# N-jettiness Subtractions

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[mostly based on
Jonathan Gaunt, Maximilian Stahlhofen, FT, Jonathan Walsh
(arXiv:1505.04794)
Ian Moult, Lorena Rothen, Iain Stewart, FT, Hua Xing Zhu
(arXiv:1612.00450)]

# Outline

# Subtractions.

# Starting Point.

$$\sigma(X) \equiv \int \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$
$$= \underbrace{\int^{\mathcal{T}_\mathrm{cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}} + \int_{\mathcal{T}_\mathrm{cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$
$$= \sigma(X, \mathcal{T}_\mathrm{cut}) + \int_{\mathcal{T}_\mathrm{cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

- $\sigma(X)$: generic N-jet cross section
  - $X$ denotes all defining Born-level measurements/cuts (mostly irrelevant and suppressed in the following)
  - $\Phi_N$ is the N-parton Born phase space (including helicity and flavor labels and possible color-singlet final state)

$$\sigma^\mathrm{LO}(X) = \int \mathrm{d}\Phi_N \, B_N(\Phi_N) \, X(\Phi_N)$$
$$B_N(\Phi_N) = f_a \, f_b \sum_\mathrm{color} \left| \mathcal{A}^\mathrm{LO}_{ab\to N}(\Phi_N) \right|^2$$

# Starting Point.

$$\sigma(X) \equiv \int \mathrm{d}\mathcal{T}_N \, \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

$$= \underbrace{\int^{\mathcal{T}_\mathrm{cut}} \mathrm{d}\mathcal{T}_N \, \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}} \quad + \quad \int_{\mathcal{T}_\mathrm{cut}} \mathrm{d}\mathcal{T}_N \, \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

$$= \qquad \sigma(X, \mathcal{T}_\mathrm{cut}) \qquad + \quad \int_{\mathcal{T}_\mathrm{cut}} \mathrm{d}\mathcal{T}_N \, \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

- $\mathcal{T}_N$: physical IR-safe N-jet resolution variable

  $$\mathcal{T}_N(\Phi_N) = 0 \qquad \mathcal{T}_N(\Phi_{\geq N+1}) > 0 \qquad \mathcal{T}_N(\Phi_{\geq N+1} \to \Phi_N) \to 0$$

- $\frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$: differential $\mathcal{T}_N$ spectrum
  - At $\mathrm{LO}_N$ $\qquad \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} = \sigma^{\mathrm{LO}}(X)\,\delta(\mathcal{T}_N) + \mathcal{O}(\alpha_s)$
  - For any $\mathcal{T}_N > 0$ given by an N+1-jet N$^{n-1}$LO calculation

# Subtractions.

Add and subtract $\displaystyle \int_{\mathcal{T}_{\rm cut}}^{\mathcal{T}_{\rm off}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\mathcal{T}_N} = \sigma^{\rm sub}(\mathcal{T}_{\rm off}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})$

$$
\begin{aligned}
\sigma &= \sigma(\mathcal{T}_{\rm cut}) \quad + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \, \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} \\
&= \sigma^{\rm sub}(\mathcal{T}_{\rm off}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \biggl[\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} - \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\mathcal{T}_N}\theta(\mathcal{T} < \mathcal{T}_{\rm off})\biggr] + \bigl[\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})\bigr] \\
&= \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} \qquad\qquad + \bigl[\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})\bigr]
\end{aligned}
$$

- $\mathcal{T}_{\rm off}$ is a priori arbitrary and exactly cancels
  - Determines $\mathcal{T}_N$ range over which subtraction acts differentially in $\mathcal{T}_N$
  - Last line: setting $\mathcal{T}_{\rm off} = \mathcal{T}_{\rm cut}$ reduces it to a global subtraction (aka slicing)

# Subtractions.

Add and subtract $\displaystyle\int_{\mathcal{T}_{\rm cut}}^{\mathcal{T}_{\rm off}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\mathcal{T}_N} = \sigma^{\rm sub}(\mathcal{T}_{\rm off}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})$

$$
\begin{aligned}
\sigma &= \sigma(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} \\
&= \sigma^{\rm sub}(\mathcal{T}_{\rm off}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \biggl[\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} - \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\mathcal{T}_N}\theta(\mathcal{T} < \mathcal{T}_{\rm off})\biggr] + \bigl[\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})\bigr] \\
&= \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} + \bigl[\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})\bigr]
\end{aligned}
$$

- Conditions on $\sigma^{\rm sub}$
  - Need to be able to explicitly calculate $\sigma^{\rm sub}(\mathcal{T})$ (and $\mathrm{d}\sigma^{\rm sub}/\mathrm{d}\mathcal{T}_N$) to N$^n$LO
  - Has to reproduce singular limit of $\sigma(\mathcal{T}_{\rm cut})$ (and $\mathrm{d}\sigma/\mathrm{d}\mathcal{T}_N$), such that for $\mathcal{T}_{\rm cut} \to 0$ we can neglect

$$\Delta\sigma(\mathcal{T}_{\rm cut}) \equiv \sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) \to 0$$

# Power Expansion.

Expand cross section in powers of $\tau_N \equiv \frac{\mathcal{T}_N}{Q}$ and $\tau_{\rm cut} \equiv \frac{\mathcal{T}_{\rm cut}}{Q}$

(where $Q$ is a typical hard scale whose precise choice is irrelevant for now)

$$\frac{d\sigma}{d\tau_N} = \frac{d\sigma^{(0)}}{d\tau_N} + \frac{d\sigma^{(2)}}{d\tau_N} + \frac{d\sigma^{(4)}}{d\tau_N} + \cdots$$

$$\sigma(\tau_{\rm cut}) = \sigma^{(0)}(\tau_{\rm cut}) + \sigma^{(2)}(\tau_{\rm cut}) + \sigma^{(4)}(\tau_{\rm cut}) + \cdots$$

