

N-jettiness Subtractions

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[for details see

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(arXiv:1505.04794)

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(arXiv:1612.00450)]



- 1 Subtractions
- 2 N-Jettiness
- 3 Subleading Power Corrections

Subtractions.

$\sigma(X)$

- $\sigma(X)$: generic N-jet cross section

- ▶ At LO_N : $\sigma^{\text{LO}}(X) = \int d\Phi_N B_N(\Phi_N) X(\Phi_N)$
- ▶ X : All defining Born-level measurements/cuts
- ▶ Φ_N : Born-level phase-space

Starting Point.

$$\sigma(X) = \int d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \underbrace{\int^{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}}_{\equiv \sigma(X, \mathcal{T}_{\text{cut}})} + \int_{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

- $\sigma(X)$: generic N-jet cross section

- ▶ At LO_N: $\sigma^{\text{LO}}(X) = \int d\Phi_N B_N(\Phi_N) X(\Phi_N)$
- ▶ X : All defining Born-level measurements/cuts
- ▶ Φ_N : Born-level phase-space

- \mathcal{T}_N : physical IR-safe N-jet resolution variable

$$\mathcal{T}_N(\Phi_N) = 0 \quad \mathcal{T}_N(\Phi_{\geq N+1}) > 0 \quad \mathcal{T}_N(\Phi_{\geq N+1} \rightarrow \Phi_N) \rightarrow 0$$

- $d\sigma(X)/d\mathcal{T}_N$: differential \mathcal{T}_N spectrum

- ▶ At LO_N: $\frac{d\sigma^{\text{LO}}(X)}{d\mathcal{T}_N} = \sigma^{\text{LO}}(X) \delta(\mathcal{T}_N)$
- ▶ $\mathcal{T}_N > 0$ defines an IR-safe physical N+1-jet cross section

Subtractions.

Add and subtract $\sigma^{\text{sub}}(\mathcal{T}_{\text{off}}) = \sigma^{\text{sub}}(\mathcal{T}_{\text{cut}}) + \int_{\mathcal{T}_{\text{cut}}}^{\mathcal{T}_{\text{off}}} d\mathcal{T}_N \frac{d\sigma^{\text{sub}}}{d\mathcal{T}_N}$

$$\begin{aligned} \sigma &= \sigma(\mathcal{T}_{\text{cut}}) + \int_{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N} \\ &= \sigma^{\text{sub}}(\mathcal{T}_{\text{off}}) + \int_{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \left[\frac{d\sigma}{d\mathcal{T}_N} - \frac{d\sigma^{\text{sub}}}{d\mathcal{T}_N} \theta(\mathcal{T} < \mathcal{T}_{\text{off}}) \right] + \sigma(\mathcal{T}_{\text{cut}}) - \sigma^{\text{sub}}(\mathcal{T}_{\text{cut}}) \end{aligned}$$

$\underbrace{\hspace{10em}}_{\text{NNLO}_N}$ $\underbrace{\hspace{15em}}_{\text{NLO}_{N+1}}$ $\underbrace{\hspace{10em}}_{\text{neglect}}$

- Subtractions $\sigma^{\text{sub}}(\mathcal{T}_{\text{cut}})$ and $d\sigma^{\text{sub}}/d\mathcal{T}_N$
 - ▶ Have to reproduce leading singular limit of $\sigma(\mathcal{T}_{\text{cut}})$ and $d\sigma/d\mathcal{T}_N$ such that we can neglect $\Delta\sigma(\mathcal{T}_{\text{cut}}) \equiv \sigma(\mathcal{T}_{\text{cut}}) - \sigma^{\text{sub}}(\mathcal{T}_{\text{cut}})$ for $\mathcal{T}_{\text{cut}} \rightarrow 0$

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$$\begin{aligned}\sigma &= \sigma(\mathcal{T}_{\text{cut}}) + \int_{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N} \\ &= \sigma^{\text{sub}}(\mathcal{T}_{\text{off}}) + \int_{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \left[\frac{d\sigma}{d\mathcal{T}_N} - \frac{d\sigma^{\text{sub}}}{d\mathcal{T}_N} \theta(\mathcal{T} < \mathcal{T}_{\text{off}}) \right] + \sigma(\mathcal{T}_{\text{cut}}) - \sigma^{\text{sub}}(\mathcal{T}_{\text{cut}}) \\ &= \underbrace{\sigma^{\text{sub}}(\mathcal{T}_{\text{cut}})}_{\text{NNLO}_N} + \underbrace{\int_{\mathcal{T}_{\text{cut}}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N}}_{\text{NLO}_{N+1}} + \underbrace{\sigma(\mathcal{T}_{\text{cut}}) - \sigma^{\text{sub}}(\mathcal{T}_{\text{cut}})}_{\text{neglect}}\end{aligned}$$

- Subtractions $\sigma^{\text{sub}}(\mathcal{T}_{\text{cut}})$ and $d\sigma^{\text{sub}}/d\mathcal{T}_N$
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- \mathcal{T}_{off} is a priori arbitrary and exactly cancels
 - ▶ Determines \mathcal{T}_N range over which subtraction acts *differentially* in \mathcal{T}_N
 - ▶ Setting $\mathcal{T}_{\text{off}} = \mathcal{T}_{\text{cut}}$ reduces it to a global subtraction (aka slicing)

Power Expansion.

