On the Thermodynamics of Bubble Chamber Expansions

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Abstract

On the basis of an adiabatic theory a number of equations is given, which allow the study of the relations between expansion ratio ε, dynamic pressure p and the parasitic boiling in a bubble chamber. A "dirt function" η is defined as the molar fraction of the liquid which is evaporated during an expansion cycle. Furthermore a method is presented to measure η under certain assumptions from pressure-volume-diagrams of bubble chamber expansions.

A number of p-v-diagrams taken in the 85 cm bubble chamber at DESY, has been studied; first results could be qualitatively compared with theory. The adiabatic compressibility of liquid hydrogen and deuterium has been measured at different temperatures.
A. Introduction

A number of theoretical and experimental investigations have been published concerning the processes of microscopic creation and macroscopic growth of bubbles in superheated liquids, which are of special interest for bubble chamber operations. All these publications describe the behaviour of bubbles under different thermodynamical conditions regardless how these conditions are realized. Fortunately, the existing bubble chambers have a certain flexibility both in adjustment of temperature as well as in the operation of the expansion system, so that it was possible to find conditions under which the chambers could be made sensitive to ionizing radiation.

The development of the next generation of larger bubble chambers and the desire for higher reproducibility of bubble chamber parameters requires a more detailed knowledge of bubble chamber operation conditions.

In order to economise mechanical design as well as power consumption, the expansion systems of most of the new large chambers will be of the free oscillating type. This means that the parameters of piston movement depend almost entirely on the mechanical and geometrical parameters of the bubble chamber and the liquid, which can hardly be changed during operation. Consequently these expansion systems will be much less flexible than the existing smaller ones. To be sure that a certain design of expansion system allows to sensitise a bubble chamber it should be possible to estimate roughly the thermodynamic conditions which can be expected in a bubble chamber.

In order to obtain a basis of informations, one of the most important parameters, i.e. the pressure change as a function of piston stroke has been investigated in the 85 cm cryogenic bubble chamber at DESY.

In Fig. 1 some early results of such measurements are represented. The curves show a typical saturation effect at higher piston amplitudes. On the other hand they could not be reproduced in consecutive experimental runs.
Starting from the well-known hypothesis that spontaneous evaporation has a remarkable influence on the pressure in a bubble chamber, a semi-phenomenological theory has been derived by one of the authors, which is here presented in detail. This theory should be compared not only with measurements of the bubble chamber at DESY but, if possible, with similar measurements in all existing large bubble chambers.

B. Theory

We consider an ideal, hermetically closed bubble chamber as shown schematically in Fig.2. The chamber is expanded by a piston of surface area $q$ which moves, as a function of time $t$, over a distance $h = h(t)$. The variation of the volume is then

$$\Delta V = \Delta V(t) = q \cdot h(t)$$

Fig.3 shows the working region of a bubble chamber in a phase diagram for hydrogen. Since a bubble chamber is expanded only by maximally some few percents, the interesting region of the diagram is shown on an enlarged scale in Fig.4 using the expansion ratio $\epsilon$ (Eq.2) instead of the volume.

Looking at Fig.4 we might start from the expanded state (point 3') of the piston, which had been held in this position for a time long enough to establish the equilibrium vapour pressure $P_{v0} = P_v(T_0)$ corresponding to the temperature $T_0$ in the liquid. If we now move the piston very slowly, reducing the volume, the vapour phase condenses at constant pressure until all vapour has been liquified. This state is reached in point 0 in the diagram. We call $V_0$ the total volume of the liquid in this point, and we will give all volumes in terms of this $V_0$. Since we are only interested in small variations of this volume, we introduce the expansion ratio

$$\epsilon = \frac{V - V_0}{V_0}$$

($\epsilon = 0$ in point 0)

($V = V_0 + \Delta V$)
If we continue to push the piston we have to compress the liquid, and consequently the pressure rises \((p = p(V))\) and \(V = V(p)\), respectively. As long as the speed of compression is infinitely slow, compression will be isothermal and, with the definition of isothermal compressibility

\[
\beta_{\text{isothermal}} = \beta = -\frac{1}{V} \left( \frac{\Delta p}{\Delta V} \right)_T
\]

we find the relation between pressure and volume

\[
\beta(p - p_{v_0}) = -\frac{V(p) - V_0}{V_0}
\]

or, using (2)

\[
\beta(p - p_{v_0}) = -\varepsilon
\]

If \(\varepsilon\) varies with time, one can calculate \(p\) as a function of time

\[
p(t) = p(\varepsilon(t)) = p_{v_0} - \frac{\varepsilon(t)}{\beta}
\]

In order to ensure a complete recondensation of all vapour bubbles before an expansion starts in a bubble chamber, one has to establish a static pressure \(p_s > p_{v_0}\) which corresponds to point 1 in Fig.4. From Eq.(4) we calculate the static \(\varepsilon\) before expansion

\[
\varepsilon_s = -\beta(p_s - p_{v_0})
\]

As has been shown before Eqs.(3) - (5) are valid exactly only in the case of isothermal expansion or compression. We have an isothermal expansion if the time necessary for the process under consideration is large compared with the time for heat transport over the linear dimensions of the liquid.
The time constant of heat diffusion is

$$\tau = \frac{d^2}{2D}$$

(7)

$$d = \text{geometrical length}$$

$$D = \frac{k_L}{c_p \rho_L} = \text{constant of heat diffusion}$$

$$k_L = \text{thermal conductivity of the liquid}$$

$$c_p = \text{specific heat of the liquid}$$

$$\rho_L = \text{density of the liquid}$$

For hydrogen under bubble chamber operating conditions one calculates with \( d = 1 \) mm,

$$\tau = 3 \text{ sec}$$

which is long compared with the period of \( 5 \cdot 10^{-2} \text{ sec.} \) of a normal bubble chamber expansion.

Thus thermal diffusion can be neglected. More exactly, the heat transport by the movement of the liquid due to the turbulences during the expansion should be considered, but this is neglected in this paper.

We approximate the behaviour of bubble chambers by an adiabatic theory. A consequence of this quasi-adiabatic expansion in bubble chambers is, that each variation of pressure results in a change of temperature in the liquid. In addition to the isothermal change of volume described by Eq.(4), this variation in temperature \( \Delta T \) causes another change of volume according to the factor of the thermal expansion. Thus the total variation of volume in the case of adiabatic expansion is

$$ (dV)_{ad} = \left(\frac{\partial V}{\partial p}\right)_T (dp)_{ad} + \left(\frac{\partial V}{\partial T}\right)_p (dT)_{ad} $$

(8)

and, after division by \( V dp \)
\[
\frac{1}{\Delta V/\Delta p} = \frac{1}{V'\partial p/\partial T} + \frac{1}{V''\partial T/\partial p} \Delta T/\Delta p
\]

(9)

With Eq. (3) and

\[
\frac{1}{\Delta V/\Delta p} = -\beta' = \text{adiabatic compressibility}
\]

(10)

\[
\frac{1}{V''/\partial T} = \alpha = \text{factor of thermal expansion}
\]

(11)

we find from Eq. (9)

\[
\beta' = \beta - \alpha \frac{dT}{dp} \Delta p
\]

(12)

and

\[
\frac{dT}{dp} \Delta p = \frac{1}{\alpha} (\beta - \beta')
\]

(13)

On the other hand the theory of thermodynamics gives for the adiabatic temperature variation with pressure

\[
\frac{dT}{dp} \Delta p = (1 - \frac{1}{\gamma}) \frac{\partial T}{\partial p} \Delta p
\]

(14)

with \(\gamma = \frac{c_p}{c_v}\).

