DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 75/44
November 1975

Duality in Quantum Field Theory

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Abstract: We formulate theses to the effect that local quantum field theory has a dual structure. Our theses are based on experience with conformal invariant quantum field theory.

The study of models has often paved the way to the discovery of general laws in physics.

In the present paper we formulate theses about local quantum field theory (QFT). If they are true then QFT has a structure much reminiscent of dual resonance models [1,2].

Our theses are abstracted from conformal invariant quantum field theory [3]. In such a theory, operator product expansions à la Wilson [4] applied to the vacuum Ω (vacuum expansions) are necessarily convergent if they hold at all as asymptotic expansions to arbitrary accuracy at short distances. This is a consequence of positivity, spectrum condition and global conformal invariance. It comes about because the vacuum expansions are identical with the partial wave expansions on the Minkowskian conformal group G^* = universal covering of SO(4,2). Partial wave expansions converge because they are nothing but the decomposition of a unitary representation into its irreducible components [5,12].

Conformal invariant QFT does not obey the asymptotic condition.

However, our thesis 1 was also shown by Schroer and Swieca to be true for massive free field theory which does satisfy the asymptotic condition [6]. Also, the asymptotic condition is a condition on the behaviour of e.g. Wightman functions at large distances. Our expansions are supposed to be valid in the distribution theoretic sense, that is after smearing with test functions. Test functions fall off fast at large distances, and the precise form of the asymptotic behaviour there should therefore not be crucial.
Nontrivial massive QFT also has many more thresholds in general than either free field theory or conformal QFT. This motivates us to formulate weaker theses than are actually true in conformal invariant QFT.

Let $\varphi(x)$ a (possibly composite) local field in a local QFT. For definiteness sake suppose until further notice that φ is hermitean scalar. To start with, consider the disconnected 4-point Wightman function,

$$W(x_1 \ldots x_4) = (\Omega, \varphi(x_1) \ldots \varphi(x_4) \Omega) \qquad (1)$$

Let $\{ O^i_\alpha(x) \}$ a complete set of nonderivative local symmetric tensor fields in the theory. In perturbation theory they can be constructed by the normal product formalism. One omits however those fields which can be written as linear combinations of other fields in the set and their space time derivatives. For instance, $\varphi(x)$ is to be included, but not $\partial^\mu \varphi$. Also the identity operator $1 = O^o_\wedge$ is included. Without loss of generality we will assume that all fields are hermitean. Indices $i = (\ell, \gamma)$ where ℓ is Lorentz spin (tensor rank for now) and γ distinguishes between fields with the same Lorentz transformation law; $\alpha = (\alpha_1 \ldots \alpha_\ell)$ is a Lorentz multi-index. (Each $\alpha_k = 0 \ldots .3$; $\alpha = \wedge$ stands for no index at all.)

We will need the following 2- and 3-point Wightman functions

$$(\Omega, O^i_\alpha(x) O^j_\beta(y) \Omega) = \Delta^{ij}_{\alpha\beta}(x-y) = \int_{\text{sptr}} dp\, e^{-ip(x-y)} \tilde{\Delta}^{ij}_{\alpha\beta}(p) \qquad (2a)$$

and

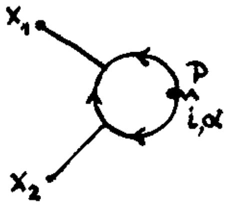
$$(\Omega, \varphi(x_1)\varphi(x_2) O^i_\alpha(z)\Omega) = W^i_\alpha(x_1 x_2; z) = (2\pi)^{-4} \int_{\text{sptr}} dp\, e^{ipz}\, \tilde{W}^i_\alpha(x_1 x_2; p) \qquad (2b)$$

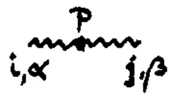
Integration is over the energy-momentum spectrum of the theory, in it $p^o \geq |\vec{p}|$. We define now a kind of amputated 3-point Wightman function B^i_α by*

$$\tilde{W}^i_\alpha(xy;p) = \sum_j B^j_\beta(xy;p)\, \tilde{\Delta}^{ji}_{\beta\alpha}(p) , \qquad p \in \text{sptr}. \qquad (3a)$$

This definition makes sense since the matrix $\tilde{\Delta}(p)$ is positive semi-definite and vanishes only if p is not in the spectrum of any $O^i\Omega$.

