The electroweak chiral Lagrangian reanalyzed

Andreas Nyffeler*
DESY, Platanenallee 6, D-15738 Zeuthen, Germany

Andreas Schenk
Hambacher Straße 14, D-64625 Bensheim-Gronau, Germany
(26 October 2000)

In this paper we reanalyze the electroweak chiral Lagrangian with particular focus on two issues related to gauge invariance. Our analysis is based on a manifestly gauge-invariant approach that we introduced recently. It deals with gauge-invariant Green's functions and provides a method to evaluate the corresponding generating functional without fixing the gauge. First we show, for the case where no fermions are included in the effective Lagrangian, that the set of low-energy constants currently used in the literature is redundant. In particular, by employing the equations of motion for the gauge fields one can choose to remove two low-energy constants which contribute to the self-energies of the gauge bosons. If fermions are included in the effective field theory analysis the situation is more involved. Even in this case, however, these contributions to the self-energies of the gauge bosons can be removed. The relation of this result to the experimentally determined values for the oblique parameters S, T, and U is discussed. In the second part of the paper we consider the matching relation between a full and an effective theory. We show how the low-energy constants of the effective Lagrangian can be determined by matching gauge-invariant Green's functions in both theories. As an application we explicitly evaluate the low-energy constants for the standard model with a heavy Higgs boson. The matching at the one-loop level and at next-to-leading order in the low-energy expansion is performed employing functional methods.

PACS number(s): 12.39.Fe, 11.15.Ex, 12.15.-y, 14.80.Bn

I. INTRODUCTION

The symmetry breaking sector of the standard model is still poorly understood from a theoretical point of view. Furthermore no direct experimental evidence of the Higgs boson has been found so far. In this situation the method of effective field theory has repeatedly been used in recent years to analyze the symmetry breaking sector [1]. It provides a convenient and model independent parametrization of various scenarios which are discussed in the literature, regarding the nature of the spontaneous breaking of the electroweak symmetry. In this approach, the unknown physics is hidden in the low-energy constants of an effective Lagrangian, which describes the effective field theory. Effective Lagrangians thereby allow a unified treatment of different parametrizations of new physics effects, such as oblique corrections to gauge bosons self-energies [2,3] and anomalous triple [4] and quartic [5] vertices of the gauge bosons.

The low-energy structure of a theory containing light and heavy particle species which are separated by a mass gap can adequately be described by an effective field theory which contains only the light fields. In the case of the standard model one can construct effective Lagrangians by introducing higher dimensional operators that preserve the $SU(2)_L \times U(1)_Y$ gauge symmetry. In the presence of a light Higgs boson, i.e. in the decoupling case [6], the symmetry is linearly realized and the corresponding effective Lagrangian, which contains the Higgs field, was presented in Ref. [7]. For a strongly interacting symmetry breaking sector, i.e. in the non-decoupling case, the effective Lagrangian can be built [8–11] in analogy to the chiral Lagrangian [12,13] for QCD and it is therefore called electroweak chiral Lagrangian. The use of effective Lagrangians might in fact be the only way, apart from lattice calculations, to gain insight into strongly interacting theories for the electroweak symmetry breaking sector, similarly to the situation with QCD at low energies. We note that by employing the electroweak chiral Lagrangian it was shown recently [14] that present electroweak precision data are still compatible with a strongly interacting model of symmetry breaking with a scale of new physics as high as 3 TeV.

The purpose of this paper is to take another look at the electroweak chiral Lagrangian and to investigate two issues related to gauge invariance where there are some subtleties involved, because one has to deal with off-shell quantities.

*Present address: Centre de Physique Théorique, CNRS-Luminy, Case 907, F-13288 Marseille Cedex 9, France. Email address: nyffeler@cpt.univ-mrs.fr

According to Refs. [12,13] the effective field theory should describe the physics of the underlying full theory at low energies. Symmetry principles thereby play a crucial role for the construction of the effective field theory and, apart from the occurrence of anomalies, the effective field theory can be described by an effective Lagrangian which respects these (possibly broken) symmetries [15]. In order to preserve the gauge symmetry even when dealing with off-shell quantities we employ a manifestly gauge-invariant approach that was introduced recently [16]. It deals with gauge-invariant Green's functions and provides a method to evaluate the corresponding generating functional without fixing the gauge.

