

Elektron-Photon-Kaskaden in Blei für Primärteilchen der Energie 6 Gev

(Electron and photon cascades in lead due to 6 Gev primaries) by
U. Völkel. [Deutsches Elektronen-Synchrotron ('DESY') (German Electron
Synchrotron)] in Hamburg. Report DESY 65/6 (July 1965) 'Theorie'.

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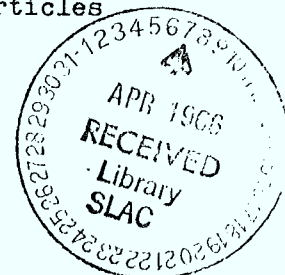
ELECTRON AND PHOTON CASCADES IN LEAD DUE TO
6 Gev PRIMARIES

by U. Völkel

A Monte Carlo method was used to calculate the spectra of the angular and the radial distribution of the particles in electron and photon cascades due to 6 Gev primaries in lead. The energy distribution in the material was also calculated. On the basis of the similarities between these results and those obtained by Nagel³ with less energetic incident electrons (100 to 1000 Mev), the normalized energy spectra and the angular distribution (with reservations) are seen to be independent of the incident energy up to 6 Gev.

I. I n t r o d u c t i o n

Nagel¹⁻³ has used the Monte Carlo method to calculate the parameters of electron and photon cascades produced in lead by particles



with incident energies up to 1000 Mev. Similar calculations have now been made for 6 Gev particles. The following cascade processes were considered in both investigations:

- 1) Electron bremsstrahlung
- 2) Moller scattering* of electrons with energy transfer $> 2 mc^2$

* Translator's note: Term unfamiliar

- 3) Multiple scattering of electron
- 4) Energy loss of electrons due to ionization with energy transfer $< 2 mc^2$
and to weak bremsstrahlung quanta with $k < 1/2 mc^2$
- 5) Photon pair formation
- 6) Compton scattering of photons
- 7) Photo-effect of the lead atom.

The reader interested in these aspects is referred to ref. 2. It should be added that by electrons we also mean positrons, which can be treated in the same manner. The electrons and the photons were monitored up to a cut-off energy of 1.5 and 0.25 Mev, respectively.

The calculations here are based on the statistical data for 200 electron cascades and 100 photon cascades. These statistics are sufficiently

representative in the case of high energies, since the results obtained from data for 100 electron cascades differ little from those obtained from the total of 200 electron cascades.

We shall deal in Sections II to VI below only with a cascade caused by primary electrons, since the relevant results can then be compared with Nagel's values. A cascade due to 6 Gev photons will be compared in Section VIII with that due to 6 Gev electrons. Energy is expressed in Mev, lengths in Pb-radiation length X_0 ($1 X_0 = 5.84 \text{ g/cm}^2 = 0.515 \text{ cm}$).

II. Cascade formation and the number of particles

To describe the cascade, let the z-axis be parallel to the axis of the cascade (plane of incidence : $z = 0$), which is also a symmetry axis. Let $\pi(E_0; E, t, r, \Theta) \cdot 2r \cdot dr \cdot d(\cos \Theta) dE$ denote the number of electrons with incident energy of E_0 which penetrate the plane $z = t \cdot X_0$ with an energy between E_0 and $E_0 + dE$, the distance from the cascade axis being between r and $r + dr$; the z-component of the impulse direction being between $\cos \Theta$ and $\cos \Theta + d(\cos \Theta)$. Further, let $\gamma(E_0; E, t, r, \Theta) \cdot 2r dr \cdot d(\cos \Theta) dE$ denote the corresponding number of photons. By the

appropriate integration of these two functions, we can obtain all the required distributions. Thus, the integral

$$\Pi(E_0; E, t) = \int_E^{E_0} dE \int_0^1 d(\cos \vartheta) \int_0^\infty 2\tau dt \pi(E_0; E, t, \tau, \vartheta)$$

gives the number of electrons with energy $> E$, that penetrate the plane

$z = t.X_0$ with an incident energy of E_0 in the 'forward' direction

($\cos \theta > 0$). The corresponding integral for photons is $\Gamma(E_0; E, t)$. The

curves obtained by plotting these two functions against t for $E_0 = 6$ Gev

are shown in Figures 1 and 2, giving an indication of the cascade formation,

Nagel's³ interpolation formulas obtained from his results for

$E_0 = 100, 200, 400$, and 1000 Mev permit the calculation of the position

and the height of the maxima of these curves from the incident energy E_0

and the cut-off energy E ; the error is about 10%. These calculations

give values of the correct order of magnitude even for $E_0 = 6$ Gev, which

is much higher than the energies so far considered. The error here is 10

to 15%. The formulas given by Nagel are as follows:

$$\Pi_{\max}(E_0, E) = 0,091 \frac{E_0 + 37,5}{E + 12,8} + 0,45 \text{ for } \begin{cases} 100 \leq E_0 \leq 1000 \\ 1,5 \leq E \leq 20 \end{cases} \quad (1a)$$

$$\pi_{\max}(E_0, E) = a(E) [E_0 + 37,5] + 0,45 \quad \text{for} \begin{cases} 100 \leq E_0 \leq 1000 \\ 20 \leq E \leq 50, \end{cases} \quad (1b)$$

For the position of the maximum:

$$t_{\max}(\pi)(E_0, E) = 1,11 \ln E_0 - 0,25 \ln E - 3,87 \quad \text{for} \begin{cases} 100 \leq E_0 \leq 1000 \\ 1,5 \leq E \leq 15 \end{cases} \quad (2a)$$

$$t_{\max}(\pi)(E_0, E) = 1,11 \ln E_0 + b(E) \quad \text{for} \begin{cases} 100 \leq E_0 \leq 1000 \\ 15 \leq E \leq 50 \end{cases} \quad (2b)$$

Nagel³ has also plotted the functions $a(E)$ and $b(E)$.

We have collated in Table 1 the values for $\pi_{\max}(E_0, E)$ and $t_{\max}(\pi)(E_0, E)$ for $E_0 = 6$ Gev, as calculated by our Monte Carlo method and from relations 1 and 2. The latter gave excessively high values for the number of particles and for the maxima at $E_0 = 6$ Gev. In fact, the particle numbers would be better described by a somewhat modified relationship for $\pi_{\max}$ (see column 4 in Table I):

$$\pi_{\max}(E_0, E) = 0,08 \frac{E_0 + 37,5}{E + 12,8} + 0,45 \quad \text{for} \begin{cases} E_0 = 6 \text{ GeV} \\ 1,5 \leq E \leq 1000 \end{cases} \quad (1')$$

The maximum depth at $E_0 = 6$ Gev and $E \geq 20$ Mev is very well described by the expression:

$$t_{\max}(\pi)(E_0, E) = \ln \frac{E_0}{3 E}, \quad (2')$$

This expression is given by the analytical theory of cascades⁴ (approximation

A) (see column 7 in Table I).

Table II contains the corresponding values for the height and the position of the maxima for the photon numbers, as calculated by the Monte Carlo method and from the following relationships³:

$$n_{\max}(E_0, E) = 0,1 \frac{E_0 + 37,5}{E + 1,33} + 0,45 \text{ for } 0,25 \leq E \leq 0,05 E_0 \quad (3)$$

(= 300)

$$t_{\max}(\gamma)(E_0, E) = 0,73 \left(\frac{E_0}{E} \right)^{0,38} \text{ for } 10 \leq E \leq 0,5 E_0 \quad (4)$$

(= 3000)

(See Table II)

As in the case of the number of electrons, Nagel's equations again lead to excessively high values (with an error of up to 20%). The following modified equations, however, give a very good description of the Monte Carlo results (see Table II, columns 4 and 7, respectively):

$$n_{\max}(E_0, E) = 0,09 \frac{E_0 + 37,5}{E + 1,33} + 0,45 \text{ for } E_0 = 6 \text{ GeV}, \quad (3')$$

$0,25 \leq E \leq 600 \text{ MeV}$

$$t_{\max}(\gamma)(E_0, E) = \ln \frac{E_0}{1,5 \cdot E} \text{ for } E_0 = 6 \text{ GeV}, \quad (4')$$

$1,5 \leq E \leq 80 \text{ MeV}$

Further, in agreement with the analytical theory of cascades (approximation A)

the maximum number of photons depends only on E_0/E for $100 \leq E_0 \leq 6000 \text{ MeV}$

and $E_0/E \leq 30$. Moreover, when $1000 \leq E_0 \leq 6000$ Mev, this relationship holds even for the case $E \geq E_0/150$ (see Figure 3).

For the sake of comparison, Figures 1 and 2 contain an exponential decrease with $e^{-\lambda_{\min} t}$, where $\lambda_{\min}$ is the minimum photon absorption coefficient in lead ($\lambda_{\min} = 0.24 X_0^{-1}$). On the basis of qualitative considerations, the curves for the number of particles should exhibit approximately the same decrease in the case of large values of t .

III. R a d i a l d i s t r i b u t i o n

The radial distribution at depth t is given by

$$\pi(E_0; t, r) = \int_{15}^{E_0} dE \int_0^1 d(\cos \vartheta) \pi(E_0; E, t, r, \vartheta)$$

for electrons, and by

$$\gamma(E_0; t, r) = \int_{0.25}^{E_0} dE \int_0^1 d(\cos \vartheta) \gamma(E_0; E, t, r, \vartheta)$$

for photons.

The expression $\pi(E_0, t, r) \cdot 2rdr$ denotes the number of electrons penetrating the plane $z = t \cdot X_0$ in the 'forward' direction ($\cos \theta > 0$); they are at a distance between r and $r + dr$ from the cascade axis.

The corresponding expression for photons is $\gamma(E_0, t, r) \cdot 2rdr$. Figures 4,

5a, and 5b show the radial distribution at various depths t , clearly indicating the gradual lateral widening of the cascades.

IV. Spectra and angular distribution

Nagel has verified the following rules for the electron spectra:

'When the depth is measured in units of the maximum thickness $t_{\max(\pi)}(E_0, mc^2)$, the form of the electron spectrum is, in a first approximation, independent of the incident energy E_0 '. The same applies to the photon spectra and to the angular distributions of both types of particle, if the unit of length is the maximum penetration thickness of the electrons $t_{\max(\mu)}(E_0, mc^2)$. When $E_0 = 6$ Gev, the following value is obtained for $t_{\max(\mu)}$ by extrapolation:

$$t_{\max(\mu)}(6 \text{ GeV}, mc^2) \approx 6 X_0.$$

From the electron spectrum

$$\pi(E_0; E, t) = \int_0^1 d(\cos \vartheta) \int_0^\infty 2r dr \pi(E_0; E, t, r, \vartheta)$$

and the photon spectrum

$$\gamma(E_0; E, t) = \int_0^1 d(\cos \vartheta) \int_0^\infty 2r dr \gamma(E_0; E, t, r, \vartheta)$$

we obtain the relative energy distributions by normalization of the particles

in the forward direction:

$$\omega_{\pi}(E_0; E, t) = \pi(E_0; E, t) / \Pi(E_0, 15, t)$$

$$\omega_{\gamma}(E_0; E, t) = \gamma(E_0; E, t) / \Gamma(E_0, 925, t)$$

The curves obtained by plotting $\omega_{\pi}(E_0; E, \tau)$ and $\omega_{\gamma}(E_0; E, \tau)$ against E at a fixed value of $\tau = t_{\max(\pi)}(E_0, mc^2)$ should therefore be independent of E_0 . Figures 6a-d and 7a-c show that this also applies for $E_0 = 6$ Gev. In each case, the actual curve shows the energy distribution found by Nagel for $100 \leq E_0 \leq 1000$. For the sake of comparison with Nagel's values for 1 Gev, our calculated values at $E_0 = 6$ Gev are shown in circles.

Figures 8, 9a and 9b show the spectra of $\pi(E_0, E, t)$ and $\gamma(E_0, E, t)$ for $E_0 = 6$ Gev and several values of the depth t . Even for larger values of t the maximum of the photon spectra lies at about 1 to 1.2 Mev, not at the energy with the smallest absorption coefficient $\lambda_{\min}$ (i.e. at about 3 to 4 Mev) as given by qualitative considerations.

To obtain the angular distribution at depth t , we assume that the probability for a particle with direction $\cos \Theta$ to penetrate the plane $z = t.X_0$ is proportional to $|\cos \Theta|$. We therefore construct the integrals:

$$\pi(E_0; t, \vartheta) = \frac{1}{140 \vartheta} \int_{1.5}^{E_0} dE \int_0^\infty 2r dr \pi(E_0, E, t, r, \vartheta)$$

$$\gamma(E_0; t, \vartheta) = \frac{1}{140 \vartheta} \int_{9.25}^{E_0} dE \int_0^\infty 2r dr \gamma(E_0, E, t, r, \vartheta)$$

The relative angular distributions

$$w_\pi(E_0; \tau, \vartheta) = \pi(E_0; \tau, \vartheta) / \overline{\pi}(E_0, 1.5, \tau)$$

$$w_\gamma(E_0; \tau, \vartheta) = \gamma(E_0; \tau, \vartheta) / \overline{\gamma}(E_0, 9.25, \tau)$$

should be independent of E_0 for a fixed value of $\tau = \lambda t_{\max}(\pi)(E_0, mc^2)$.

In this case, however, the plots for $E_0 = 6$ Gev systematically lie above Nagel's curves (see Figures 10, 11a and 11b). It should be borne in mind, however, that, as normalization is carried out for a particle in the forward direction, the proportion of particles traveling backward must be greater at high incident energies than at low ones.

Figures 12a, b and 13a and b show the angular distributions corresponding to $\pi(E_0, t, \theta)$ and $\gamma(E_0, t, \theta)$ at $E_0 = 6$ Gev and for several depths t .

V. Calculation of the primary energy from the path length of the electrons.

Nagel has obtained the following expression for the mean overall

path length $\overline{S(E_0, 1.5)}$ of all the electrons in the cascade:

$$\overline{S(E_0, 1.5)} = 0.0905 E_0 \pm 0.3 \% [X_0]$$

having the half-width

$$\Delta S(E_0, 1.5) = 0.125 \sqrt{E_0} [X_0] .$$

We therefore obtain for $E_0 = 6 \text{ GeV}$ that

$$\overline{S(6 \text{ GeV}, 1.5)} = (543 \pm 1.5) X_0$$

$$\Delta S(6 \text{ GeV}, 1.5) = 9.7 X_0$$

whilst the Monte Carlo method leads to

$$\begin{aligned} \overline{S(6 \text{ GeV}, 1.5)} &= 545 X_0 \\ \Delta S(6 \text{ GeV}, 1.5) &= 9 X_0 \end{aligned}$$

VI. Spatial distribution of energy in the lead cylinder

In Nagel's system, the energy losses incurred during the cascade formation (the ionization losses of the electrons and the separation of particles below the cut-off energies) are represented in a spatial 'block diagram'. The expression $E(r, t)/E_0$ gives the percentage of the incident energy that is accumulated at a distance r from the cascade axis at a depth t in a cylindrical tube of height and thickness X_0 (i.e. in a volume of $2\pi r X_0^2$). Figure 14 shows the curves for

$$\left(\frac{E(r)}{E_0} \right)_{t=\infty} = \frac{1}{E_0} \int_0^{\infty} E(r, t') dt' , \quad \left(\frac{E(t)}{E_0} \right)_{r=\infty} = \frac{1}{E_0} \int_0^{\infty} E(r', t) dr'$$

and

$$\left(\frac{1}{E_0} \int_t^{\infty} E(t') dt' \right)_{r=\infty} = \frac{1}{E_0} \int_t^{\infty} dt' \int_0^{\infty} dr' E(r', t')$$

The decrease for large t strictly obeys the $e^{-\lambda_{\min} t}$ law.

We can calculate from the 'block diagram' the percentage of primary energy that leaves a lead cylinder with a given radius r and length t (in X) (see Figure 15). We must further add to the values shown in Figure 15 the energy that has been back-scattered to the plane of incidence, this energy amounting on average to 0.175%. It appears that no further gain occurs at a given length t , if r exceeds a maximum $r(t)$.

Further, we can read off from the isoenergetic curves obtained the possible values of r and t necessary for the accumulation of a given percentage of energy in the lead cylinder. Nagel has found that these isoenergetic curves become, in a first approximation, independent of the primary energy E_0 when r is plotted against $t/\ln E_0$. Figure 16 shows Nagel's interpolated curves for 80, 90, and 93% of the incident

energy Nagel's points for 1000 Mev and our values for 6 Gev. It can be seen that the minimum length of t increases more slowly than $\ln E_0$, and that the minimum radius decreases at higher energies. The cascade becomes, therefore, relatively narrow. Figure 16 also shows the curves for 98, 99, and 99.5% of incident energy (for $E_0 = 6$ Gev).

VI. Cascade produced by 6 Gev incident photons

The evaluation of 100 cascades produced by 6 Gev photons basically confirms the qualitative considerations as expressed e.g. by Heitler⁴ (p. 393): A high energy photon leads to one pair formation after an average of 1.4 path lengths. After about $1 X_0$, we have therefore the same initial conditions as in the case of an electron cascade in the plane of incidence. It follows therefore that a photon cascade is displaced in the direction of the cascade axis by about $1 X_0$ with respect to the electron cascade. This is clearly shown by a comparison between Figures 1_γ , 2_γ and 14_γ on the one hand, and Figures 1, 2, and 14 on the other; the former set is generally similar to the latter.

Comparison of the radial distributions, the spectra, and the angular distributions shows that the distribution curve for $z = t.X_0$ in

the electron cascade approximately coincides with the distribution curve for $z = (t + 1)X_0$ in the photon cascade, particularly when t is large. However, differences arise for the first three or four path lengths. Furthermore, the energy back-scattered to the plane of incidence is 0.12% in the photon cascade, that is, less than in the electron cascade (0.175%).

The statistics are generally worse in the case of photon cascades (for the same number of cascades), since the 'fate' of the incident particles differs more here. A high-energy electron traversing a distance of $1 X_0$ undergoes about 12 bremsstrahlung processes (distributed over the whole path) and loses on average two-thirds of its energy. On the other hand, a high-energy photon traverses a distance of $1 X_0$ without incurring any energy loss, after which its energy is divided by pair formation (e^+e^- pair).

R e f e r e n c e s

1. H.-H. Nagel and Ch. Schlier, Z.f. Phys. 174 (1963) 464.
2. H.-H. Nagel, Dissertation, Bonn 1964, Z.f. Phys. 186 (1965) 319.
3. H.-H. Nagel, Z.f. Phys. 186 (1965) 319 and private communication.
4. W. Heitler, Quantum Theory of Radiation, Oxford, 3rd Ed., p.392.
5. K. Ott, Z. Naturf. 9a (1954) 488.