- Singular (leading-power) terms

$$\frac{d\sigma^{\rm sing}}{d\tau_N} \equiv \frac{d\sigma^{(0)}}{d\tau_N} \sim \delta(\tau_N) + \left[\frac{\mathcal{O}(1)}{\tau_N}\right]_+$$

$$\sigma^{\rm sing}(\tau_{\rm cut}) \equiv \sigma^{(0)}(\tau_{\rm cut}) \sim \mathcal{O}(1)$$

- Nonsingular (subleading-power) terms

$$\tau_N \frac{d\sigma^{(2k)}}{d\tau_N} \sim \mathcal{O}(\tau_N^k) \qquad \sigma^{(2k)}(\tau_{\rm cut}) \sim \mathcal{O}(\tau_{\rm cut}^k)$$

# Putting Everything Together.

$$\sigma = \underbrace{\sigma^{\rm sub}(\tau_{\rm cut})}_{\text{NNLO}_N} + \underbrace{\int_{\tau_{\rm cut}} \mathrm{d}\tau_N \frac{\mathrm{d}\sigma}{\mathrm{d}\tau_N}}_{\text{NLO}_{N+1}} + \underbrace{\Delta\sigma(\tau_{\rm cut})}_{\text{neglect}}$$

where we have to choose

$$\sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{\rm sing}(\tau_{\rm cut})\,[1 + \mathcal{O}(\tau_{\rm cut})]$$

So neglecting $\Delta\sigma(\tau_{\rm cut})$ we only miss $\mathcal{O}(\tau_{\rm cut})$ power-suppressed terms

$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut}) - \sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{(2)}(\tau_{\rm cut}) + \cdots \sim \mathcal{O}(\tau_{\rm cut})$$

The tradeoff: Lowering $\tau_{\rm cut}$ ...

- ... reduces size of missing power corrections
- ... increases numerical cancellations between first two terms
  - Requires numerically more precise calculation of $\mathrm{d}\sigma/\mathrm{d}\tau_N$ in a region where the N+1-jet NLO calculation quickly becomes much less stable
  - Computational cost increases substantially

# Estimating Size of Missing Power Corrections.

There is one more important caveat

- Power suppression gets weaker at higher orders in $\alpha_s$ due to stronger log enhancement

$$\sigma^{(2)}(\tau_{\rm cut}) = \sum_{n=0} \sigma^{(2,n)}(\tau_{\rm cut}) \Big(\frac{\alpha_s}{4\pi}\Big)^n$$

$$\sigma^{(2,n)}(\tau_{\rm cut}) = \tau_{\rm cut} \sum_{m=0}^{2n-1} A_m^{(2,n)} \ln^m \tau_{\rm cut}$$

$\Rightarrow$ Dominant missing $\mathcal{O}(\alpha_s^n)$ terms actually scale as

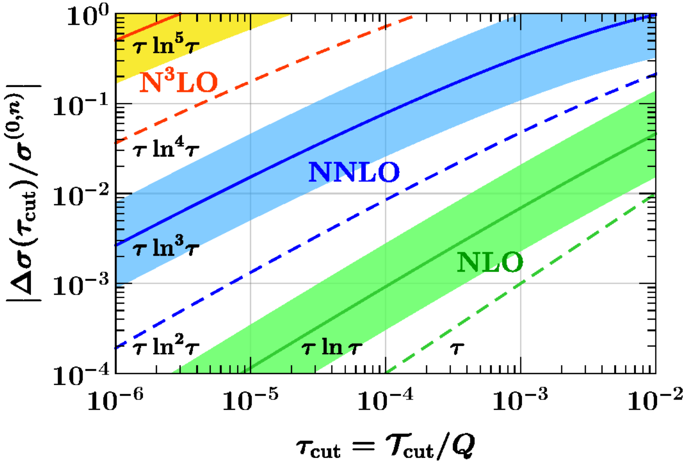
$$\Delta\sigma(\tau_{\rm cut}) \sim \alpha_s^n\, \tau_{\rm cut}\, \ln^{2n-1} \tau_{\rm cut}$$

- Can use this to get a rough order of magnitude estimate of their size by taking $A^{(2,n)} = \sigma^{(0,n)} \times [1/3, 3]$
- Works quite well for the cases we have checked

# Estimating Size of Missing Power Corrections.

## Estimate $\Delta\sigma(\tau_{\text{cut}})$ at N$^n$LO

- relative to full N$^n$LO coefficient

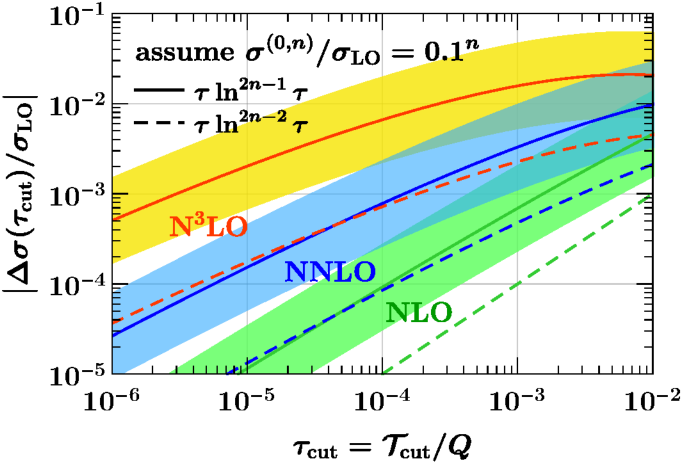



Typical values in current implementations are in $\tau_{\text{cut}} \simeq 10^{-4} \ldots 10^{-3}$ range

# Estimating Size of Missing Power Corrections.

## Estimate $\Delta\sigma(\tau_{\rm cut})$ at N$^n$LO

- relative to $\sigma_{\rm LO}$, assuming a 10% correction at each $\alpha_s$ order