From

\[
V = V(p, T)
\]

(15)

one finds

\[
dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT
\]

(16)
and for constant volume \( (dV = 0) \)

\[
\left( \frac{\partial V}{\partial p} \right)_T \left( \frac{\partial p}{\partial V} \right)_T + \left( \frac{\partial V}{\partial T} \right)_p = 0
\]

which gives with Eqs. (10) and (11)

\[
\left( \frac{\partial T}{\partial p} \right)_V = \frac{\delta}{\alpha}
\]

and

\[
\left( \frac{\partial T}{\partial p} \right)_{ad} = (1 - \frac{1}{\gamma}) \frac{\delta}{\alpha}
\]

If one combines Eq. (19) with Eq. (12), one gets the following relation between \( \delta' \) and \( \delta \) which is well known from thermodynamics,

\[
\delta' = \frac{\delta}{\gamma}
\]

If three of the four parameters \( \alpha; \delta; \delta'; \) and \( \gamma \) are known, one can use either Eqs. (13) or (19) to calculate the temperature drop in the liquid due to the expansion. (See also Appendix II).

Every expansion is performed by a fast movement of the piston which results in a variation of the volume. So we chose \( \epsilon(t) \) as independent variable in all our equations. Starting from point 1 in Fig. 4 with the static pressure \( p_s \) and \( \epsilon_s < 0 \) according to Eq. (6) we have now the relations for the fast expansion

\[
\delta'[p(t) - p_s] = \epsilon(t) - \epsilon_s
\]

and

\[
p(t) = p_s - \frac{1}{\alpha_s}[\epsilon(t) - \epsilon_s]
\]
In point 2 of Fig. 4 p(t) is equal to the initial vapour pressure \( P_{vo} \).

We remember that in consequence of Eq. (13) 2 is now a point in the p-V-diagram completely situated in the liquid phase region at the lower temperature \( T = T_o - \Delta T \). The volume at \( p(t) = P_{vo} \) is

\[
\varepsilon_2 = (\beta - \beta')(p_s - P_{vo})
\]  

(23)

In point 2* the pressure curve intersects the phase boundary \( P_v(c) \) which separates the phase regions of pure liquid and liquid plus vapour in equilibrium*. If \( p_v = P_v(T) \) is the equilibrium-vapour-pressure as function of temperature, we find for the pressure in point 2, in linear approximation,

\[
P_{v}^* = P_{vo} + \frac{dp_v}{dT} \bigg|_{T = T_o} \Delta T
\]  

(24)

and, with Eq. (13)

\[
P_{vo} = p_s + \left( \frac{dp_v}{dT} \right) \bigg|_{T = T_o} \cdot \frac{1}{\alpha} \cdot (\beta - \beta')
\]  

(25)

\[
P_{v}^* = \frac{p_{vo} - p_s \frac{dp_v}{dT} \bigg|_{T = T_o}}{1 - \frac{dp_v}{dT} \bigg|_{T = T_o} \cdot \frac{1}{\alpha} (\beta - \beta')}
\]

If

\[
\frac{dp_v}{dT} \bigg|_{T = T_o} \frac{1}{\alpha} (\beta - \beta') \ll 1
\]  

(26)

we find approximately

\[
P_{v}^* = p_{vo} - \frac{1}{\alpha} (\beta - \beta') (p_s - P_{vo}) \cdot \left( \frac{dp_v}{dT} \right) \bigg|_{T = T_o}
\]  

(27)

*A definition of \( P_v(c) \) is given in Appendix II.
Eqs. (6), (21) and (25) give for $\epsilon^*$

$$
\epsilon^* = \left( \frac{1}{\beta} - \frac{dp}{\alpha \frac{d\rho}{\rho}} \right) \frac{T = T_o}{1 - \frac{\beta - \beta'}{\alpha} \frac{dp}{d\rho} = T_o}.
$$

(28)

If one now continues to expand to $\epsilon > \epsilon^*$, the further behaviour of $p(\epsilon)$ depends on the amount of vapour which is produced anywhere within the volume of the liquid. In the case of a very fast expansion the quantity of vapour produced during the time of expansion can be neglected. Then function $p(\epsilon)$ still follows Eq. (20) until the maximum position of the piston is reached (point 3 in Fig. 4). This gives the theoretical minimum of pressure$^*$

$$(p_{\text{min}})_{\text{theor.}} = p_s - \frac{1}{\beta} \left( \epsilon_{\text{max}} - \epsilon_s \right).
$$

(29)

We now consider the case of evaporation which starts at a moment $\epsilon' > \epsilon^*$ after $\epsilon(\tau)$ has passed the value $\epsilon = \epsilon^*$.++

In Eq. (21) $\epsilon(\tau) - \epsilon_s$ is the volume which the moving piston adds to the chamber volume during the expansion.

For $p(\tau) < p_v^*$ this volume is always occupied completely by the expanding liquid.

For $\tau > \tau'$ a certain amount of liquid evaporates and occupies a volume $v(\tau)$ which varies with time. So we have instead of Eq. (21)

$$
- \beta' [p(\tau) - p_s] + \frac{v(\tau)}{\rho_0} = \epsilon(\tau) - \epsilon_s
$$

(30)

$^*$Beta' is considered to be constant. This is practically valid for hydrogen and deuterium within the pressure range used in bubble chambers.