E (MeV)	π_{max} Monte Carlo	π_{max} (1a) from	π_{max} (1')	$t_{\text{max}}(\pi) [L^{\lambda_0}]$ Monte Carlo	$t_{\text{max}}(\pi)$ from (2)	$t_{\text{max}}(\pi)$ from (2')
1,5	34	38,85	34,2	5,5	5,65	(7,2)
3,0	30	35,2	31	5,5	5,48	(6,5)
6,0	25,5	29,65	26,1	5,0	5,27	(5,8)
10,0	21,3	24,55	21,6	4,8	5,18	(5,3)
20,0	15	17,2	15,2	4,5	4,9	4,6
30,0	11,6	13,25	11,7	4,2	4,6	4,2
40,0	9,5	10,85	9,58	4,0	4,3	3,9
50,0	8,0	9,2	8,13	3,8	3,9	3,68
80,0	5,5	6,37	5,65	3,3	--	3,2
100,0	4,6	5,31	4,72	3,0	--	3,0
200,0	2,7	2,93	2,71	2,4	--	2,3
400,0	1,6	1,78	1,72	1,4	--	1,6
1000,0	1,05	0,99	0,93	0,8	--	0,7

TABLE I : $\pi_{\text{max}}(E_0, E)$, $t_{\text{max}}(\pi)(E_0, E)$ for $E_0 = 6$ GeV

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E (MeV)	$T_{\max}$ Monte Carlo	$T_{\max}$ from (3)	$T_{\max}$ from (3')	$t_{\max}(\gamma)$ Monte Carlo	$t_{\max}(\gamma)$ from (4)	$t_{\max}(\gamma)$ from (4')
0,25	332	370	333	7,8		7,9
1,0	245	260	234	7,8		7,6
1,5	200	224	201	7,7		7,2
2,0	168	181	163	7,5		7,0
3,0	129	140	126	7,3		6,7
5,0	89,1	113	102	7,0		6,0
10	49,3	53,8	48,5	6,0	(8,3)	5,3
20	26,4	28,9	26,1	5,5	6,4	4,9
30	17,8	19,8	17,9	4,8	5,5	4,6
40	13,6	15,1	13,6	4,5	4,9	4,2
60	9,33	10,4	9,4	4,0	4,2	3,9
80	7,26	7,9	7,1	3,5	3,8	3,7
100	5,90	6,5	5,85	3,2	3,5	
200	3,4	3,5	3,15	2,4	2,7	
300	2,4	2,45	2,25	2,1	2,3	
400	1,9	(1,95)	1,80	1,8	2,1	
600	1,27	(1,50)	1,40	1,6	1,8	
800	0,92	(1,20)	1,13	1,6	1,6	
1000	0,76	(1,05)	0,99	1,5	1,5	
2000	0,36		0,72	1,0	1,1	
3000	0,21		0,63	0,5	(0,94)	

TABELLE II: $T_{\max}(E_0, E)$, $t_{\max}(\gamma)(E_0, E)$ for $E_0 = 6$ GeV

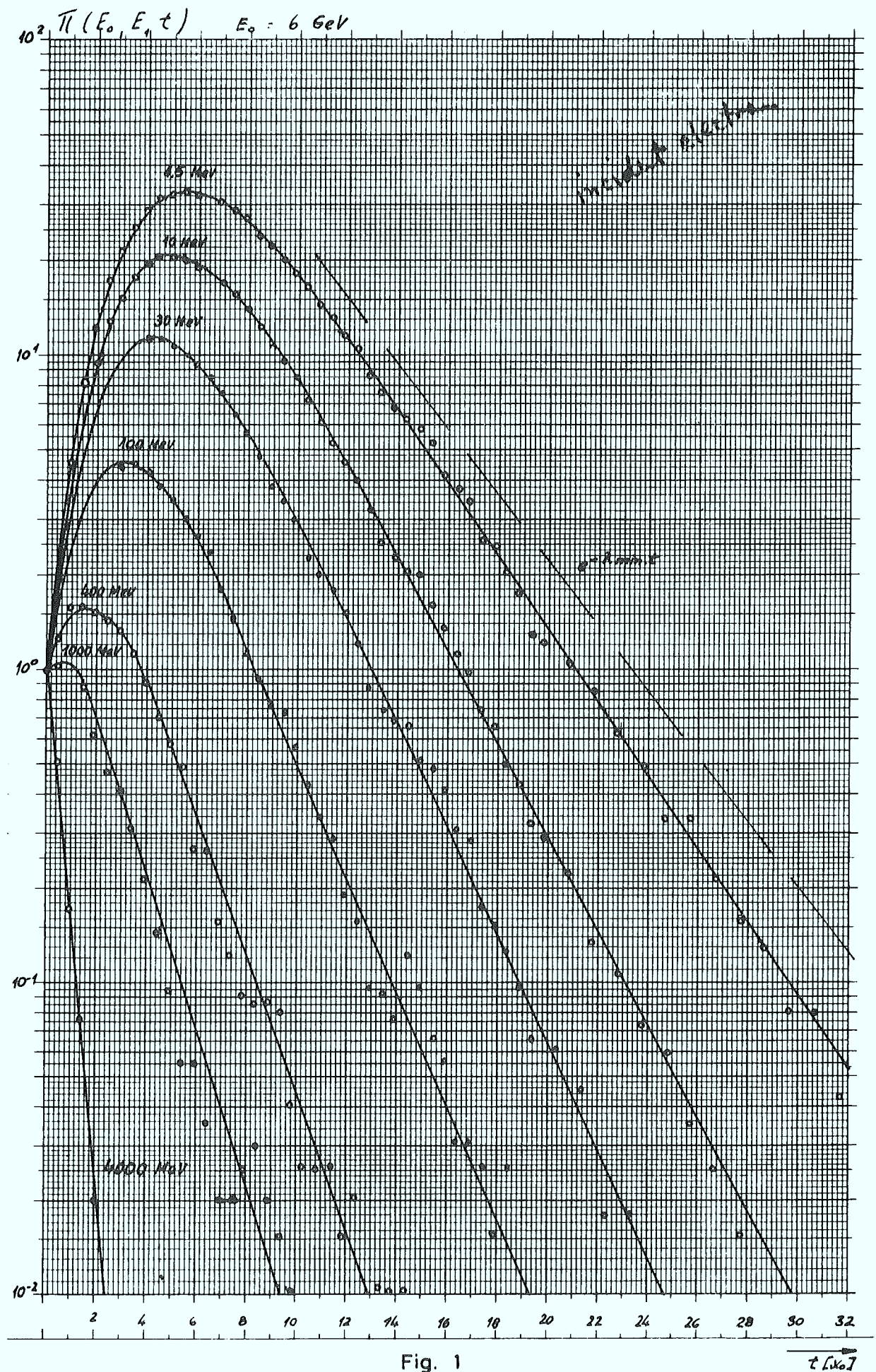


Fig. 1

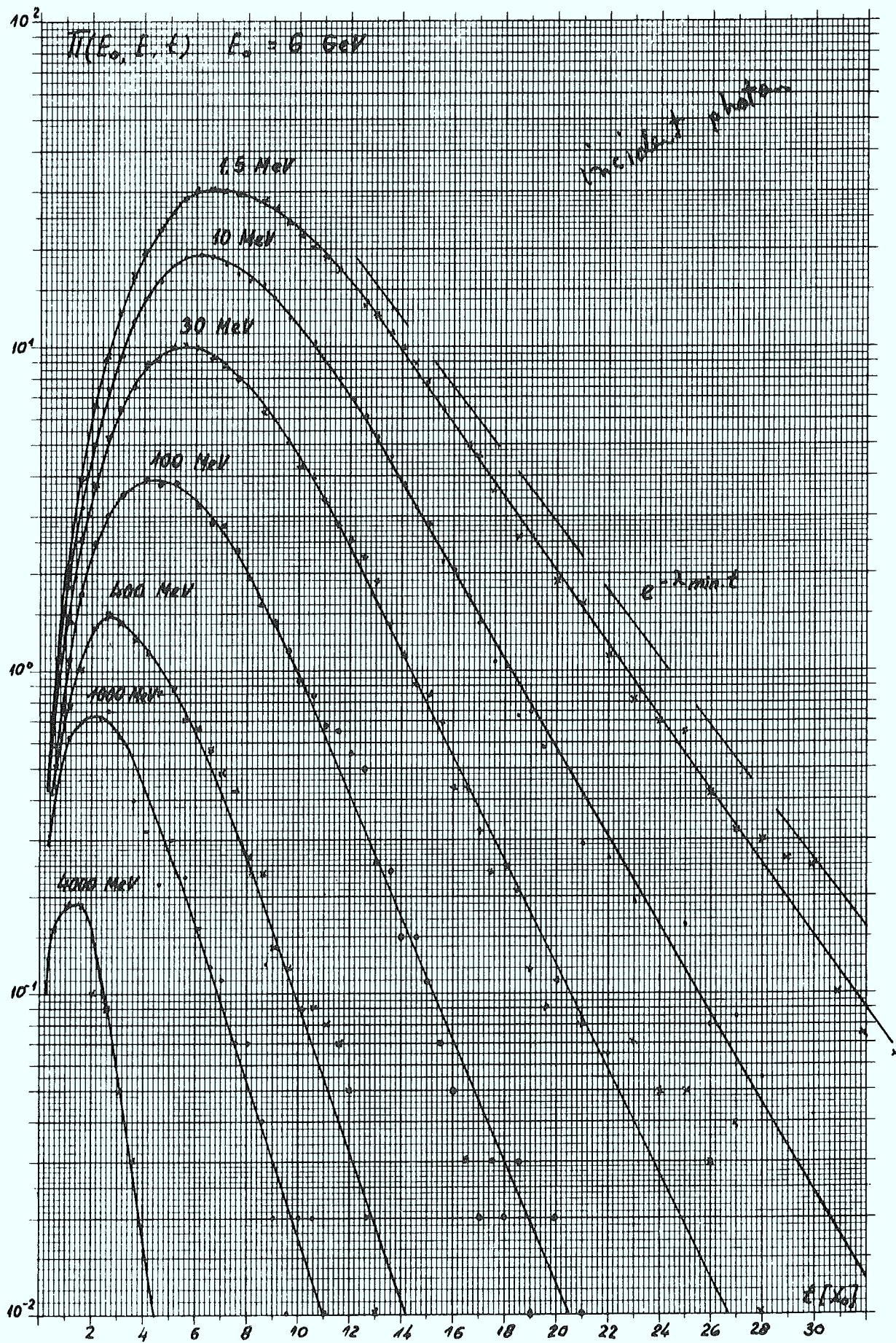
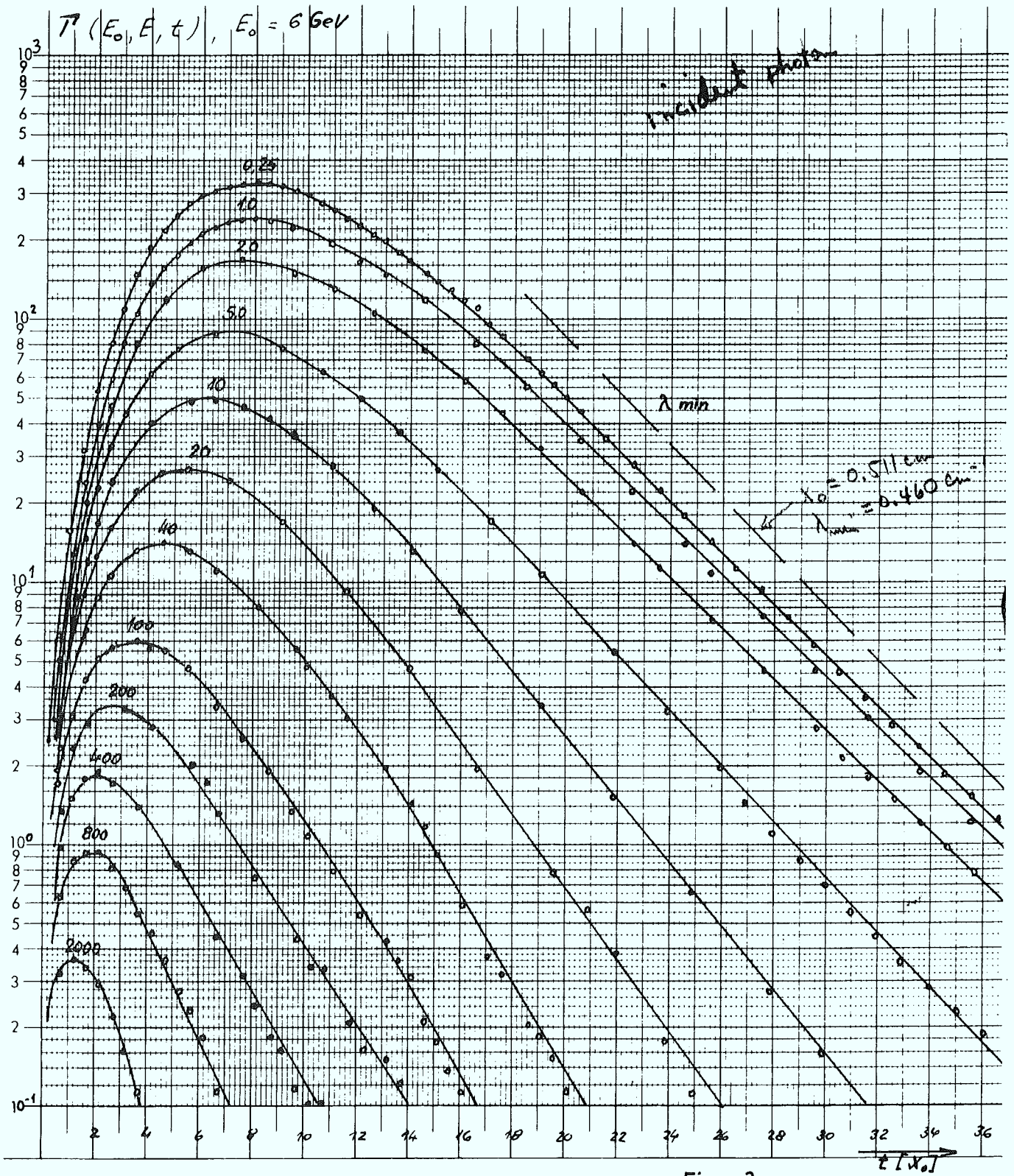
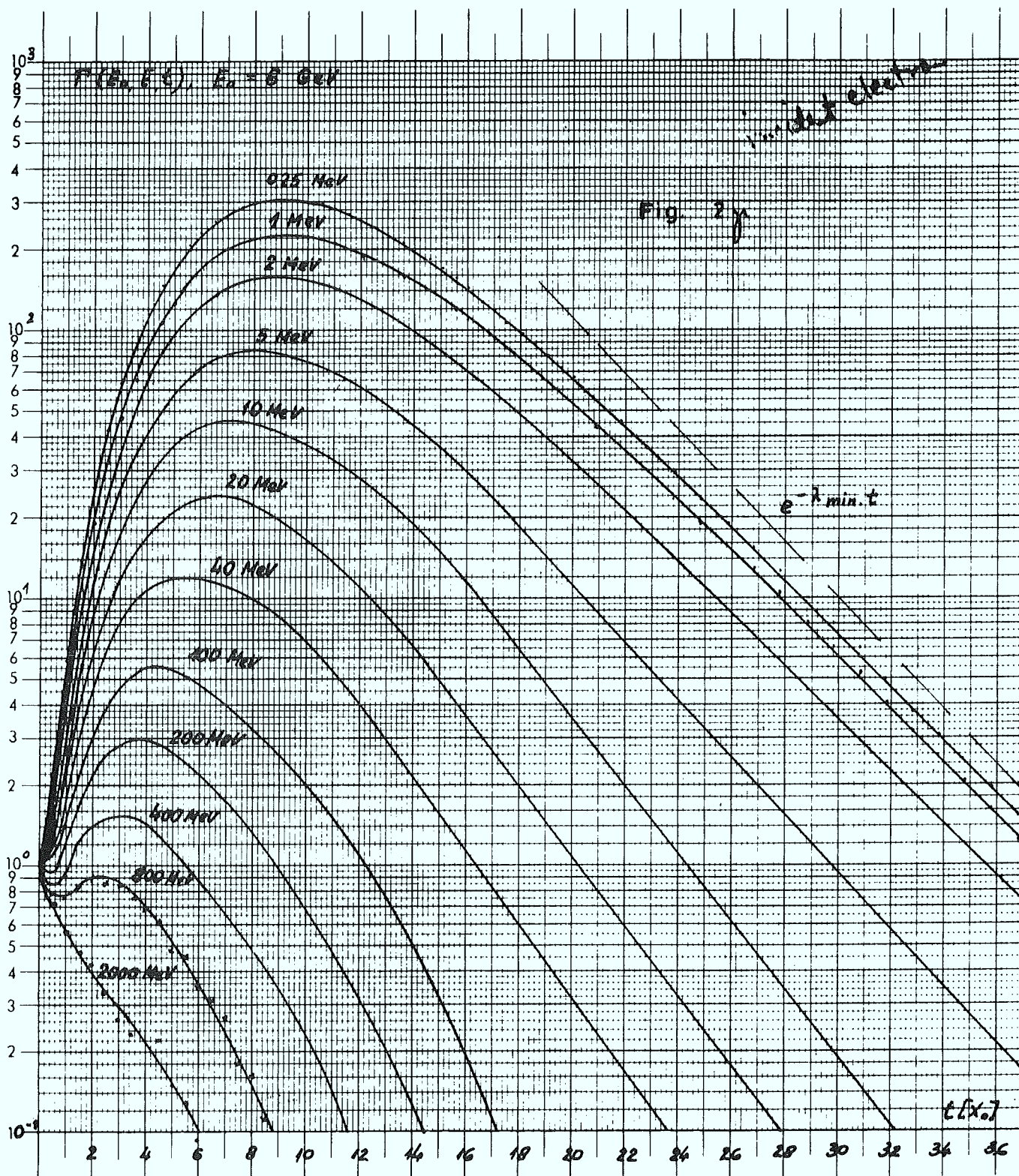


