. Schultz and H. Wessels, Proc. XIII Int. Conf. on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley (1967)). We are grateful to Professor Knop...
for pointing out to us that the photoproduction cross sections have a normalization uncertainty of 5%, implying a normalization error of 2.5% in $G_W^2(0)$. We have, therefore, replaced the error ±0.01 given in ref. 5 by ±0.08.

10) H.L. Lynch, J.V. Allaby and D.M. Hitson,

11) F.W. Brasse, J. Engler, E. Ganssauge and M. Schweizer,
Nuovo Cim. 55A (1968) 679. An extension of this work to larger $q^2$ has been presented at the XIV Int. Conf. on High Energy Physics at Vienna (1968) and is to be published by W. Albrecht, F.W. Brasse, H. Doerner, W. Flauger, K. Frank, E. Ganssauge, J. Gayler, H. Hultschig and J. May.
FIGURE CAPTIONS

Fig. 1 Inelastic cross section before application of radiative corrections. The momentum transfer at the peak of the $A(1236)$ isobar was $q^2 = 0.63$ (GeV/c)$^2$.

Fig. 2 Some of the measured spectra in the region of the $A(1236)$ isobar. The scattering angle was $\theta = 13.33^\circ$ in all cases. See text for the meaning of the curves.

Fig. 3 a) The transition form factor $G_M(q^2)$ as a function of the momentum transfer $q^2$.

b) The $q^2$-dependence of the ratio of $G_M(q^2)/G(q^2)$ with $G(q^2) = 3(1+q^2/0.71)^{-2}$. 
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<th>$\theta$ [deg]</th>
<th>$q^2$ [(GeV/c)$^2$]</th>
<th>$G_M^p(q^2)$</th>
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<tr>
<td>13.33</td>
<td>0.20</td>
<td>1.77 (±3.5%)</td>
</tr>
<tr>
<td>13.33</td>
<td>0.30</td>
<td>1.38 (±3.5%)</td>
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<tr>
<td>13.33</td>
<td>0.40</td>
<td>1.17 (±3.0%)</td>
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<td>0.47</td>
<td>0.978 (±3.0%)</td>
</tr>
<tr>
<td>35.00</td>
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<td>0.961 (±3.5%)</td>
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<tr>
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<td>0.50</td>
<td>0.964 (±3.0%)</td>
</tr>
<tr>
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<tr>
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<td>0.735 (±3.0%)</td>
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