++ See Chapt. C,1 and C,2.
with \( v(t) \equiv 0 \) for \( t < t' \). \((30a)\)

Now we introduce

\[
v(t) = n(t) \cdot v_m(T'; p(t))
\]

and

\[
v_o = N_o \cdot v_m(T_o)
\]

with

\[
\begin{align*}
n(t) & \quad = \text{number of evaporated moles at time } t \\
N_o & \quad = \text{total number of moles in the bubble chamber} \\
v_m(T'; p(t)) & \quad = \text{molar volume of vapour at a temperature } T' \\
& \quad \text{and pressure } p(t) \\
v_m(T_o) & \quad = \text{molar volume of liquid at equilibrium} \\
& \quad \text{temperature } T_o
\end{align*}
\]

To be independent of the special geometry of a bubble chamber, we define the dimensionless functions

\[
\eta(t) = \frac{n(t)}{N_o}
\]

and

\[
u[p(t); T'; T_o] = \frac{v_m(T'; p(t))}{v_m(T_o)}
\]

and so we get instead of (30)

\[
\varepsilon(t) - \varepsilon_s = -\beta'[p(t) - p_s] + u[p(t); T'; T_o] \eta(t)
\]

from which we calculate the pressure

\[
p(t) = p_s - \frac{1}{\beta'}[\varepsilon(t) - \varepsilon_s - u[p(t); T'; T_o] \eta(t)]
\]
and the time-derivative

\[
\frac{dp}{dt} = -\frac{1}{\beta} \left[ \frac{dc}{dt} - \eta(t) \frac{\partial p}{\partial p} \frac{dp}{dt} + \frac{\partial u}{\partial T'} \frac{dT'}{dt} - \omega[p(t), T', T_c] \frac{dn}{dt} \right]
\]

\[
\frac{dp}{dt} = -\frac{dc(t) - \omega[p(t), T', T_c] \frac{dn}{dt} - \eta(t) \frac{\partial u}{\partial T'} \frac{dT'}{dt}}{\beta' - \eta(t) \frac{\partial u}{\partial p}}.
\]

Eq. (38) is valid for all combinations of \( T'(t) \) and \( p(t) \) within the vapour. Now we use the special assumption that the temperature \( T'(t) \) within the vapour phase is the equilibrium vapour pressure temperature which corresponds to the instantaneous pressure \( p(t) \). In this case we have

\[
T'(t) = T'(p(t))
\]

\[
\omega[p(t), T', T_c] = \omega[p(t), T_c]
\]

\[
\frac{\partial u}{\partial T'} = 0
\]

\[
\frac{\partial u}{\partial p} = \frac{du}{dp}
\]

and we get the simpler form

\[
\frac{dp}{dt} = \frac{dc(t) - \omega[p(t); T_c] \frac{dn}{dt}}{\beta' - \eta(t) \frac{du}{dp}}.
\]

which is zero in the minimum for
\[
\frac{dc(t)}{dt} = \omega (p(t), T) \frac{dn}{dt}
\]  
(44)

From Eq. (37) the pressure in a bubble chamber as a function of time can be calculated, provided the "dirt function" \( \eta(t) \) is known. There exist two limits for the pressure-curves in the p-c-diagram, one for infinitely fast expansion (from point 1 in Fig. 4 passing 2 to point 3), the other one for infinitely slow expansion (from point 1 passing 0 to 3'). Between them a large number of pressure-volume-functions is possible. Their shape depends on the functions \( \eta(t) \) and \( c(t) \). With the assumption that bubble chamber expansions are practically adiabatic, all expansion curves starting from the same point 1 in Fig. 4 will normally follow the line from point 1 to point 2' and begin to separate beyond point 2'. This gives a possibility to measure the adiabatic compressibility \( B' \) of a liquid in a bubble chamber.

In the liquid phase of the p-v-diagram one has (differentiation of Eq. (21))

\[
B' = -\frac{dc}{dp}
\]  
(45)

for \( \epsilon < \epsilon < c' \).

As can be seen from Eqs. (43) and (44) the limit in pressure reduction depends on two parameters, the speed of expansion \( \frac{dc}{dt} \) and the rate of evaporation \( \frac{dn}{dt} \). The "dirt function" \( \eta(t) \) is a complicated statistical function which depends on all effects which have influence on spontaneous boiling. While a theoretical prediction seems rather hopeless at present (a very simplified model is given in Appendix I), \( \eta(t) \) can easily be measured in a p-c-diagram, as shown in Fig. 4.

For each value of pressure \( P_A \) we find in point A the theoretical \( \epsilon \)-value without evaporation, while we have \( \epsilon \) with evaporation in point B.

From Eq. (22) follows

\[
P(t)_A = p_\epsilon - \frac{1}{B'} [c(t)_A - \epsilon_\epsilon]
\]  
(46)

and from Eq. (37)

"The expression "dirt function" refers to "dirty bubble chambers" which are characterized by evaporation."
\[ p(t)_B = P_s - \frac{1}{\beta} \{ \varepsilon(t)_B - \varepsilon_s - \omega(p(t); T'; T_o) \eta(t) \} \tag{47} \]

For \( p(t)_A = p(t)_B \), one gets

\[ \omega(p(t); T'; T_o) \eta(t) = \varepsilon(t)_B - \varepsilon(t)_A \] \tag{48}

Since in a \( p-\varepsilon \)-diagram the time is eliminated, we primarily get \( \varepsilon_A, \varepsilon_B \) and consequently \( \eta \) as functions of \( p \)

\[ \omega(p; T'; T_o) \eta(p) = \varepsilon(p)_B - \varepsilon(p)_A \] \tag{49}

but this can easily be transformed into (48) if either \( p \) or \( \varepsilon \) had been measured simultaneously as a function of time. Then one has

\[ \eta(t) = \frac{\varepsilon[p(t)]_B - \varepsilon[p(t)]_A}{\omega(p(t); T'; T_o)} \] \tag{50}

\( \omega \) can be calculated as a function of \( p \) from the mole volumes (Eq. 35). For the case of vapour and liquid in equilibrium \( \omega(p) \) has been plotted for hydrogen at different temperatures \( T_o \) (Fig. 3).

As can be seen from this figure, \( \omega \) increases very rapidly with decreasing pressure \( p \).

Consequently according to Eq. (43), the speed of pressure variation \( \frac{dp}{dt} \) decreases with pressure even at constant \( \frac{d\varepsilon}{dt} \) and at small values of \( \frac{d\eta}{dt} \). In every case there exists a lower limit in pressure given by Eq. (44). If this limit is not sufficient for the desired applications, the only way to overcome the influence of \( \varepsilon \) and \( \eta \) is to increase \( \frac{d\varepsilon}{dt} \), i.e. the speed of the piston. (In a bubble chamber \( \frac{d\varepsilon}{dt} \) is not constant, but it decreases at the end of the piston stroke).
A very important feature is the question how parasitic boiling depends on the special geometry and size of a bubble chamber. There are four different effects by which parasitic boiling is created:

1. Rough parts and edges on the surfaces of the container,
2. Impurities within the liquid
3. Ionizing radiation such as cosmic rays or radioactive material on the surface or in the liquid (e.g., tritium contamination)
4. Statistical temperature variations.

Under normal operating conditions of present bubble chambers it seems, that point 1 dominates. If this will prove to be true, the number of nuclei for parasitic boiling is proportional to the surface of the container, and since the ratio of surface to volume decreases with increasing sizes of bubble chambers, \( \eta \) will decrease too. Unfortunately, in bubble chambers the sources of boiling are not distributed homogeneously over all parts of the surface, but there are several regions where they are concentrated, such as joints, expansion system etc.

Thus at the present state of knowledge it is difficult to extrapolate the behaviour of \( \eta \) from one bubble chamber to another. Rather, it should be measured in different chambers.