Typical values in current implementations are in $\tau_{\rm cut} \simeq 10^{-4} \ldots 10^{-3}$ range

# The Upshot (or an early summary).

All IR-singular contributions are projected onto the physical observable $\mathcal{T}_N$

## The drawback

- Subtractions are nonlocal, i.e. not point-by-point in the real emission phase-space
  - Phase-space slicing in $\mathcal{T}_N =$ global (maximally nonlocal) subtraction
- In practice, it is a question of numerical stability whether this is a disadvantage or not
  - Naively expect larger numerical cancellations (since they happen later)
  - On the other hand, simpler structure and fewer subtration terms

## The advantage

- Subtractions are given by singular limit of a physical cross section
  - By choosing the “right” observable they can be computed using a factorization theorem
  - Also allows computing power corrections, giving significant improvements
- All nonsingular contributions are immediately given in terms of existing lower-order Born+1-jet calculations

# Resolution Variables for Physical Subtractions.

## In principle, any IR-sensitive resummable variable could be used

In fact, in the context of resummation, the singular terms are routinely obtained as a "by-product" of the resummation and used as subtraction to get the nonsingular terms.

## Other variables used as subtractions for NNLO calculations

- Color-singlet production: $q_T$ subtractions utilize $q_T$ of color-singlet system [Catani, Grazzini '07]
    - Very sucessfully applied to Higgs, Drell-Yan, and essentially any combination of diboson production
      [Catani et al. '07, '09, '11; Ferrera, Grazzini, Tramontano '11, '14; Cascioli et al. '14; Gehrmann et al. '14; Grazzini, Kallweit, Rathlev, Torre '13, '15; several more implementations]
    - Primarily used as global subtraction (as far as I know)
- Top-quark decay rate: inclusive jet mass (global) [Gao, Li, Zhu '12]
- $e^+e^- \to t\bar{t}$: Total radiation energy (global) [Gao, Zhu '14]

# Resolution Variables for Physical Subtractions.

N-jettiness event shape is explicitly designed as N-jet resolution variable with simplest possible factorization/resummation properties

- Differential 0-jettiness subtractions are implemented in GENEVA Monte-Carlo (as basis of its NNLO+NNLL′+PS matching) [Alioli et al. '13, '15]
- Global 0-jettiness (beam thrust)
  - Drell-Yan and Higgs [Gaunt, Stahlhofen, FT, Walsh '15]
  - $VH$, diphoton [Campbell, Ellis, Li, Williams '16]
  - NNLO Color-singlet in MCFM 8 [Boughezal et al. '16]
- Global 1-jettiness
  - $pp \to V/H + j$ [Boughezal, Focke, Liu, Petriello + Campbell, Ellis, Giele '15, '16]
  - $pp \to \gamma + j$ [Campbell, Ellis, Williams '16]

# N-Jettiness.

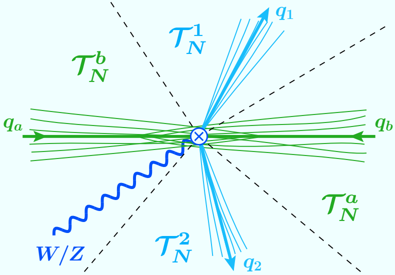
# N-Jettiness Event Shape.

[Stewart, FT, Waalewijn, '10]

$$\mathcal{T}_N = \sum_k \min\left\{\frac{2q_a\cdot p_k}{Q_a}, \frac{2q_b\cdot p_k}{Q_b}, \frac{2q_1\cdot p_k}{Q_1}, \frac{2q_2\cdot p_k}{Q_2}, \ldots, \frac{2q_N\cdot p_k}{Q_N}\right\}$$
$$\equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$$

- Partitions phase space into N jet regions and 2 beam regions
- $Q_{a,b}$, $Q_j$ determine distance measure
  - Geometric measures: $Q_i = 2\rho_i E_i$
- Born reference momenta $q_i$

$$q_{a,b} = x_{a,b}\frac{E_{\rm cm}}{2}(1, \pm\hat{z})$$
$$q_j = E_j(1, \vec{n}_j)$$




Specifying them corresponds to choosing an (IR-safe) Born projection

- Specific choice is part of N-jettiness definition but only affects the power-suppressed terms and is therefore not needed for singular terms

# All-order Singular Structure.

$$\frac{d\sigma^{\text{sing}}(X)}{d\tau_N} = \int d\Phi_N \, \frac{d\sigma^{\text{sing}}(\Phi_N)}{d\tau_N} \, X(\Phi_N)$$

$$\begin{aligned} \frac{d\sigma^{\text{sing}}(\Phi_N)}{d\tau_N} &= \mathcal{C}_{-1}(\Phi_N)\,\delta(\tau_N) + \sum_{m\geq 0} \mathcal{C}_m(\Phi_N)\,\mathcal{L}_m(\tau_N) \\ &= \sum_{n\geq 0} \Big[\mathcal{C}_{-1}^{(n)}(\Phi_N)\,\delta(\tau_N) + \sum_{m=0}^{2n-1} \mathcal{C}_m^{(n)}(\Phi_N)\,\mathcal{L}_m(\tau_N)\Big] \Big(\frac{\alpha_s}{4\pi}\Big)^n \end{aligned}$$

- Singular only depend on Born phase space $\Phi_N \equiv \{q_i, \lambda_i, \kappa_i\}$
  - Subtractions are FKS-like in this respect
- Plus distributions encode cancellation of real and virtual IR divergences

$$\mathcal{L}_m(\tau_N) = \Big[\frac{\theta(\tau_N)\ln^m(\tau_N)}{\tau_N}\Big]_+ \qquad \int^{\tau^{\text{cut}}} d\tau_N\,\mathcal{L}_m(\tau_N) = \frac{\ln^{m+1}(\tau^{\text{cut}})}{m+1}$$