We shall use a graphical notation,

$B^i_\alpha(x_1 x_2; p)$ = 

; 

$= \tilde{\Delta}^{ij}_{\alpha\beta}(p)$ (3b)

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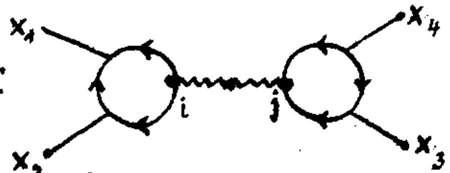
* We adopt a summation convention over repeated Lorentz multi-indices, viz. $x_\alpha y_\alpha = x_o y_o - \vec{x}\cdot\vec{y}$ etc.

The arrows indicate the orders of the arguments, cp. Eqs (2). Redundant arrows will be omitted later on. In particular, if $(x_1-x_2)^2 < 0$ the arrow between the two long external legs is immaterial because of locality. Reversal of all arrows will mean complex conjugation.

<u>Thesis 1.</u> The disconnected Wightman 4-point function (1) admits a convergent expansion

$$W(x_1 \ldots x_4) = \sum_{ij} \int_{sptr} dp \; B^i_\alpha(x_1x_2;p)\, \tilde{\Delta}^{ij}_{\alpha\beta}(p)\; \bar{B}^j_\beta(x_4x_3;p) \qquad .$$

$$\equiv \sum_{ij} \qquad (4)$$



with 2-point functions $\tilde{\Delta}^{ij}$ and amputated 3-point functions B^j defined by Eqs. (2) and (3).

A bar stands for complex conjugation. In the graphical notation, integration over p and summation over Lorentz indices not shown is understood. It is also understood that a term is included coming from the identity operator $\mathbf{1} = O^o_\kappa$. This term has the form

$$W(x_1-x_2)W(x_3-x_4) =$$  where $W(x-y) = (\Omega, \varphi(x)\varphi(y)\Omega)$.

Convergence of expansion (4) is meant in the distribution theoretic sense, i.e. it holds after smearing with test functions $f(x_1x_2)g(x_3x_4)$.

Formula (4) incorporates the spectrum condition. Also positivity of the 4-point Wightman function is incorporated; it follows from (4) and positivity of the 2-point matrix $\tilde{\Delta}(p)$ for $p \in$ sptr. Thus

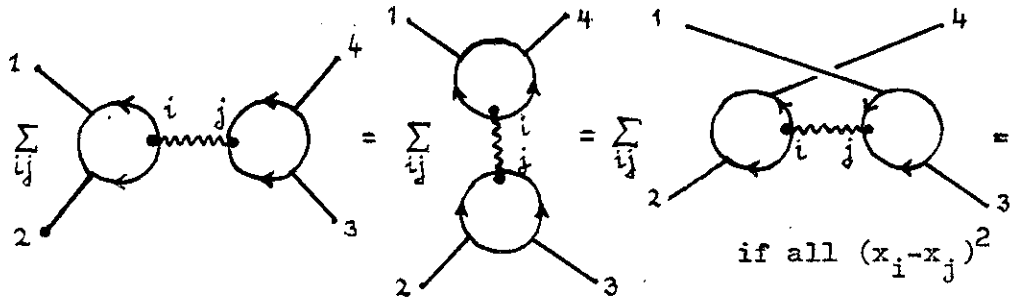
$$W[f^* * f] = \int dx_1 \ldots dx_4 \; \bar{f}(x_2x_1) W(x_1 \ldots x_4) f(x_3x_4) \geq 0 \; .$$

Further restrictions on the amplitudes come from locality, however. The Wightman function $W(x_1 \ldots x_4)$ is completely specified by its values for relatively spacelike arguments $x_1 \ldots x_4$. Its values for all other arguments can be obtained by analytic continuation through the extended tube [7]. Analyticity in the extended tube follows from Lorentz invariance and spectrum condition, it is therefore also true for the individual terms in the expansion (4). The analytic continuation can therefore be performed term by term.

