

Collinear and Transverse Momentum Dependent parton densities obtained with a Parton Branching Method

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We present a solution of the DGLAP evolution equations, written in terms of Sudakov form factors to describe the branching and no-branching probabilities, using a parton branching Monte Carlo method. We demonstrate numerically that this method reproduces the semi-analytical solutions. We show how this method can be used to determine Transverse Momentum Dependent (TMD) parton distribution functions, in addition to the usual integrated parton distributions functions. We discuss numerical effects of the boundary of soft gluon resolution scale parameter on the resulting parton distribution functions. We show that a very good fit of the integrated TMDs to high precision HERA data can be obtained over a large range in x and Q^2 .

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1. Motivation

Appropriate use of parton distribution functions (PDFs) (see e.g. [1, 2, 3]) and a proper treatment of the transverse momentum kinematics in the parton showers (see e.g. [4, 5, 6]) are important for comparisons of theoretical predictions with experimental measurements at the Large Hadron Collider and future colliders experiments. One of the approaches to address these issues is the transverse momentum dependent (TMD) PDFs formalism (see e.g. [7]). The goal of our project is to determine TMD PDFs sets for all flavours, applicable over a broad kinematic region in x , μ^2 and k_T . We study in particular the effects of the soft gluon resolution scale in QCD radiation [8] on the obtained distributions. Previous results were reported in [9], [10].

2. Introduction to the method

We start the discussion from the DGLAP evolution equation for momentum weighted parton densities $xf_a(x, \mu^2) = \tilde{f}_a(x, \mu^2)$

$$\frac{d\tilde{f}_a(x, \mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz P_{ab}(z, \mu^2) \tilde{f}_b\left(\frac{x}{z}, \mu^2\right) \quad (2.1)$$

where a, b denote quarks (n_f flavours, $n_f \equiv n_f(\mu^2)$) or gluons, x is the longitudinal momentum fraction of the proton carried by a parton a , z is the splitting variable and μ is the evolution variable. The splitting functions P_{ab} have the following structure

$$P_{ab}(z, \mu^2) = D_{ab}(\mu^2) \delta(1-z) + K_{ab}(\mu^2) \frac{1}{(1-z)_+} + R_{ab}(z, \mu^2) \quad (2.2)$$

where $D_{ab}(\mu^2) = \delta_{ab}d_a(\mu^2)$, $K_{ab}(\mu^2) = \delta_{ab}k_a(\mu^2)$ and the function R_{ab} does not contain any power divergences (no pieces $\sim (1-z)^{-n}$ for $n = 1, 2, \dots$) when $z \rightarrow 1$ but it can contain logarithmic divergences $\ln(1-z)$ which can be however integrated and give finite result. The piece with a delta function and plus prescription corresponds to the virtual and non-resolvable emissions. As long as $P_{ab}(z, \mu^2)$ has this structure, the formalism presented here can be applied (LO, NLO, NNLO). Introducing the soft gluon resolution scale z_M [8], defining the *Sudakov form factor* as

$$\Delta_a(\mu^2) = \exp\left(-\int_{\ln\mu_0^2}^{\ln\mu^2} d(\ln\mu'^2) \sum_b \int_0^{z_M} dz z P_{ba}^R(z, \mu'^2)\right) \quad (2.3)$$

and using *momentum sum rule* ($\sum_c \int_0^1 dz z P_{ca}(z, \mu^2) = 0$), Eq.2.1 can be rewritten

$$\frac{d\tilde{f}_a(x, \mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} dz P_{ab}^R(z, \mu^2) \tilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \tilde{f}_a(x, \mu^2) \frac{1}{\Delta_a(\mu^2)} \frac{d\Delta_a(\mu^2)}{d\ln\mu^2} \quad (2.4)$$

where $P_{ab}^R(z, \mu^2) = R_{ab}(z, \mu^2) + K_{ab}(\mu^2) 1/(1-z)$ is the *real* part of the splitting function. Terms skipped by introducing z_M are of order $\mathcal{O}(1-z_M)$.

Eq.2.4 has an iterative solution which can be solved by a parton branching method. More details can be found in [8] and [11].