The first topic is the analysis of the general effective field theory which describes a strongly interacting electroweak symmetry breaking sector. We are particularly interested in the question of how many independent, physically relevant parameters are contained in the effective Lagrangian. It is well known from chiral perturbation theory [13,17] that one can use the equations of motion which are derived from the lowest order effective Lagrangian to remove redundant terms that appear at higher orders in the low-energy expansion. This procedure is well defined within a functional approach where one performs an expansion around the solutions of the classical equations of motion in the path-integral representation of the generating functional of suitably chosen Green's functions. It is only in this framework where we will use the equations of motion later on. Equivalently, one can also remove terms in the effective Lagrangian by performing appropriate reparametrizations of the fields and external sources in the path integral [18].

In the usual gauge-dependent framework the equations of motion for the gauge fields are gauge-dependent. For instance, contributions from the gauge-fixing terms and from the non-gauge-invariant source terms would appear in Eqs. (2.34)–(2.36) below. It is doubtful whether these equations can then be used to eliminate redundant gauge-invariant terms from the effective Lagrangian. As a matter of fact, we do not know of any reference where this has been tried. The equations of motion in our approach are gauge-invariant. Employing them we first show for a purely bosonic effective field theory, i.e. when no fermions are included in the effective Lagrangian, that the set of parameters currently used in the literature contains two redundant low-energy constants which can be removed. In particular, one can choose to remove two low-energy constants which contribute to the self-energies of the gauge bosons which are not observable anyway. If fermions are present, the situation is more involved. We will show that these two parameters renormalize the coupling of the massive gauge bosons to charged and neutral currents and, thus, have no physical meaning in a full effective Lagrangian analysis. The relation of this result to the experimentally determined values for the oblique parameters S, T, and U [2] as quoted by the particle data group will be discussed.

The second topic of this paper is to study the evaluation of the low-energy constants in the effective Lagrangian for a given underlying theory. Comparing the theoretical predictions for the low-energy constants for different models with experimental constraints might help to rule out some of the underlying theories under consideration before direct effects become visible. This point motivates to determine the values of the low-energy constants in the effective theory for various models. At low energies, the standard model with a heavy Higgs boson in the spontaneously broken phase can adequately be described by such an effective field theory. In order to determine the effective Lagrangian one can require, for instance, that corresponding Green's functions in both theories have the same low-energy structure. One can take this matching condition as the definition of the effective field theory. At this point the issue of gauge invariance is crucial. If gauge-dependent Green's functions are used in this matching procedure one has to make sure that no gauge artifacts enter the low-energy constants of the effective Lagrangian.

Several groups [19–21] have performed such a matching calculation for the standard model with a heavy Higgs boson in recent years, thereby extending the results which were obtained long time ago [8,9]. Gradually the importance to maintain gauge invariance in the matching procedure was recognized. Whereas the matching was performed with gauge-dependent Green's functions in Ref. [19], the authors of Refs. [20,21] proposed new methods to overcome these gauge artifacts. See Ref. [22] for a more detailed account of the development. The extension of the method proposed in Ref. [20] to the two-loop level was discussed in Ref. [23]. Nevertheless, the problems with gauge dependencies have not yet been fully resolved. In the meantime similar matching calculations have been performed for various models [24–26] without considering the issue of gauge invariance any further.

To avoid any problems with gauge dependencies one should in fact match only gauge-invariant quantities, such as S-matrix elements [20]. As it turns out, however, matching S-matrix elements is quite cumbersome because one has to deal with the whole infrared physics. Techniques which involve Green's functions are much easier to use. We therefore propose to match Green's functions of gauge-invariant fields in order to determine the effective Lagrangian. In this way no gauge artifacts can appear through the matching procedure and one can employ functional methods [27]. For the Abelian Higgs model such a manifestly gauge-invariant matching calculation has been performed in Ref. [22]. In the present paper we show how one can determine the effective Lagrangian for the standard model with a heavy Higgs boson by matching gauge-invariant Green's functions in the full and the effective theory at low energies at the one-loop level. For this purpose we can use a generating functional of gauge-invariant Green's functions for the bosonic sector of the standard model which was discussed in a recent paper [16]. In this way the starting point of the matching procedure is well defined and gauge invariance is manifestly preserved throughout the whole calculation.