Expand cross section in powers of $\tau_N \equiv \frac{\mathcal{T}_N}{Q}$ and $\tau_{\text{cut}} \equiv \frac{\mathcal{T}_{\text{cut}}}{Q}$
(where Q is a typical hard scale whose precise choice is irrelevant for now)

$$\frac{d\sigma}{d\tau_N} = \frac{d\sigma^{(0)}}{d\tau_N} + \frac{d\sigma^{(2)}}{d\tau_N} + \frac{d\sigma^{(4)}}{d\tau_N} + \dots$$
$$\sigma(\tau_{\text{cut}}) = \sigma^{(0)}(\tau_{\text{cut}}) + \sigma^{(2)}(\tau_{\text{cut}}) + \sigma^{(4)}(\tau_{\text{cut}}) + \dots$$

- Singular (leading-power) terms

$$\frac{d\sigma^{\text{sing}}}{d\tau_N} \equiv \frac{d\sigma^{(0)}}{d\tau_N} \sim \delta(\tau_N) + \left[\frac{\mathcal{O}(1)}{\tau_N} \right]_+$$
$$\sigma^{\text{sing}}(\tau_{\text{cut}}) \equiv \sigma^{(0)}(\tau_{\text{cut}}) \sim \mathcal{O}(1)$$

- ▶ Distributional structure encodes real-virtual cancellation of IR singularities

- Nonsingular (subleading-power) terms

$$\tau_N \frac{d\sigma^{(2k)}}{d\tau_N} \sim \mathcal{O}(\tau_N^k) \quad \sigma^{(2k)}(\tau_{\text{cut}}) \sim \mathcal{O}(\tau_{\text{cut}}^k)$$

Putting Everything Together.

$$\sigma = \underbrace{\sigma^{\text{sub}}(\tau_{\text{cut}})}_{\text{NNLO}_N} + \underbrace{\int_{\tau_{\text{cut}}} d\tau_N \frac{d\sigma}{d\tau_N}}_{\text{NLO}_{N+1}} + \underbrace{\Delta\sigma(\tau_{\text{cut}})}_{\text{neglect}}$$

Subtractions have to satisfy

$$\sigma^{\text{sub}}(\tau_{\text{cut}}) = \sigma^{\text{sing}}(\tau_{\text{cut}}) [1 + \mathcal{O}(\tau_{\text{cut}})]$$

such that neglecting $\Delta\sigma(\tau_{\text{cut}})$ only misses $\mathcal{O}(\tau_{\text{cut}})$ power-suppressed terms

$$\Delta\sigma(\tau_{\text{cut}}) = \sigma(\tau_{\text{cut}}) - \sigma^{\text{sub}}(\tau_{\text{cut}}) = \sigma^{(2)}(\tau_{\text{cut}}) + \dots \sim \mathcal{O}(\tau_{\text{cut}})$$

Tradeoff: Lowering τ_{cut} ...

- ... reduces size of missing power corrections
- ... increases numerical cancellations between first two terms
 - ▶ Requires numerically more precise calculation of $d\sigma/d\tau_N$ in a region where the N+1-jet NLO calculation quickly becomes much less stable
 - ▶ Computational cost increases substantially

The Upshot (or an early summary).

All IR-singular contributions are projected onto the physical observable \mathcal{T}_N

Potential drawback

- Subtractions are nonlocal (i.e. not point-by-point in real emission phase space)
 - ▶ Phase-space slicing in $\mathcal{T}_N =$ global (maximally nonlocal) subtraction
- In practice, it is a question of numerical stability whether this is a disadvantage or not
 - ▶ Naively expect larger numerical cancellations (since they happen “later”)
 - ▶ Most relevant is numerical stability of real-virtual and double-real matrix elements in deep unresolved limit which are always needed regardless of subtraction method

Key advantages

- Subtractions are given by singular limit of a physical cross section
 - ▶ For the “right” observable can be systematically computed using a factorization theorem
 - ▶ Also allows computing power corrections, giving significant improvements
 - ▶ Much simpler structure and fewer subtraction terms
- All nonsingular contributions are immediately given in terms of existing lower-order Born+1-jet calculations

Resolution Variables for Physical Subtractions.

In principle, any IR-sensitive resumable variable could be used

In fact, in the context of resummation, the singular terms are routinely obtained as a “by-product” of the resummation and used as subtraction to get the nonsingular terms.

Other variables used as subtractions for NNLO calculations

- Color-singlet production: q_T subtractions utilize q_T of color-singlet system [Catani, Grazzini '07]
 - ▶ Very successfully applied to Higgs, Drell-Yan, and essentially any combination of diboson production
[Catani et al. '07, '09, '11; Ferrera, Grazzini, Tramontano '11, '14; Cascioli et al. '14; Gehrmann et al. '14; Grazzini, Kallweit, Rathlev, Torre '13, '15; several more implementations]
 - ▶ Primarily used as global subtraction (as far as I know)
- Top-quark decay rate: inclusive jet mass (global) [Gao, Li, Zhu '12]
- $e^+e^- \rightarrow t\bar{t}$: Total radiation energy (global) [Gao, Zhu '14]

Resolution Variables for Physical Subtractions.

N-jettiness event shape is explicitly designed as N-jet resolution variable with simplest possible factorization/resummation properties [Stewart, FT, Waalewijn '10]

- Differential 0-jettiness subtractions are implemented in GENEVA Monte Carlo (basis of its NNLO+NNLL'+PS matching) [Alioli et al. '13, '15]
- Global 0-jettiness (aka beam thrust)
 - ▶ Drell-Yan and Higgs [Gaunt, Stahlhofen, FT, Walsh '15]
 - ▶ VH , diphoton [Campbell, Ellis, Li, Williams '16]
 - ▶ NNLO color-singlet in MCFM 8 [Boughezal et al. '16]
- Global 1-jettiness
 - ▶ $pp \rightarrow V/H + j$ [Boughezal, Focke, Liu, Petriello + Campbell, Ellis, Giele '15, '16]
 - ▶ $pp \rightarrow \gamma + j$ [Campbell, Ellis, Williams '16]

N-Jettiness.