Fig 1 γ





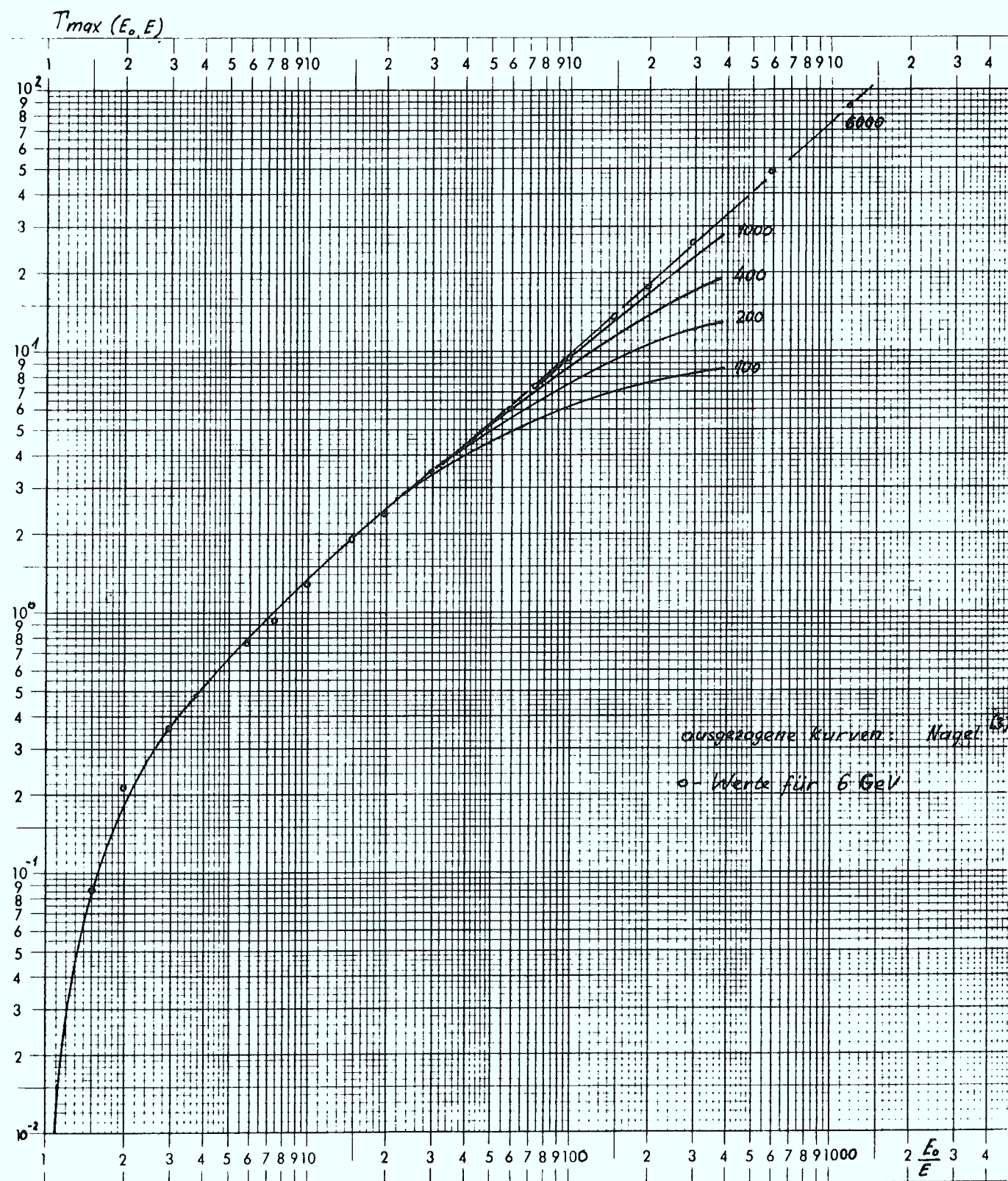


Fig. 3

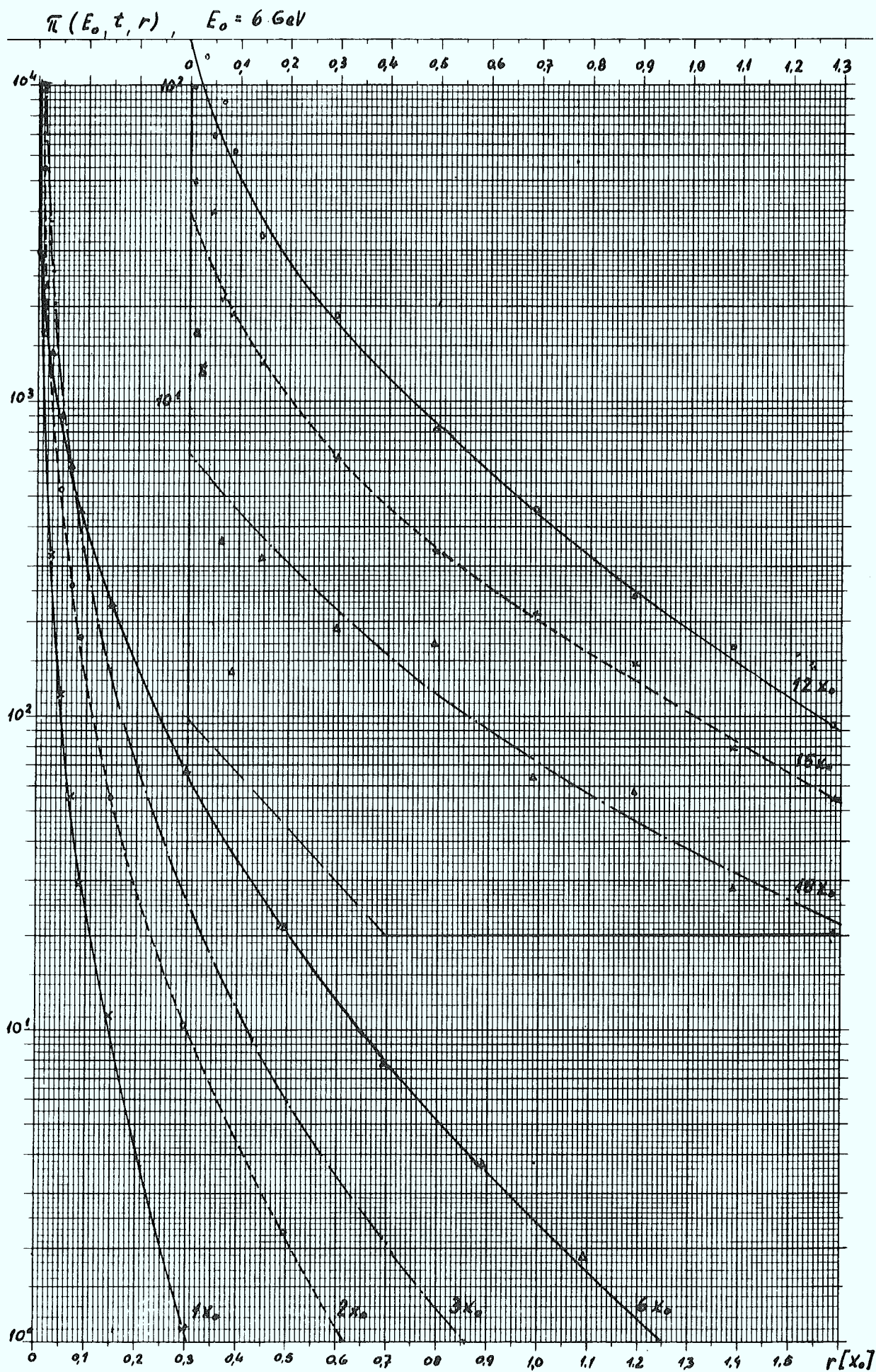
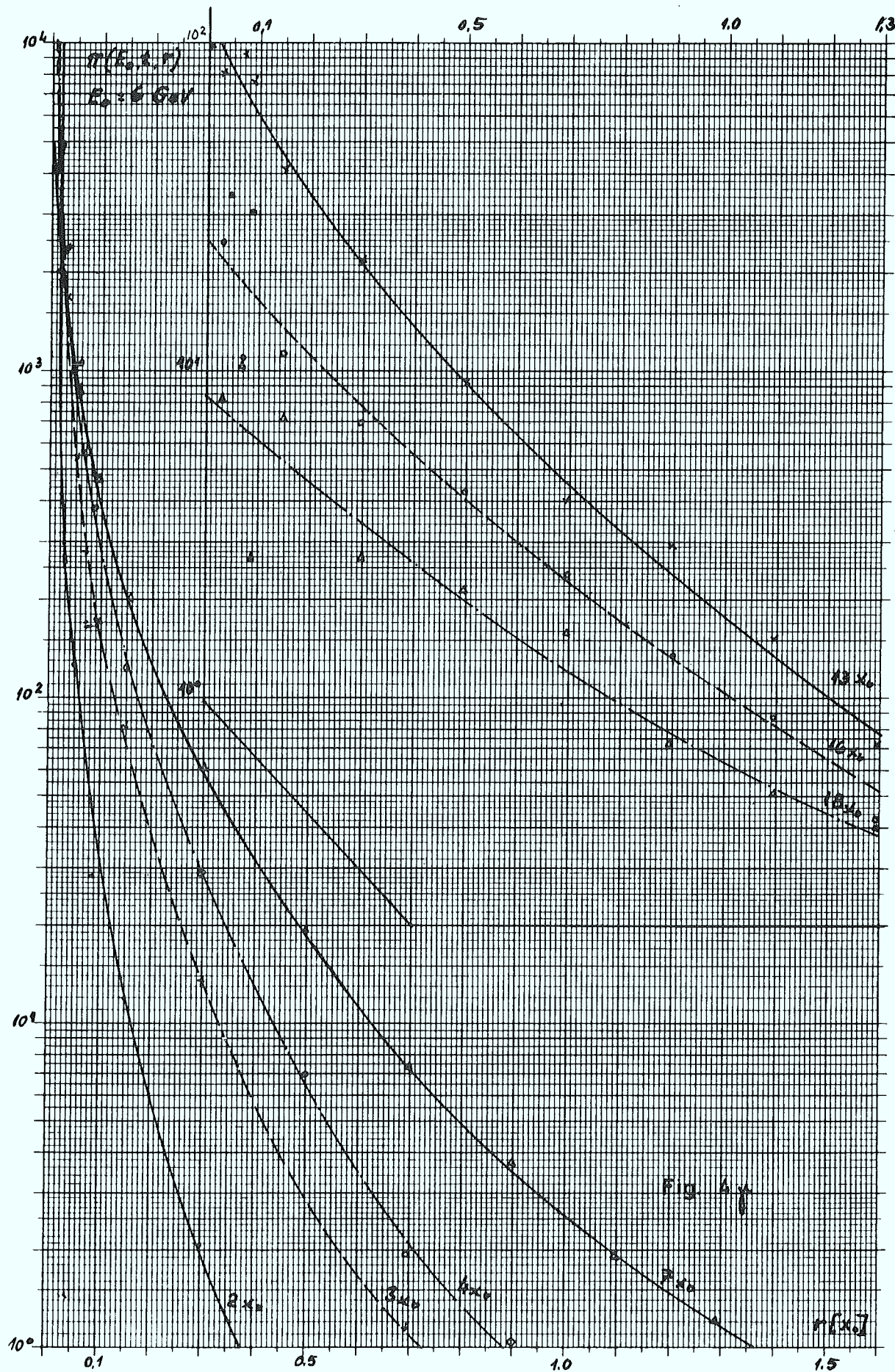


Fig. 4



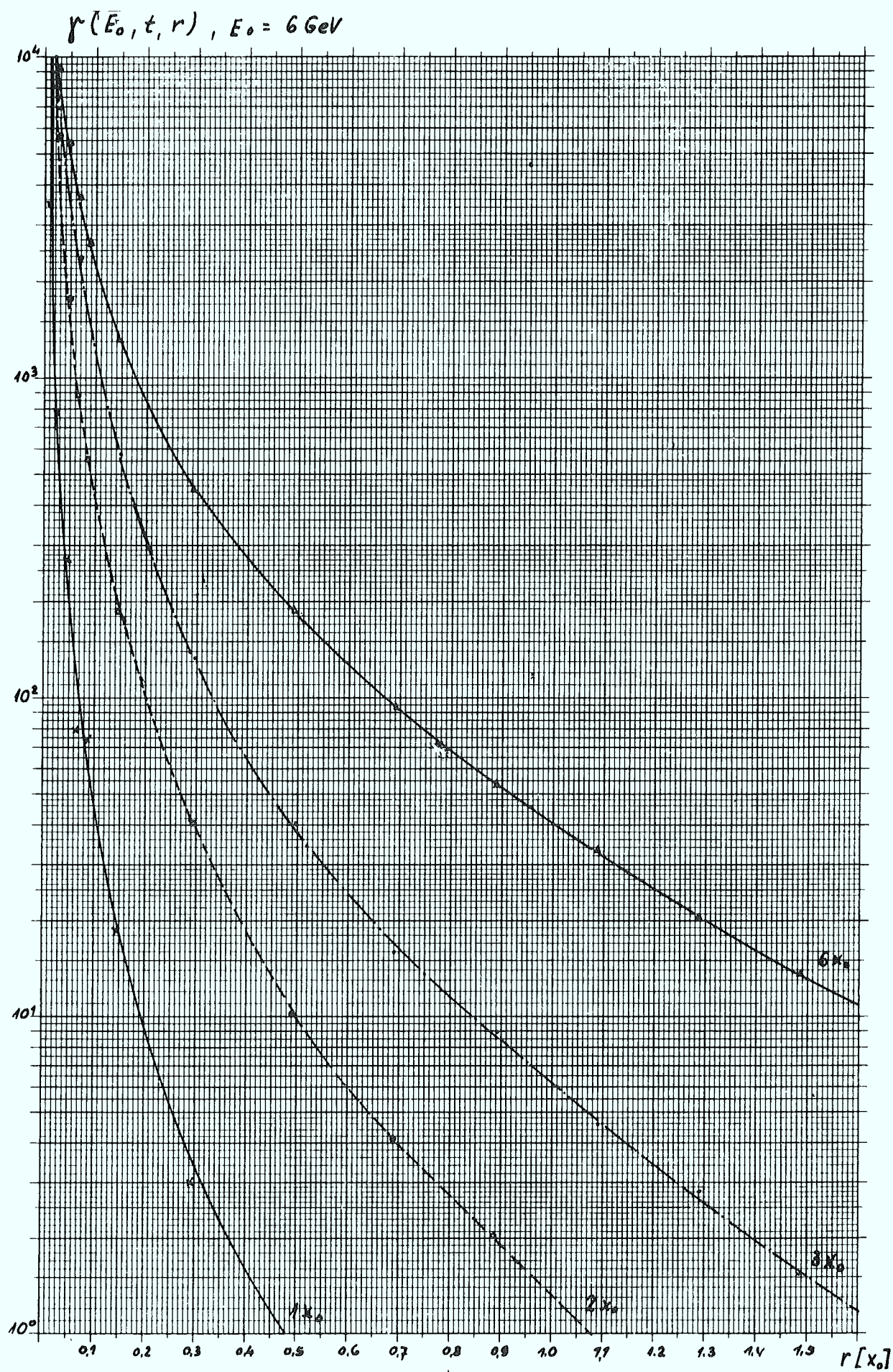
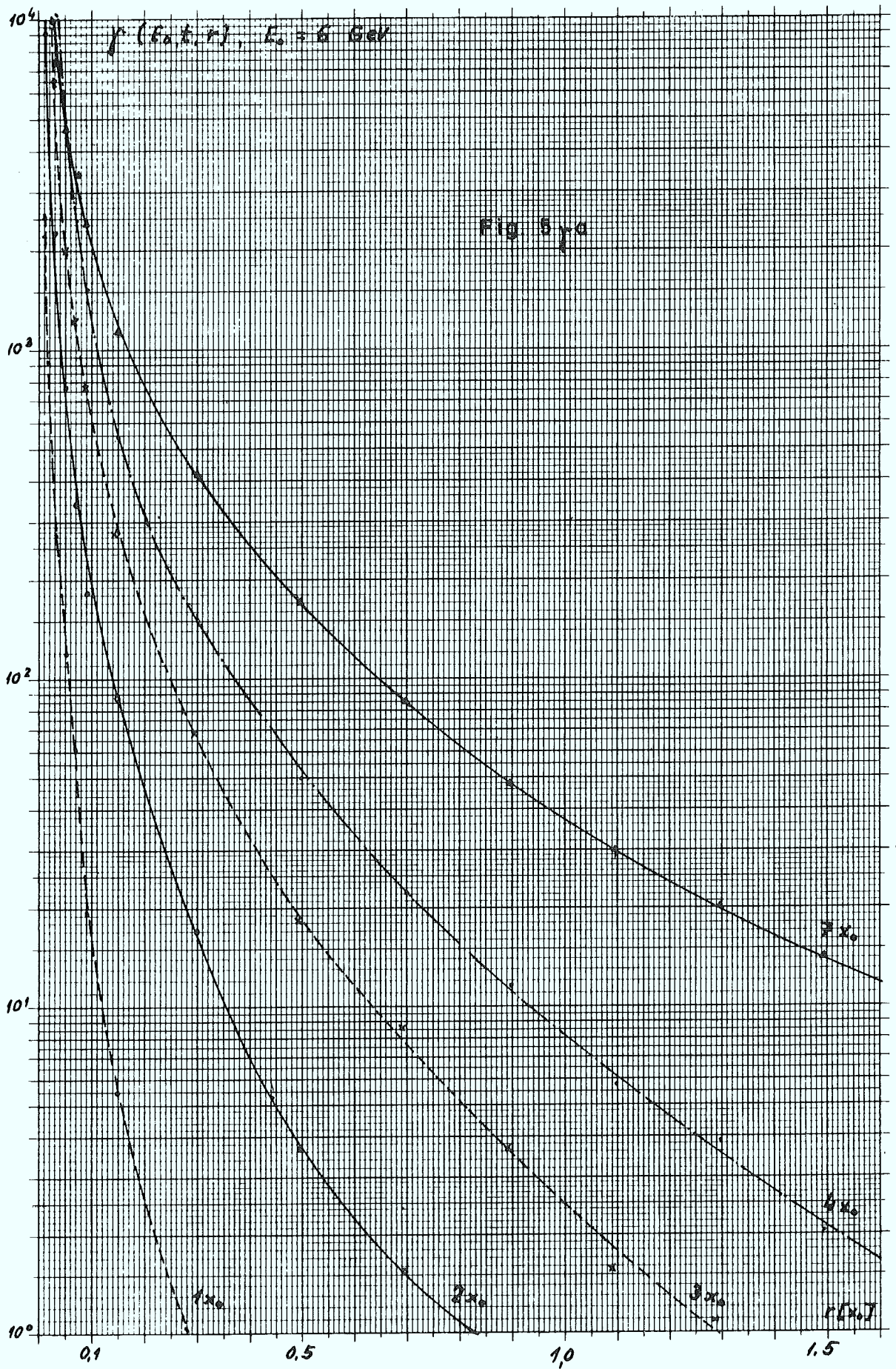