As to points 2-4 these effects are entirely proportional to the volume (apart from radioactive materials in the chamber walls which should be avoided in bubble chamber constructions). The influence of impurities is experimentally investigated in this experiment (Chapter C, 4). Ionizing radiation can usually be neglected (we found that even beam-intensities which were \( 10^3 \) times higher than normally used in high energy physics experiments did not cause any measurable change of the shape of the p-v-diagrams).
Statistical temperature variations seem to become important only in the case of either extremely high temperatures (near the critical point) or extremely low pressures, where they give rise to the foam limit well known in overexpanded bubble chambers.

C. Experimental Arrangement and Preliminary Results

The purpose of our present measurements was to gather experimental material as a basis for a more detailed knowledge of the expansion in bubble chambers, especially of the function \( n(t) \). The experimental arrangement was as follows (Fig.6):

The pressure was picked up by a piezoelectric quartz gauge, which had been calibrated under operation conditions in the chamber. The movement of the piston was measured by means of an inductive speed sensitive transducer (a magnet core mounted on the piston and moving within a coil). The signal of the coil was integrated. Both signals of pressure and displacement have been displayed and photographed simultaneously on two oscilloscopes. The first oscilloscope gave \( p \) and \( h \) as functions of time, the second one gave \( p \) as a function of \( h \).

Both signal amplitudes have been measured with a compensation method (using the Tektronix compensation amplifier type Z). The coordinates of \( p \) and \( h \) were measured on the photographs and using the corresponding calibration factors, the \( p-v \)-diagrams could be plotted. In Fig.7 some of the original photographs are shown.

A number of about 140 \( p-v \)-diagrams have been photographed both for hydrogen and deuterium. The evaluation of the diagrams is not yet completed, but some preliminary results can be presented here.

1. Pressure Threshold for Parasitic Boiling

In the previous chapters the starting time \( t' \) of parasitic boiling was assumed to be close to the moment when the pressure in the bubble chamber has passed the vapour pressure \( p_v \). Our experiment should give some informations about the validity of this assumption.
Starting from a static pressure $p_s > p_{vo}$ the pressure was reduced to different minimum values $p_{min}$. For each $p_{min}$ the irreversible heat per expansion cycle $I = \frac{1}{pdV}$ was measured. The value of $I$ is relative small if the expansion occurs in the liquid phase only, but it increases very rapidly if vapour bubbles exist within the volume of the liquid.

In Figs. 8 and 9 $\frac{1}{pdV}$ is plotted versus the minimum pressure for different static pressures $p_s$ at the two temperatures $T_0 = 25 \, \Omega K$ (Fig. 8) and $T_0 = 27 \, \Omega K$ (Fig. 9). Within the limits of errors, the three curves in each diagram converge in a region very close to the vapour pressure $p_{vo}$ (exactly at different static pressures $p_s$ the points $p_{vo}^* \neq p_{vo}$ should be different and $p_{vo}^* \neq p_{vo}$ according to Eq. (27), but the precision of this measurement is not sufficient to find that out).

It should be mentioned that the starting pressure for parasitic boiling is much higher than the pressure $p_L$ at which bubbles are produced by ionizing particles (limit of sensitivity$^{1,5}$). This is due to the completely different creation processes.

2. The Functions $p(c); p(t), \tau(t)$ and $\eta(t)$

Even without the knowledge of $\eta(t)$ one can draw some important conclusions from the previous equations. Eqs. (40) and (41) show that, in consequence of the rate of evaporation, the minimum of the pressure is reached before the piston has stopped its movement in the maximum position. In other words: a further increase of $\gamma$ (i.e. of the piston stroke) has no further influence on the pressure minimum in the chamber. In the plots of Figs. 8 and 9 this limit of pressure reduction is indicated by an increase of the irreversible heat to infinite values.

A comparison between the $p-c$-diagrams of Figs. 10 and 11 demonstrates this effect. In Fig. 10 an increase of $c_{max}$ from curve (a) to curve (b) results in a lower minimum of pressure. In Fig. 11 the increased maximum of the expansion ratio did not reduce the minimum pressure $p_{min}$. Only the value of $\frac{1}{pdV}$ was increased from 137 Joule to 174 Joule.
Figs. 12 and 13 show the time dependence of the functions $\eta(t)$, $\varepsilon(t)$ and $p(t)$. In qualitative agreement with the theory of Chapter B the minimum of the pressure is reached earlier than the maximum of the volume. Although the results of Figs. 8 and 9 indicate that parasitic boiling begins already very close to the pressure $p = p_v^*$, we observe in most of the diagrams of Figs. 12 and 13 a delay of the order of one millisecond between the time $t^*$ and the time $t'$ at which evaporation starts. With the present errors of the experiment it is not possible to decide whether this delay really exists or not.

Furthermore it can be seen that evaporation continues even after the extreme values of $p$ and $\varepsilon$ are passed.* The delay for the stop of evaporation can be explained in the following way: We have treated the expansion cycle theoretically as an adiabatic-isentropic process. This would mean that the vapour-liquid system in the bubbles passes only states of thermodynamical equilibrium. Since vapour in equilibrium is always saturated, no further evaporation could occur. For a real process a small amount of non-equilibrium is necessary, i.e. we have to assume a vapour density in the bubbles which is by a small amount lower than the equilibrium density corresponding to the pressure $p(t)$ and the temperature of the liquid surface $T'' \neq T'$.

On the other hand, during the recompression phase we need a higher vapour density compared with the equilibrium density. It takes a certain time before the conditions for evaporation have been changed into those for recompression. During this time the piston moves already in the opposite direction while the vapour is heated adiabatically.

3. Influence of Impurities

The influence of impurities on the rate of evaporation is demonstrated in Figs. 14 and 15. In both diagrams a and b the movement of the piston was the same ($\varepsilon_a(t) \equiv \varepsilon_b(t)$). After the first diagram (a) was photo-

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*This effect can also be observed in the equivalent plots of Ref. 7 (evaporated mass vs. pressure).
graphed, a certain quantity of nitrogen was injected into the chamber which caused an equal distribution of nitrogen crystals over all the volume of the chamber. These crystals acted as additional nuclei of evaporation. A plot of \( \eta \) as a function of \( \tau \) is given in the lower part of Fig.15 for both cases (a) and (b).

The results of this experiment agree qualitatively with the theory of Chapter B. Because of the higher rate of evaporation in case (b) the pressure reduction is stopped earlier and the minimum pressure is about 0.4 kPa higher than in case (a). On the other hand it takes more time to reach the conditions for recondensation in the higher vapour volume.

The irreversible heat per expansion cycle increases from 77 Joules to 266 Joules, i.e. approximately proportional to \( \eta_{\text{max}} \).