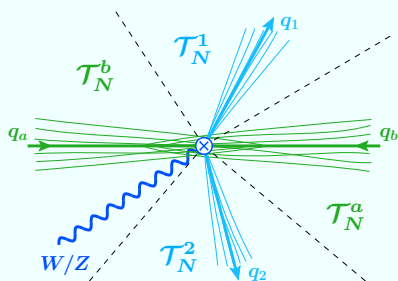
N-Jettiness Event Shape.

[Stewart, FT, Waalewijn, '10]

$$\mathcal{T}_N = \sum_k \min \left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \frac{2q_2 \cdot p_k}{Q_2}, \dots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$
$$\equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \dots + \mathcal{T}_N^N$$

- Partitions phase space into N jet regions and 2 beam regions
- $Q_{a,b}, Q_j$ determine distance measure
 - ▶ Geometric measures: $Q_i = 2\rho_i E_i$
- Born reference momenta q_i

$$q_{a,b} = x_{a,b} \frac{E_{\text{cm}}}{2} (1, \pm \hat{z})$$
$$q_j = E_j (1, \vec{n}_j)$$



Specifying them corresponds to choosing an (IR-safe) Born projection

- ▶ Specific choice is part of N-jettiness definition but only affects the power-suppressed terms and is therefore not needed for singular terms

All-order Singular Structure.

$$\frac{d\sigma^{\text{sing}}(X)}{d\tau_N} = \int d\Phi_N \frac{d\sigma^{\text{sing}}(\Phi_N)}{d\tau_N} X(\Phi_N)$$

$$\begin{aligned} \frac{d\sigma^{\text{sing}}(\Phi_N)}{d\tau_N} &= \mathcal{C}_{-1}(\Phi_N) \delta(\tau_N) + \sum_{m \geq 0} \mathcal{C}_m(\Phi_N) \mathcal{L}_m(\tau_N) \\ &= \sum_{n \geq 0} \left[\mathcal{C}_{-1}^{(n)}(\Phi_N) \delta(\tau_N) + \sum_{m=0}^{2n-1} \mathcal{C}_m^{(n)}(\Phi_N) \mathcal{L}_m(\tau_N) \right] \left(\frac{\alpha_s}{4\pi} \right)^n \end{aligned}$$

- Singular only depend on Born phase space $\Phi_N \equiv \{q_i, \lambda_i, \kappa_i\}$
 - ▶ Subtractions are FKS-like in this respect
- Plus distributions encode analytic cancellation of real and virtual IR divergences

$$\mathcal{L}_m(\tau_N) = \left[\frac{\theta(\tau_N) \ln^m(\tau_N)}{\tau_N} \right]_+ \quad \int^{\tau^{\text{cut}}} d\tau_N \mathcal{L}_m(\tau_N) = \frac{\ln^{m+1}(\tau^{\text{cut}})}{m+1}$$

All-order Singular Structure.

$$\frac{d\sigma^{\text{sing}}(X)}{d\tau_N} = \int d\Phi_N \frac{d\sigma^{\text{sing}}(\Phi_N)}{d\tau_N} X(\Phi_N)$$

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- Integrated subtractions

$$\sigma^{\text{sing}}(\Phi_N, \tau_{\text{cut}}) = \mathcal{C}_{-1}(\Phi_N) + \sum_{m \geq 0} \mathcal{C}_m(\Phi_N) \frac{\ln^{m+1}(\tau_{\text{cut}})}{m+1}$$

- $\mathcal{C}_{-1}(\Phi_N)$ contains finite remainder of N-parton virtuals

- ▶ At LO: $\mathcal{C}_{-1}^{(0)}(\Phi_N) = B_N(\Phi_N)$
- ▶ Most nontrivial piece, corresponds to virtual plus integrated subtraction in other subtraction schemes

Factorization Theorem.

[Stewart, FT, Waalewijn, '09, '10]

$$\frac{d\sigma^{\text{sing}}(\Phi_N)}{d\mathcal{T}_N} = \int dt_a B_a(t_a, x_a, \mu) \int dt_b B_b(t_b, x_b, \mu) \left[\prod_{i=1}^N \int ds_i J_i(s_i, \mu) \right] \\ \times \vec{C}^\dagger(\Phi_N, \mu) \hat{S}_\kappa \left(\mathcal{T}_N - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \sum_{i=1}^N \frac{s_i}{Q_i}, \{\hat{q}_i\}, \mu \right) \vec{C}(\Phi_N, \mu)$$

- All functions are IR finite and have an operator definition in SCET
- Simplifying features of N-jettiness
 - ▶ No dependence on jet algorithm (jet clustering, jet radius, etc.)
 - ▶ No recoil effects from soft radiation
 - ▶ No additional \vec{p}_T dependence or convolutions, no rapidity divergences
- To obtain subtraction coefficients simply expand and collect terms, e.g.,

$$\mathcal{C}_{-1}^{(2)} = f_a f_b [\vec{C}^{\dagger(0)} \vec{C}^{(2)} + \vec{C}^{\dagger(2)} \vec{C}^{(0)}] \\ + \vec{C}^{\dagger(0)} [B_a^{(2)} f_b + f_a B_b^{(2)} + f_a f_b \hat{S}^{(2)}] \vec{C}^{(0)} \\ + \text{1-loop cross terms}$$

Hard Matching Coefficients.