Fig. 5a



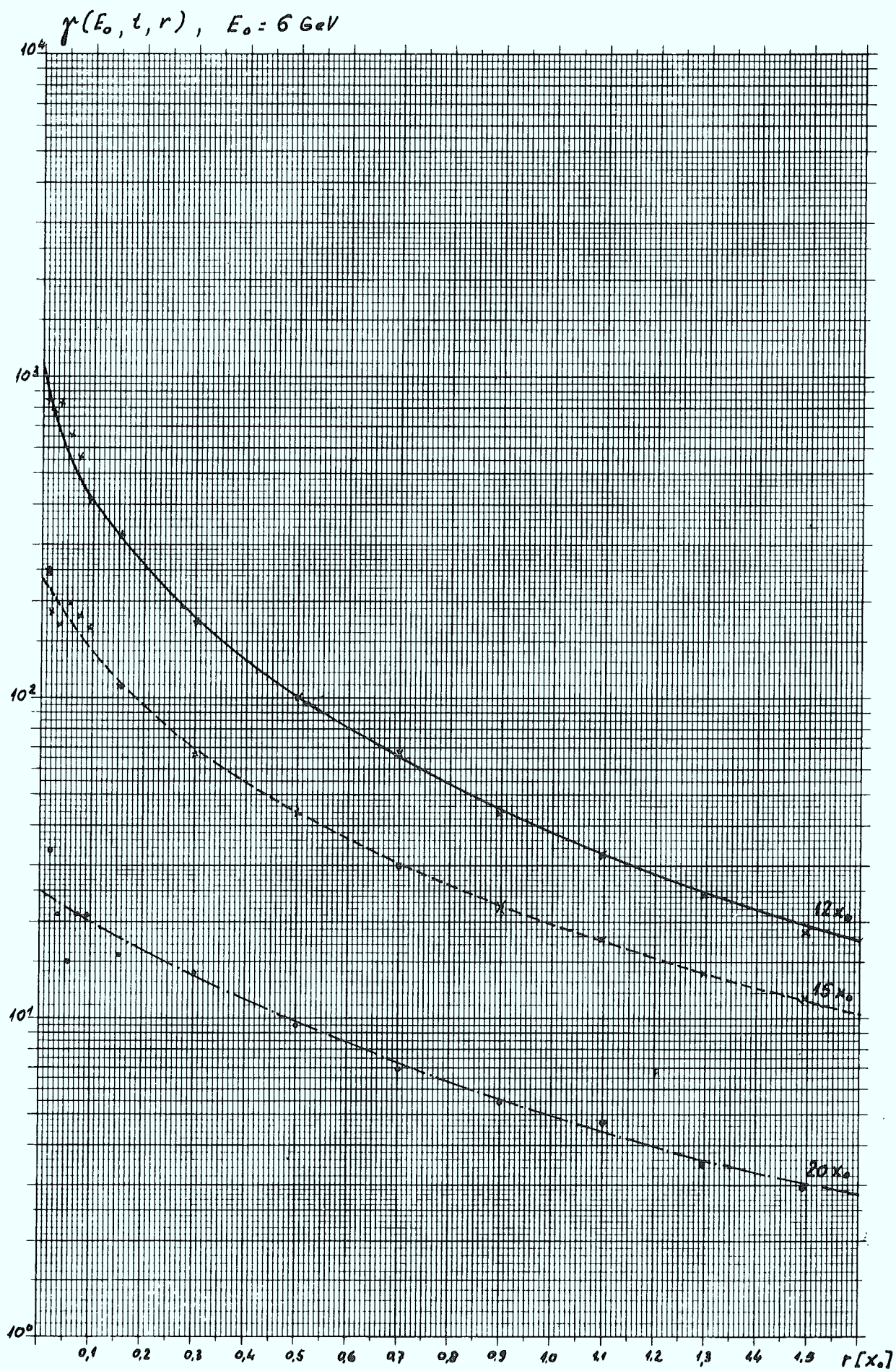
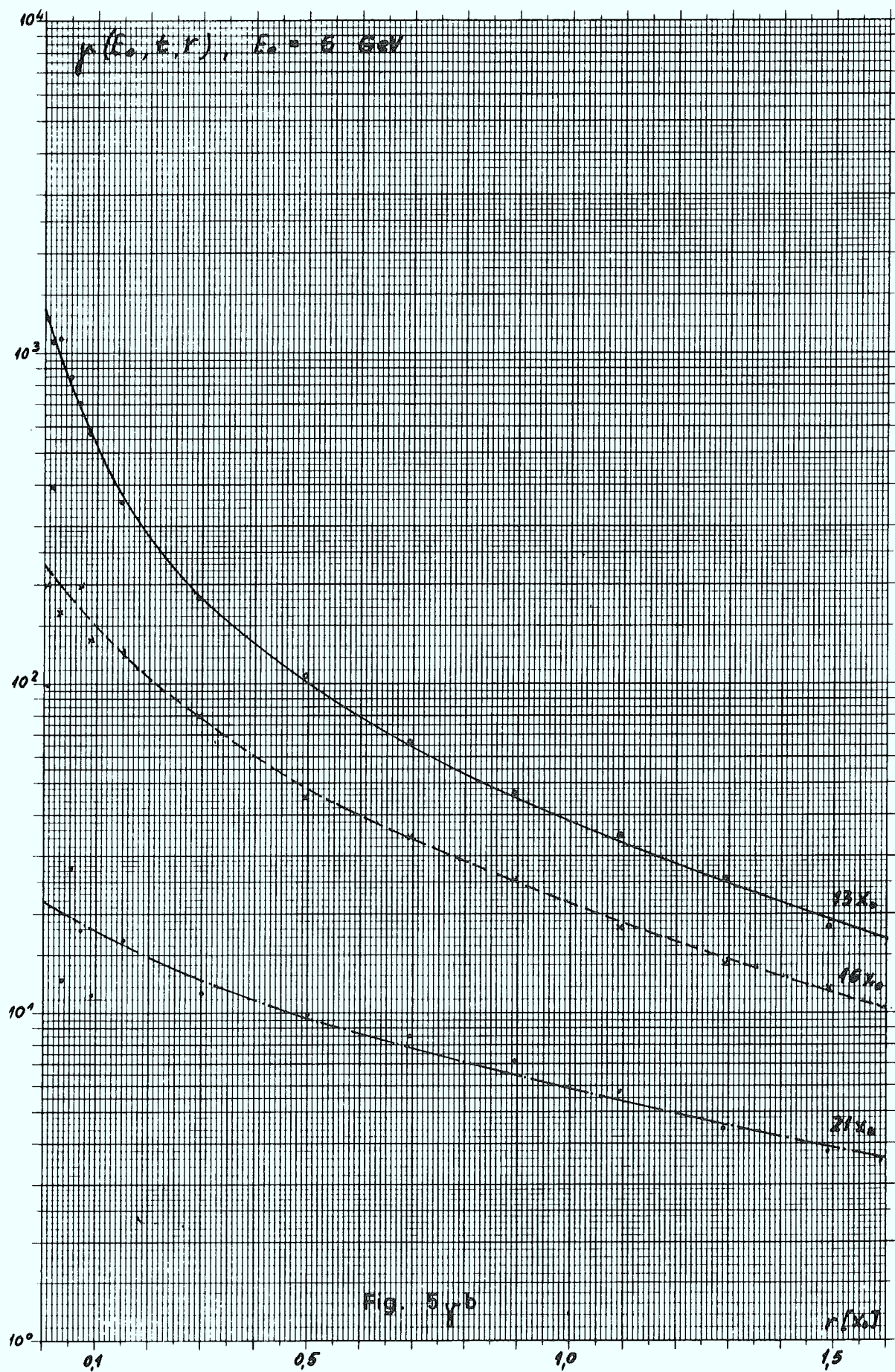
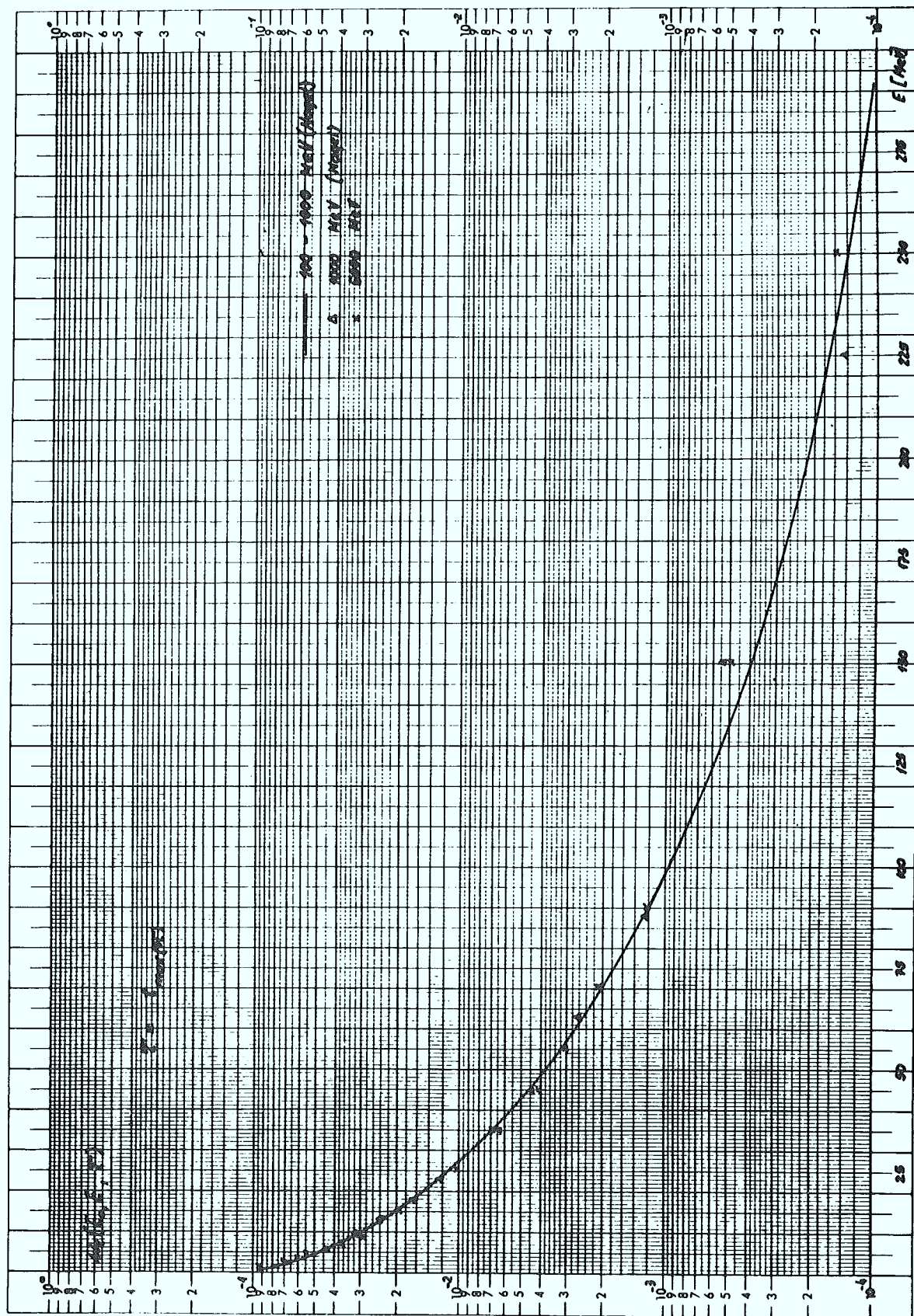


Fig. 5 b





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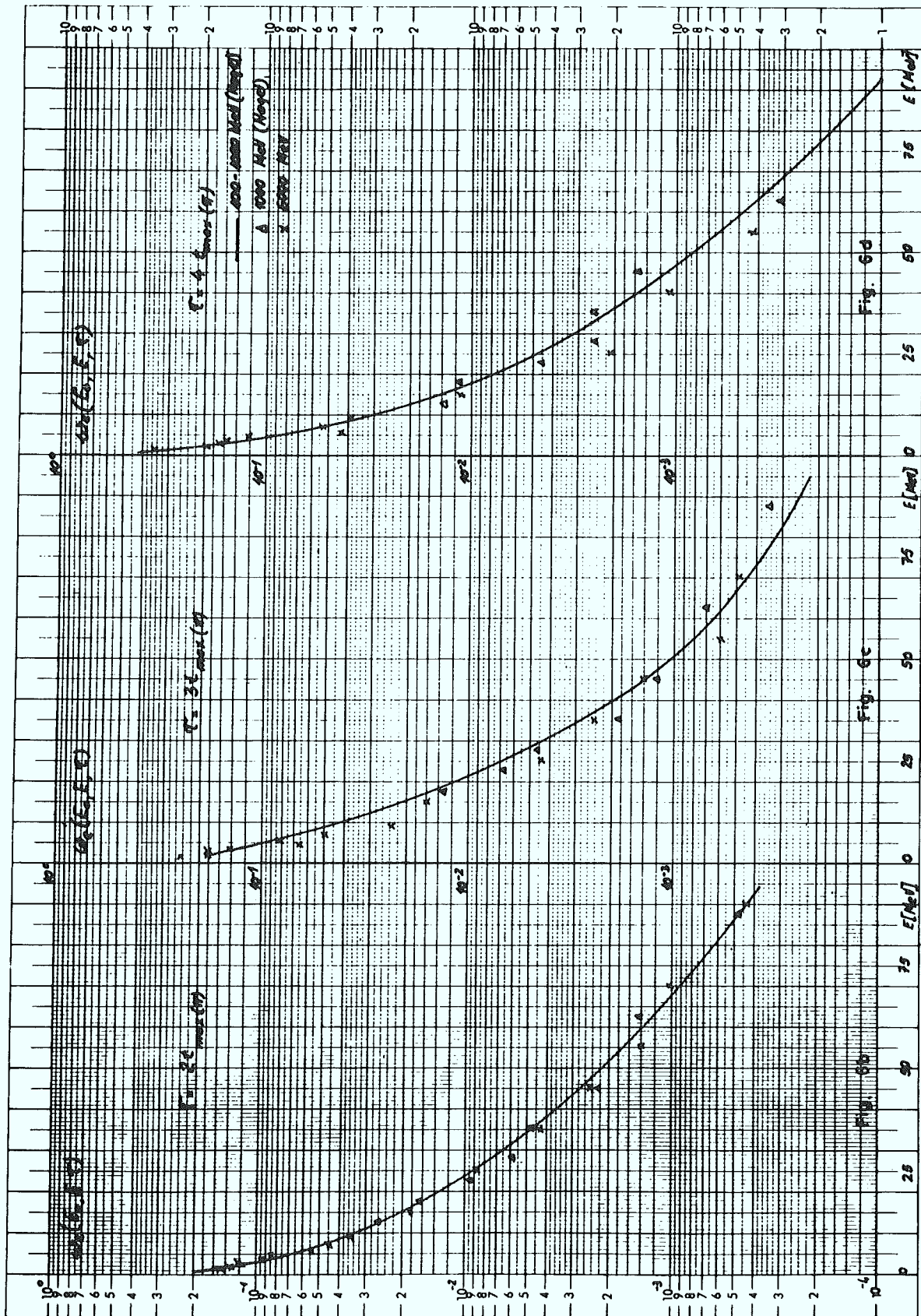
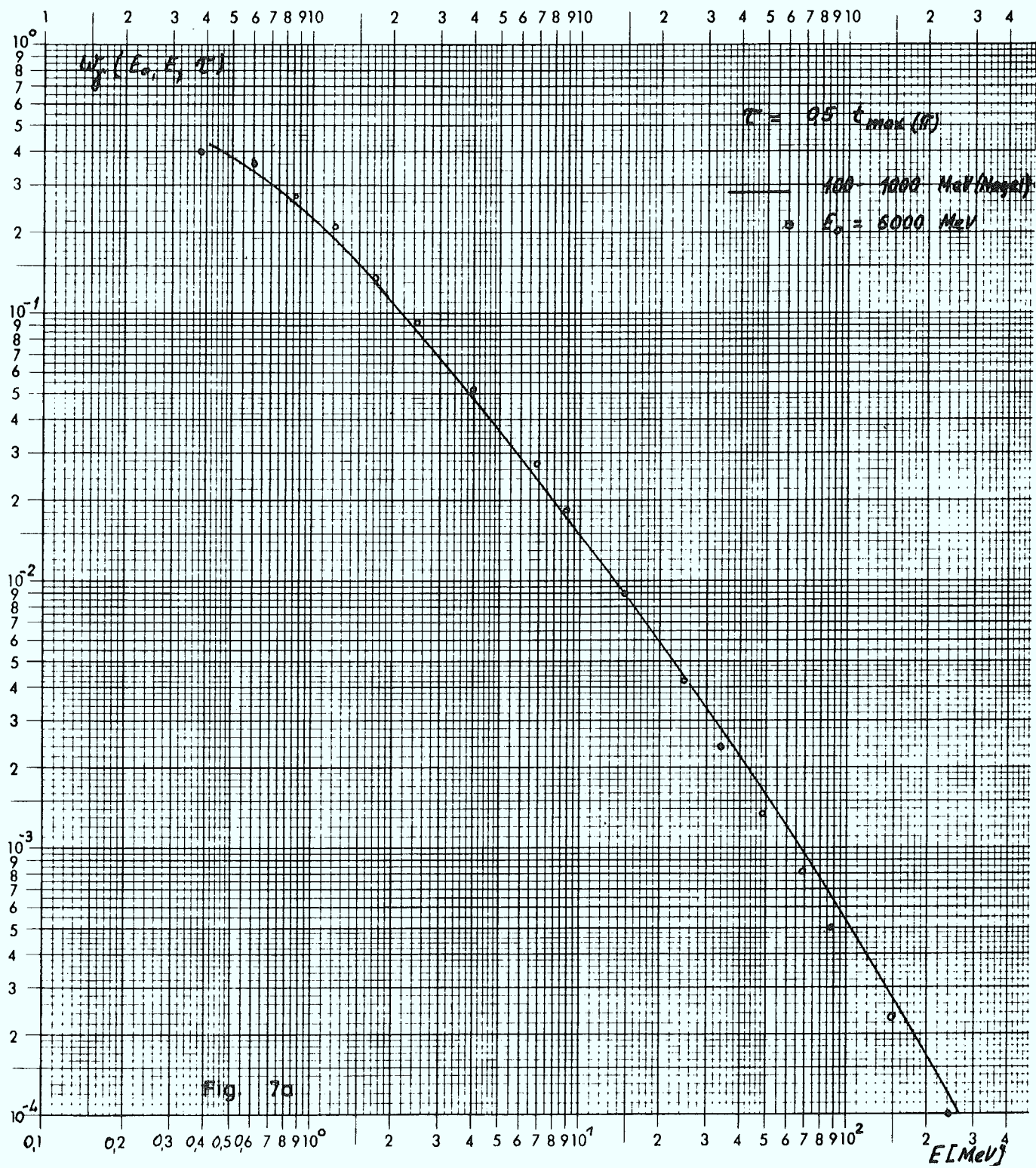
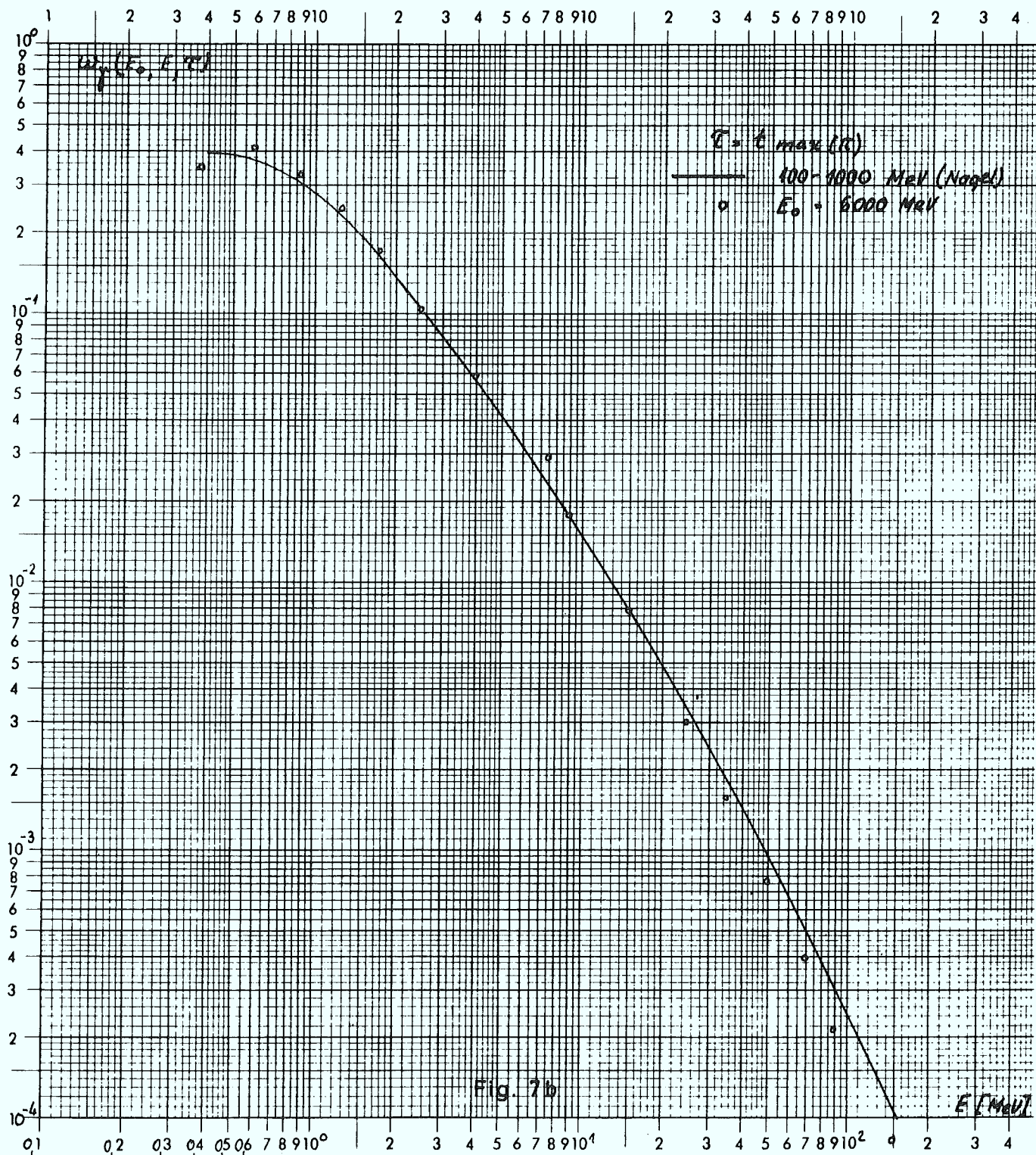