4. Influence of the Piston Region

The measuring results reported in C 1 - C 3 represent the integral effect of all bubbles produced anywhere in the chamber. Measurements in other chambers indicate a relative high production rate of parasitic bubbles in the piston region. In order to get a first information whether we have the same effect in our chamber, we used an additional heat exchanger above the piston (E in Fig.2) to cool this region separately. In Figs.16 and 17 the values of \( \eta_{\text{min}} \), \( \eta_{\text{max}} \) (for \( H_2 \)), resp. \( (\omega \cdot \eta)_{\text{max}} \) (for \( D_2 \)) are plotted versus the cooling time \( t_{\text{cool}} \). The conditions of the expansion system as well as the cooling power in the heat exchanger E were kept constant. Within the limits of errors no influence of piston cooling could be found in hydrogen. In deuterium a small difference between the values of \( \eta_{\text{max}} \) and \( (\omega \cdot \eta)_{\text{max}} \), before cooling and after a cooling time of about 40 minutes was measurable. From these results we conclude that in our bubble chamber a preference of the piston region, as observed in other chambers, is practically negligible in hydrogen. In deuterium this effect is measurable, but it seems to be very moderate.

5. Measurements of the Adiabatic Compressibility \( \beta' \) in \( H_2 \), \( D_2 \), and in a \((5.0 \pm 0.7)/94.3\) Mole Percent \((H_2 + D_2)/\text{He Mixture}\)

\( \beta' \) was derived from the measured p-v-diagrams using Eq.(45). Table I gives the results for hydrogen compared with data published by other
authors. The agreement with the values of which are calculated from the data of , is very good. In the case of deuterium (Table 2) no data for comparison are available.

In Table 3 we give the adiabatic compressibility measured in a mixture of 94.3 mole percent neon, 5 mole percent hydrogen and 0.7 mole percent deuterium.* For comparison the value of in pure neon is indicated. Both compressibilities have the same order of magnitude. It seems reasonable that the contamination of hydrogen "softens" the neon, i.e., \( \beta'_{\text{Mix}} > \beta'_{\text{Ne}} \).

### Table 1

<table>
<thead>
<tr>
<th>( T_0 )</th>
<th>( 10^3 \cdot \beta' )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 K</td>
<td>1.75 ± 0.1</td>
<td>this experiment</td>
</tr>
<tr>
<td>27 K</td>
<td>2.19 ± 0.08</td>
<td></td>
</tr>
</tbody>
</table>

\[
\left[ \frac{\text{kp}}{\text{cm}^2} \right]^{-1}
\]

<table>
<thead>
<tr>
<th>( \frac{\text{kp}}{\text{cm}^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.71</td>
</tr>
<tr>
<td>2.18</td>
</tr>
<tr>
<td>1.2</td>
</tr>
<tr>
<td>1.48</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>( T_0 )</th>
<th>( 10^3 \cdot \beta' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 °</td>
<td>1.25 ± 0.1</td>
</tr>
<tr>
<td>32 °</td>
<td>1.44 ± 0.06</td>
</tr>
</tbody>
</table>

\[
\left[ \frac{\text{kp}}{\text{cm}^2} \right]^{-1}
\]

*This mixture was used in our bubble chamber during another experiment.
TABLE 3
Adiabatic compressibility $\beta'$ in a $(5.0 + 0.7)/94.3$ mole % $(H_2 + D_2)/Ne$ mixture at $T = 30.5$ K

\[
\begin{array}{ccc}
\text{Mixture} & \beta' \times 10^4 & \text{Pure Neon} \\
\hline
& 5.5 & 4.9 \\
& [ \frac{kp}{cm^2} ]^{-1} & \\
\end{array}
\]

B. Summary and Conclusions

The results of our experiments may be summarized as follows:

1. Parasitic boiling in a bubble chamber can occur at every pressure $p$ which is smaller than the vapour pressure $p^*_v$.

2. If $p^*_v$ is passed rapidly, there seems to exist a delay between the time $t^*$ at which $p(t) = p^*_v$ and the time $t'$ when parasitic boiling starts.

3. The behaviour of the pressure as a function of time during an expansion can be described by introducing a "dirt-function" $\eta(t)$ into the equations for the expansion of pure compressible liquids. $\eta(t)$ is the molar fraction of the bubble chamber liquid which has been evaporated up to the time $t$.

4. Some consequences of the existence of $\eta$ could be qualitatively proved by experimental results:
   a) The minimum of pressure is always passed before the time at which the maximum of piston stroke is reached.
   b) At a given expansion speed the pressure drop in a bubble chamber is limited by the amount of vapour produced in the time before the minimum.
   c) An increase of the number of nuclei for parasitic boiling increases the value of $p_{\text{min}}$. 
5. The total volume of the vapour in the bubble chamber can directly be measured in a p-v-diagram. \( \eta \) can be calculated, if the molar volume of the vapour is known. From our measurements we found \( \eta_{\text{max}} \) to be of the order of \( 10^{-4} \) for expansions under normal operating conditions and \( 4 \cdot 10^{-4} \) for expansions which occurred under extreme unusual conditions.

6. The adiabatic compressibility \( \beta' \) can be measured directly in a bubble chamber. In the case of hydrogen \( \beta' \) agrees very well with values calculated from other data.

Similar measurements should be done also in other bubble chambers. Mainly the following points seem to be of special interest:

a) Dependence of \( \eta \) on the size of bubble chambers

b) Dependence of \( \eta \) and \( \frac{d\eta}{dt} \) on the speed of expansion

c) Dependence of \( \frac{d\eta}{dt} \) on the instantaneous pressure \( p(t) \)

d) Dependence of the delay time \( t' - t^* \) (if it exists) on expansion speed, pressure, and impurities. Its study seems to be of great importance for the design of fast cycling bubble chambers. An ideal chamber for fast cycling would be a chamber where the total period of the expansion cycle is shorter than the delay time \( t' - t^* \). Such a bubble chamber would have completed the cycle before parasitic boiling could start.
References


6. V. P. Kenney, W. B. Shephard, W. B. Madden, E. A. Harrington, Argonne National Laboratory ANL-9892 (1967).


Acknowledgement

We are greatly indebted to Professor M. W. Teucher for his continued interest and encouragement in the course of these investigations. We also thank Miss S. Nolte for many numerical evaluations. Furthermore, we have to thank our bubble chamber team, especially W. Eschricht, H. J. Fiebig, U. Knopf, K. D. Nowakowski, O. Peters, and W. Stahlischmidt.
APPENDIX I

The "Dirt Function" \( \eta(t) \)

The function \( \eta(t) \) was introduced and defined in Chapter B. A simplified model which might be useful for qualitative theoretical considerations is given in this appendix.