Encode the process-dependent N-parton virtual QCD corrections

Arise from matching QCD onto SCET

- In pure dimensionless regularization with $\overline{\text{MS}}$ given in terms of IR-finite ($\overline{\text{MS}}$ -subtracted) N-parton QCD amplitudes
- General formalism using SCET helicity operator basis

[Moult, Stewart, FT, Waalewijn '15]

- ▶ Using same color basis $\bar{T}^{a_1 \dots a_n}$ as in QCD calculation, directly given by corresponding color-ordered helicity amplitudes

$$\bar{T}^{a_1 \dots a_n} i\vec{C}_{\pm \dots \pm} = \mathcal{A}_{\text{fin}}(g_1^\pm \dots q_n^\pm) \equiv \frac{\bar{T}^{a_1 \dots a_n} \hat{Z}_C^{-1} \vec{\mathcal{A}}_{\text{ren}}(g_1^\pm \dots q_n^\pm)}{Z_\xi^{n_q/2} Z_A^{n_g/2}}$$

- ▶ \hat{Z} , Z_ξ , Z_A are SCET $\overline{\text{MS}}$ renormalization constants (in pure dimensional regularization equivalent to QCD $1/\epsilon_{\text{IR}}$ divergences)
- ▶ QCD helicity amplitudes should be UV-renormalized in CDR or HV

Beam and Jet Functions.

Encode cancellation of IR singularities between collinear real and virtual radiation and corresponding IR-finite remainder

- Inclusive virtuality-dependent (SCET-I) beam and jet functions
 - ▶ Universal for any N, only depend on parton type (quark vs. gluon)
 - ▶ Important: Overlap with soft contributions (known as zero bins in SCET) is scale-less and vanishes in pure dimensionless regularization

Jet functions

- (Straight)forward IR-finite vacuum matrix element of collinear quark or gluon operator

[NLO: Bauer, Manohar '03, Fleming, Leibovich, Mehen '03, Becher, Schwartz '06;

NNLO: Becher, Neubert '06, Becher, Bell '10]

Beam functions

- Require matching onto PDFs in terms of IR-finite matching coefficients

[NLO: Stewart, FT, Waalewijn '09, '10; NNLO: Gaunt, Stahlhofen, FT '14]

$$B_i(t, x) = \sum_j \int \frac{dz}{z} \mathcal{I}_{ij}(t, z) f_j\left(\frac{x}{z}\right)$$

- ▶ NNLO beam functions are key ingredient for color-singlet production

Encodes cancellation of IR singularities between soft real and virtual radiation and corresponding IR-finite remainder

- Matrix element of $N+2$ lightlike soft Wilson lines along collinear directions
 - ▶ Matrix acting on external color space, accounts for all color correlations in soft IR divergences
- Explicitly depends on N -jettiness measurement and partitioning
 - ▶ with respect to fixed collinear directions (no soft recoil effects)
- NLO: Known for any number of Wilson lines (and any Q_i) using on hemisphere decomposition [Jouttenus, Stewart, FT, Waalewijn '11]
- NNLO
 - ▶ 2 partons: Hemisphere soft function [Kelley, Schwartz, Schabinger, Zhu '11; Monni, Gehrmann, Luisoni '11; Hornig, Lee, Stewart, Walsh, Zuberi '11; Kang, Labun, Lee '15]
 - ▶ 3 partons: Numerically for $pp \rightarrow L + 1j$ [Boughezal, Liu, Petriello '15] recently for massive 3rd parton [Li, Wang '16]
 - ▶ Not yet known for general N

Subleading Power Corrections.

[Moult, Rothen, Stewart, FT, Zhu; arXiv:1612.00450 + more work in progress]

Missing Power Corrections.

There is one more important caveat

- Power suppression gets weaker at higher orders in α_s due to stronger log enhancement

$$\sigma^{(2)}(\tau_{\text{cut}}) = \sum_{n=0} \sigma^{(2,n)}(\tau_{\text{cut}}) \left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\sigma^{(2,n)}(\tau_{\text{cut}}) = \tau_{\text{cut}} \sum_{m=0}^{2n-1} A_m^{(2,n)} \ln^m \tau_{\text{cut}}$$

⇒ Dominant missing $\mathcal{O}(\alpha_s^n)$ terms actually scale as

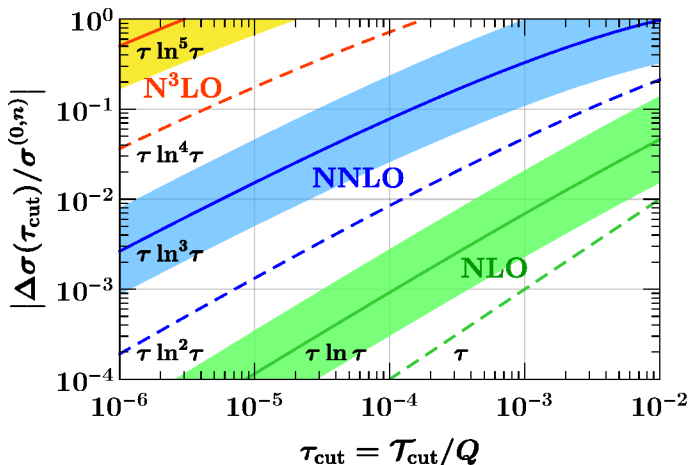
$$\Delta\sigma(\tau_{\text{cut}}) \sim \alpha_s^n \tau_{\text{cut}} \ln^{2n-1} \tau_{\text{cut}}$$

- ▶ Can use this to get a rough order of magnitude estimate of their size by taking $A^{(2,n)} = \sigma^{(0,n)} \times [1/3, 3]$
- ▶ Works quite well for the cases we have checked

Estimating Size of Missing Power Corrections.