Suppose then that the arguments $x_1 \dots x_4$ are all relatively spacelike, viz. $(x_i - x_j)^2 < 0$. Then by locality

$$W(x_1 \dots x_4) = W(x_{\pi 1} \dots x_{\pi 4}) \text{ for all 4! permutations } \pi$$

We can insert the expansion (4) for each of these amplitudes to obtain



$$= \text{c.c.} \qquad \text{if all } (x_i - x_j)^2 \;\; 0 \tag{5}$$

c.c. stands for complex conjugate; omitting an arrow in the bubble means that the order of the external legs does not matter [cp. after (3b)] .

Next we will briefly discuss the relation of thesis 1 to operator product expansions. Let us say right away that it is not strong enough to imply convergence of operator product expansions.

We note that one summation in (4) can be carried out with (3a) to give

$$W(x_1 \dots x_4) = \sum_j \int_{sptr} dp \; \bar{B}^j_\alpha(x_4 x_3; p) \tilde{W}^j_\alpha(x_1 x_2; p) \tag{4'}$$

In conformal invariant QFT, 2- and 3-point functions $\Delta^{ij}_{\alpha\beta}$, B^i_α are determined up to normalization by spin and dimensions of the fields O^i and φ . One finds (by inspection) that $B^i_\alpha(xy;p)$ is an entire function of p. It may therefore be expanded in a convergent power series in p. Using this, equation (4') may be rewritten as

$$(\Omega, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\Omega) = (2\pi)^4 \sum_i \bar{B}^i_\alpha(x_4 x_3; -i\nabla_z)(\Omega, \varphi(x_1)\varphi(x_2) O^i_\alpha(z)\Omega)_{z=0} \tag{6}$$

B^i_α should be read as a power series in $-i\nabla_z$. Restriction to 4-point amplitude can be lifted (s. below) so that one gets more generally a "vacuum expansion" of the form

$$\varphi(x)\varphi(y)\Omega = \sum_j (2\pi)^4 B^j_\alpha(yx;-i\nabla_z)O^j_\alpha(z)\Omega\Big|_{z=0} \tag{6'}$$

One may put y = 0 in this . In conclusion, thesis 1 together with analyticity of B^j_α in p imply convergence of the operator product expansion of $\varphi(x)\varphi(y)$ when applied to the vacuum.

Analyticity of B^j_α in p may not be true generally though. We have no argument to rule out the possibility that amputation (3a) removes some but not all thresholds from the 3-point functions. Vacuum expansions (6) need not converge then even if one assumes that they continue to hold as asymptotic expansions.

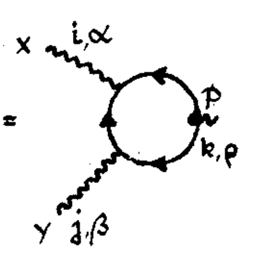
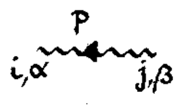
Let us now go on to the consideration of arbitrary n -point functions. Let $\{O^i_\alpha(z)\}$ a complete set of nonderivative hermitean local fields, including the unit operator $O^o_\wedge(x) = 1$. Put

$$(\Omega, O^i_\alpha(x)O^j_\beta(y)O^k_\rho(z)\Omega) = W^{ijk}_{\alpha\beta\rho}(xyz) = (2\pi)^{-4}\int dp\, e^{ipz}\, \tilde{W}^{ijk}_{\alpha\beta\rho}(xy;p) \tag{7a}$$

Define amputated 3-point functions **B** by

$$\tilde{W}^{ijk}_{\alpha\beta\rho}(xy;p) = \sum_\ell B^{ij\ell}_{\alpha\beta\sigma}(xy;p)\tilde{\Delta}^{\ell k}_{\sigma\rho}(p) \tag{7b}$$

We use a graphical notation as before

$B^{ijk}_{\alpha\beta\rho}(xy;p)$ =  ;  = $\tilde{\Delta}^{ij}_{\alpha\beta}(p)$ (7c)

Reversing all arrows will mean complex conjugation; omitting an arrow means that its direction is immaterial.