# All-order Singular Structure.

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}\tau_N} = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\tau_N} \, X(\Phi_N)$$

$$\begin{aligned}\frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\tau_N} &= \mathcal{C}_{-1}(\Phi_N)\,\delta(\tau_N) + \sum_{m\geq 0} \mathcal{C}_m(\Phi_N)\,\mathcal{L}_m(\tau_N) \\ &= \sum_{n\geq 0}\Big[\mathcal{C}_{-1}^{(n)}(\Phi_N)\,\delta(\tau_N) + \sum_{m=0}^{2n-1} \mathcal{C}_m^{(n)}(\Phi_N)\,\mathcal{L}_m(\tau_N)\Big]\Big(\frac{\alpha_s}{4\pi}\Big)^n\end{aligned}$$

- Integrated subtractions

$$\sigma^{\mathrm{sing}}(\Phi_N, \tau_{\mathrm{cut}}) = \mathcal{C}_{-1}(\Phi_N) + \sum_{m\geq 0} \mathcal{C}_m(\Phi_N)\,\frac{\ln^{m+1}(\tau^{\mathrm{cut}})}{m+1}$$

- $\mathcal{C}_{-1}(\Phi_N)$ contains finite remainder of N-parton virtuals
  - At LO: $\mathcal{C}_{-1}^{(0)}(\Phi_N) = B_N(\Phi_N)$
  - Most nontrivial piece, corresponds to virtual plus integrated subtraction in other subtraction schemes

# Factorization Theorem.

[Stewart, FT, Waalewijn, '09, '10]

$$\frac{d\sigma^{\rm sing}(\Phi_N)}{d\mathcal{T}_N} = \int dt_a\, B_a(t_a, x_a, \mu) \int dt_b\, B_b(t_b, x_b, \mu) \Big[\prod_{i=1}^{N} \int ds_i\, J_i(s_i, \mu)\Big]$$

$$\times\, \vec{C}^\dagger(\Phi_N, \mu)\, \widehat{S}_\kappa\Big(\mathcal{T}_N - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \sum_{i=1}^{N} \frac{s_i}{Q_i}, \{\hat{q}_i\}, \mu\Big) \vec{C}(\Phi_N, \mu)$$

- All functions are IR finite and have an operator definition in SCET
- Simplifying features of N-jettiness
  - No dependence on jet algorithm (jet clustering, jet radius, etc.)
  - No recoil effects from soft radiation
  - No additional $\vec{p}_T$ dependence or convolutions, no rapidity divergences
- To obtain subtraction coefficients simply expand and collect terms, e.g.,

$$\mathcal{C}_{-1}^{(2)} = f_a f_b \big[\vec{C}^{\dagger(0)} \vec{C}^{(2)} + \vec{C}^{\dagger(2)} \vec{C}^{(0)}\big]$$
$$+ \vec{C}^{\dagger(0)} \big[B_a^{(2)} f_b + f_a B_b^{(2)} + f_a f_b\, \widehat{S}^{(2)}\big] \vec{C}^{(0)}$$
$$+ \text{1-loop cross terms}$$

# Hard Matching Coefficients.

Encode the process-dependent N-parton virtual QCD corrections

Arise from matching QCD onto SCET

- In pure dimensionless regularization with $\overline{\text{MS}}$ given in terms of IR-finite ($\overline{\text{MS}}$-subtracted) N-parton QCD amplitudes
- General formalism using SCET helicity operator basis
  [Moult, Stewart, FT, Waalewijn '15]
  - Using same color basis $\bar{T}^{a_1\cdots\alpha_n}$ as in QCD calculation, directly given by corresponding color-ordered helicity amplitudes

$$\bar{T}^{a_1\cdots\alpha_n}\,\mathrm{i}\vec{C}_{\pm\cdots\pm} = \mathcal{A}_{\mathrm{fin}}(g_1^\pm \cdots q_n^\pm) \equiv \frac{\bar{T}^{a_1\cdots\alpha_n}\,\widehat{Z}_C^{-1}\,\vec{\mathcal{A}}_{\mathrm{ren}}(g_1^\pm \cdots q_n^\pm)}{Z_\xi^{n_q/2}\,Z_A^{n_g/2}}$$

  - $\widehat{Z}$, $Z_\xi$, $Z_A$ are SCET $\overline{\text{MS}}$ renormalization constants (in pure dimensional regularization equivalent to QCD $1/\epsilon_{\mathrm{IR}}$ divergences)
  - QCD helicity amplitudes should be UV-renormalized in CDR or HV

# Beam and Jet Functions.

Encode cancellation of IR singularities between collinear real and virtual radiation and corresponding IR-finite remainder

- Inclusive virtuality-dependent (SCET-I) beam and jet functions
  - Universal for any N, only depend on parton type (quark vs. gluon)
  - Important: Overlap with soft contributions (known as zero bins in SCET) is scale-less and vanishes in pure dimensionless regularization

## Jet functions

- (Straight)forward IR-finite vacuum matrix element of collinear quark or gluon operator

  [NLO: Bauer, Manohar '03, Fleming, Leibovich, Mehen '03, Becher, Schwartz '06;

  NNLO: Becher, Neubert '06, Becher, Bell '10]

## Beam functions

- Require matching onto PDFs in terms of IR-finite matching coefficients

  [NLO: Stewart, FT, Waalewijn '09, '10; NNLO: Gaunt, Stahlhofen, FT '14]

$$B_i(t, x) = \sum_j \int \frac{\mathrm{d}z}{z} \mathcal{I}_{ij}(t, z)\, f_j\Big(\frac{x}{z}\Big)$$

  - NNLO beam functions are key ingredient for color-singlet production

# Soft Function.

Encodes cancellation of IR singularities between soft real and virtual radiation and corresponding IR-finite remainder