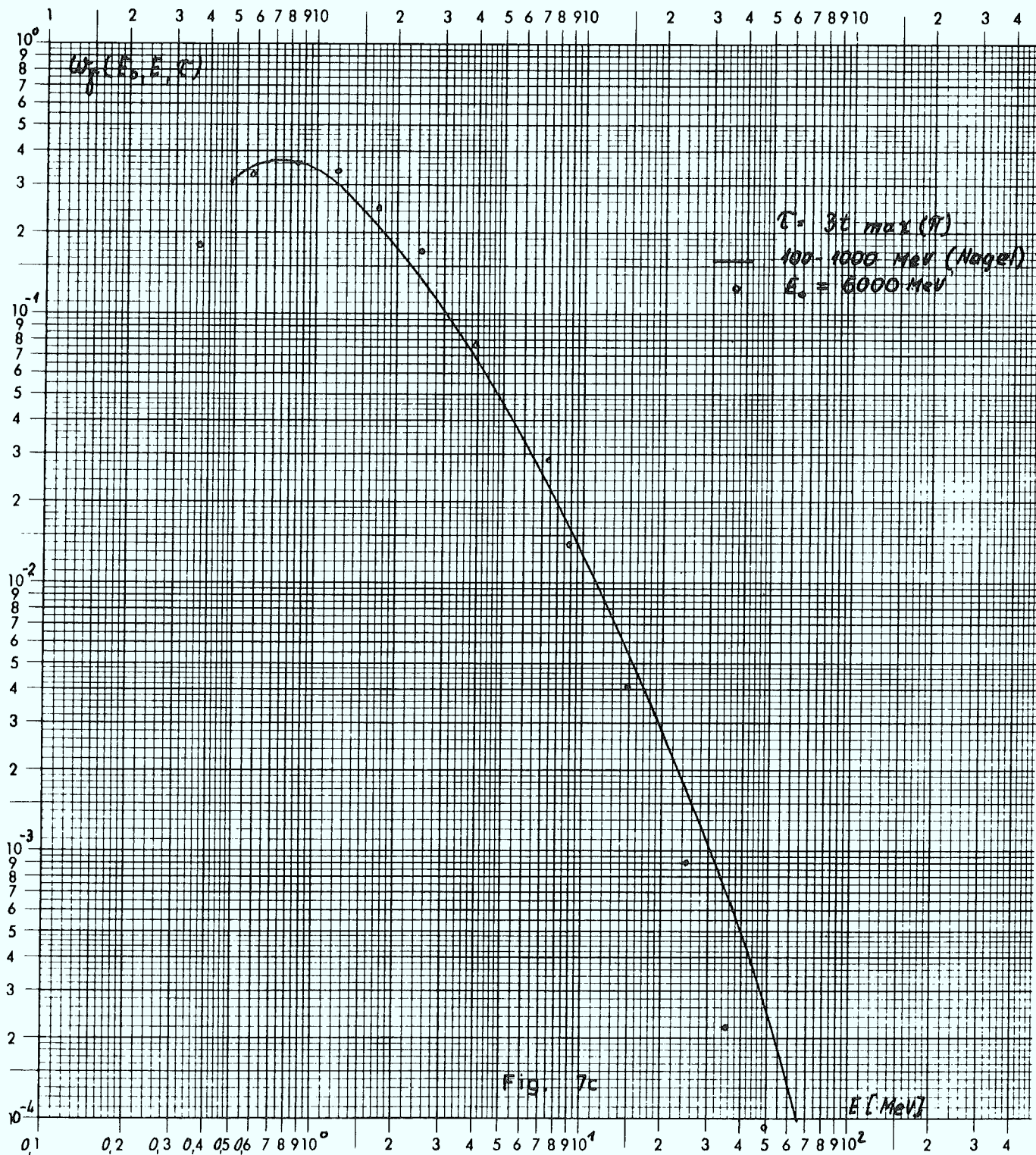
Fig. 6d

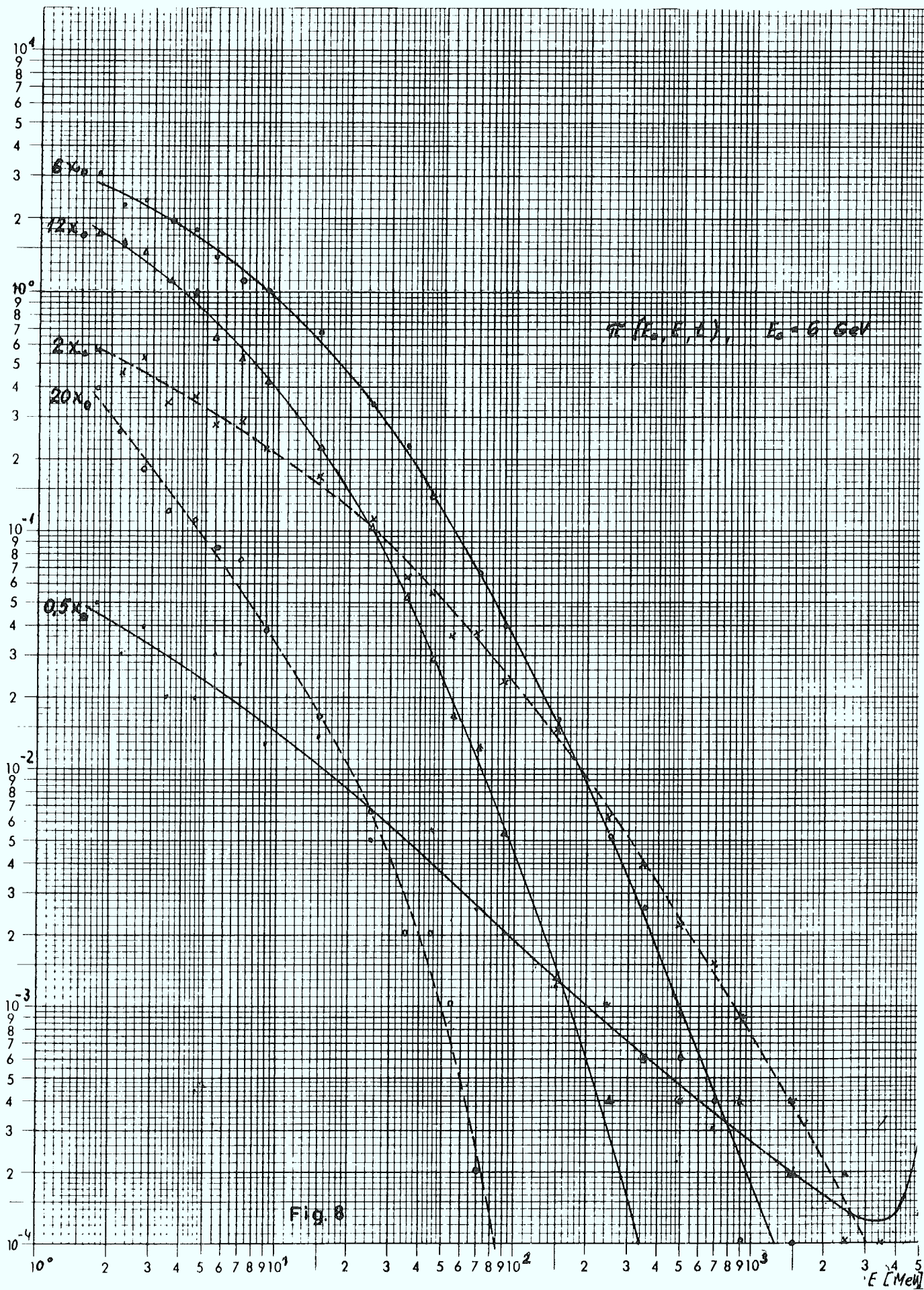
Fig. 6c

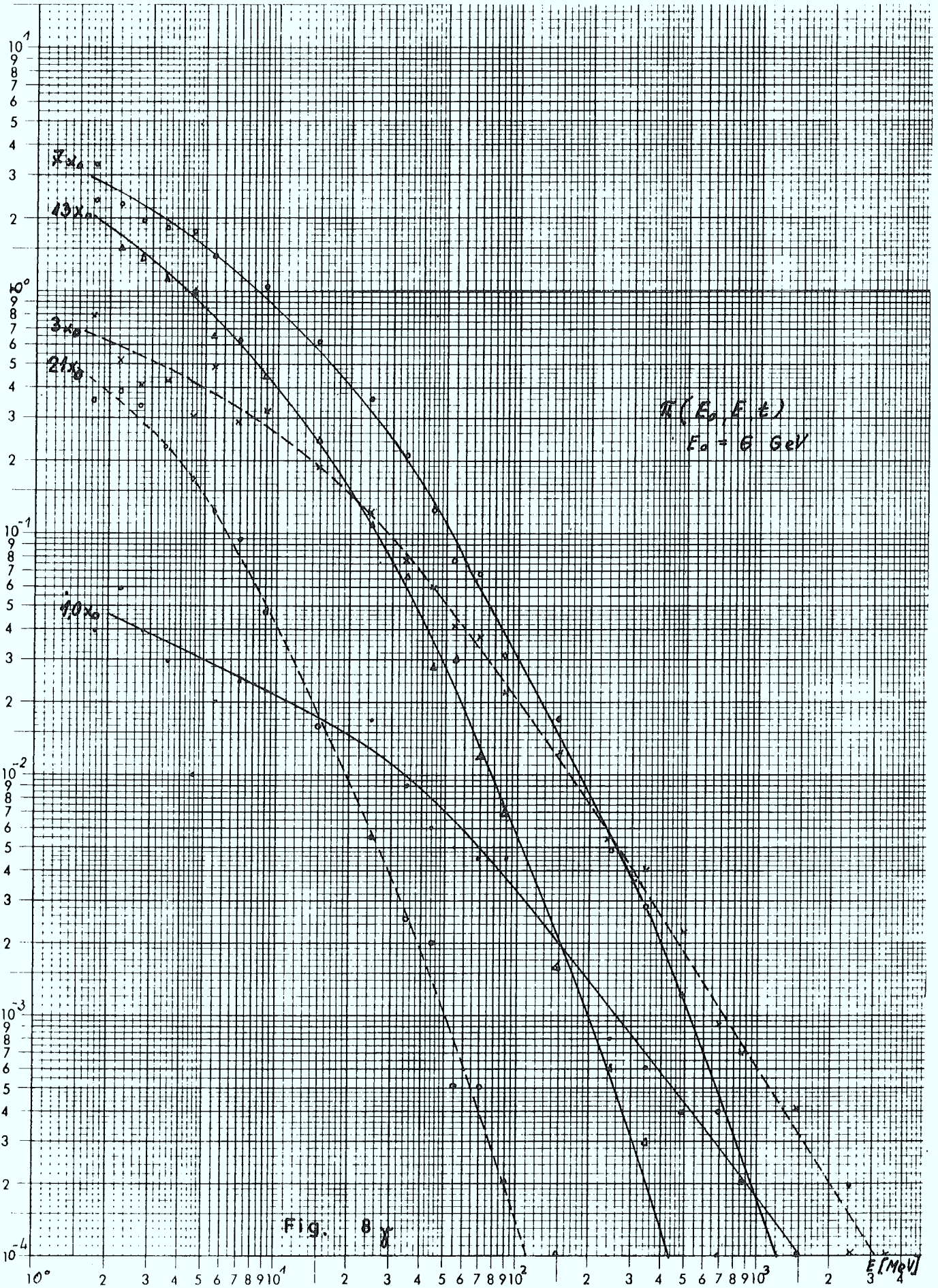
Fig. 6b

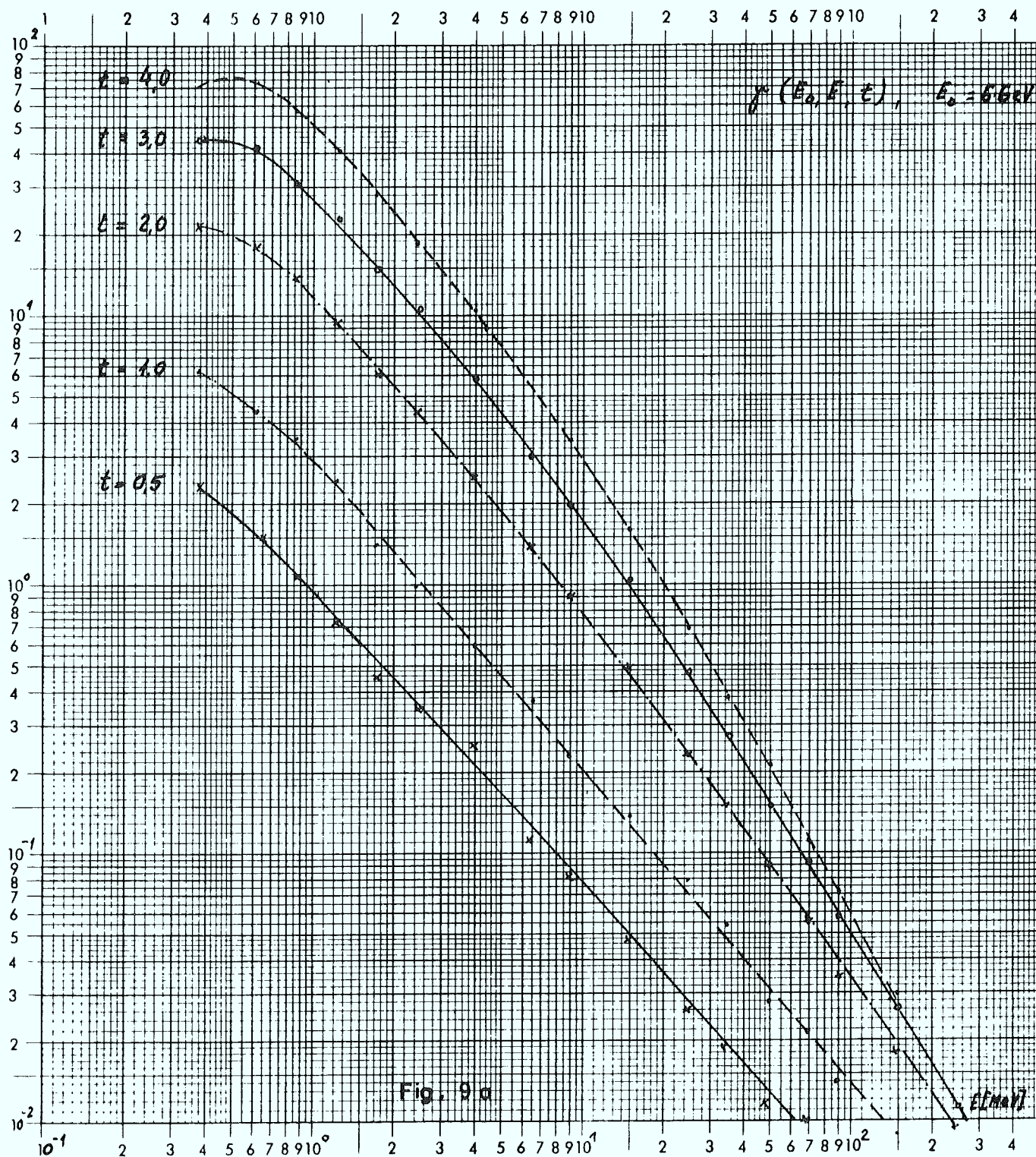


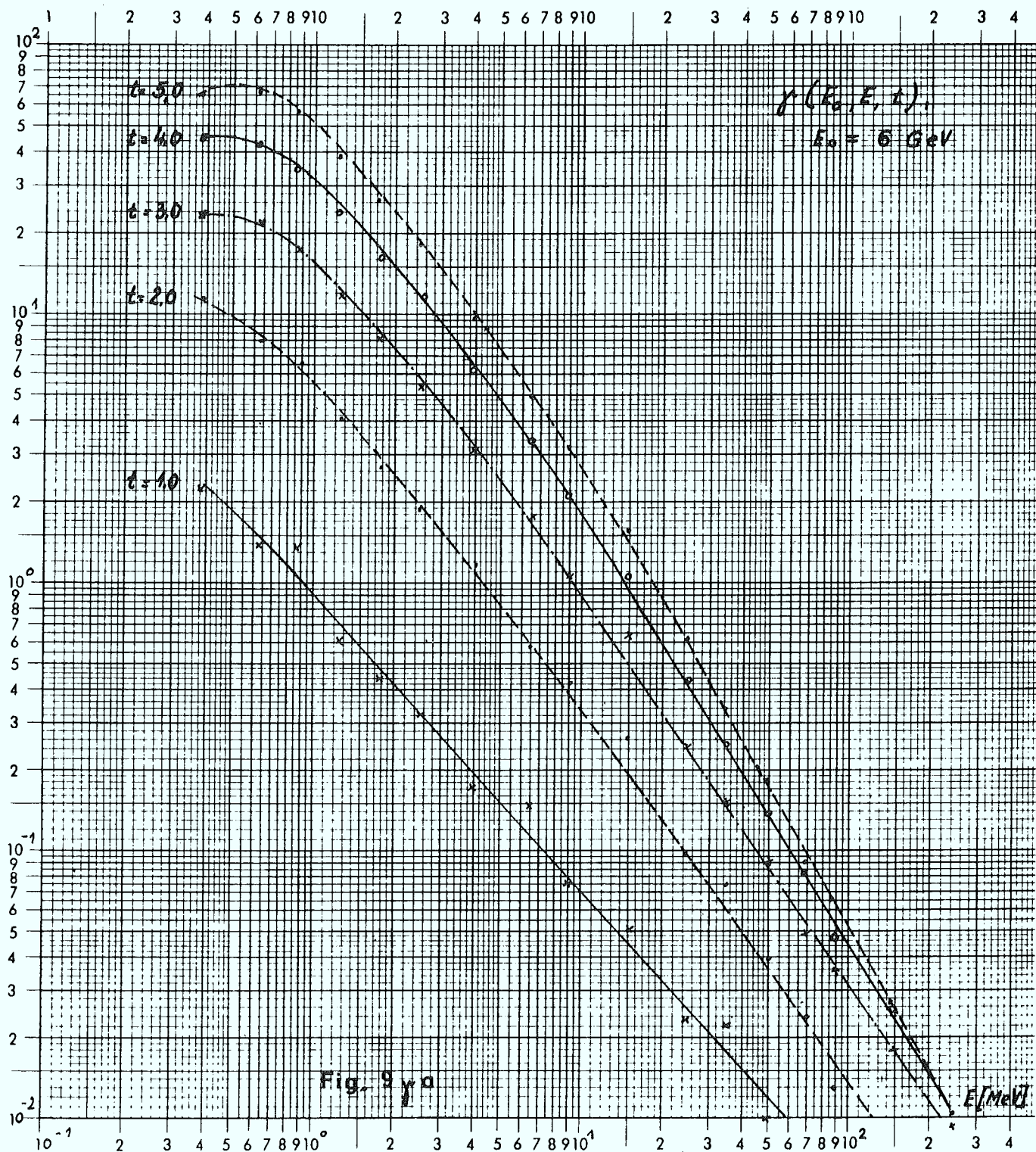