In view of the fact that all fits to electroweak precision data over the last couple of years tend to prefer a light

Higgs boson[1], we will regard the standard model with a heavy Higgs boson merely as a model of a strongly interacting symmetry breaking sector, where, however, perturbation theory can still be applied if the coupling constant is not too strong. Thus, it serves as a testing ground for our gauge-invariant method of matching. The corresponding values for the low-energy constants will also represent a reference point for other strongly interacting models. As pointed out in Ref. [29], it is very difficult to get any reliable estimate for the low-energy parameters for genuinely strongly interacting models of the electroweak symmetry breaking sector.

This paper is organized as follows: In the next section we introduce the general effective field theory for a strongly interacting electroweak symmetry breaking sector within the gauge-invariant functional framework presented in Ref. [16]. We discuss our choice of gauge-invariant operators and the corresponding source terms which emit one-particle states of the gauge bosons. We then determine the number of independent low-energy constants by employing the equations of motion to remove redundant terms from the effective Lagrangian. We sketch the inclusion of fermions in the effective field theory and relate our findings to the experimentally determined oblique parameters S, T, and U. In order to prepare the matching calculation in the second part of this paper we briefly recapitulate in Sec. III the main results from our manifestly gauge-invariant approach to the standard model [16]. We calculate the generating functional for the gauge-invariant Green's functions in the bosonic sector up to the one-loop level. In this section we also present the renormalization prescriptions for the fields, the mass parameter and the coupling constants of the model. In Sec. IV we evaluate the matching condition between gauge-invariant Green's functions in the full and the effective theory at low energies at the one-loop level for the case of the standard model with a heavy Higgs boson. The effective Lagrangian for the bosonic sector is determined up to order p^4 in the low-energy expansion. In Sec. V we express the result for the effective Lagrangian in terms of the physical masses of the Higgs and the gauge bosons and the electric charge. Finally, we compare our results with those obtained by other groups. We summarize our findings in Sec. VI. The source terms which appear in the general effective Lagrangian at order p^4 are listed in Appendix A. The relations between our set of operators for the electroweak chiral Lagrangian and the basis which is usually used in the literature can be found in Appendix B. Some technical details needed for the calculation of the one-loop generating functional in the standard model are presented in Appendix C.

II. EFFECTIVE FIELD THEORY

A. The general effective Lagrangian

In this section we will discuss the general effective field theory for the bosonic[2] part of a strongly interacting electroweak symmetry breaking sector, closely following the functional approach to the standard model introduced in Ref. [16]. The relation of our approach to the one that is usually adopted in the literature [8–11] will be discussed below. According to Refs. [12,13] the effective field theory should describe the physics of the underlying full theory at low energies. We assume that

$$p^2, M_W^2, M_Z^2 \ll M^2 \,, \tag{2.1}$$

where p is a typical momentum and M is the mass scale for heavy particles in the underlying theory, e.g. a heavy Higgs boson in the standard model or a technirho in some technicolor model [31]. In general, symmetry principles are crucial for the construction of the effective field theory and, apart from the occurrence of anomalies, the effective field theory can be described by an effective Lagrangian which respects these (possibly broken) symmetries [15]. In our case this Lagrangian is gauge-invariant and depends on the Goldstone boson field $\bar{U}$, confined to the sphere $\bar{U}^\dagger \bar{U} = 1$, the $SU(2)_L$ gauge fields $\bar{W}_\mu^a\, (a = 1, 2, 3)$, the $U(1)_Y$ gauge field $\bar{B}_\mu$, and external sources $\bar{K}_{\mu\nu}, \bar{J}_\mu^a, (a = 1, 2, 3)$

$$\mathcal{L}_{eff} = \mathcal{L}_{eff}\left(\bar{W}_{\mu\nu}^a, \bar{B}_{\mu\nu}, \bar{U}, \bar{D}_\mu \bar{U}, \bar{D}_\mu \bar{D}_\nu \bar{U}, \ldots; \bar{K}_{\mu\nu}, \bar{J}_\mu^a\right) \,, \tag{2.2}$$

where the Goldstone boson doublet $\bar{U}$ is coupled to the gauge fields through the covariant derivative

$$\bar{D}_\mu \bar{U} = \left(\partial_\mu - i\frac{\tau^a}{2}\bar{W}_\mu^a - i\frac{1}{2}\bar{B}_\mu\right)\bar{U} \,. \tag{2.3}$$

---

[1]For instance, at the Moriond 2000 meeting the value $M_H = \left(67\,^{+60}_{-33}\right)$ GeV was presented [28].