Simple estimate of $\Delta\sigma(\tau_{\text{cut}})$ at $N^n\text{LO}$

- relative to full $N^n\text{LO}$ coefficient

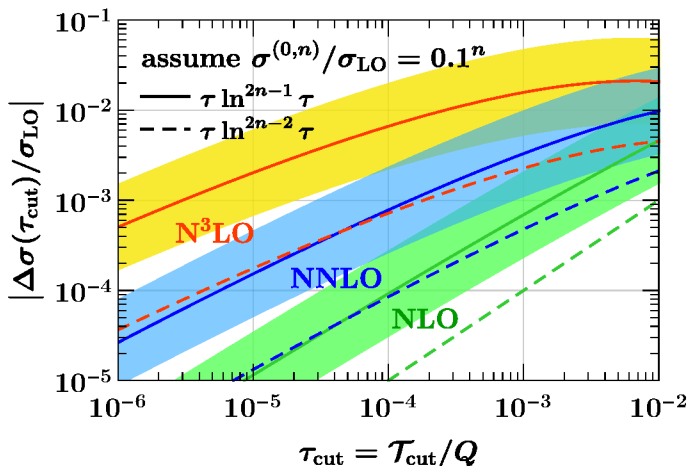


Typical values in current implementations are in $\tau_{\text{cut}} \simeq 10^{-4} \dots 10^{-3}$ range

Estimating Size of Missing Power Corrections.

Simple estimate of $\Delta\sigma(\tau_{\text{cut}})$ at $N^n\text{LO}$

- relative to σ_{LO} , assuming a 10% correction at each α_s order



Typical values in current implementations are in $\tau_{\text{cut}} \simeq 10^{-4} \dots 10^{-3}$ range

Improving the Subtractions.

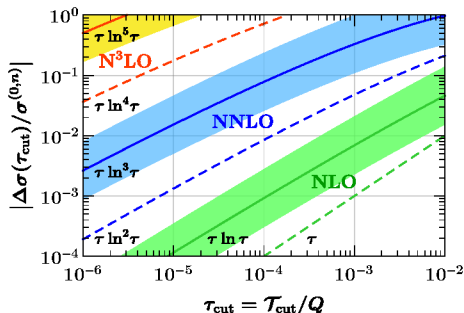
Recall

$$\sigma^{\text{sub}}(\tau_{\text{cut}}) = \sigma^{\text{sing}}(\tau_{\text{cut}}) [1 + \mathcal{O}(\tau_{\text{cut}})]$$

$$\Delta\sigma(\tau_{\text{cut}}) = \sigma(\tau_{\text{cut}}) - \sigma^{\text{sub}}(\tau_{\text{cut}})$$

Calculating dominant power corrections
they can be included in $\sigma^{\text{sub}}(\tau_{\text{cut}})$
to reduce the size of missing $\Delta\sigma(\tau_{\text{cut}})$ terms

- Each factor of log can potentially give an order of magnitude numerical improvement
 - ▶ Even the LL next-to-leading power (NLP) terms are very interesting
- Many things that could be ignored at leading power start to matter at subleading power.
 - ▶ Choice of N-jettiness definition can strongly impact size of power corrections



SCET at Subleading Power.

SCET is explicitly constructed to maintain manifest power counting at all stages of a calculation

Provides natural organization of different sources of power corrections

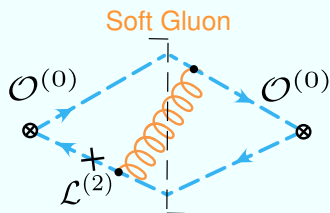
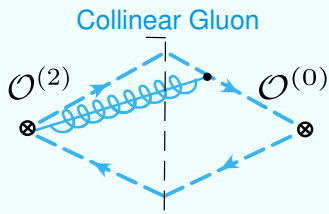
- Insertions of subleading SCET Lagrangian
 - ▶ Corrects dynamics of propagating soft and collinear particles
- Subleading hard-scattering operators
 - ▶ SCET helicity operator basis extended to subleading power
[Feige, Kolodrubetz, Moulton, Stewart; Moulton, Vita, Stewart '17]
- Subleading corrections to the measurement/observable

Since we don't care about resummation, we don't actually need a full factorization theorem at subleading power

Instead, we can perform the calculation at fixed order with SCET as organizational principle, focusing on the highest logarithmic terms

Alternative approach: Analytically expand NLO_{N+1} calculation "brute-force"
[Boughezal, Liu, Petriello '16]

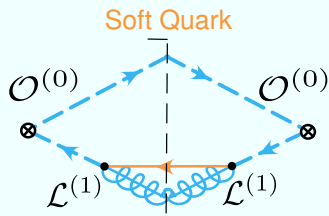
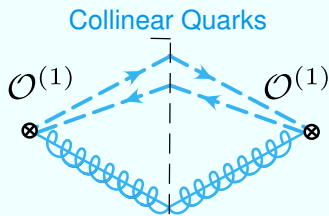
Simplest Example: Subleading Thrust at NLO.



$$\frac{1}{\sigma_0} \frac{d\sigma^{(2,1)}}{d\tau} = 8C_F \left[\left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2\tau} \right) - \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2\tau^2} \right) \right] = 8C_F \ln \tau ,$$

- Result gives directly (no additional expansions) the NLP contribution
- Total NLP result reproduces known thrust result
- $1/\epsilon$ poles must cancel between collinear and soft contributions
 - ▶ In SCET these are UV poles arising from the scale separation between different sectors
 - ▶ From full-theory point of view these are IR poles and must cancel because there are no nontrivial IR divergences at subleading power

Simplest Example: Subleading Thrust at NLO.