- Matrix element of N+2 lightlike soft Wilson lines along collinear directions
  - Matrix acting on external color space, accounts for all color correlations in soft IR divergences
- Explicitly depends on N-jettiness measurement and partitioning
  - with respect to fixed collinear directions (no soft recoil effects)
- NLO: Known for any number of Wilson lines (and any $Q_i$) using on hemisphere decomposition [Jouttenus, Stewart, FT, Waalewijn '11]
- NNLO
  - 2 partons: Hemisphere soft function [Kelley, Schwartz, Schabinger, Zhu '11; Monni, Gehrmann, Luisoni '11; Hornig, Lee, Stewart, Walsh, Zuberi '11; Kang, Labun, Lee '15]
  - 3 partons: Numerically for $pp \to L + 1j$ [Boughezal, Liu, Petriello '15] recently for massive 3rd parton [Li, Wang '16]
  - Not yet known for general N

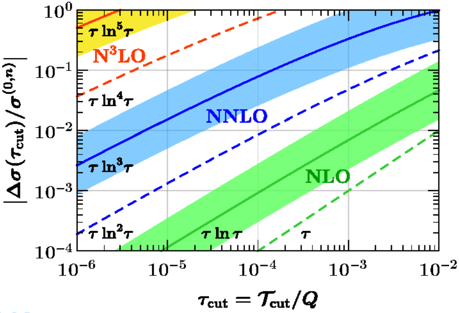
# Subleading Power Corrections.

# Basic Idea.

Recall

$$\sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{\rm sing}(\tau_{\rm cut})\,[1 + \mathcal{O}(\tau_{\rm cut})]$$

$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut}) - \sigma^{\rm sub}(\tau_{\rm cut})$$
$$\sim \tau_{\rm cut} \ln^n \tau_{\rm cut}$$




Calculating the dominant power corrections
they can be included in the subtraction to reduce the size of the missing terms

- Each factor of log can potentially give an order of magnitude numerical improvement
  - Even the LL next-to-leading power (NLP) terms are very interesting
- Many things that could be ignored at leading power start to matter at subleading power.
  - Choice of N-jettiness definition can strongly impact size of power corrections

# SCET at Subleading Power.

SCET is explicitly constructed to maintain manifest power counting at all stages of a calculation

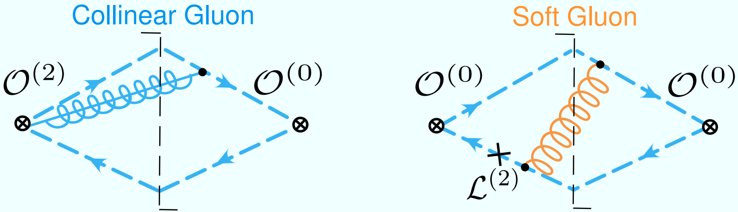
Provides natural organization of different sources of power corrections

- Insertions of subleading SCET Lagrangian
  - Corrects dynamics of propagating soft and collinear particles
- Subleading hard-scattering operators
  - Helicity operator basis extended to subleading power
- Subleading corrections to the measurement

Since we don't care about resummation, we don't actually need a full factorization theorem at subleading power

Instead, we can perform the calculation at fixed order with SCET as organizational principle, focusing on the highest logarithmic terms

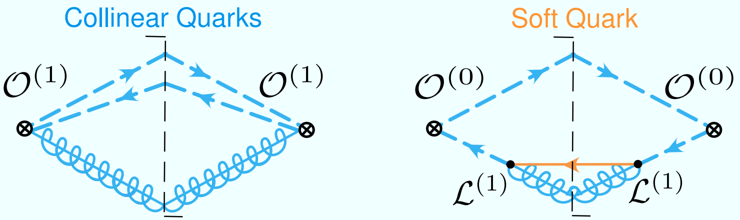
# Example: Thrust at NLO.




$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,1)}}{d\tau} = 8C_F\left[\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2\tau}\right) - \left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2\tau^2}\right)\right] = 8C_F \ln\tau \,,$$

- Result gives directly (no additional expansions) the NLP contribution
- Total NLP result reproduces known thrust result
- $1/\epsilon$ poles must cancel between collinear and soft contributions
  - In SCET these are UV poles arising from the scale separation between different sectors
  - From full-theory point of view these are IR poles and must cancel because there are no nontrivial IR divergences at subleading power

# Example: Thrust at NLO.




$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2,1)}}{\mathrm{d}\tau} = 4C_F\left[-\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{Q^2\tau}\right)+\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{Q^2\tau^2}\right)\right] = -4C_F\ln\tau$$

- Result gives directly (no additional expansions) the NLP contribution
- Total NLP result reproduces known thrust result
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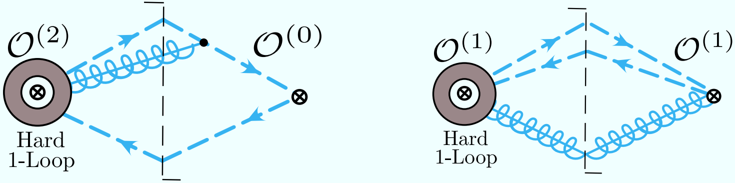
# Going to NNLO.