Starting from Eq. (B,34) the variation of \( \eta(t) \) with time is given by

\[
\frac{dn}{dt} = \frac{1}{n_0} \frac{dn}{dt}
\]  

\( \frac{dn}{dt} \) can be represented as a sum over all bubbles in the liquid:

\[
\frac{dn}{dt} = \sum_{\nu=1}^{\nu} \frac{m(t)}{n_0} \frac{dn_{\nu}}{dt}
\]  

where \( \frac{dn_{\nu}}{dt} \) is the rate of evaporation in the bubble number \( \nu \) and \( m(t) \) is the total number of bubbles in the liquid at the time \( t \). In order to evaporate this amount of vapour per unit of time, an equivalent current of heat \( dq_{\nu} \) has to arrive at the surface of this bubble. So we have

\[
\frac{1}{m(t)} \sum_{\nu=1}^{\nu} \frac{m(t)}{n_0} L(T') \frac{dn_{\nu}}{dt} \]

\[
\frac{1}{m(t)} \sum_{\nu=1}^{\nu} \frac{m(t)}{n_0} dq_{\nu}
\]

with

\( L(T') = \text{molar heat of evaporation at the temperature } T' \text{ of the bubble surface.} \)

If the distribution of temperature \( T \) around each bubble is known, one can calculate \( dq_{\nu} \) and consequently \( \frac{dn_{\nu}}{dt} \) under the assumption that the
temperature $T'$ within each bubble is uniform and equal to the
temperature of the surface of the bubble. Then is

$$\frac{dq}{dt} = -k(T') \int (\text{grad } T; \overrightarrow{df}_v)$$ (I 4)

$k(T')$ = thermal conductivity of the liquid near the bubble surface at
temperature $T'$
$\overrightarrow{df}_v$ = vectorial surface element of the bubble No. $v$

Eqs. (I 3) and (I 4) give

$$\frac{dn}{dt} = \frac{1}{N_L(T')} \sum_{v=1}^{m(t)} k(T') \int (\text{grad } T; \overrightarrow{df}_v)$$ (I 5)

$k(T')$; $L(T')$; $m; T'$ vary with time, as well as shapes and sizes of the
bubble surfaces. For a numerical calculation of (I 5) the knowledge of
these time functions for each single bubble would be necessary.

In a simple model we make the assumption that for all bubbles $\frac{dq}{dt}$
varys with time in the same way. This means that at the moment $t = 0$
the function

$$\frac{dq}{dt} (0) = \frac{dq}{dt} (0, \tau_v) = \psi(0, \tau_v)$$

has the same value for all bubbles of the same creation time $\tau_v$.

From the results of Chapter C we conclude that bubbles are created per-
manently once the pressure having passed the vapour pressure $p_v^*$. So the
total number of bubbles changes its value with time and the number of
bubbles $\Delta m(\tau_o)$ produced at the time $\tau_o$ during an interval $\Delta \tau$ is

$$\Delta m(\tau_o) = \frac{dm}{dt}(\tau_o) \Delta \tau .$$  \hfill (I 7)

If we introduce Eqs. (I 6) and (I 7) into Eq. (I 3) we get for the time $t = \Theta$ instead of Eq. (I 5)

$$\frac{dn}{dt}(\Theta) = \frac{1}{N_o L(T')} \sum_{\nu=1}^{\Delta T} \frac{dm}{dt}(\tau, \nu) \cdot \Delta \tau \cdot \psi(\Theta, \tau, \nu)$$  \hfill (I 8)

and, approaching the sum by an integral

$$\frac{dn}{dt}(\Theta) = \frac{1}{N_o L(T(\Theta))} \int_{t'}^{\Theta} \frac{dm}{dt}(\tau) \cdot \psi(\Theta, \tau) \, d\tau$$  \hfill (I 9)

$t'$ is the time at which evaporation starts.

Finally the total amount of vapour at time $t$ is given by

$$\eta(t) = \frac{1}{N_o} \int_{t'}^{t} \int_{t'}^{\Theta} \frac{d\Theta}{L(T(\Theta))} \frac{dn}{dt}(\tau) \cdot \psi(\Theta, \tau) \, d\tau$$  \hfill (I 10)

For spherical bubbles $\psi(\Theta, \tau)$ can be calculated. In analogy to Eq. (I 6) we find

$$\psi(\Theta, \tau) = \frac{dm}{dt}(\Theta, \tau) .$$  \hfill (I 11)
We consider a bubble which has been created at a time \( \tau \). Its radius \( R \) at time \( t \) is a function of \( t \) and \( \tau \)

\[ R = R(t, \tau) \quad (I.12) \]

In the volume of this bubble we have \( n(t, \tau) \) moles of vapour.

\[ n(t, \tau) = \frac{4\pi}{3M} \rho_v(t) R^3(t, \tau) \quad (I.13) \]

\( M \) = molecular weight.

The amount of heat which is necessary to evaporate this quantity of vapour, is

\[ q(t, \tau) = \frac{4\pi}{3M} L(t) \rho_v(t) R^3(t, \tau) \quad (I.14) \]

From (I.14) follows

\[ \frac{dq}{dt}(\Theta, \tau) = \psi(\Theta, \tau) = \frac{4\pi}{3M} \left[ 3 R^2(\Theta, \tau) \frac{dR}{dt}(\Theta, \tau) + L(\Theta) \rho_v(\Theta) + 
\right.

\left. + R^3(\Theta, \tau) \left\{ L(\Theta) \frac{d\rho_v}{dt}(\Theta) + \rho_v(\Theta) \frac{dL}{dt}(\Theta) \right\} \right] \quad (I.15) \]

From the theory\(^1\) we know the law of bubble growth in superheated liquids. We use the differential form

\[ \frac{dR}{dt} = \frac{A}{\sqrt{t}} \quad (I.16) \]

with

\[ A = 2 \sqrt{\frac{2}{\pi}} \left( \frac{k_c \rho_p}{L_v} \right) (T_0 - T') \quad (I.17) \]
At constant pressure $p$ in a liquid $A$ can be approximated by a constant value. If $p$ varies with time one has to consider $A$ as a function of $t$. In this case the radius of a bubble at time $t_0$, which has been created at $t = t_0$, becomes

$$R(t_0, t) = \frac{A(t)}{\sqrt{t_0 - t}} \int \frac{d\theta}{t} \quad (I \ 18)$$

and

$$\frac{dR}{dt}(t_0, t) = \frac{A(t)}{\sqrt{t_0 - t}} \quad (I \ 19)$$

Now we introduce Eqs. (I 18) and (I 19) into (I 15) and then we replace in Eq. (I 9) the function $\psi(t_0, t)$ by Eq. (I 15). We get

$$\frac{dn}{d\tau}(t) = \frac{4\pi}{3M_o} \left( 3\rho \frac{dR}{dt}(t) + \left[ \frac{dR}{dt}(t) + \frac{P(t)}{\rho(t)} \right] \frac{dL}{dt}(t) \right) F_2(t) \quad (I \ 20)$$

with

$$F_1(t) = \int_{t'}^{t} \left[ \frac{A(t)}{\sqrt{t_0 - t}} d\theta \right]^2 \frac{A(t)}{\sqrt{t_0 - t}} dt \quad (I \ 21)$$

and

$$F_2(t) = \int_{t'}^{t} \left[ \int_{t'}^{t} \frac{A(t)}{\sqrt{t_0 - t}} d\theta \right]^3 dt \quad (I \ 22)$$
Finally the integration of Eq.(1.20) gives $\eta$ as function of time

$$\eta(t) = \frac{4\pi \sigma}{3N_0} \int_{t'}^{t} \left[ 3 \rho_v(\theta) \varphi'(\theta) + \left( \frac{d\rho_v}{dt}(\theta) + \frac{\rho_v(\theta)}{L(\theta)} \frac{dL}{dt}(\theta) \right) \varphi(\theta) \right] \varphi(\theta) \, d\theta$$

Eq.(1.23) could be integrated if $\frac{dm}{dt}$ as a function of time were known.