As a consequence of the Wightman axioms of local quantum field theory, the 2- and 3-point functions Δ^{ij}, W^{ijk} have the following <u>axiomatic properties:</u> i) Lorentz invariance .

ii) positivity and spectrum condition of 2-point matrix:

$\tilde{\Delta}^{ij}_{\alpha\beta}(p) = 0$ for $p \notin \bar{V}_+$, i.e. outside forward lightcone.

$\sum_{ij} \bar{z}^i_\alpha \Delta^{ij}_{\alpha\beta}(p) z^j_\beta \geq 0$ for all finite sequences of complex (AP)

numbers $\{z^i_\alpha\}$ and all p.

iii) The 3-point functions W^{ijk} resp. Fourier transforms $\tilde{W}^{ijk}$ satisfy:

hermiticity condition $\bar{W}^{ijk}_{\alpha\beta\rho}(xyz) = W^{kji}_{\rho\beta\alpha}(zyx)$

spectrum condition $W^{ijk}_{\alpha\beta\rho}(xy;p) = 0$ for $p \notin \bar{V}_+$.

locality $W^{i_1 i_2 i_3}_{\alpha_1\alpha_2\alpha_3}(x_1x_2x_3)$ is invariant under interchange of indices 1 and 2 if $(x_1-x_2)^2 < 0$, etc.

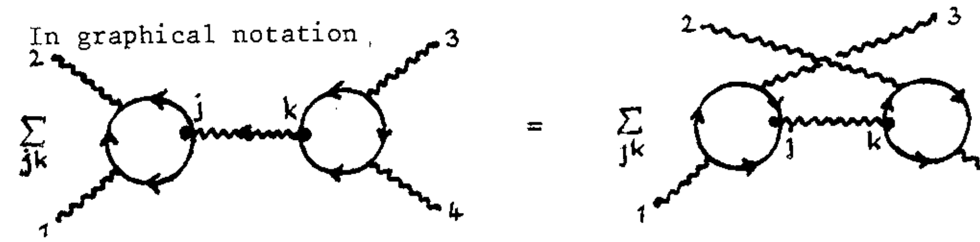
Locality of 2-point functions has not been listed as it is automatic consequence of i) and ii) . Also omitted are distribution theoretic axioms (temperedness), they should be added according to taste.(The 3-point functions $\tilde{W}(x_1x_2;p)$ should be measures in p after smearing in x_1, x_2.) Let us now assume that expansion (4) holds for arbitrary disconnected 4-point Wightman functions involving any four fields O^i in place of φ . Locality will impose crossing conditions as before. In particular

$$(\Omega, O^{i_1}_{\alpha_1}(x_1) O^{i_2}_{\alpha_2}(x_2) O^{i_3}_{\alpha_3}(x_3) O^{i_4}_{\alpha_4}(x_4)\Omega) = \sum_{jk} \int dp\, B^{i_1 i_2 j}_{\alpha_1\alpha_2\beta}(x_1x_2;p) \tilde{\Delta}^{jk}_{\beta\rho}(p) \bar{B}^{i_4 i_3 k}_{\alpha_4\alpha_3\rho}(x_4x_3;p)$$

$$= \sum_{jk} \int dp B^{i_1 i_3 j}_{\alpha_1\alpha_3\beta}(x_1x_3;p) \tilde{\Delta}^{jk}_{\beta\rho}(p) \bar{B}^{i_4 i_2 k}_{\alpha_4\alpha_2\rho}(x_4x_2;p) = (\Omega, O^{i_1}_{\alpha_1}(x_1) O^{i_3}_{\alpha_3}(x_3) O^{i_2}_{\alpha_2}(x_2) O^{i_4}_{\alpha_4}(x_4)\Omega)$$