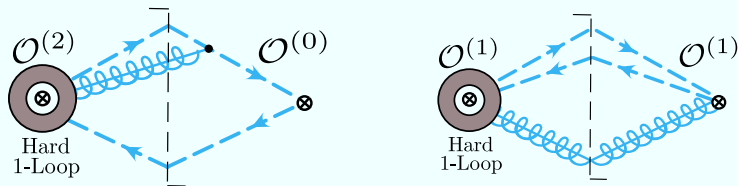
$$\frac{1}{\sigma_0} \frac{d\sigma^{(2,1)}}{d\tau} = 4C_F \left[-\left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2 \tau} \right) + \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2 \tau^2} \right) \right] = -4C_F \ln \tau$$

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Subleading Thrust at NNLO.

Same cancellation of $1/\epsilon$ poles must happen at NNLO

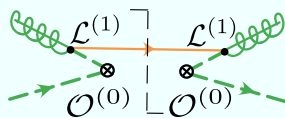
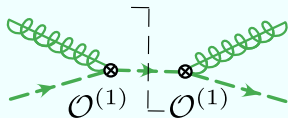
- Yields nontrivial constraints (consistency relations) on the different contributions from hard, collinear, and soft sectors
 - ▶ Significantly reduces number of NNLO coefficients that must be calculated
 - ▶ Equivalently provides for powerful cross checks
- The LL NNLO result is determined by a single coefficient
 - ▶ hard-collinear (easiest) or collinear-soft or soft-softor



$$\frac{1}{\sigma_0} \frac{d\sigma^{(2,2)}}{d\tau} = \left[-32C_F^2 + 8C_F(C_F + C_A) \right] \ln^3 \tau + \dots$$

- ▶ New color structure compared to leading power from quark channel

0-Jettiness for Drell-Yan at NLP.



Crossing the thrust calculation and taking into account differences in measurement, phase space, and PDFs

Results for coefficients of the partonic cross section

with $\delta_a \equiv \delta(\xi_a - x_a)$ and $\delta'_a \equiv x_a \delta'(\xi_a - x_a)$ and $\tau \equiv T_0/Q$

- NLO

$$C_{q\bar{q}}^{(2,1)}(\xi_a, \xi_b) = 8C_F \left(\delta_a \delta_b + \frac{\delta'_a \delta_b}{2} + \frac{\delta_a \delta'_b}{2} \right) \ln \tau + \dots$$

$$C_{qg}^{(2,1)}(\xi_a, \xi_b) = -2T_F \delta_a \delta_b \ln \tau + \dots$$

- NNLO

$$C_{q\bar{q}}^{(2,2)}(\xi_a, \xi_b) = -32C_F^2 \left(\delta_a \delta_b + \frac{\delta'_a \delta_b}{2} + \frac{\delta_a \delta'_b}{2} \right) \ln^3 \tau + \dots$$

$$C_{qg}^{(2,2)}(\xi_a, \xi_b) = 4T_F(C_F + C_A) \delta_a \delta_b \ln^3 \tau + \dots$$

► qg channel already contributes at leading-log, in contrast to leading power.

Numerical Results for Drell-Yan.

We can obtain the full nonsingular numerically

$$\frac{1}{\sigma_{\text{LO}}} \frac{d\sigma^{\text{nons}}}{d \ln \mathcal{T}_0} = \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma}{d \ln \mathcal{T}_0} - \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma^{\text{sing}}}{d \ln \mathcal{T}_0}$$

- Use $Z + j$ NLO calculation from MCFM 8 for $d\sigma/d \ln \mathcal{T}_0$
- Perform a χ^2 fit to (with $\tau \equiv \mathcal{T}_0/m_Z$)

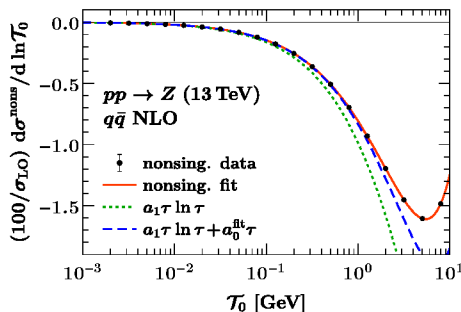
$$F_{\text{NLO}}(\tau) = \frac{d}{d \ln \tau} \left\{ \tau \left[(a_1 + b_1 \tau + c_1 \tau^2) \ln \tau + a_0 + b_0 \tau + c_0 \tau^2 \right] \right\}$$

$$F_{\text{NNLO}}(\tau) = \frac{d}{d \ln \tau} \left\{ \tau \left[(a_3 + b_3 \tau) \ln^3 \tau + (a_2 + b_2 \tau) \ln^2 \tau + a_1 \ln \tau + a_0 \right] \right\}$$

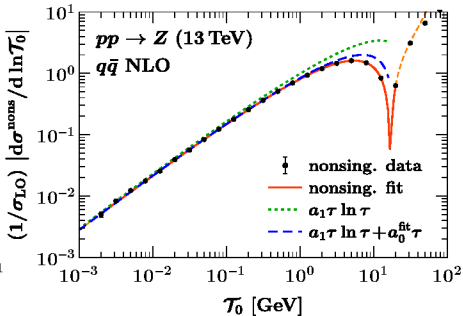
- ▶ Requires high MC statistics to get precise enough nonsingular data to be able to distinguish different terms of similar shape
- ▶ Important to include b_i, c_i coefficients in the fit to avoid biasing the fit result for the NLP a_i coefficients we are interested in
- ▶ Important to carefully select fit range in \mathcal{T}_0 and validate fit stability

Numerical Results at NLO.