Same cancellation of $1/\epsilon$ poles must happen at NNLO

- Yields nontrivial constraints (consistency relations) on the different contributions from hard, collinear, and soft sectors
  - Significantly reduces number of NNLO coefficients that must be calculated
  - Equivalently provides for nontrivial cross checks

- The LL NNLO result is determined by a single coefficient
  - hard-collinear (easiest) or collinear-soft or soft-softor




$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2,2)}}{\mathrm{d}\tau} = \Big[-32C_F^2 + 8C_F(C_F + C_A)\Big]\ln^3\tau + \cdots$$

  - New color structure compared to leading power from quark channel

# 0-Jettiness for Drell-Yan at NLP.

Crossing the thrust calculation and taking into account differences in measurement, phase space, and PDFs

Coefficients of the partonic cross section
with $\delta_a \equiv \delta(\xi_a - x_a)$ and $\delta'_a \equiv x_a\, \delta'(\xi_a - x_a)$ and $\tau \equiv \mathcal{T}_0/Q$

- NLO

$$C^{(2,1)}_{q\bar{q}}(\xi_a, \xi_b) = 8C_F \left(\delta_a\delta_b + \frac{\delta'_a\delta_b}{2} + \frac{\delta_a\delta'_b}{2}\right) \ln\tau + \cdots$$

$$C^{(2,1)}_{qg}(\xi_a, \xi_b) = -2T_F\, \delta_a\delta_b\, \ln\tau + \cdots$$

- NNLO

$$C^{(2,2)}_{q\bar{q}}(\xi_a, \xi_b) = -32C_F^2 \left(\delta_a\delta_b + \frac{\delta'_a\delta_b}{2} + \frac{\delta_a\delta'_b}{2}\right) \ln^3\tau + \cdots$$

$$C^{(2,2)}_{qg}(\xi_a, \xi_b) = 4T_F(C_F + C_A)\, \delta_a\delta_b\, \ln^3\tau + \cdots$$

  - $qg$ channel already contributes at leading-log, in contrast to leading power

# Numerical Results for Drell-Yan.

We can obtain the full nonsingular numerically

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma^{\rm nons}}{d\ln\mathcal{T}_0} = \frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{d\ln\mathcal{T}_0} - \frac{1}{\sigma_{\rm LO}}\frac{d\sigma^{\rm sing}}{d\ln\mathcal{T}_0}$$

- Use $Z+j$ NLO calculation from MCFM 8 for $d\sigma/d\ln\mathcal{T}_0$
- Perform a $\chi^2$ fit to (with $\tau \equiv \mathcal{T}_0/m_Z$)

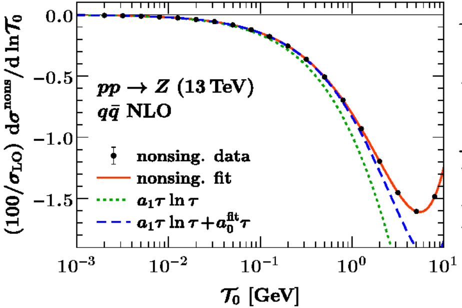
$$F_{\rm NLO}(\tau) = \frac{d}{d\ln\tau}\Big\{\tau\big[(a_1 + b_1\tau + c_1\tau^2)\ln\tau + a_0 + b_0\tau + c_0\tau^2\big]\Big\}$$

$$F_{\rm NNLO}(\tau) = \frac{d}{d\ln\tau}\Big\{\tau\big[(a_3 + b_3\tau)\ln^3\tau + (a_2 + b_2\tau)\ln^2\tau + a_1\ln\tau + a_0\big]\Big\}$$

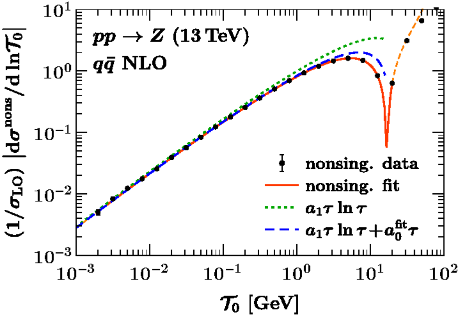
  - Requires high MC statistics to get precise enough nonsingular data to be able to distinguish different terms of similar shape
  - Important to include $b_i$, $c_i$ coefficients in the fit to avoid biasing the fit result for the NLP $a_i$ coefficients we are interested in
  - Important to carefully select fit range in $\mathcal{T}_0$ and validate fit stability

# Numerical Results at NLO.

linear scale



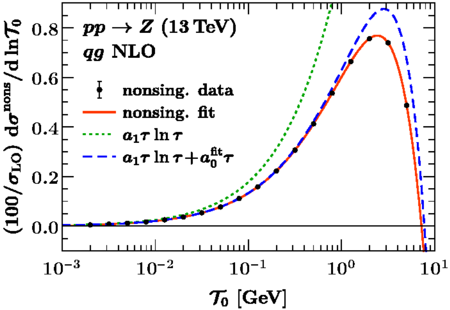

log scale



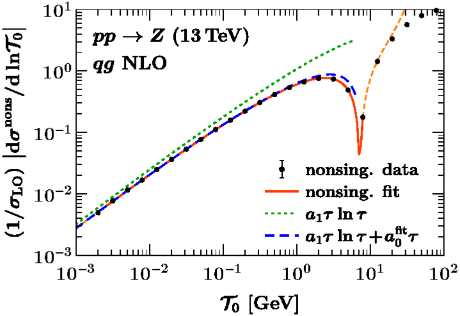

| channel and coefficient | | fitted | calculated |
|---|---|---|---|
| $q\bar{q}$ NLO | $a_1$ | $+0.25366 \pm 0.00131$ | $+0.25509$ |
| $qg$ NLO | $a_1$ | $-0.27697 \pm 0.00113$ | $-0.27720$ |
| $q\bar{q}$ NLO | $a_0$ | $+0.13738 \pm 0.00057$ | |
| $qg$ NLO | $a_0$ | $-0.40062 \pm 0.00052$ | |

# Numerical Results at NLO.

linear scale




log scale




| channel and coefficient | | fitted | calculated |
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| $q\bar{q}$ NLO | $a_1$ | $+0.25366 \pm 0.00131$ | $+0.25509$ |
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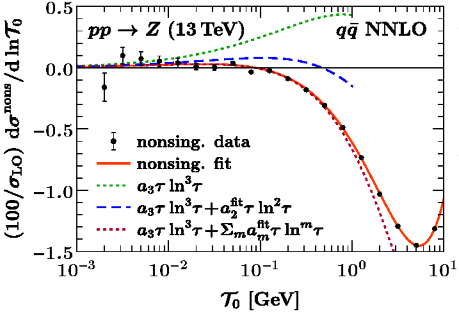
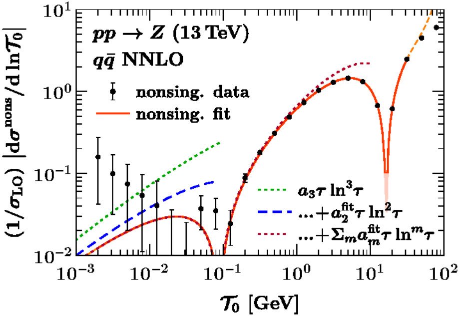
# Numerical Results at NNLO.