[2]The electroweak chiral Lagrangian including matter fields was presented in Ref. [30].

Note that we have absorbed the coupling constants $\bar{g}$ and $\bar{g}'$ into the gauge fields $\bar{W}^a_\mu$ and $\bar{B}_\mu$, respectively. The field strengths are given by

$$\bar{W}^a_{\mu\nu} = \partial_\mu \bar{W}^a_\nu - \partial_\nu \bar{W}^a_\mu + \varepsilon^{abc}\bar{W}^b_\mu \bar{W}^c_\nu \,, \tag{2.4}$$

$$\bar{B}_{\mu\nu} = \partial_\mu \bar{B}_\nu - \partial_\nu \bar{B}_\mu \,. \tag{2.5}$$

The fields and the sources in the effective theory have been denoted with a bar in order to distinguish them from those occurring in the standard model which will be discussed below. The Goldstone boson field $\bar{U}$ and the gauge fields $\bar{W}^a_\mu, \bar{B}_\mu$ transform under $SU(2)_L$ gauge transformations in the following way:

$$\begin{aligned} &\bar{U} \to \mathcal{V}\bar{U} \,, \quad \mathcal{V} \in SU(2) \,, \\ &\bar{W}_\mu \to \mathcal{V}\bar{W}_\mu\mathcal{V}^\dagger - i(\partial_\mu\mathcal{V})\mathcal{V}^\dagger \,, \quad \bar{W}_\mu \equiv \bar{W}^a_\mu \frac{\tau^a}{2} \,, \end{aligned} \tag{2.6}$$

and under $U(1)_Y$ gauge transformations as follows:

$$\begin{aligned} &\bar{U} \to e^{-i\omega/2}\, \bar{U} \,, \\ &\bar{B}_\mu \to \bar{B}_\mu - \partial_\mu \omega \,. \end{aligned} \tag{2.7}$$

The effective Lagrangian in Eq. (2.2) describes the dynamics of the massive gauge bosons $\bar{W}^\pm_\mu, \bar{Z}_\mu$, and the massless photon $\bar{A}_\mu$. In order to have nontrivial solutions of the equations of motion, we furthermore couple external sources, denoted by $\bar{K}_{\mu\nu}$ and $\bar{J}^a_\mu$ in Eq. (2.2), to the gauge fields. In the applications that we will discuss below we will be forced to deal with off-shell quantities. Therefore, we want to preserve the gauge symmetry, which is imposed in the construction of the effective Lagrangian, even in the presence of these external sources.

As discussed in detail for the Abelian Higgs model in Ref. [22], for QED in Ref. [32], and for the standard model in Ref. [16], the appropriate choice of the source terms is crucial for a manifestly gauge-invariant analysis. The sources will only respect the gauge symmetry, if they do not couple to the gauge degrees of freedom. Otherwise, one has to impose constraints on the fields in order to solve the equations of motion. Usually, this problem is cured by fixing a gauge. However, one can also turn the argument around and consider only those external sources which couple to gauge-invariant operators. As we will see below, such a manifestly gauge-invariant treatment is in fact possible at the classical level as well as when quantum corrections are taken into account.

In this respect our approach to the effective field theory description of a strongly interacting electroweak symmetry breaking sector differs from the one that is usually adopted in the literature [8–11]. Although the authors of these references also start with a gauge-invariant effective Lagrangian they then add gauge-fixing and Faddeev-Popov terms. Since these terms break the gauge symmetry these authors, as well as those of Refs. [19–21,23–26], are then working with gauge-dependent Green's functions.