linear scale



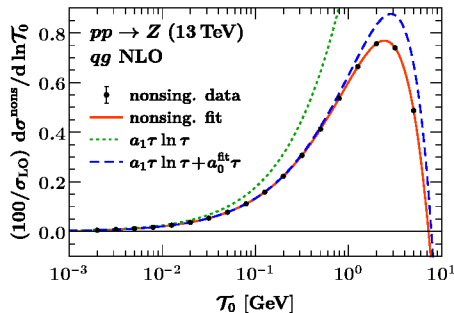
log scale



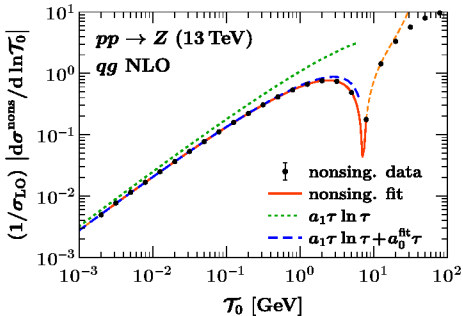
channel and coefficient		fitted	calculated
$q\bar{q}$ NLO	a_1	$+0.25366 \pm 0.00131$	$+0.25509$
qg NLO	a_1	-0.27697 ± 0.00113	-0.27720
$q\bar{q}$ NLO	a_0	$+0.13738 \pm 0.00057$	
qg NLO	a_0	-0.40062 ± 0.00052	

Numerical Results at NLO.

linear scale



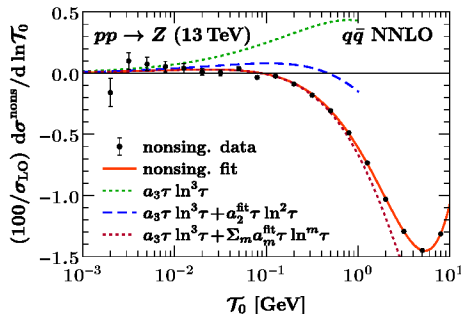
log scale



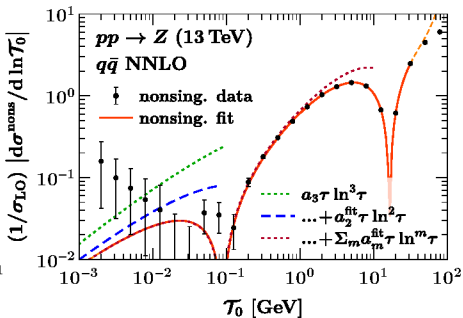
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Numerical Results at NNLO

linear scale



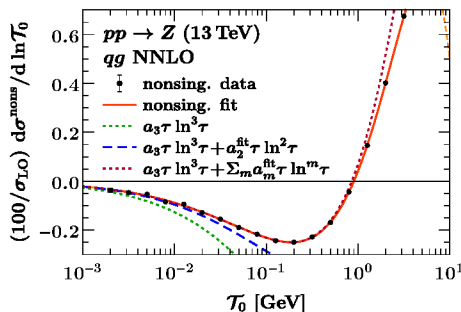
log scale



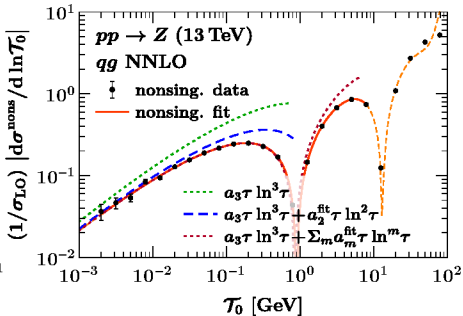
channel and coefficient		fitted	calculated
$q\bar{q}$ NNLO	a_3	-0.01112 ± 0.00150	-0.01277
qg NNLO	a_3	$+0.02373 \pm 0.00247$	$+0.02256$
$q\bar{q}$ NNLO	a_2	-0.04662 ± 0.00180	
qg NNLO	a_2	$+0.04234 \pm 0.00242$	

Numerical Results at NNLO.

linear scale



log scale



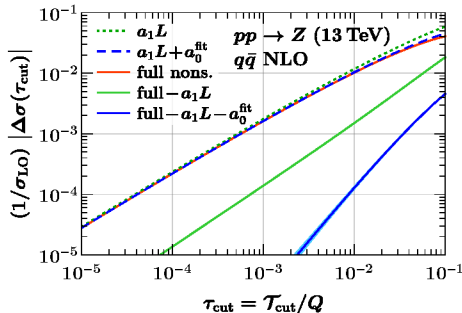
channel and coefficient		fitted	calculated
$q\bar{q}$ NNLO	a_3	-0.01112 ± 0.00150	-0.01277
qg NNLO	a_3	$+0.02373 \pm 0.00247$	$+0.02256$
$q\bar{q}$ NNLO	a_2	-0.04662 ± 0.00180	
qg NNLO	a_2	$+0.04234 \pm 0.00242$	

Results for $\Delta\sigma$.

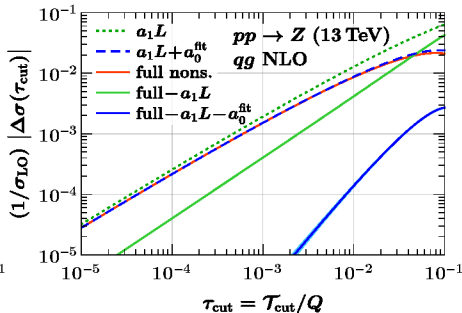
Once we have a precise fit of the full nonsingular spectrum, can integrate it to get the true power corrections

$$\Delta\sigma(\tau_{\text{cut}}) = \int^{\tau_{\text{cut}}} d\tau \frac{d\sigma^{\text{nons}}}{d\tau}$$

$q\bar{q}$ channel



qg channel



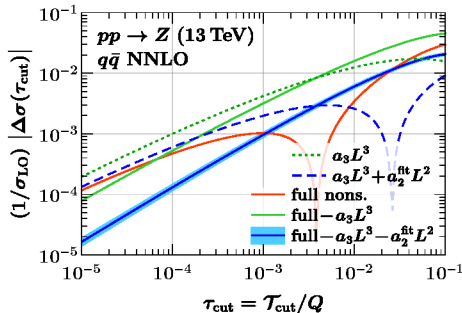
- Agrees well with naive scaling estimate (apart from where it goes through 0)

Results for $\Delta\sigma$.