linear scale



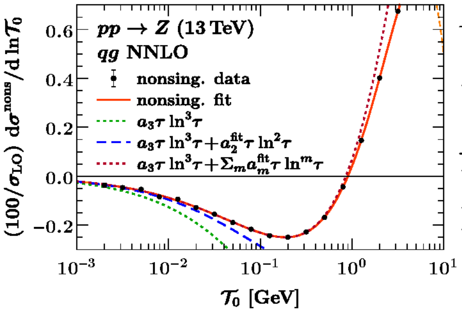

log scale



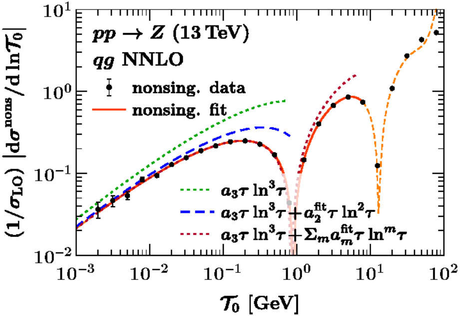

| channel and coefficient | | fitted | calculated |
|---|---|---|---|
| $q\bar{q}$ NNLO | $a_3$ | $-0.01112 \pm 0.00150$ | $-0.01277$ |
| $qg$ NNLO | $a_3$ | $+0.02373 \pm 0.00247$ | $+0.02256$ |
| $q\bar{q}$ NNLO | $a_2$ | $-0.04662 \pm 0.00180$ | |
| $qg$ NNLO | $a_2$ | $+0.04234 \pm 0.00242$ | |

# Numerical Results at NNLO.

linear scale



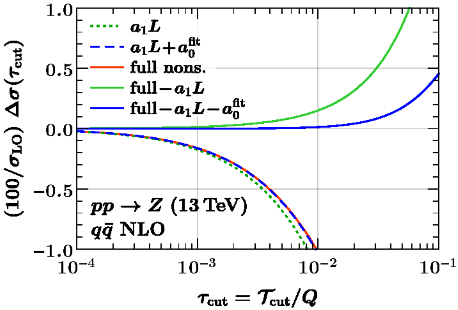

log scale



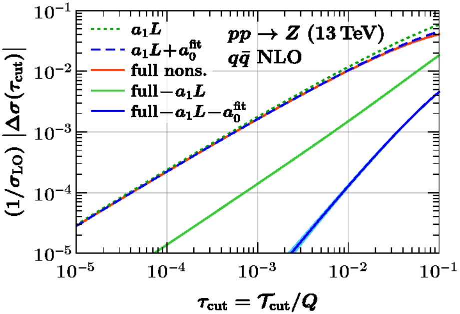

| channel and coefficient | | fitted | calculated |
|---|---|---|---|
| $q\bar{q}$ NNLO | $a_3$ | $-0.01112 \pm 0.00150$ | $-0.01277$ |
| $qg$ NNLO | $a_3$ | $+0.02373 \pm 0.00247$ | $+0.02256$ |
| $q\bar{q}$ NNLO | $a_2$ | $-0.04662 \pm 0.00180$ | |
| $qg$ NNLO | $a_2$ | $+0.04234 \pm 0.00242$ | |