In order to write down appropriate source terms we will first introduce fields for the dynamical degrees of freedom which are already invariant under the non-Abelian group $SU(2)_L$ and, in parts, under the Abelian group $U(1)_Y$ as well. It has been known for a long time [33–35] that all fields in the standard model Lagrangian can be written, in the spontaneously broken phase, in a gauge-invariant way up to the unbroken $U(1)_{\rm em}$. A similar approach can be employed for the effective field theory description. Defining the Y-charge conjugate doublet by

$$\tilde{\bar{U}} = i\tau_2 \bar{U}^* \,, \tag{2.8}$$

we can introduce the following fields, see also Ref. [16]:

$$\bar{\mathcal{W}}^+_\mu = \frac{i}{2}\left(\tilde{\bar{U}}^\dagger(\bar{D}_\mu\bar{U}) - (\bar{D}_\mu\tilde{\bar{U}})^\dagger\bar{U}\right) \,, \tag{2.9}$$

$$\bar{\mathcal{W}}^-_\mu = \frac{i}{2}\left(\bar{U}^\dagger(\bar{D}_\mu\tilde{\bar{U}}) - (\bar{D}_\mu\bar{U})^\dagger\tilde{\bar{U}}\right) \,, \tag{2.10}$$

$$\bar{\mathcal{Z}}_\mu = i\left(\tilde{\bar{U}}^\dagger(\bar{D}_\mu\tilde{\bar{U}}) - \bar{U}^\dagger(\bar{D}_\mu\bar{U})\right) \,, \tag{2.11}$$

$$\bar{\mathcal{A}}_\mu = \bar{B}_\mu + \bar{s}^2\bar{\mathcal{Z}}_\mu \,, \tag{2.12}$$

$$\bar{\mathcal{W}}^\pm_\mu = \frac{1}{2}(\bar{\mathcal{W}}^1_\mu \mp i\bar{\mathcal{W}}^2_\mu) \,, \tag{2.13}$$

which are invariant under the $SU(2)_L$ gauge transformations from Eq. (2.6). In Eq. (2.12) we used the following definition of the weak mixing angle:

$$\bar{c}^2 \equiv \cos^2 \bar{\theta}_W = M_W^2/M_Z^2 \, , \; \bar{s}^2 \equiv 1 - \bar{c}^2 \; . \tag{2.14}$$

In order to calculate Green's functions from which we then can extract physical masses, coupling constants and S-matrix elements, we have to introduce external sources which emit one-particle states of the gauge bosons. In analogy to our effective field theory analysis of the Abelian Higgs model [22] we couple a source to the field strength $\bar{B}_{\mu\nu}$. For the massive gauge bosons the situation is more involved. Whereas the field $\bar{\mathcal{Z}}_\mu$ is fully $SU(2)_L \times U(1)_Y$ gauge-invariant, the charged gauge fields $\bar{\mathcal{W}}_\mu^\pm$ have a residual gauge dependence under the $U(1)_Y$ gauge transformations from Eq. (3.4)[3]:

$$\bar{\mathcal{W}}_\mu^\pm \to e^{\mp i\omega} \bar{\mathcal{W}}_\mu^\pm \; . \tag{2.15}$$

We can, however, compensate this gauge dependence by multiplying the charged fields $\bar{\mathcal{W}}_\mu^\pm$ by a phase factor [36,37,32,16]. Appropriate $SU(2)_L \times U(1)_Y$ gauge-invariant source terms for all the fields can then be written in the following way:

$$\bar{K}_{\mu\nu} \bar{B}_{\mu\nu} \, , \quad \bar{J}_\mu^+ \bar{\varphi}^+ \bar{\mathcal{W}}_\mu^- + \bar{J}_\mu^- \bar{\varphi}^- \bar{\mathcal{W}}_\mu^+ \, , \quad \bar{J}_\mu^{\mathcal{Z}} \bar{\mathcal{Z}}_\mu \, , \tag{2.16}$$

with external sources $\bar{K}_{\mu\nu}, \bar{J}_\mu^\pm$, and $\bar{J}_\mu^{\mathcal{Z}}$. The phase factor in Eq. (2.16) is defined by

$$\bar{\varphi}^\pm(x) = \exp\left(\mp i \int d^d y \, \mathcal{G}_0(x-y) \, \partial_\mu \bar{B}_\mu(y) \right) . \tag{2.17}$$

with

$$\mathcal{G}_0(x-y) = \langle x | \frac{1}{-\Box} | y \rangle . \tag{2.18}$$

For computational convenience we are working in Euclidean space-time.