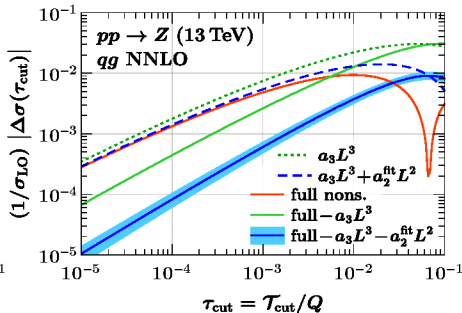
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qg channel

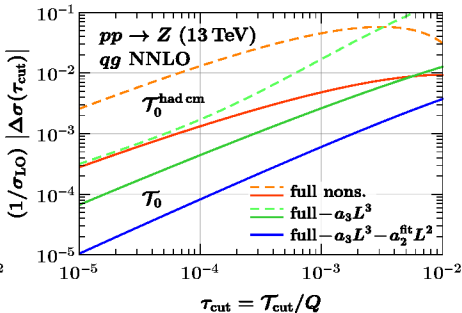
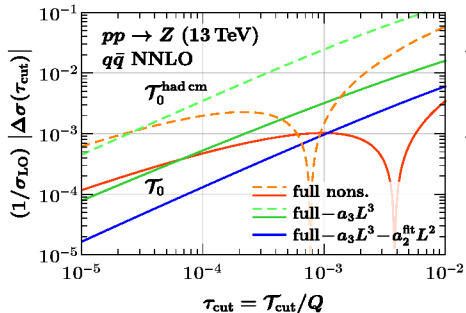


- Agrees well with naive scaling estimate (apart from where it goes through 0)

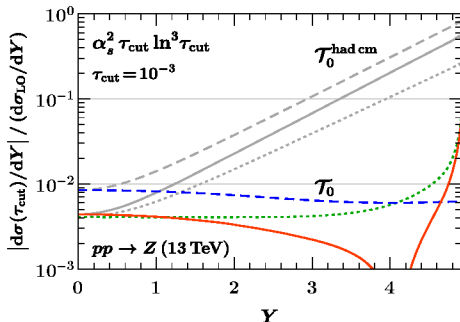
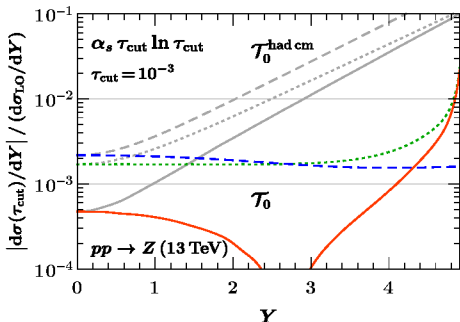
Dependence on 0-Jettiness Definitions.

General definition: $\mathcal{T}_0^x = \sum_k \min\{\lambda_x p_k^+, \lambda_x^{-1} p_k^-\}$ [see Stewart, Waalewijn, FT '09]

- leptonic: $\lambda = \sqrt{q^-/q^+} = e^Y$
 - ▶ Accounts for boost between leptonic (Born) and hadronic cm frame
 - ▶ Default original definition (used e.g. in GENEVA)
 - ▶ Most natural definition \rightarrow power expansion in \mathcal{T}_0/Q
- hadronic: $\lambda_{\text{had cm}} = 1$ (currently used in MCFM8)
 - ▶ Defines $\mathcal{T}_0^{\text{had cm}}$ in hadronic cm frame
 - ▶ Power expansion effectively in $\mathcal{T}_0^{\text{had cm}}/(Qe^{\pm Y})$ deteriorates for large Y



Rapidity Dependence of Power Corrections.



- Exponential enhancement of power corrections

$$\tilde{C}_{q\bar{q}}^{(2,2)}(\xi_a, \xi_b) = -16C_F^2 \left[e^Y \delta_a (\delta_b + \delta'_b) + e^{-Y} (\delta_a + \delta'_a) \delta_b \right] \ln^3 \tau + \dots$$

$$\tilde{C}_{qg}^{(2,2)}(\xi_a, \xi_b) = 4T_F(C_F + C_A) e^Y \delta_a \delta_b \ln^3 \tau + \dots$$

- Explains need for rapidity cuts in MCFM 8 to obtain stable predictions
- Same arguments hold for beam contributions for any N
 - Need to choose $\rho_{a,b} = 1$ in Born frame or $\rho_{a,b} = e^{\pm Y}$ in hadronic frame

Features of physical subtractions

- All IR singularities are projected onto physical observable
 - ▶ Also possible to make it more differential, e.g., separating into N-jettiness contributions from individual regions, going double-differential, ...
- Subtraction terms are given by singular contributions of a physical cross section
 - ▶ N-jettiness observable and factorization theorem available for any N
 - ▶ Extension to massive quarks possible [see e.g. Pietrulewicz, Samitz, Spiering, FT '17]
 - ▶ Immediate combination with resummation and matching to parton shower
- The other key ingredient is a Born+jet NLO calculation that remains stable deep into the IR-singular region
- Can analyze and compute power corrections in SCET
 - ▶ Systematic and significant improvements in numerical implementations
- Subtractions will be publicly available in C++ library SCETlib
[watch <http://scetlib.desy.de>]