# Impact On $\Delta\sigma$

linear scale

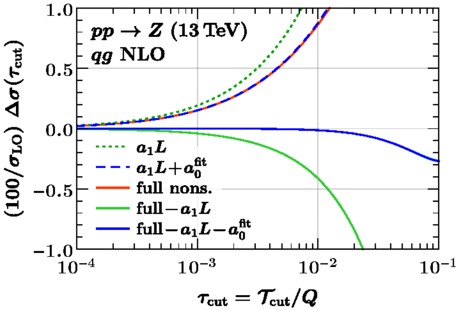



log scale

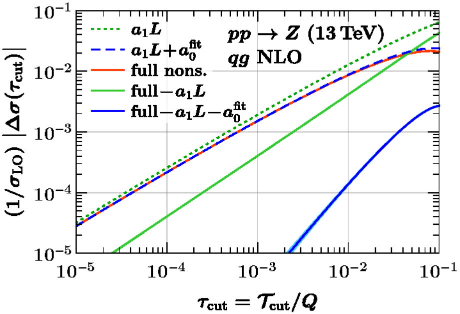


# Impact On $\Delta\sigma$.

linear scale

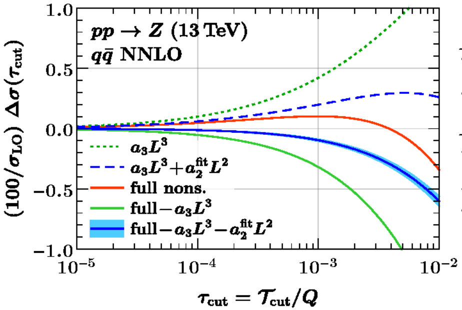



log scale

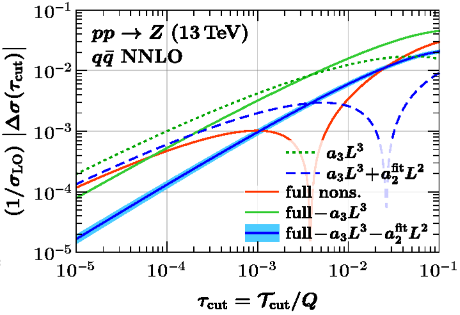


# Impact On $\Delta\sigma$

linear scale



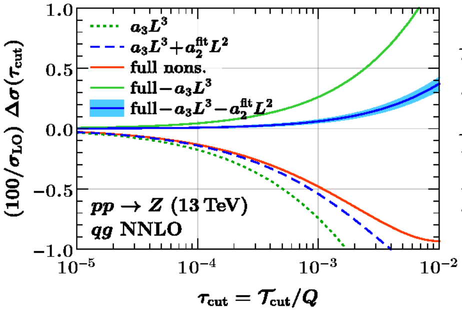

log scale

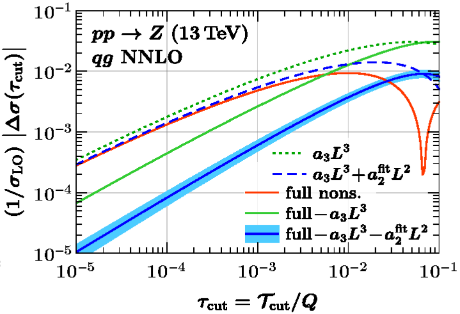


# Impact On $\Delta\sigma$.

linear scale

log scale

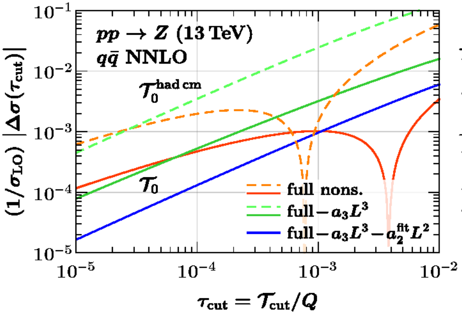


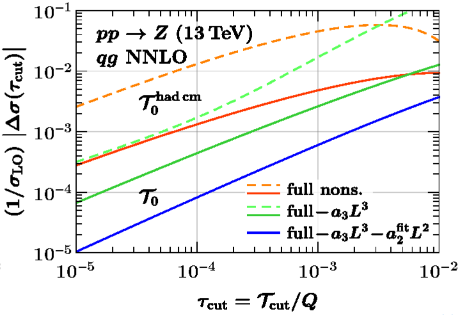



# Dependence on Definition.

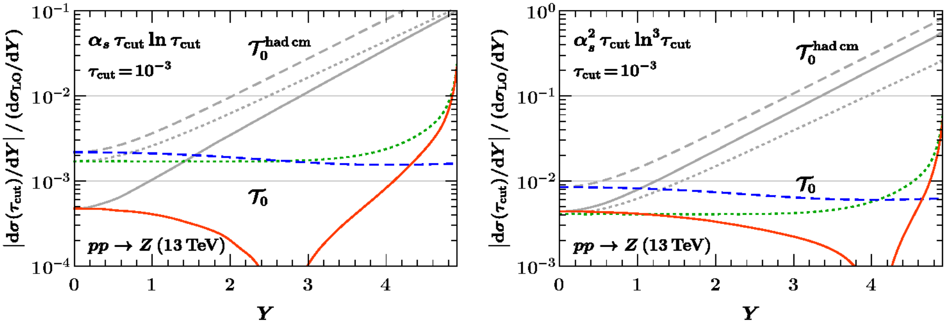
$$\mathcal{T}_0^x = \sum_k \min\{\lambda_x\, p_k^+, \lambda_x^{-1}\, p_k^-\}$$

- leptonic: $\lambda = \sqrt{q^-/q^+} = e^Y$
  - Accounts for boost between leptonic (Born) and hadronic cm frame
  - Most natural (original) definition, power expansion in $\mathcal{T}_0/Q$
- hadronic: $\lambda_{\text{had cm}} = 1$ (currently used in MCFM [Boughezal et. al '16])
  - Defines $\mathcal{T}_0^{\text{had cm}}$ in hadronic cm frame
  - Power expansion effectively in $\mathcal{T}_0^{\text{had cm}}/(Qe^{\pm Y})$ deteriorates for large $Y$



# Rapidity Dependence of Power Corrections.




- Exponential enhancement of power corrections

$$\widetilde{C}^{(2,2)}_{q\bar q}(\xi_a,\xi_b) = -16C_F^2\Big[e^{Y}\delta_a(\delta_b+\delta_b') + e^{-Y}(\delta_a+\delta_a')\delta_b\Big]\ln^3\tau + \cdots$$

$$\widetilde{C}^{(2,2)}_{qg}(\xi_a,\xi_b) = 4T_F(C_F+C_A)e^{Y}\delta_a\delta_b\ln^3\tau + \cdots$$

  - Explains rapidity cuts needed in MCFM 8 to obtain stable predictions

- Same arguments hold for beam contributions for any N
  - Need to choose $\rho_{a,b}=1$ in Born frame or $\rho_{a,b}=e^{\pm Y}$ in hadronic frame

# Summary and Outlook.

## Features of physical subtractions

- All IR singularities are projected onto physical observable
  - Also possible to make it more differential, e.g., separating into N-jettiness contributions from individual regions, going double-differential, ...
- Subtraction terms are given by singular contributions of a physical cross section
  - N-jettiness observable and factorization theorem available for any N
  - Extension to massive quarks is also possible
- The other key ingredient is a Born+jet NLO calculation that remains stable deep into the IR-singular region
- Can analyze and compute power corrections in SCET
  - Significant improvements in numerical implementations possible
  - Currently looking at other (gluon-initiated) color-singlet channels
  - TODO: Universality of NLP terms, extension to more final-state jets
  - Similar recent work (without SCET, also for $gg$) in [Boughezal, Liu, Petriello '16]
- Planning to make subtractions publicly available in C++ library SCETlib [http://scetlib.desy.de]