Using identities of the form

$$\begin{aligned}
\bar{D}_\mu \bar{U} &= \frac{i}{2} \bar{\mathcal{Z}}_\mu \bar{U} - i \bar{\mathcal{W}}_\mu^+ \tilde{\bar{U}} \, , \\
\bar{D}_\mu \tilde{\bar{U}} &= -i \bar{\mathcal{W}}_\mu^- \bar{U} - \frac{i}{2} \bar{\mathcal{Z}}_\mu \tilde{\bar{U}} \, , \\
\bar{D}_\mu \bar{D}_\nu \bar{U} &= \left(\frac{i}{2} (\partial_\mu \bar{\mathcal{Z}}_\nu) - \frac{1}{4} \bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\nu - \bar{\mathcal{W}}_\mu^- \bar{\mathcal{W}}_\nu^+ \right) \bar{U} + \left(-i \bar{d}_\mu \bar{\mathcal{W}}_\nu^+ + \frac{1}{2} \bar{\mathcal{W}}_\mu^+ \bar{\mathcal{Z}}_\nu - \frac{1}{2} \bar{\mathcal{Z}}_\mu \bar{\mathcal{W}}_\nu^+ \right) \tilde{\bar{U}} \, , \\
\bar{D}_\mu \bar{D}_\nu \tilde{\bar{U}} &= \left(-i \bar{d}_\mu \bar{\mathcal{W}}_\nu^- + \frac{1}{2} \bar{\mathcal{Z}}_\mu \bar{\mathcal{W}}_\nu^- - \frac{1}{2} \bar{\mathcal{W}}_\mu^- \bar{\mathcal{Z}}_\nu \right) \bar{U} + \left(-\frac{i}{2} (\partial_\mu \bar{\mathcal{Z}}_\nu) - \frac{1}{4} \bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\nu - \bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^- \right) \tilde{\bar{U}} \, ,
\end{aligned} \tag{2.19}$$

where

$$\bar{d}_\mu \bar{\mathcal{W}}_\nu^\pm = \left(\partial_\mu \mp i \bar{B}_\mu \right) \bar{\mathcal{W}}_\nu^\pm \, , \tag{2.20}$$

one can express the Lagrangian in terms of the fields $\bar{\mathcal{W}}_\mu^\pm, \bar{\mathcal{Z}}_\mu, \bar{B}_\mu$, and covariant derivatives thereof

$$\mathcal{L}_{eff} = \mathcal{L}_{eff} \left(\bar{\mathcal{W}}_\mu^\pm, \bar{\mathcal{Z}}_\mu, \bar{B}_\mu, \ldots ; \bar{K}_{\mu\nu}, \bar{J}_\mu^\pm, \bar{J}_\mu^{\mathcal{Z}}, \right) \; . \tag{2.21}$$

As a matter of convenience we write the field $\bar{B}_\mu$ in Eq. (2.21) instead of the photon field $\bar{\mathcal{A}}_\mu$.

The generating functional in the effective field theory is given by the path integral

$$e^{-W_{eff}[\bar{K}_{\mu\nu}, \bar{J}_\mu^\pm, \bar{J}_\mu^{\mathcal{Z}}]} = \int \mathrm{d}\mu[\bar{U}, \bar{W}_\mu^a, \bar{B}_\mu] e^{-\int \mathrm{d}^d x \mathcal{L}_{eff}} \; . \tag{2.22}$$

Note that we still integrate over the original fields $\bar{U}, \bar{W}_\mu^a$, and $\bar{B}_\mu$ in Eq. (2.22). Furthermore, we have absorbed an appropriate normalization factor into the measure $\mathrm{d}\mu[\bar{U}, \bar{W}_\mu^a, \bar{B}_\mu]$. Derivatives of this functional with respect to the

[3]Note that the $SU(2)_L$ invariant field $\bar{\mathcal{A}}_\mu$ from Eq. (2.12) transforms under $U(1)_Y$ as $\bar{\mathcal{A}}_\mu \to \bar{\mathcal{A}}_\mu - \partial_\mu \omega$, i.e. like an Abelian gauge field.

source $\bar{K}_{\mu\nu}$ generate Green's functions of the field strength $\bar{B}_{\mu\nu}$, while derivatives with respect to $\bar{J}^{\pm}_{\mu}$ and $\bar{J}^{\mathcal{Z}}_{\mu}$ generate Green's functions for the gauge-invariant fields $\bar{\varphi}^{\mp}\bar{\mathcal{W}}^{\pm}_{\mu}$ and $\bar{\mathcal{Z}}_{\mu}$, respectively. As was pointed out in Refs. [22,32,16] it is possible to evaluate the path integral in Eq. (2.22) without the need to fix a gauge as will be shown below.

