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Addendum to "Charm and Bottom Quark Masses: An Update"

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We update the experimental moments for the charm quark as computed in Ref. [1] and used in Refs. [2] and [3] for the determination of the charm-quark mass. The new value for the $\overline{\text{MS}}$ charm-quark mass reads $m_c(3\,\text{GeV}) = 0.993 \pm 0.008$ GeV.

In Ref. [2] the $\overline{\text{MS}}$ charm- and bottom-quark masses have been determined using relativistic sum rules which relate theoretically calculated moments of the photon vacuum polarization function to experimentally measured moments. The latter are determined from measurements of the R-ratio and properties of the narrow resonances. The moments of the vacuum polarization function can be computed in perturbative QCD. In this note we update the experimental input and re-evaluate the corresponding moments. New results for the charm-quark mass m_c are presented which are about 35% more precise than those of our previous determination.
For convenience we briefly present the formalism which is used in order to obtain m_c. The n-th theory moment is obtained from

$$\mathcal{M}_n^{\text{th}} = \frac{12\pi^2}{n!}\left(\frac{\text{d}}{\text{d}q^2}\right)^n \Pi_c(q^2)\bigg|_{q^2=0}\,, \tag{1}$$

where $\Pi_c(q^2)$ is the vector-current correlator with virtual charm-quark loops which can be cast into the form

$$\Pi_c(q^2) = Q_c^2\frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n z^n\,, \tag{2}$$

with $z = q^2/(4m_c^2)$. Here $m_c = m_c(\mu)$ is the $\overline{\text{MS}}$ heavy quark mass at the scale μ and $Q_c = 2/3$ is the electric charge of the charm quark in units of the elementary charge. The results which we use for the first four coefficients $\bar{C}_n$ are known up to four-loop accuracy analytically [4]. For applications and calculational techniques of the determination of the related massive tadpole

	J/Ψ	$\Psi(2S)$
M_Ψ(GeV) [8]	3.096900(6)	3.686097(25)
Γ_{ee}(keV) [6, 8]	5.57(8)	2.34(4)
$(\alpha/\alpha(M_\Psi))^2$	0.957785	0.95554

Table 1: Updated input values for the resonance parameters.

diagrams we refer to the review [5]. Equating the theory moments $\mathcal{M}_n^{\rm th}$ with the experimentally measured moments,

$$\mathcal{M}_n^{\rm exp} = \int \frac{{\rm d}s}{s^{n+1}} R_c(s)\,, \tag{3}$$

where $R_c = \sigma(e^+e^- \to c\bar{c})/\sigma(e^+e^- \to \mu^+\mu^-)$, leads to

$$m_c = \frac{1}{2}\left(\frac{9Q_c^2}{4}\frac{\bar{C}_n}{\mathcal{M}_n^{\rm exp}}\right)^{\frac{1}{2n}}\,, \tag{4}$$

which can be used in order to extract the charm-quark mass. The experimental moments $\mathcal{M}_n^{\rm exp}$ receive contributions from the narrow resonances, the charm-threshold region and the continuum region above a center of mass energy $\sqrt{s}$ of about 5 GeV. Even for small values of n the contributions from the J/Ψ and $\Psi(2S)$ resonances are dominant.

There is essential new input from measurements of the electronic decay width Γ_{ee} of the J/Ψ [6] and $\Psi(2S)$ [7] resonances from BES III which shall be used in the following. The latter value is incorporated into the latest PDG result [8], whereas $\Gamma_{ee}(J/\Psi)$ of Ref. [6] is not included. We thus combine the results from Refs. [6] and [8] and obtain the updated resonance input parameters as listed in Tab. 1. We also update the mass values for the resonances using the recent PDG values [8]. Note, however, that their improvement has no influence on the results for the moments. Furthermore we update the value of the strong coupling constant and use $\alpha_s(M_Z) = 0.1181 \pm 0.0011$ [8] (instead of $\alpha_s(M_Z) = 0.1189 \pm 0.002$ as in Ref. [2]).

For the moments we obtain

n	$\mathcal{M}_n^{\rm res}$	$\mathcal{M}_n^{\rm cc}$	$\mathcal{M}_n^{\rm cont}$	$\mathcal{M}_n^{\rm exp}$	$\mathcal{M}_n^{\rm np}$
	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$	$\times 10^{(n-1)}$
1	0.1191(14)	0.0318(15)	0.0645(10)	0.2154(23)	−0.0001(3)
2	0.1169(15)	0.0178(8)	0.0143(3)	0.1490(17)	−0.0002(5)
3	0.1165(15)	0.0101(5)	0.0042(1)	0.1308(16)	−0.0004(8)
4	0.1176(16)	0.0058(3)	0.0014(0)	0.1248(16)	−0.0006(12)

and the updated table for the charm-quark mass reads

n	m_c(3 GeV)	exp	α_s	μ	np$_{\rm LO}$	total	$m_c(m_c)$
1	0.993	0.007	0.004	0.002	0.001	0.008	1.279
2	0.982	0.004	0.007	0.005	0.001	0.010	1.269
3	0.982	0.003	0.008	0.006	0.001	0.010	1.269
4	1.003	0.002	0.005	0.028	0.001	0.029	1.288

where np$_{\rm LO}$ indicates that we use the leading order (LO) approximation for the gluon condensate contribution (see also the discussion in Ref. [2]).

One observes a noteworthy reduction of the uncertainty in the experimental moments. As compared to the results from Ref. [2] there is an increase in the charm-quark mass by 7 MeV for $n = 1$, by 6 MeV for $n = 2$, by 4 MeV for $n = 3$ and a decrease by 1 MeV for $n = 4$. For $n = 1$, which constitutes our final result, the uncertainty decreases from 13 MeV to 8 MeV. Within the uncertainty all results in the above table are consistent with each other and with the results obtained in Ref. [2]. Our final result for the $\overline{\rm MS}$ charm-quark mass reads $m_c(3\,{\rm GeV}) = 0.993 \pm 0.008$ GeV and $m_c(m_c) = 1.279 \pm 0.008$ GeV.

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