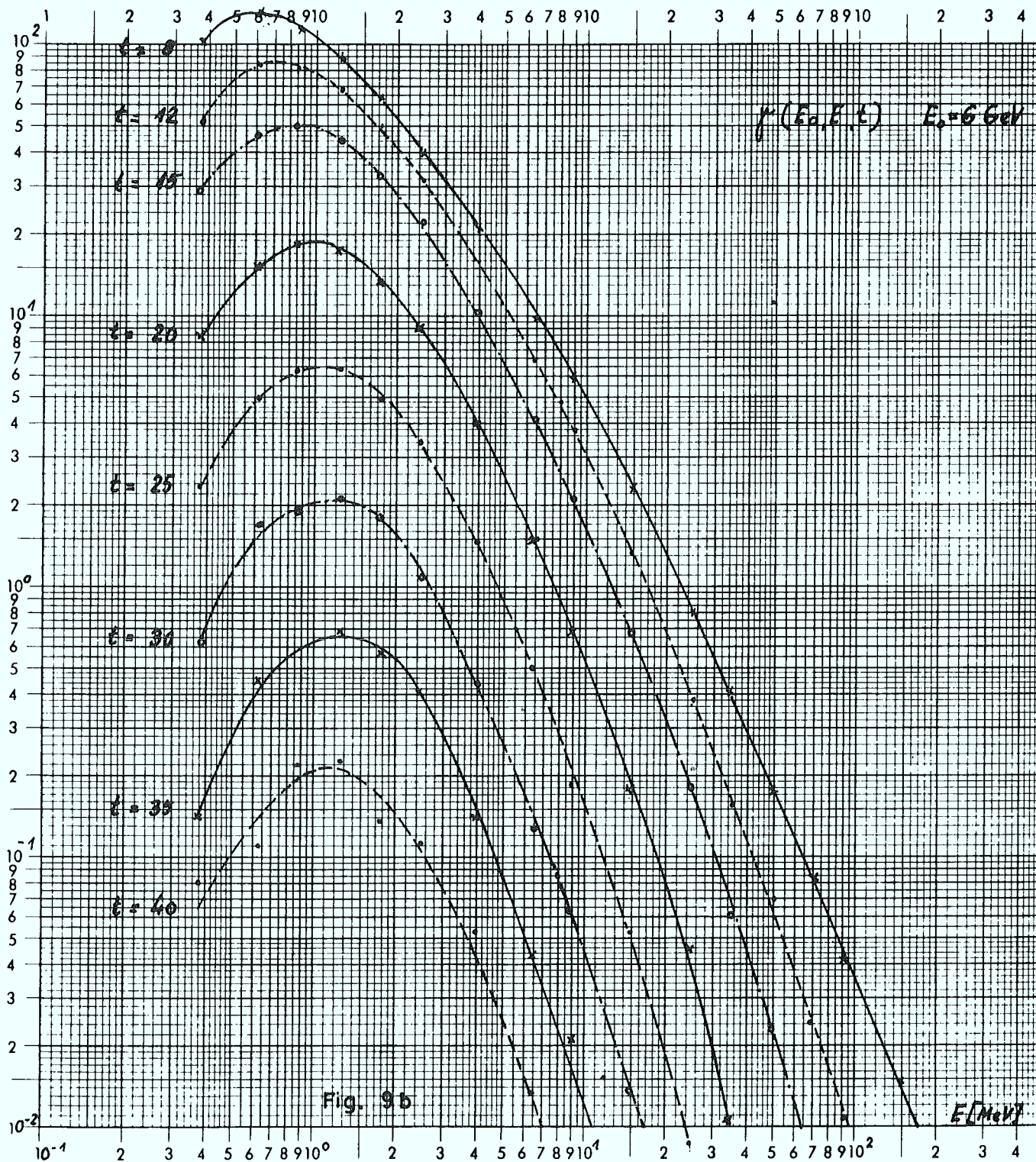


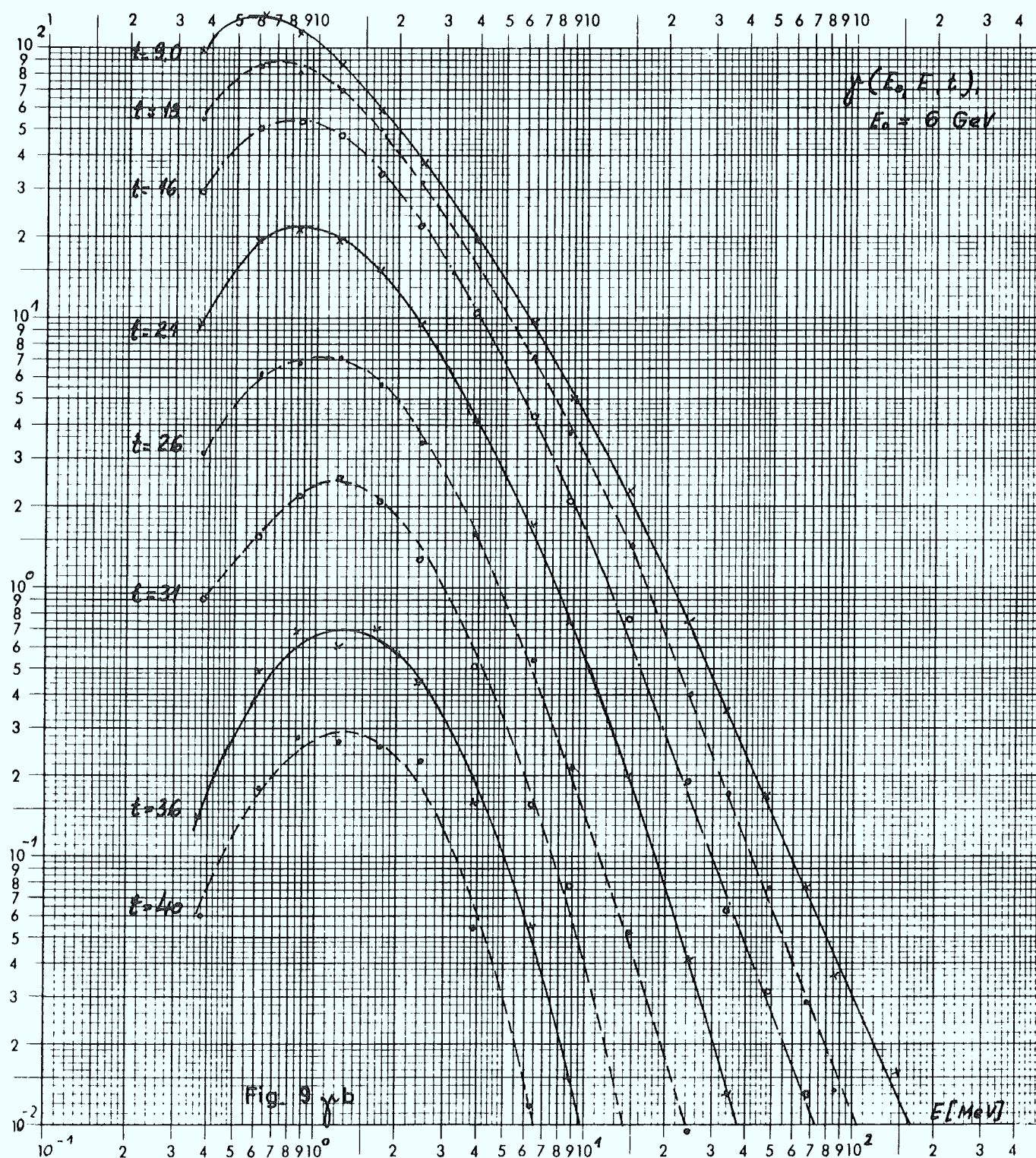


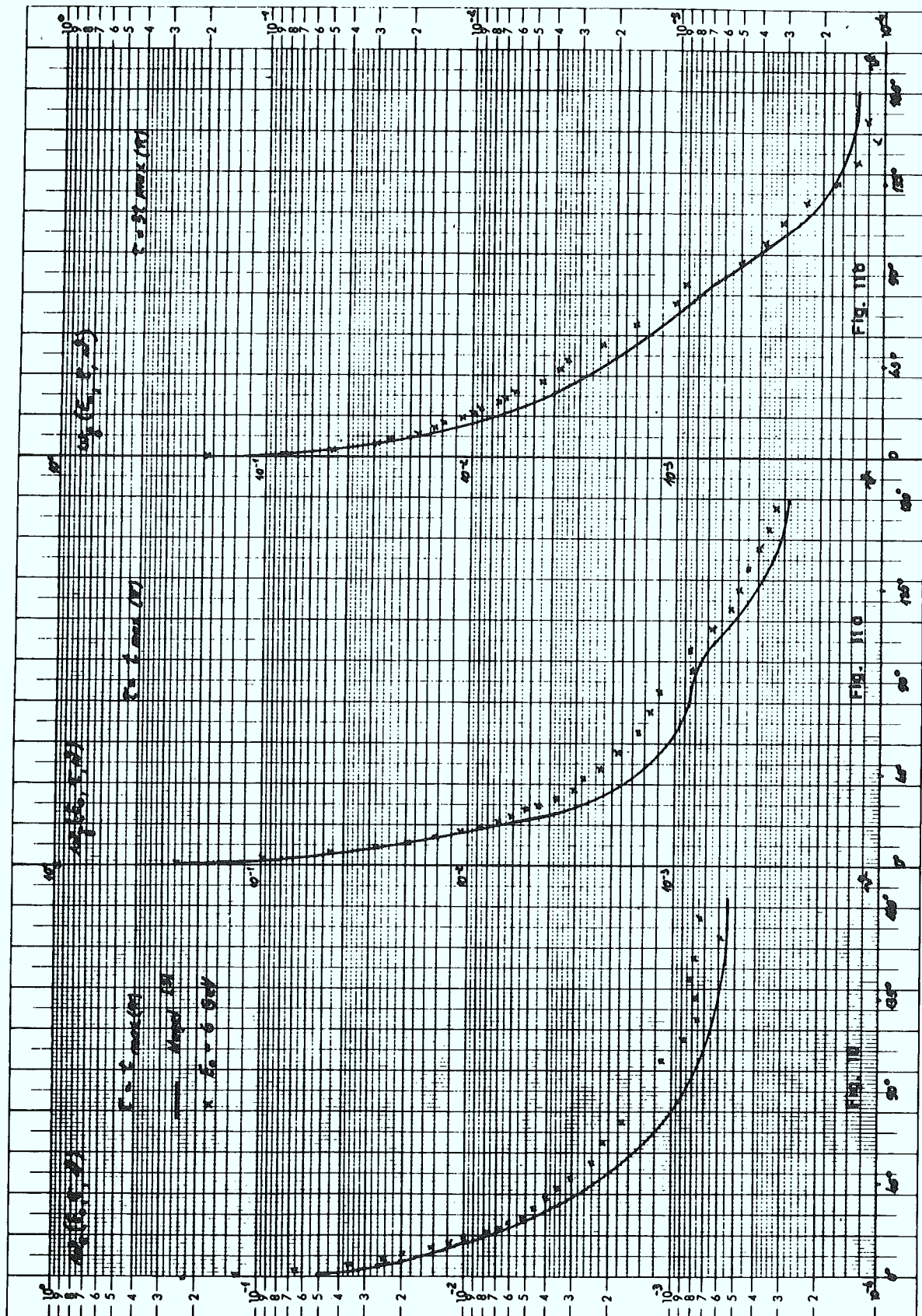


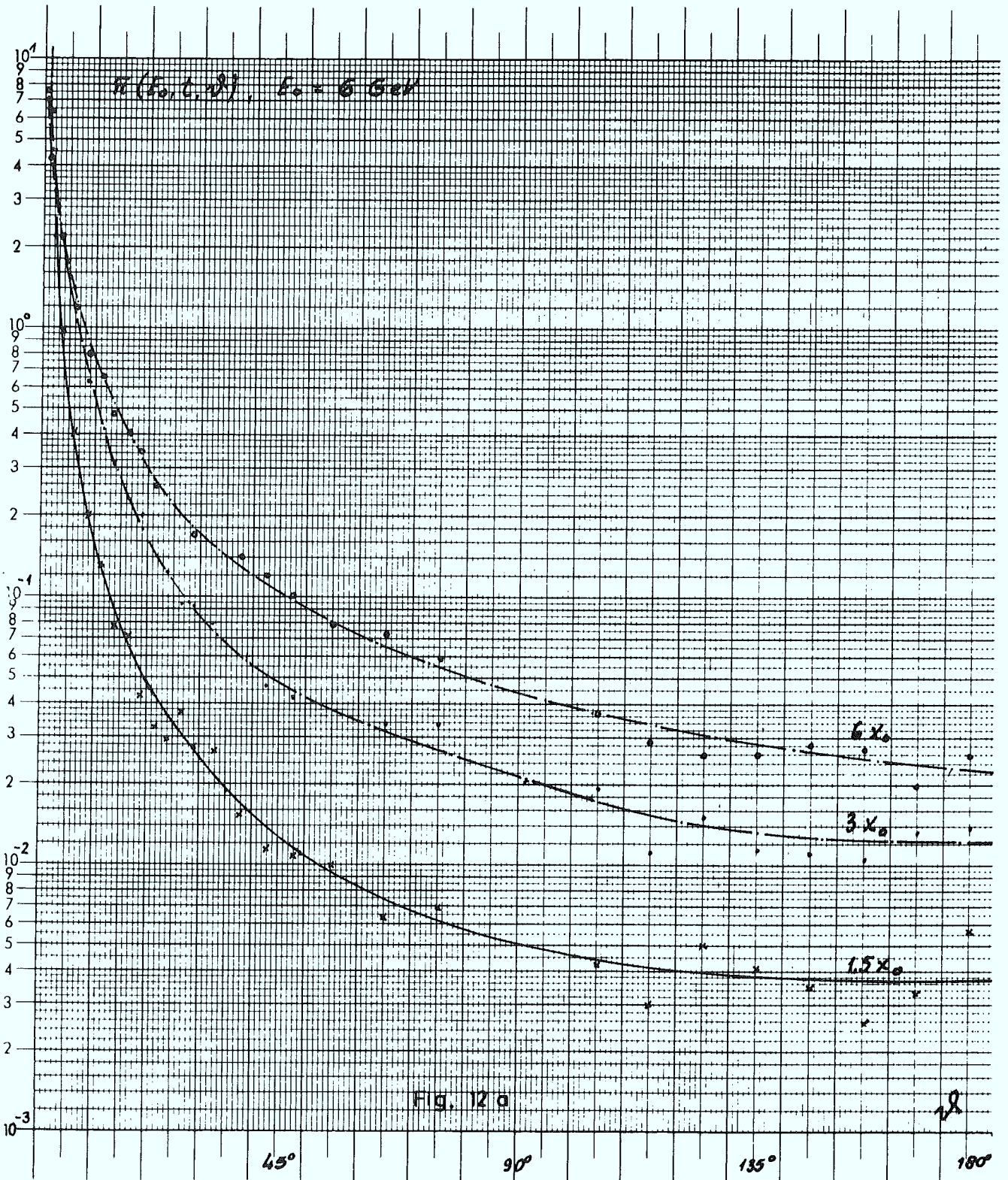


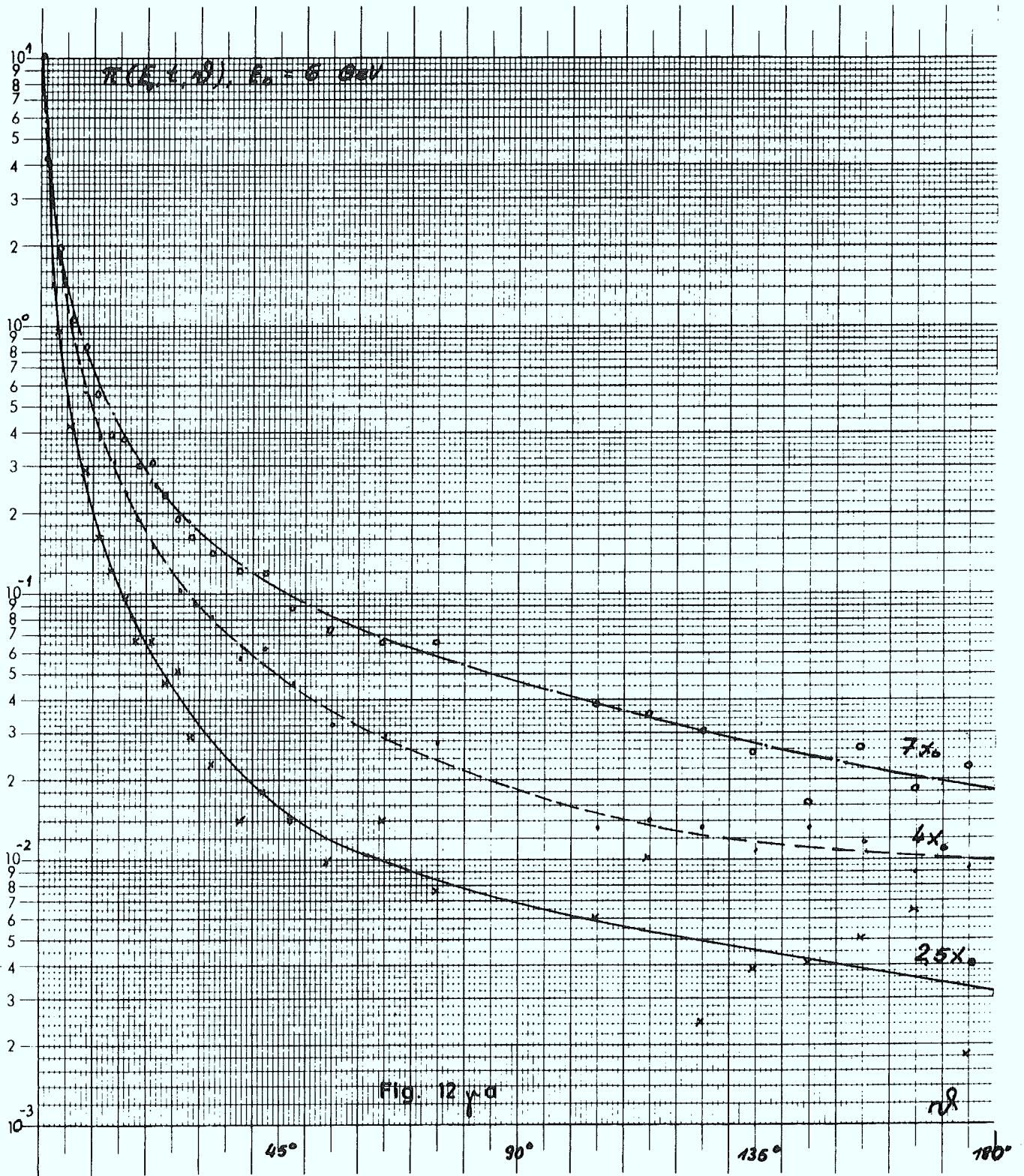


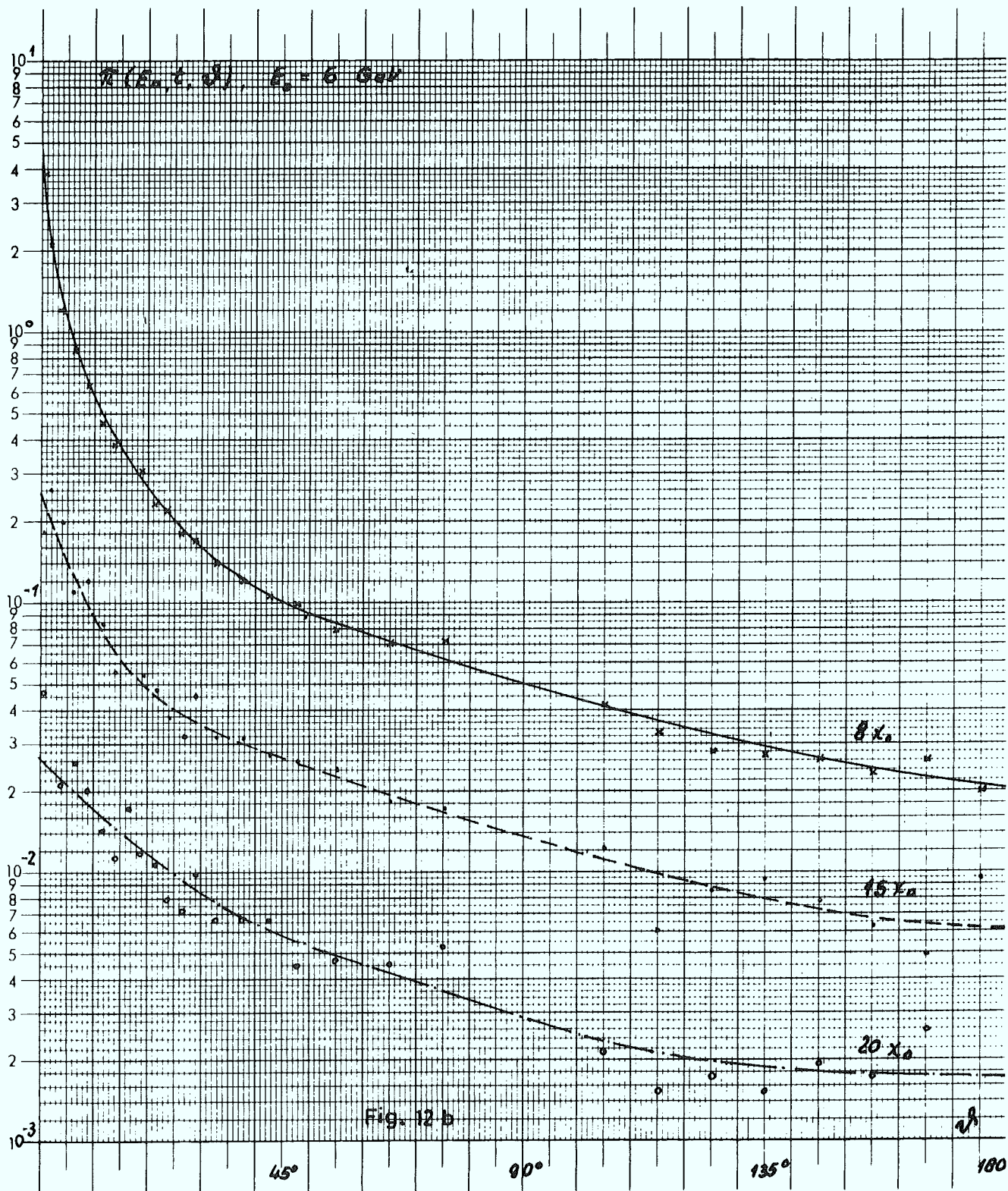