The effective Lagrangian $\mathcal{L}_{eff}$ in Eq. (2.21) is a sum of terms with an increasing number of derivatives, mass factors, and powers of external sources, corresponding to an expansion in powers of the momenta and the masses,

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots \,, \tag{2.23}$$

where $\mathcal{L}_k$ is of order p^k and has the general form

$$\mathcal{L}_k = \sum_i l_i^{(k)} \mathcal{O}_i^{(k)} \,. \tag{2.24}$$

The coefficients $l_i^{(k)}$ in Eq. (2.24) represent the low-energy constants of the effective theory and count as order p^0. The operators $\mathcal{O}_i^{(k)}$ involve the light fields and the sources in such a way that they respect the $SU(2)_L \times U(1)_Y$ gauge symmetry.

In order to evaluate the low-energy expansion up to a given order, we follow the counting rules usually adopted in chiral perturbation theory [12,13] for the bookkeeping of the terms in the effective Lagrangian. These rules are necessary for the internal consistency of the effective field theory. We note that they are formulated completely within the framework of the effective field theory. In particular, there is no expansion with respect to some heavy mass scale in the underlying theory involved. We thus treat the covariant derivative $\bar{D}_\mu$, the gauge boson masses M_W and M_Z and the momenta as quantities of order p, while the Goldstone boson field $\bar{U}$ is of order p^0. In counting the masses M_W and M_Z as order p, the low-energy expansion is carried out at a fixed ratio p^2/M_W^2 and p^2/M_Z^2, and correctly reproduces all singularities associated with the gauge bosons. The consistency of these rules requires that the coupling constants $\bar{g}, \bar{g}'$ and therefore the electromagnetic coupling constant $\bar{e}$, defined in Eq. (2.32) below, are also treated as quantities of order p. Note that this is different from the usual dimensional analysis: the coupling constants have dimension (mass)^0, yet they count as order p in the low-energy expansion. This is similar to chiral perturbation theory where the quark masses m_q are quantities of order p^2 [13] and where the electromagnetic coupling constant e is counted as order p if virtual photons are included [38]. Our counting rules furthermore imply that $\cos\bar{\theta}_W$ and $\sin\bar{\theta}_W$ are treated as quantities of order p^0, whereas the gauge fields $\bar{W}^a_\mu, \bar{B}_\mu$ and therefore also $\bar{\mathcal{W}}^{\pm}_\mu, \bar{\mathcal{Z}}_\mu$, and $\bar{\mathcal{A}}_\mu$ count as quantities of order p. Finally, the external sources $\bar{J}^{\pm}_\mu$ and $\bar{J}^{\mathcal{Z}}_\mu$ count as quantities of order p, while the source $\bar{K}_{\mu\nu}$ and the phase factor $\bar{\varphi}^{\pm}$ are of order p^0.

In general, there are two different kinds of contributions to the generating functional. On the one hand, one has tree-level contributions given by the integral $\int d^dx \mathcal{L}_{eff}$, which has to be evaluated at the stationary point, i.e., with the solutions of the equations of motion. On the other hand there are contributions from loops, which ensure unitarity. General power counting arguments show that n-loop corrections are suppressed by at least $2n$ powers of the momentum [12]. For instance, tree-level contributions with one vertex from $\mathcal{L}_k$ and any number of vertices from $\mathcal{L}_2$ are of order p^k, while one-loop corrections with one vertex from $\mathcal{L}_k$ and any number of vertices from $\mathcal{L}_2$ are of order p^{k+2}. On the other hand, graphs with more vertices from $\mathcal{L}_{k'}$ where $k' > 2$ or with more loops are suppressed by additional powers of the momentum. The corresponding expansion of the generating functional is denoted by

$$W_{eff} = W_2 + W_4 + W_6 + \cdots \,, \tag{2.25}$$

where W_k is of order p^k.