Up to now there do not exist any theoretical ideas about the individual number $m$ of bubbles and its variation with time. In principle, an inversion of Eq.(1.9) can be used to calculate $\frac{dm}{dt}$ if $\eta(t)$ is measured in a p-v-diagram.
APPENDIX II

The Rate of Adiabatic Temperature Change $\frac{dT}{dp}$

In Eqs. (13) and (19) of Chapter B, the temperature drop for adiabatic pressure variations is given as a function of $\alpha, \beta, \beta'$ resp. $\alpha, \beta, \gamma$. In the case of deuterium these values are not completely available from published data tables. A linear approximation for $\frac{dT}{dp}$ based on $\gamma = c_p/c_v$ the equilibrium vapour pressure function $p_v(T)$, the molar volume of the liquid at equilibrium conditions $V_m(p_v)$ and the adiabatic compressibility $\beta'$ measured in this experiment can be derived in the following way:

In Fig. 4 the curve $p = p_v(T)$ is the phase boundary between pure liquid ($p > p_v(T)$) and liquid-vapour in equilibrium ($p < p_v(T)$). It can be calculated if the vapour pressure as well as the equilibrium molar volume $V_m(p_v)$ of the liquid are known as functions of temperature.

In this case one has

$$p_v = p_v(T) \quad (II\ 1)$$

and

$$V_{nv} = V_{nv}(T) \quad (II\ 2)$$

with the inverse function

$$T_v = T_v(V_m) \quad (II\ 3)$$

If we replace $T$ in (II 1) by (II 3) we get

$$p_v = p_v(V_m) \quad (II\ 4)$$
For small variations of \( V = V_o \cdot V_m \) we introduce the expansion ratio \( \epsilon \) defined in Eq.(B 2) and we get with \( V = V_o (1 + \epsilon) \)

\[
p_v = p_v [V_o (1 + \epsilon)] = p_v (\epsilon)
\]

(II 5)

With the definitions of Chapter B for the points 0; 2; 2* in Fig.4 we calculate from Eq.(B 22), following the adiabatic expansion curve

\[
p_v^* - p_{vo} = -\frac{1}{\beta'} (\epsilon^* - \epsilon_2)
\]

(II 6)

Following the phase boundary \( p = p_v (\epsilon) \) from point 0 (with \( \epsilon = 0 \)) to 2*, one gets for the same difference

\[
p_v^* - p_{vo} = \left( \frac{dp_v}{dc} \right)_{T = T_o} \cdot \epsilon^*
\]

(II 7)

and from (II 6) and (II 7), eliminating \( \epsilon^* \)

\[
p_v^* - p_{vo} = \frac{\epsilon_2}{\beta' + \left( \frac{dc}{dp_v} \right)_{T = T_o}}
\]

(II 8)

Finally we replace \( \epsilon_2 \) by Eq.(B 23) which gives

\[
p_v^* - p_{vo} = \frac{(\gamma - 1)(p_g - p_{vo})}{\beta' + \left( \frac{dc}{dp_v} \right)_{T = T_o}}
\]

or, using Eq.(B 20),

\[
\Delta p_v = p_v^* - p_{vo} = \frac{\gamma - 1}{1 + \frac{1}{\beta'} \left( \frac{dc}{dp_v} \right)_{T = T_o}} (p_g - p_v)
\]

(II 9)
If now $T_v(p)$ is the equilibrium vapour pressure temperature corresponding to a given pressure $p$ we can easily calculate the temperature change:

$$\Delta T = \left( \frac{dT_v}{dp} \right)_T = T_o \delta p_v \tag{II 10}$$

and, with Eq.(II 9)

$$\Delta T = -\left( \frac{\gamma - 1}{1 + \frac{1}{\beta} \frac{dc}{dp}} \right) \frac{dT_v}{dp} \frac{T}{T_o} \quad (p_s - p_{vo}) \tag{II 11}$$

Division of (II 11) by $(p_s - p_{vo})$ gives the rate of adiabatic temperature change with pressure in linear approximation:

$$\frac{\Delta T}{\delta p} = -\left( \frac{\gamma - 1}{1 + \frac{1}{\beta} \frac{dc}{dp_v}} \right) \frac{dT_v}{dp} \frac{T}{T_o} \tag{II 12}$$
Figure Captions

Fig. 1 Minimum Pressure as a Function of Piston Stroke.

Fig. 2 Schematic View of a Bubble Chamber.

Fig. 3 Phase Diagram for Hydrogen (The 26-degrees isothermal and the operating region of a bubble chamber are indicated).

Fig. 4 p-v-Diagram (schematic).

Fig. 5 The Function

\[
\omega(p_v; T_0) = \frac{V_m(p_v)}{V_m(T_0)}
\]

(The curves have been calculated for the special case that the temperature \( T' \) of the vapour is equal to the equilibrium vapour pressure temperature corresponding to the instantaneous pressure \( p(t) \)).

Fig. 6 Experimental Arrangement.

Fig. 7 p-v-Diagram, \( h(t) \) and \( \epsilon(t) \) in Deuterium (Original Photographs).

Fig. 8 Irreversible Heat per Expansion Versus Minimum Pressure \( p_{\text{min}} \) in Hydrogen at \( T_0 = 25 \, ^\circ K \).

Fig. 9 Irreversible Heat per Expansion Versus Minimum Pressure \( p_{\text{min}} \) in Hydrogen at \( T_0 = 27 \, ^\circ K \).

Fig. 10 p-v-Diagrams in \( H_2 \) at Moderate Piston Amplitudes.

Fig. 11 p-c-Diagrams in \( H_2 \) at Large Piston Amplitudes.

Fig. 12 Expansion Ratio \( \epsilon \); Pressure \( p \) and "Dirty Function" \( \eta \) of the Diagrams of Fig.10 as Functions of Time.
Fig. 13 Expansion Ratio $\epsilon$, Pressure $p$ and "Dirt Function" $\eta$ of the Diagrams of Fig. 11 as Functions of Time.

Fig. 14 Influence of Impurities on Parasitic Boiling in $H_2$ (Original Photograph).