$$\text{if } (x_2 - x_3)^2 < 0 \qquad (8)$$

In graphical notation



if $(x_2-x_3)^2 < 0$

We can now attempt to build up n - point functions out of $\tilde{\Delta}^{ij}$, B^{ijk} by generalizing the procedure for the 4-point function. For any m = 2... n-2

$$(\Omega, O^{i_1}_{\alpha_1}(x_1)\dots O^{i_n}_{\alpha_n}(x_n)\Omega) = \sum_{jk} dp \int_{Sptr} B^{i_1 \dots i_m j}_{\alpha_1 \dots \alpha_m \beta}(x_1 \dots x_m; p) \tilde{\Delta}^{jk}_{\beta\rho}(p) \bar{B}^{i_n \dots i_{m+1} k}_{\alpha_n \dots \alpha_{m+1} \rho}(x_n \dots x_{m+1}; p) \tag{9a}$$

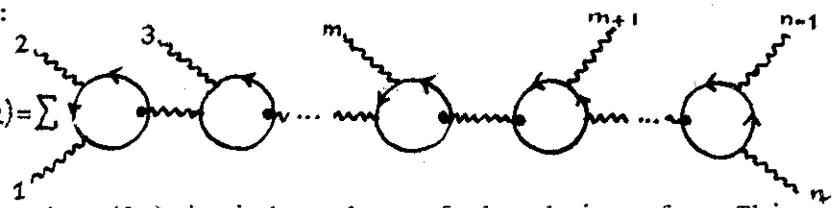
with

$$B^{j_1 i_2 \dots i_{m-1} j_m}_{\beta_1 \alpha_2 \dots \alpha_{m-1} \beta_m}(y_1 x_2 \dots x_m; p_m) = \sum_{j_2 \dots j_{m-1}} \int \dots \int \prod_{r=2}^{m-1} (dp_r dy_r e^{ip_r y_r}) \cdot \tag{9b}$$

$$\cdot B^{j_1 i_2 j_2}_{\beta_1 \alpha_2 \beta_2}(y_1 x_2; p_2) \dots B^{j_{m-1} i_m j_m}_{\beta_{m-1} \alpha_m \beta_m}(y_{m-1} x_m; p_m)$$

for $m \geq 2$

In graphical notation:

$$(\Omega, O^{i_1}_{\alpha_1}(x_1)\dots O^{i_n}_{\alpha_n}(x_n)\Omega) = \sum$$  (9c)

Let us note that expression (9a) is independent of the choice of m: This follows from hermiticity condition (AP iii) upon reshuffling the amputations.

We have from (7) the identities

$$\sum_k \int dy \; e^{ipy} B^{ijk}_{\alpha\beta\sigma}(yx; p') \tilde{\Delta}^{k\ell}_{\sigma\rho}(p') = \int dy' \; e^{-ip'y'} \tilde{\Delta}^{ik}_{\alpha\sigma}(p) \bar{B}^{\ell jk}_{\rho\beta\sigma}(y'x; p) \tag{10}$$

since by hermiticity of matrix $\tilde{\Delta}^{k\ell}_{\alpha\beta}(p)$,

$$\text{lhs.} = \int dy \, dy' \; e^{i(py - p'y')} W^{ij\ell}_{\alpha\beta\rho}(yxy') = \int dy \, dy' \; e^{i(py-p'y')} \bar{W}^{\ell ji}_{\rho\beta\alpha}(y'xy) = \text{rhs.}$$

Inserting (9b) into (9a) and using identity (10) for the factor involving $B(\dots; p_m)\tilde{\Delta}(p_m)$ we see that expression (9a) remains unchanged when m-1 is substituted for m.