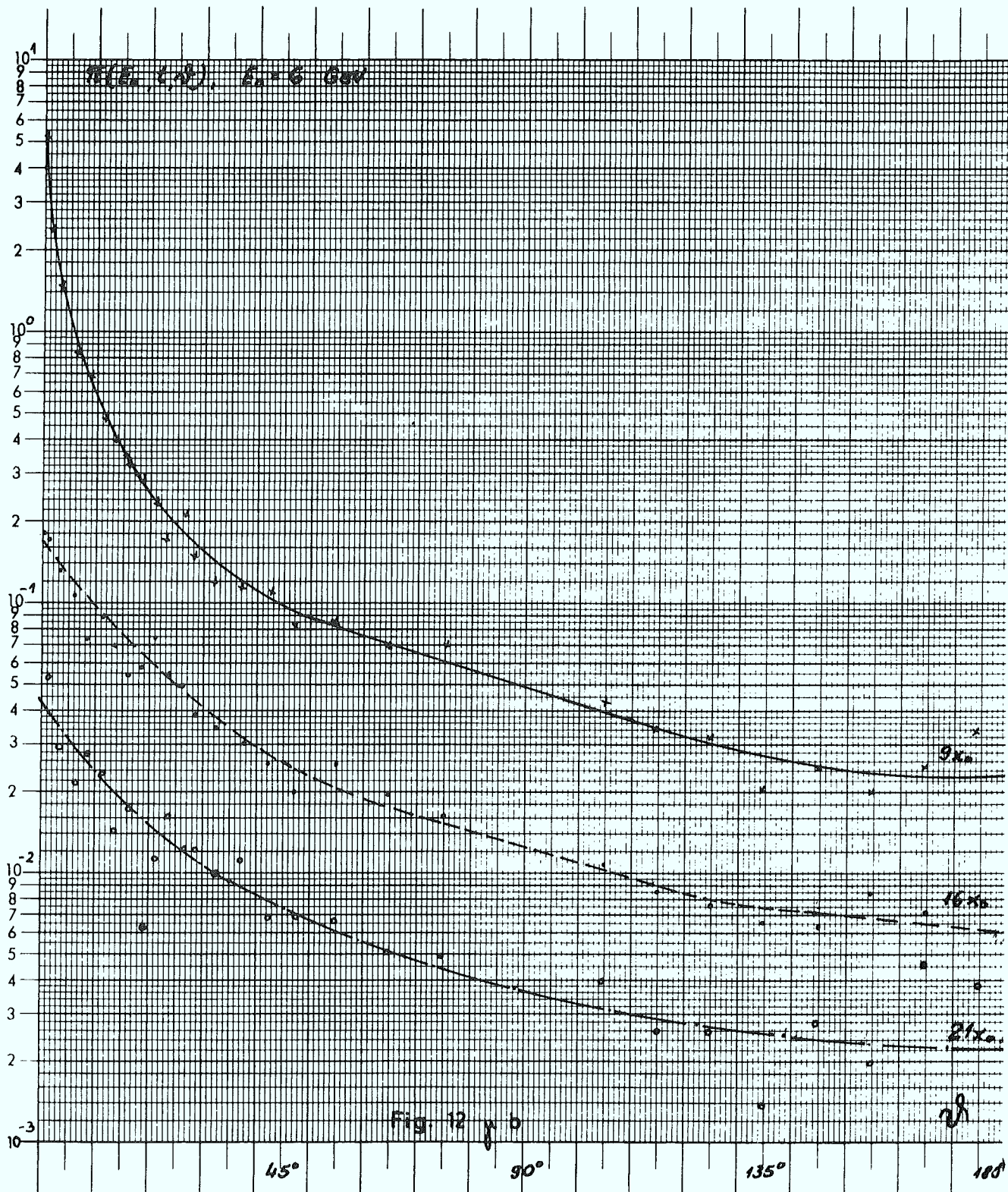


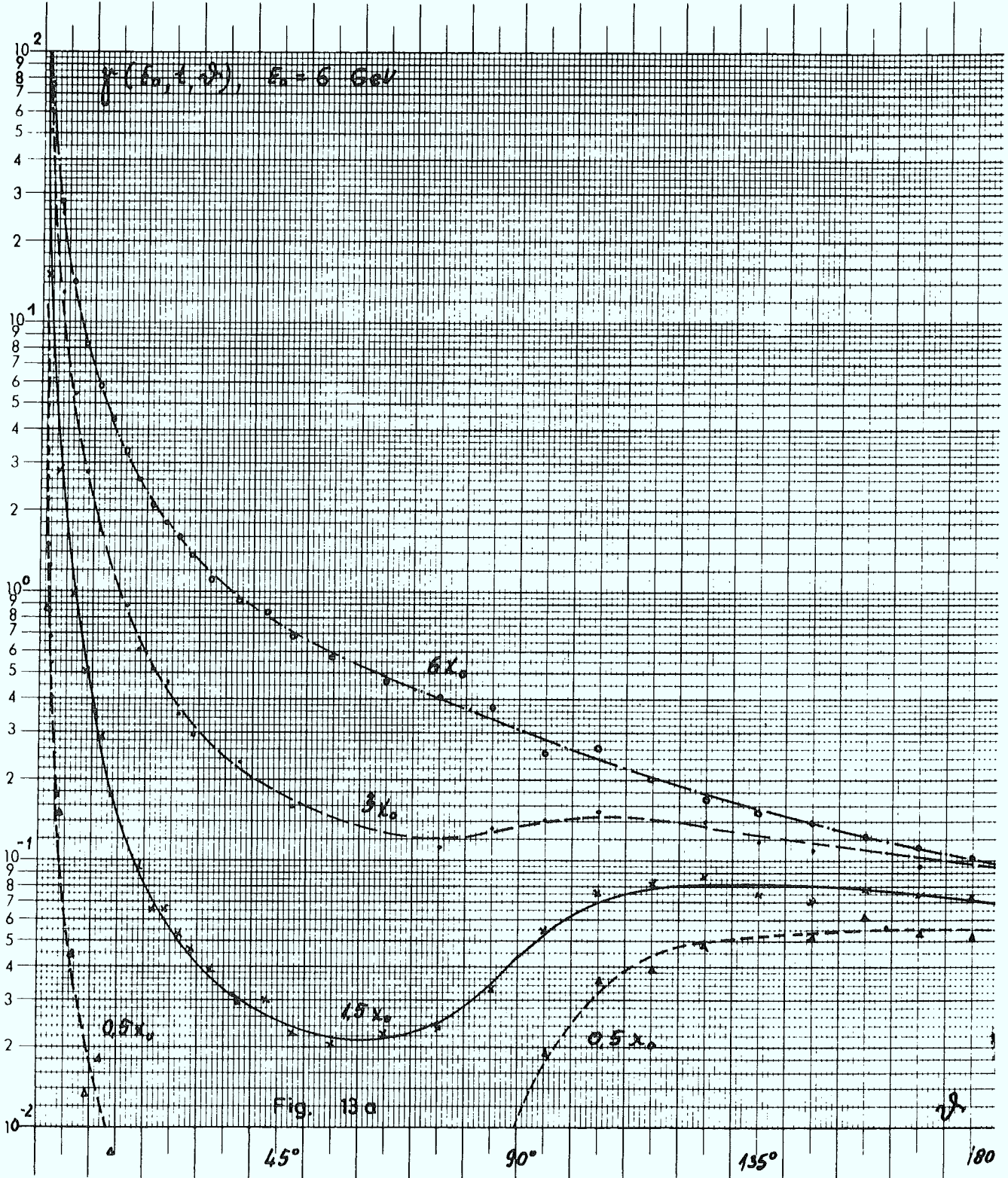


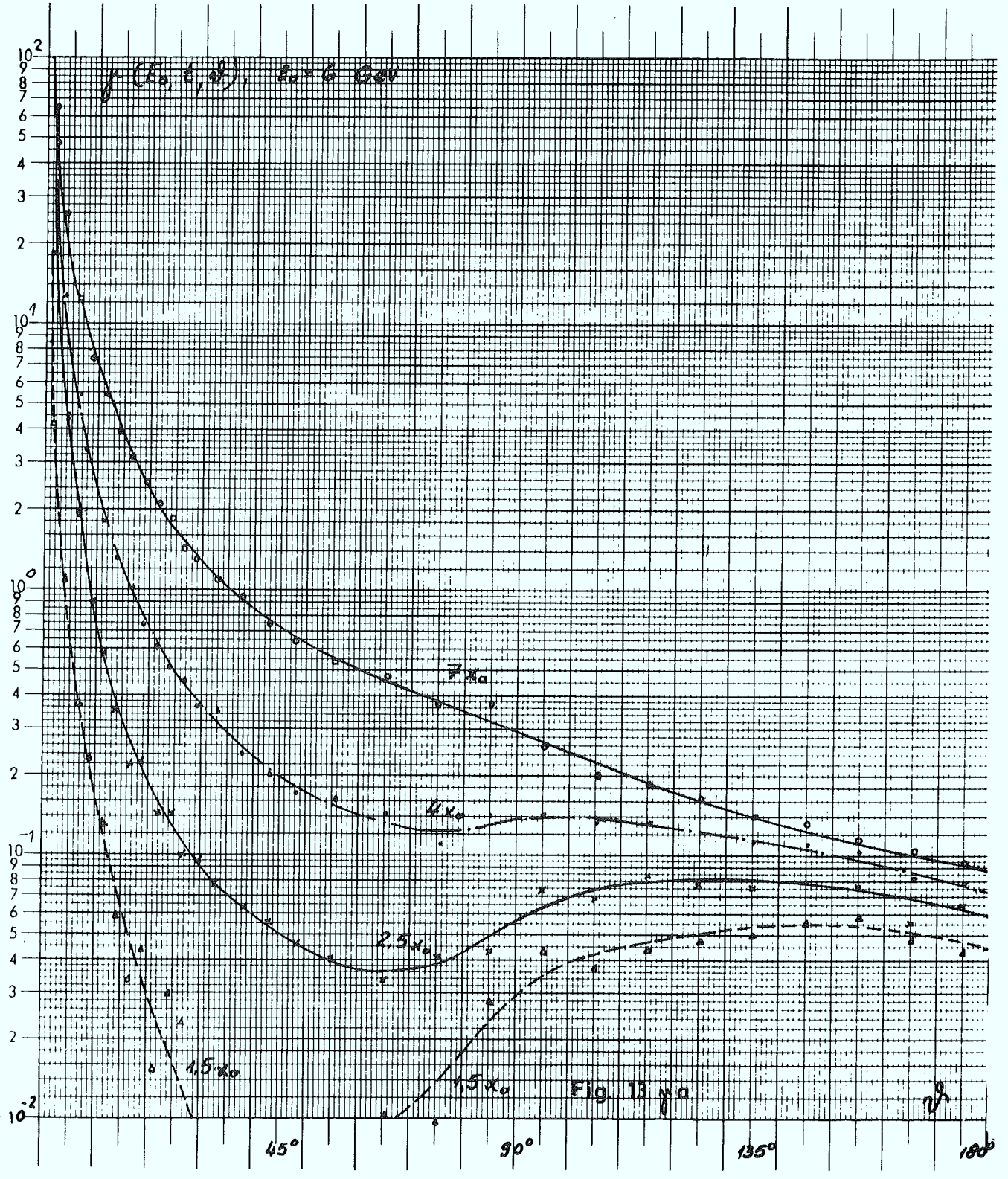


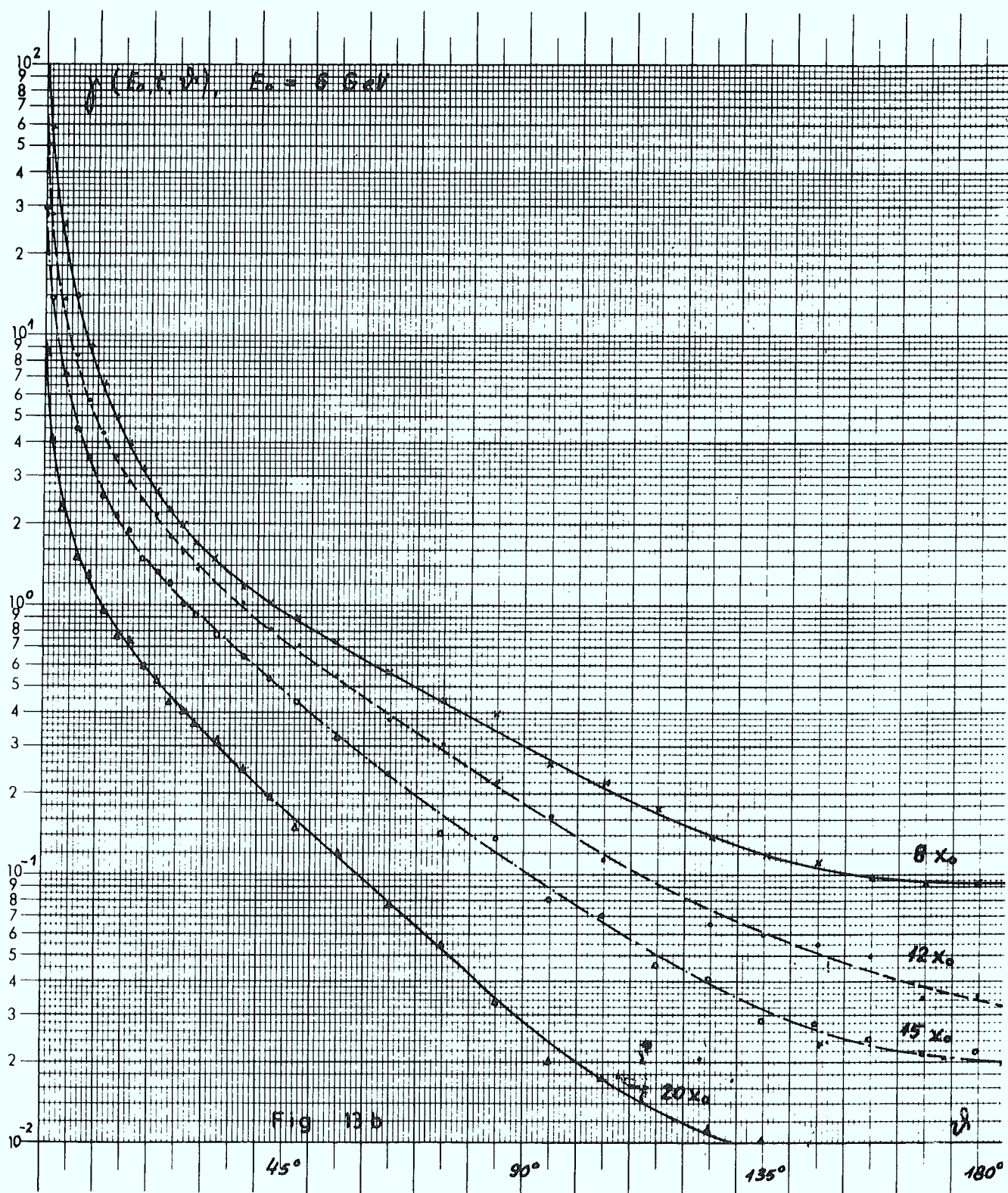


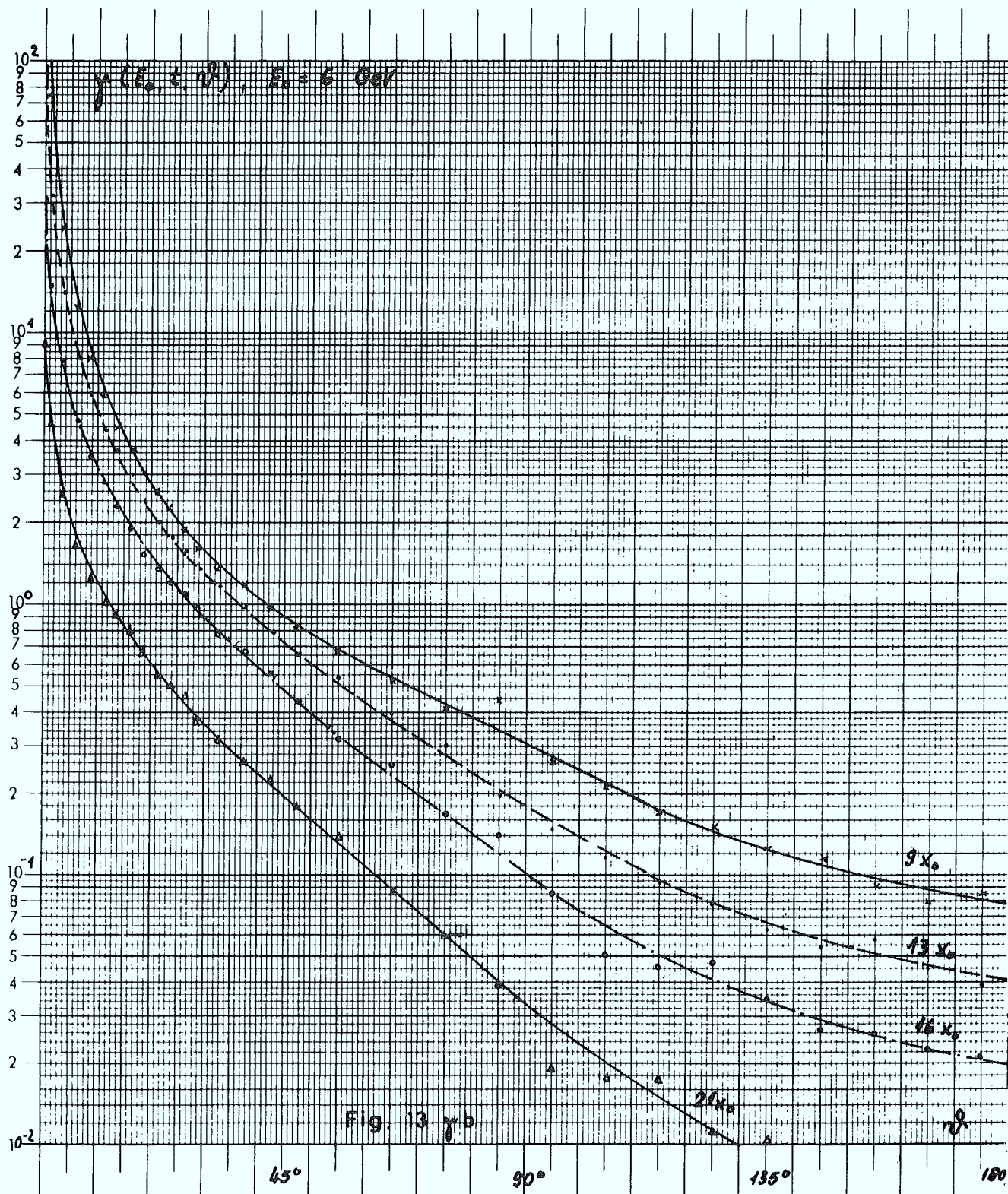