3. Collinear PDFs from parton branching method

In this section we show the results for collinear PDFs (or integrated TMDs - iTMDs) coming from the parton branching method. In the left hand side of the Fig.1 we show the results for the gluon density using NLO splitting functions. The evolution is performed with the parton branching method up to the evolution scale $\mu^2 = 10, 10^3, 10^5 \text{ GeV}^2$ for $z_M = 1 - 10^{-5}$. The obtained distributions are compared with the pdfs calculated from QCDNUM [12] using the same initial distributions. We illustrate a very good agreement between these two methods. In the right hand side of the Fig.1 we show the results for the down quark density with the NLO splitting functions for different values of the parameter $z_M = 1 - 10^{-3}, 1 - 10^{-5}, 1 - 10^{-8}$ at the evolution scale $\mu^2 = 10^5 \text{ GeV}^2$. We show that as long as z_M is close to 1, the results are independent of z_M .

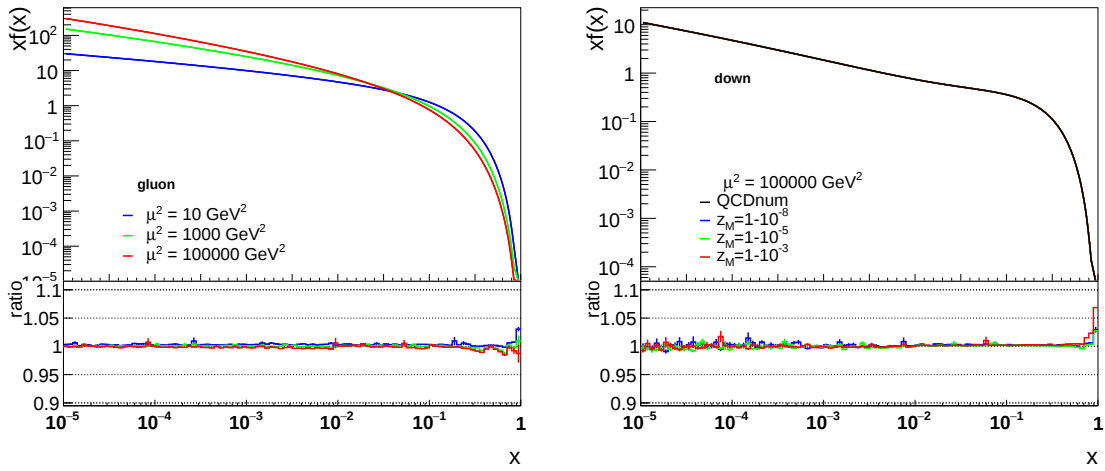


Figure 1: Upper part of the plots: pdfs from the parton branching method for the gluon density at the evolution scale $\mu^2 = 10, 10^3, 10^5 \text{ GeV}^2$ for $1 - z_M = 10^{-5}$ (left) and for the down quark density at $\mu^2 = 10^5 \text{ GeV}^2$ for $z_M = 1 - 10^{-3}, 1 - 10^{-5}, 1 - 10^{-8}$ (right). Lower part of the plots: ratios of the results from the parton branching method and QCDNUM.

4. TMD densities

With the iterative procedure, each resolvable branching is generated and the kinematics in each branching is calculated. Thanks to that, the parton branching method has the great advantage that the transverse momentum k_t of the propagating parton can be calculated and a TMD $A_a(x, k_t, \mu)$ can be determined. However, a prescription is needed to relate the evolution variable μ with the transverse momentum of the emitted ($q_{t,c}$) and propagating ($k_{t,a}$) parton (notation is explained in the Fig.2). It has been observed [8] that when q_t -ordering is used

$$q_{t,c}^2 = \mu^2, \quad (4.1)$$

the obtained TMDs depend on the z_M parameter. z_M plays an important role in the large z region-region where soft gluons are emitted. The z_M dependence is a signal that the soft gluons are not

treated properly. However, if angular ordering condition is used to associate μ and $q_{t,c}$

$$q_{t,c}^2 = (1 - z)^2 \mu^2, \quad (4.2)$$

the soft gluons emissions are treated properly - there is no dependence on z_M . This effect is shown in the Fig.3.

It is important to stress that the collinear pdf distributions are independent of z_M parameter regardless of the choice of prescription to associate μ and $q_{t,c}$.

In the Fig.4 we illustrate that the TMDs for all flavours can be obtained from this method.