It is useful to extend validity of expansion (9) to m = 0,1 , n-1, n

This can be done by supplementing definitions (9b) by

$$B^{ij}_{\alpha\beta}(x; p) = e^{-ipx} \delta_{ij} \delta_{\alpha\beta} \quad ; \quad B^{i}_{\wedge}(p) = \delta_{io} \tag{9d}$$

We have assumed here that all fields except $1 = O^o$ have zero vacuum expectation value. In the expansion (9) there are terms coming from the identity operator. The corresponding 2- and 3-point functions are

$$\tilde{\Delta}^{oj}(p) = \delta_{oj}\,\delta(p) \qquad (7d) \qquad B^{ijo}_{\alpha\beta\wedge}(xy;p) = \Delta^{ij}_{\alpha\beta}(x-y) \qquad (7e)$$

We are now ready to formulate

Thesis 2: Disconnected Wightman n-point functions admit convergent expansions (9). The 2-and 3-point functions appearing therein are defined by Eqs (7a-e).

Of course thesis 1 is merely a special case of this. It was stated separately for pedagogical reasons.

We note that Eqs (9) remain valid for $n < 4$. They reduce in this case to Eqs (7) when they are not altogether trivial. Eqs. (7), which were used to define 2- and amputated 3-point functions originally, are thus actually themselves part of the system of Eqs (9).

Assertion: Suppose that one can find a set of 2-point functions $\tilde{\Delta}^{ij}(p)$ and amputated 3-point functions $B^{ijk}_{\alpha\beta\rho}(xy;p)$ such that

i) axiomatic properties (AP) are true for the 2-and 3-point functions when W^{ijk} is defined by (7b) in terms of $\tilde{\Delta}^{ij}$, B^{ijk}.

ii) Expansions (9) converge.

iii) Crossing relation (8) is fulfilled.

Then the Wightman functions defined by (9) satisfy the usual postulates of local quantum field theory: Lorentz invariance, spectrum condition, positivity and locality.

If a theory can be constructed in this way we call it dual [because of (8)]. In this language, our thesis 2 postulates duality of local QFT.

Let us verify the assertion. Write

$$(\Omega, O^{i_1}_{\alpha_1}(x_1)\ldots O^{i_n}_{\alpha_n}(x_n)\Omega) = W(X_1\ldots X_n) \;;\quad X_k = (x_k, i_k, \alpha_k)$$

Lorentz-invariance and spectrum condition are clear. Positivity: Let $f_0, f_1(X_1)\ldots f_N(X_1\ X_N)$ a finite sequence of test functions. Define

$$z^j_\beta(p) = \sum_{m=0}^{N}\sum_{\{i\}}\int dx_1\ldots dx_m \bar{f}_m(x_1 i_1\alpha_1\ldots x_m i_m\alpha_m) B^{i_1\ldots i_m j}_{\alpha_1\ldots\alpha_m\beta}(x_1\ldots x_m;p) \qquad (11)$$

Then, using (9a),

$$\sum_{r,s}\sum\int dx_1\ldots dx_{r+s}\bar{f}_r(X_1\ldots X_r)W(X_1\ldots X_{r+s})f(X_{r+s}\ldots X_{r+1}) = \sum_{jk}\int dp\ \bar{z}^j_\alpha(p)\tilde{\Delta}^{jk}_{\alpha\beta}(p)z^k_\beta(p) \geq 0$$

by positivity (APii) of the 2-point matrix.

Locality: Suppose that $(x_m - x_{m+1})^2 < 0$ for some m. Consider first the case $2 \leq m \leq n-2$. Inserting crossing relation (8) for the center piece of the expansion (9c) we see that $W(X_1\ldots X_m X_{m+1}\ldots X_n) = W(X_1\ldots X_{m+1}X_m\ldots X_n)$ as required by locality. If instead $(x_1-x_2)^2 < 0$, the relation $W(X_1X_2\ldots) = W(X_2X_1\ldots)$ follows from locality of the 3-point function $W^{i_1 i_2 j}_{\alpha_1\alpha_2\beta}(x_1x_2y)$ and (9c),(7b). The case m = n - 1 is analogous. □

Let us conclude the paper with a few remarks.