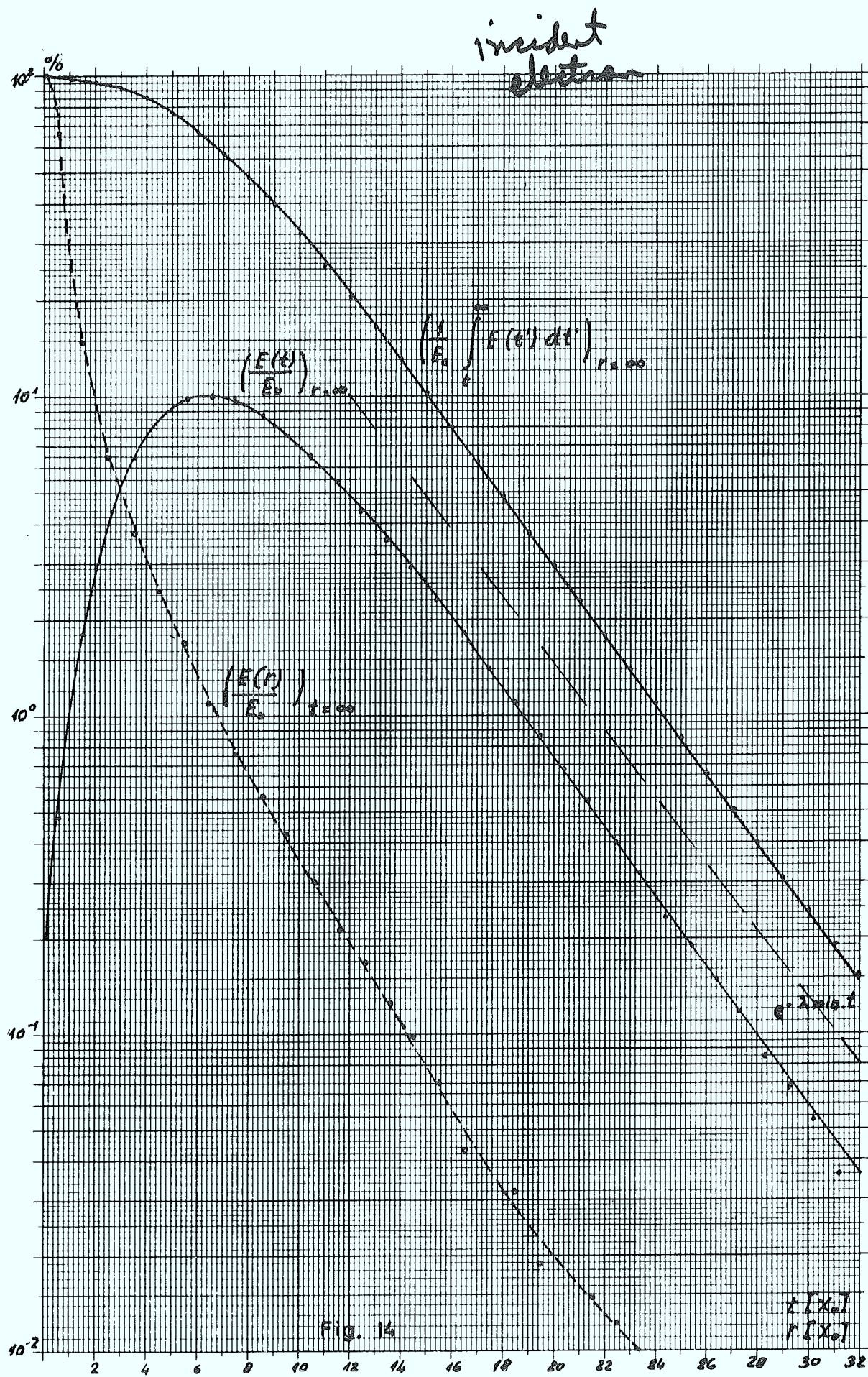


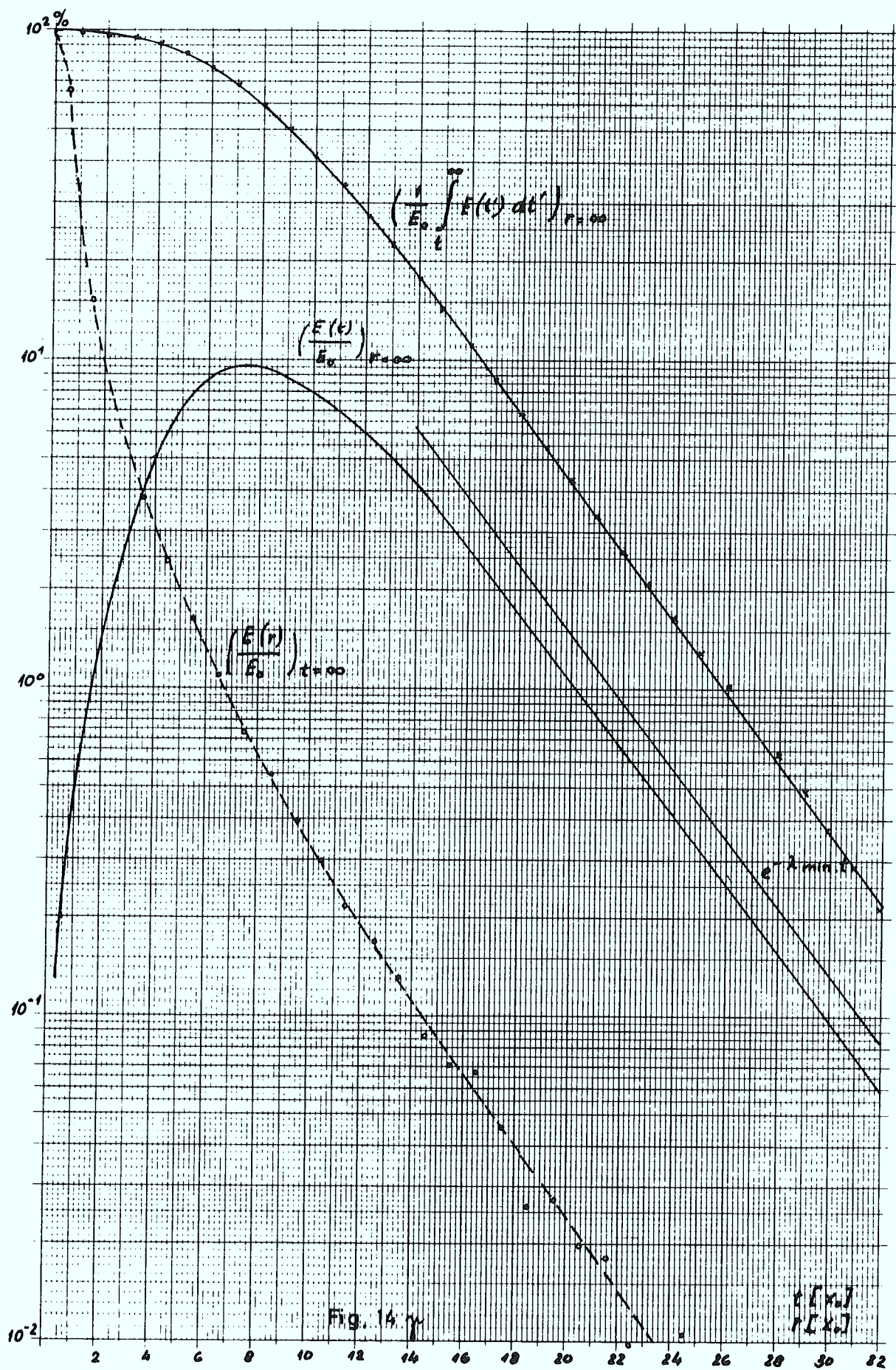


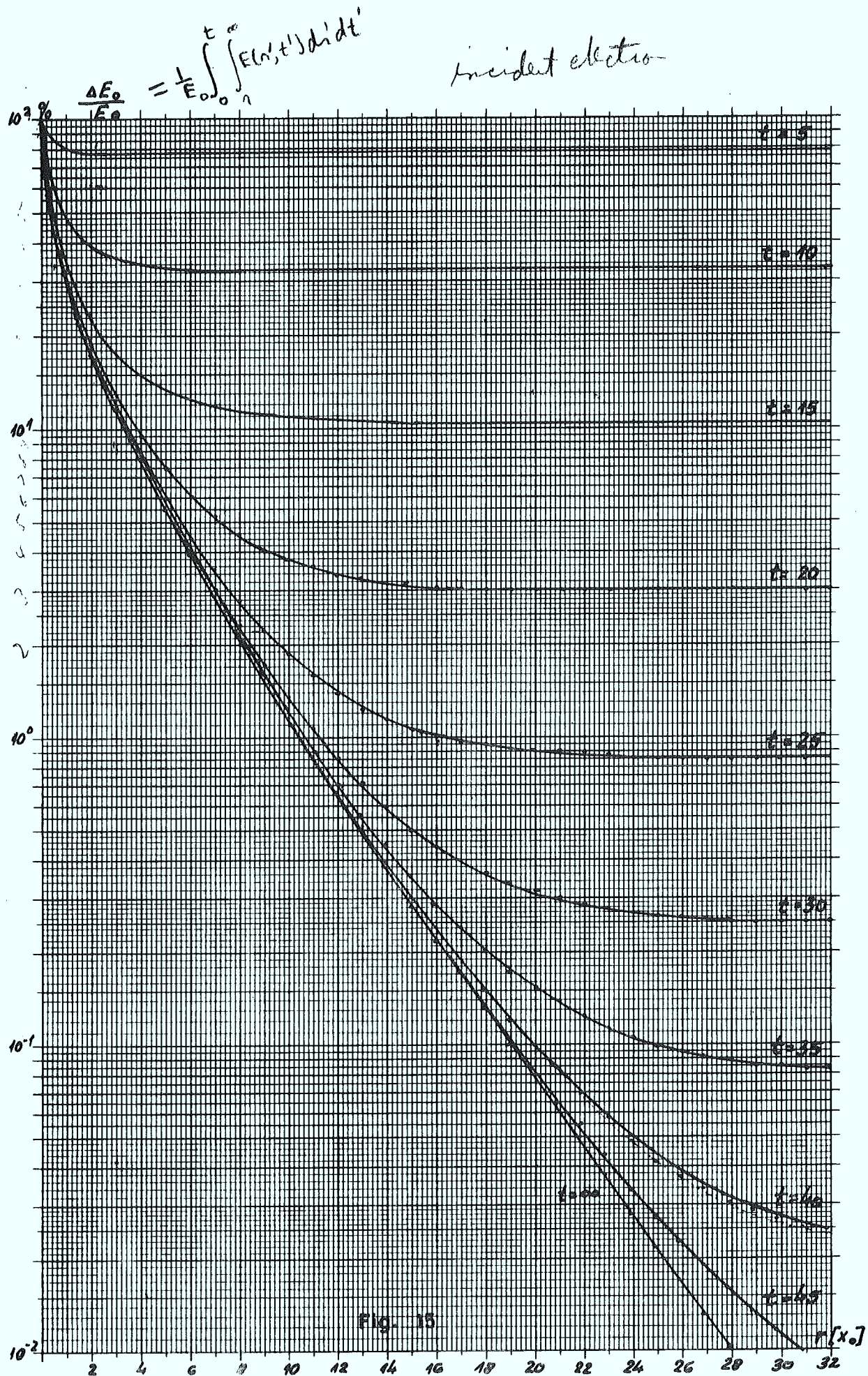












incident photo

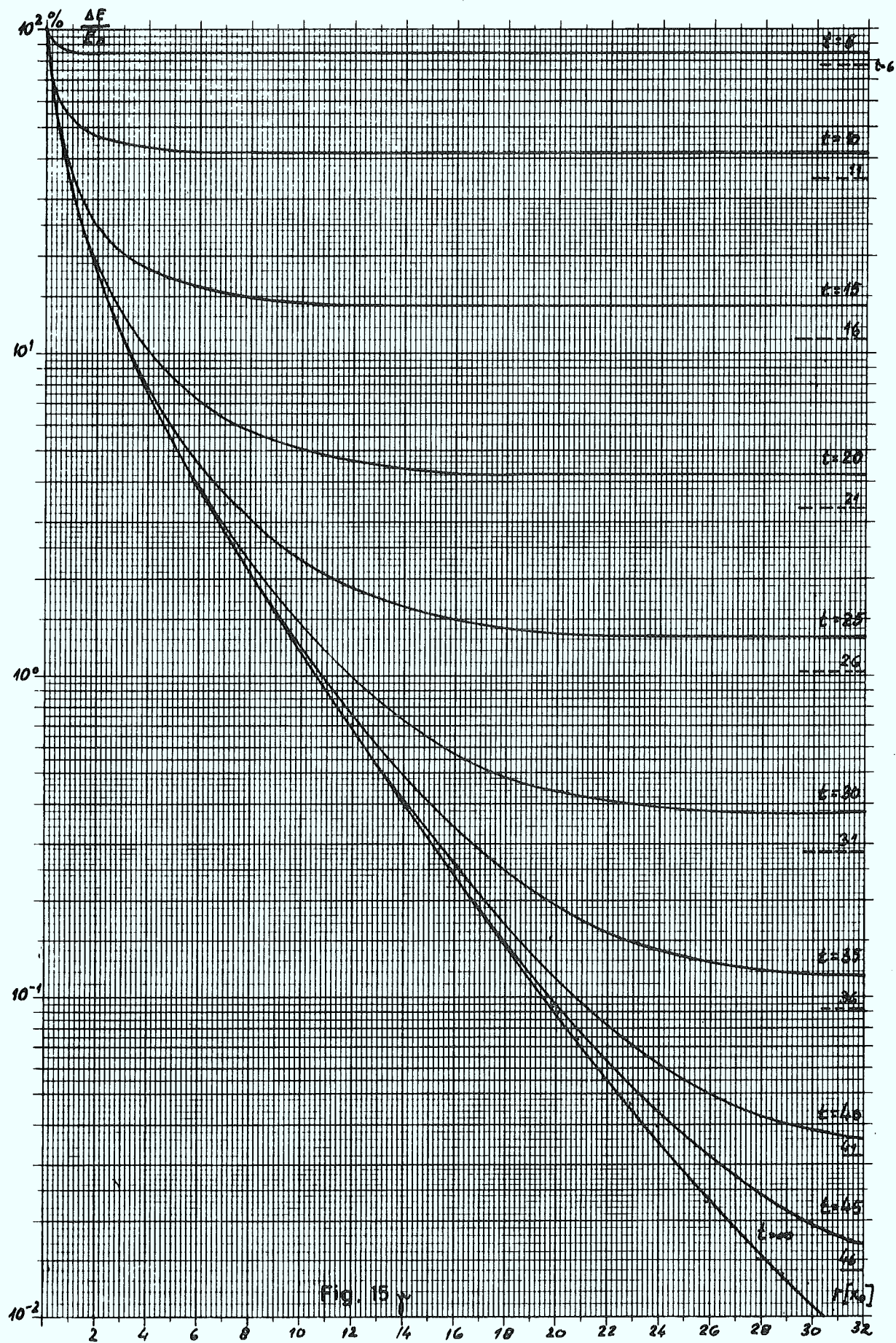


Fig. 15 γ

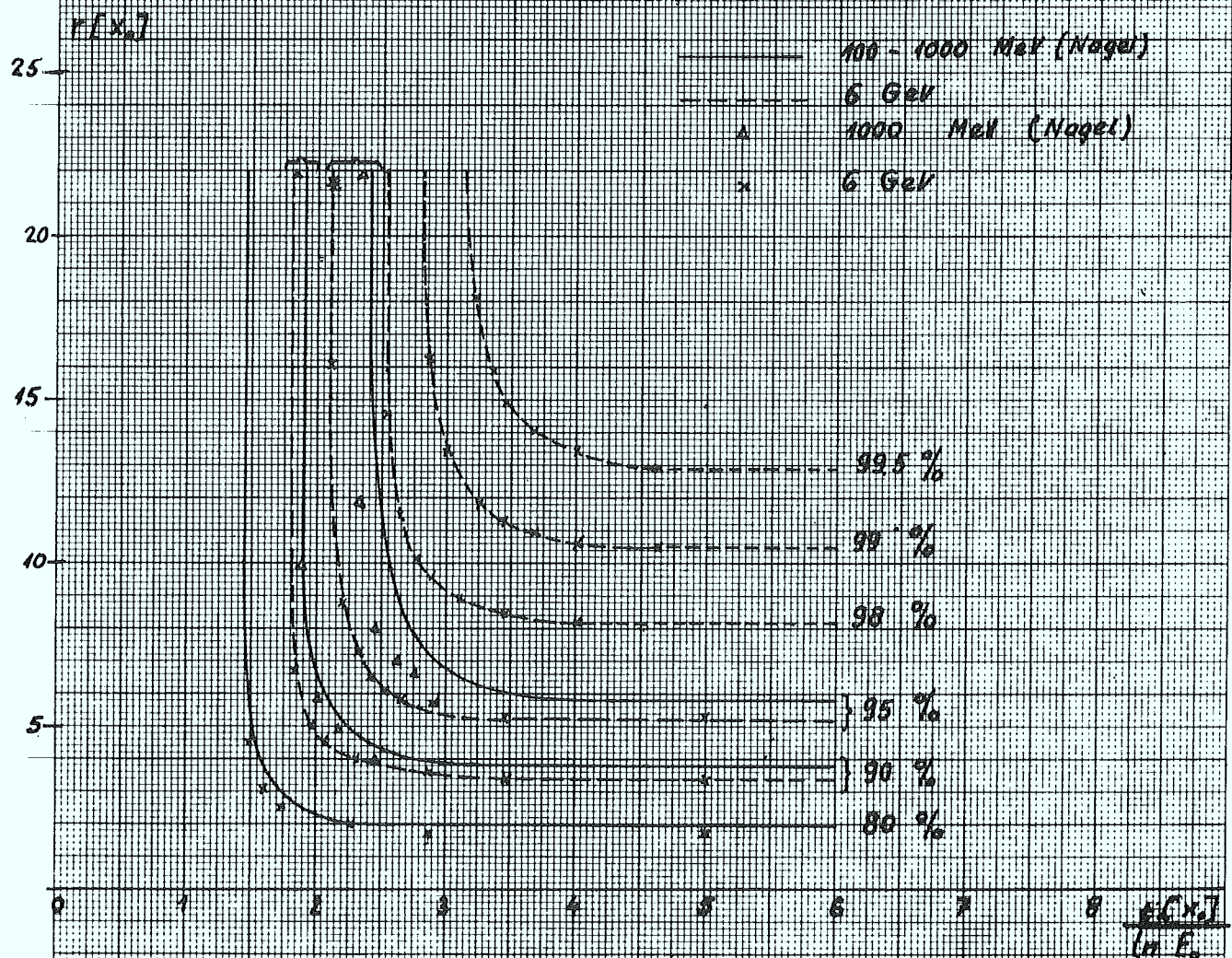


Fig. 16