Fig. 15 Influence of Impurities on Parasitic Boiling in $H_2$ (Plots Derived from Fig. 14).

Fig. 16 Influence of Forced Cooling of the Expansion Region on Irreversible Heat $\int pdV$, Maximum Amount of Vapour $n_{max}$ and Minimum Pressure $p_{min}$ in Hydrogen.

Fig. 17 Influence of Forced Cooling of the Expansion Region on Irreversible Heat $\int pdV$, Maximum Volume of Vapour $(\omega \cdot n)_{max}$ and Minimum Pressure $p_{min}$ in Deuterium.
Fig. 1

Minimum pressure as a function of piston stroke.

- $p_s = 3.6 \ \text{kp/cm}^2$
- $p_s = 4.6 \ \text{kp/cm}^2$

$T_0 = 25^\circ\text{K}$
\( T_0 \) = temperature of the liquid
\( p_{V_0} = p_V(T_0) \) = vapour pressure at \( T = T_0 \)
\( p_s \) = static pressure before expansion (\( p_s > p_{V_0} \))
\( V_s \) = total volume of the liquid under pressure \( p_s \)
\( V_o \) = total volume of the liquid under saturated vapour pressure \( p_{V_0} \)
\( q \) = surface area of the piston
\( h_s \) = position of the piston before expansion
\( h(t) \) = position of the piston at the time \( t \)
\( E \) = separate heat exchanger
Phase Diagram for H₂

Fig. 3

Critical point

Starting point (p₀, v₀)

\[ T_c = 32.976 \, ^\circ\text{K} \]
\[ p_c = 13.19 \, \text{kp/cm}^2 \]
\[ v_c = 31.61 \, \text{cm}^3 \]

Temperature: T = 26 \, ^\circ\text{K}

Working region

Pressure vs. Molar Volume
Fig. 4

p-V Diagram (schematic)
Fig. 5

The Function $\omega(p_v, T_0) = \frac{Y_m(p_v)}{Y_m(T_0)}$

- $T = 24^\circ K$
- $T = 26^\circ K$
- $T = 28^\circ K$

Equilibrium vapour pressure $p_v [\text{kPa/cm}^2]$
Fig. 7
p-V Diagram, Piston Displacement $h(t)$
and Pressure $p(t)$ in Deuterium
(Original Photographs)

$T_0 = 32^\circ K$

$P_{Vo} = 6.51 \text{ kP/cm}^2$

$P_s = 6.91 \text{ kP/cm}^2$

$h_{max.} = 29 \text{ mm}$

$p_{min.} = 2.46 \text{ kP/cm}^2$
Fig. 8
Irreversible Heat per Expansion versus Minimum Pressure $P_{\text{min}}$.

$T = 25^\circ \text{K}$; $P_{V_0} = 3.3 \frac{\text{kp}}{\text{cm}^2}$

$\times \times \times P_S = 3.6 \frac{\text{kp}}{\text{cm}^2}$

$\circ \circ \circ P_S = 4.6 \frac{\text{kp}}{\text{cm}^2}$

$\bigtriangleup \bigtriangleup \bigtriangleup P_S = 5.6 \frac{\text{kp}}{\text{cm}^2}$
Fig. 9
Irreversible Heat per Expansion versus Minimum Pressure $p_{\text{min}}$.

$T = 27^\circ K$; $p_0 = 50 \ \text{kp/cm}^2$

- $p_s = 5.5 \ \text{kp/cm}^2$
- $p_s = 6.5 \ \text{kp/cm}^2$
- $p_s = 7.5 \ \text{kp/cm}^2$

Irreversible heat per expansion

Minimum pressure $p_{\text{min}} [\text{kp/cm}^2]$
Fig. 10

$p-\epsilon$ Diagrams in $H_2$ at

Moderate Piston Amplitudes

$T_0 = 25^\circ K$

$p_{vo} = 3.3 \, \text{kPa cm}^2$

$p_s = 5.6 \, \text{kPa cm}^2$

$p_v^* = 3.17 \, \text{kPa cm}^2$

Pressure

Expansion ratio $(\epsilon - \epsilon_s) \cdot 10^3$
Fig. 11

\( p - \epsilon \) Diagrams in \( \text{H}_2 \) at Large Piston Amplitudes

\[ T_0 = 25^\circ K \]
\[ p_{vo} = 3.3 \ \frac{\text{kp}}{\text{cm}^2} \]
\[ p_s = 3.6 \ \frac{\text{kp}}{\text{cm}^2} \]
\[ p_V^* = 3.29 \ \frac{\text{kp}}{\text{cm}^2} \]
Fig. 12a

The Functions $\varepsilon(t)$, $p(t)$ and $\eta(t)$ of Fig. 10 Curve a

---

- $\eta(t)$
- $\varepsilon(t)$
- $p(t)$

Pressure

Expansion ratio $(\varepsilon - \varepsilon')10^4$

Time [ms]
Fig. 12b
The Functions $\varepsilon(t)$, $p(t)$ and $\eta(t)$ of Fig. 10 Curve b
Fig. 13a
The Functions $\epsilon(t)$, $p(t)$ and $\eta(t)$ of Fig.11 Curve a
Fig. 13b
The Functions $\varepsilon(t)$, $p(t)$ and $\eta(t)$ of Fig.11 Curve b

Pressure $p$ in $\text{kp/cm}^2$

Expansion ratio $(\varepsilon - \varepsilon_0)^{10}$

Time $t$ in $\text{ms}$
Fig. 14

Influence of Impurities on Parasitic Boiling in $H_2$

(Original Photograph)

$T_0 = 27^\circ K$  \hspace{1cm} $p_{vo} = 5.0 \frac{kp}{cm^2}$  \hspace{1cm} $p_2 = 5.5 \frac{kp}{cm^2}$

$h_{max.} = 28 \text{mm}$

$P_{min.} = 1.92 \frac{kp}{cm^2}$ before injection of $N_2$

$P_{min.} = 2.26 \frac{kp}{cm^2}$ after injection of $N_2$
Fig. 15
Influence of Impurities on Parasitic Boiling in H₂

a before injection of N₂
b after injection of N₂
T = 27°K
\( P_0 = 4.99 \text{ kPa cm}^{-1} \); \( P_s = 5.5 \text{ kPa cm}^{-1} \)
Fig. 16
Influence of Forced Cooling of the Expansion
Region on $\int p \, dv$, $\eta_{\text{max}}$, and $p_{\text{min}}$ in H$_2$

\[ \int p \, dv \quad [W/\text{sec}] \]

\[ p \quad [\text{kPa} / \text{cm}^2] \]

\[ \eta_{\text{max}} \times 10^6 \]

Time [min]
Fig. 17
Influence of Forced Cooling of the Expansion Region on \( \int p \, dv \), \((\omega \cdot \eta)_{\text{max}}\) and \(p_{\text{min}}\) in \(D_2\)