1) It is interesting to compare our thesis with the assertion of the Reeh Schlieder theorem that the polynomial algebra of an open set in space time creates a dense subspace of the physical state space H out of the vacuum Ω [7]. Our thesis asserts that linear combinations of states $\phi^i(f)\Omega = \int dx\, f_\alpha(x)O^i_\alpha(x)\Omega$ create a dense subspace of H out of Ω.

If we picture composite local fields as some kind of normal products (defined eg. by operator product expansions) we could say that in comparison with Reeh Schlieder the range of integration over relative coordinates is shrunk to a point while integration over center of mass coordinate goes over all space time.

2) More than a decade ago, the bootstrap idea emerged in analytic S-matrix theory [11]. It says for instance that the ρ is bound together from π's by forces that come largely from the exchange of ρ's again (or of its Regge trajectory). Veneziano duality is a further developement of such ideas. In S-matrix language, crossed channel exchanges produce direct channel poles. Crossing relations (5), (8) may be looked upon as a QFT-pendant of this. Let us then try to make a connection as good as we can.

Consider the elastic scattering of two identical scalar particles which are their own antiparticles

$$1 + 2 \mapsto 3 + 4 \tag{12}$$

Consider the off mass shell scattering amplitude $A(p_1 \cdots p_4) = A(stu; p_1^2 \cdots p_4^2)$ $s = (p_1+p_2)^2$, $t = (p_1+p_3)^2$, $s+t+u = \Sigma p_i^2$. Its absorptive part in the channel (12) is [8]

$$\mathrm{Abs}_{12} A(p_1 \ldots p_4)(2\pi)^4 \delta(\Sigma p_i) = \frac{1}{16\pi^3} \int dx_1 \ldots dx_4 \exp(-i \Sigma p_j x_j) \cdot (\Omega, R'\{\varphi(x_3)\varphi(x_4)\} R'\{\varphi(x_1)\varphi(x_2)\}\Omega) \tag{13}$$

with $R'\{\varphi(x)\varphi(y)\} = -iK_x K_y \Theta(x-y)[\varphi(x),\varphi(y)]_-$; $K_x = \Box_x + m^2$,

i.e. R' is a retarded product. The <u>partially</u> retarded function appearing here can also be expanded as in (4). Define

$$C^j_\beta(xy;p) = -iK_x K_y \Theta(x-y)\{B^j_\beta(xy;p) - B^j_\beta(yx;p)\}$$

Then we can expand

$$(\Omega, R'\{\varphi(x_3)\varphi(x_4)\} R'\{\varphi(x_1)\varphi(x_2)\}\Omega) = \sum_{ij} \int_{Sptr} dp\, C^i_\alpha(x_3 x_4;p)\, \tilde{\Delta}^{ij}_{\alpha\beta}(p) \bar{C}^j_\beta(x_1 x_2;p) \tag{14}$$

This can be inserted in (13). Convergence is still in the distribution theoretic sense, i.e. after smearing with test functions. This means essentially that one must sum first before going to sharp masses on the mass shell $p_i^2 = m^2$. To extract information from crossing relations one must determine the absorptive part in the crossed channel $1 + 3 \to 2 + 4$ from $\mathrm{Abs}_{12} A(p_1 \ldots p_4)$. In the QFT context we did this by use of axiomatic analyticity in x-space, cp. discussion before (5). In analytic S-matrix theory one uses p-space analyticity instead. One would first have to recover the scattering amplitude from its absorptive part by dispersion relations - hopefully they converge without subtraction for some range of t. Then one would have to analytically continue in s, t to the crossed channel assuming e.g. Mandelstam analyticity. Finally one could then take the absorptive part in the crossed channel and write down its expansion analog to (14).