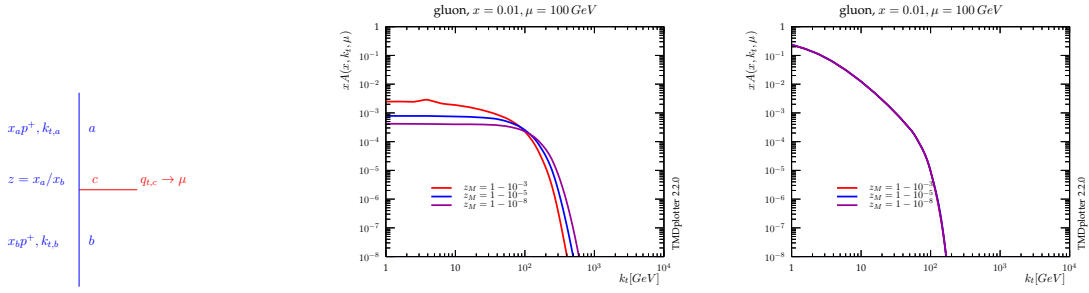


Figure 2: Illustration of a general splitting process: $b \rightarrow a + c$. **Figure 3:** TMDs from the parton branching methods for different values of the z_M parameter for q_t -ordering (left) and angular ordering (right).

5. Fit to precision DIS data

The initial parton density distributions have to be determined from a fit of the free parameters to describe inclusive cross sections. We performed the fit within xFitter package [13] with a method developed in [14, 15, 16, 17].

We used precision measurements in neutral and charged current interactions at various beam energies from HERA 1+2 [18] of $\sigma_{red} = d^2 \sigma^{ep} / dx dQ^2 \cdot Q^4 x / (2\pi \alpha^2 (1 + (1 - y)^2))$ (which is the DIS cross section where the photon flux is removed) in the range $3.5 < Q^2 < 30000 \text{ GeV}^2$, with $\alpha_s(m_Z) = 0.118$, at a starting scale $\mu_0^2 = 2 \text{ GeV}^2$, masses for heavy quarks as $m_c = 1.73 \text{ GeV}$, $m_b = 5.0 \text{ GeV}$ and $m_t = 175 \text{ GeV}$ together with a fixed $z_M = 1 - 10^{-5}$. The fit of integrated TMDs was performed at LO and NLO with experimental uncertainties, for all flavours. A very good χ^2/ndf was obtained for $3.5 < Q^2 < 30000 \text{ GeV}^2$. In the Fig.5 we show an example of the comparison of the σ_{red} , obtained from a fit using xFitter, compared to the precision measurements from HERA.

6. Summary

We presented a new approach to solve the DGLAP evolution equation with a parton branching method. We showed that it can reproduce exactly collinear pdfs. We explained how the TMD

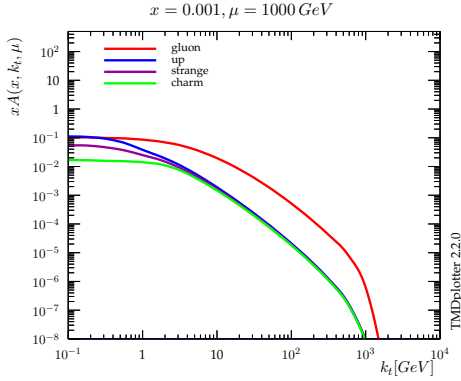


Figure 4: TMDs for different flavours from the parton branching method using angular ordering condition from Eq.(4.2).

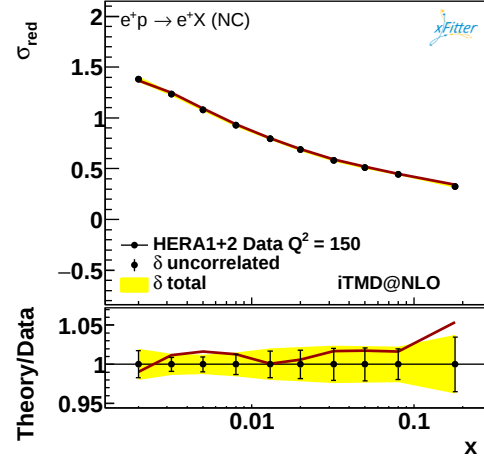


Figure 5: Example of a result of the fit of iTMDs from parton branching method to precision DIS data within xFitter.

distributions can be obtained in our method. We discussed how one could associate the evolution variable μ with transverse momentum of the emitted and propagating partons to treat properly soft gluon emissions. We showed that with q_t -ordering TMD PDFs are not consistently defined whereas a consistent set of TMD PDFs can be obtained from the parton branching with angular ordering. We determined TMDs with xFitter package from the fit of integrated TMDs to σ_{red} precision measurements from HERA 1+2 data at LO and NLO, with experimental uncertainties, for all flavours, in a wide kinematic range of x , Q^2 and k_t .

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