In conclusion, it it were not for a questionable interchange of limits (expanding and going to the mass shell) we would get a relation between the expansions of the on-shell absorptive parts of scattering amplitudes in crossed channels at the cost of having to use analyticity in p-space in place of x-space. If one is willing to make the further hypothesis (approximation) that the 2-point function $\Delta^{ij}_{\alpha\beta}(p)$ can be written as a sum of δ-functions supported at physical particle masses then the result would look much like a dual resonance model.

3) Let us add few remarks on Regge trajectories and fundamental fields. Consider temporarily QFT in an arbitrary not necessarily integer number D of space time dimensions. In models, local fields come in families. We call them towers. For instance, in conformal invariant ϕ^3-theory in $D = 6 + \varepsilon$ dimensions [9] the fundamental field ϕ has anomalous dimension $d = \frac{1}{2}D - 1 + \Delta$ with anomalous part $\Delta = \frac{1}{18}\varepsilon + \dots$. Then there is a tower of traceless symmetric tensor fields $O_{\alpha_1 \dots \alpha_s}$ of even rank $s = 2,4, \dots$ with dimension $d_s = D-2+s+\sigma_s$ whose anomalous part

$$\sigma_s = \left[\frac{1}{9} - \frac{1}{3(s+2)(s+1)}\right]\varepsilon + \dots \quad ; \sigma_s \to 2\Delta \quad \text{as } s \to \infty .$$

They are composite fields quadratic in ϕ. The dimensions of the component fields become addition in the limit $s \to \infty$, so we have asymptotically straight trajectories in dimension. One would think that there is also a scalar field - call it ϕ^2 - with dimension $d_0 = D-2+\sigma_0$. But such a field does actually not exist, it simply does not appear in operator product expansions of $\phi(x)\phi(0)$ or in expansions (4) .[We see here the ghost of the field equations, remember that we count only nonderivative fields, they transform differently]. Instead the "shadow" of the missing point on the trajectory appears as the fundamental field with dimension $d = D - d_0$. To order ε this relation between dimensions may be checked from the explicit formulae given above. Because of the normal product algorithm of renormalized perturbation theory one would expect that towers of fields exist whether or not there is conformal symmetry. Moreover, one will speculate that in the real world there is a connection between towers of fields and Regge trajectories - the fields of suitable rank would serve as interpolating fields for particles on

the Regge trajectories [10] .
We have seen above a model with a fundamental field. It is not a member of a tower but appears instead as a shadow of a missing member in the tower. Typically it has a dimension $d < \frac{1}{2}D$. In the present frame work one could however very well imagine theories without any fundamental fields.

If crossing relations (8) have any solutions such that expansions (9) converge, they will usually also have solutions which do not involve any fundamental field. This is not the whole issue though. Take for instance $\varphi=\Theta_{\mu\nu}$, the stress tensor. Everybody believes in that local field. Consider vacuum expectation values of stress tensors. Their expansions (9) will only involve fields that are singlets under all exact internal symmetries, and fundamental fields will usually not appear in them at all. Nevertheless we have at hand a perfectly respectable Wightman QFT and the crossing relation (8) will be satisfied. We call this the rudimentary theory. Its main short-coming is that it will usually not furnish interpolating fields for all particles. Nevertheless one should see these particles in stress-tensor correlation functions, because particles could in principle be detected in a laboratory by the gravitational forces which they exert.
The question of fundamental fields and quanta associated with them is thus a rather subtle one. Nevertheless it may be useful to remark that one could in principle start the bootstrap - i.e. try to solve crossing relations (8) - without knowing whether there are fundamental fields, or even what the internal symmetry group is, by considering the rudimentary theory first.
The rudimentary theory will fix an algebra of observables (measurements based on gravitational forces). According to ideas of Doplicher, Haag and Roberts in algebraic QFT this algebra will in turn fix the complete theory uniquely , including internal symmetries [13].

Acknowledgement: The author is much indebted to R. Haag, J. Jersak, M. Lüscher, B.Schroer and K. Symanzik for helpful and stimulating discussions.

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