1. The generating functional at order p^2

At order p^2 the effective Lagrangian can be written in the form

$$\mathcal{L}_2 = \mathcal{L}_2^0 + \mathcal{L}_2^s \,, \tag{2.26}$$

with

$$\mathcal{L}_2^0 = \frac{\bar{v}^2}{2}\left(\bar{\mathcal{W}}^+_\mu \bar{\mathcal{W}}^-_\mu + \bar{\rho}\frac{1}{4}\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu\right) + \frac{1}{4\bar{g}^2}\bar{\mathcal{W}}^a_{\mu\nu}\bar{\mathcal{W}}^a_{\mu\nu} + \frac{1}{4\bar{g}'^2}\bar{B}_{\mu\nu}\bar{B}_{\mu\nu} \,, \tag{2.27}$$

and

$$\mathcal{L}_2^s = -\frac{1}{2}\bar{K}_{\mu\nu}\bar{B}_{\mu\nu} + 2\bar{v}^2(\bar{j}_\mu^+\bar{\mathcal{W}}_\mu^- + \bar{j}_\mu^-\bar{\mathcal{W}}_\mu^+) + \bar{v}^2\bar{J}_\mu^{\mathcal{Z}}\bar{\mathcal{Z}}_\mu + 4\bar{c}_{\mathcal{W}}\bar{v}^2\bar{J}_\mu^+\bar{J}_\mu^- + \bar{c}_{\mathcal{Z}}\bar{v}^2\bar{J}_\mu^{\mathcal{Z}}\bar{J}_\mu^{\mathcal{Z}}\,, \tag{2.28}$$

where

$$\bar{\mathcal{W}}_{\mu\nu}^a = \partial_\mu\bar{\mathcal{W}}_\nu^a - \partial_\nu\bar{\mathcal{W}}_\mu^a + \varepsilon^{abc}\bar{\mathcal{W}}_\mu^b\bar{\mathcal{W}}_\nu^c\,,\ a = 1,2,3\,, \tag{2.29}$$

$$\bar{\mathcal{W}}_\mu^3 = \bar{\mathcal{Z}}_\mu + \bar{B}_\mu\,, \tag{2.30}$$

$$\bar{j}_\mu^\pm = \bar{\varphi}^\pm\bar{J}_\mu^\pm\,. \tag{2.31}$$

The Lagrangian $\mathcal{L}_2^0$ contains only the mass terms and the kinetic terms of the gauge bosons in the effective theory. Note that in the general effective Lagrangian $\mathcal{L}_2^s$ in Eq. (2.28) there appear additional contact terms involving the external sources only. The masses of the gauge bosons, the weak mixing angle and the electric charge can be expressed through the quantities $\bar{v}, \bar{\rho}, \bar{g}$, and $\bar{g}'$ as follows:

$$M_W^2 = \frac{\bar{v}^2\bar{e}^2}{4\bar{s}^2}\,,\ M_Z^2 = \bar{\rho}\frac{\bar{v}^2\bar{e}^2}{4\bar{s}^2\bar{c}^2}\,,\ \bar{c}^2 = \frac{\bar{g}^2}{\bar{g}^2+\bar{g}'^2}\,, \bar{e}^2 = \frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\,. \tag{2.32}$$

The expression for the weak mixing angle $\bar{c}^2$ follows from the requirement that the field $\bar{\mathcal{Z}}_\mu = \bar{\mathcal{W}}_\mu^3 - \bar{B}_\mu$ is invariant under gauge transformations. Similarly, the electric charge $\bar{e}$ is determined by the coupling of the charged gauge boson $\bar{\mathcal{W}}_\mu^\pm$ to the photon field $\bar{\mathcal{A}}_\mu$. The low-energy constants $\bar{v}$ and $\bar{\rho}-1$ are of order p^0. Note that $\bar{\rho} \equiv M_Z^2\bar{c}^2/M_W^2$ is the inverse of the usual ρ-parameter. In allowing $\bar{\rho} \neq 1$ we do not assume that custodial symmetry breaking effects vanish at leading order in the low-energy expansion. Hence, we follow the first paper of Ref. [9] and Ref. [10]. Note that in the recent literature it became customary to include such a custodial symmetry breaking term only at order p^4, following the conventions used in the second paper of Ref. [9] and the second paper of Ref. [19]. Since $\bar{\rho}-1$ is very small [39] this might indeed be justified, if the low-energy expansion is carried out up to order p^4 or higher.

At order p^2, the generating functional of the effective field theory is given by

$$W_2[\bar{K}_{\mu\nu}, \bar{J}_\mu^\pm, \bar{J}_\mu^{\mathcal{Z}}] = \int d^dx\mathcal{L}_2\left(\bar{\mathcal{W}}_\mu^a, \bar{B}_\mu; \bar{K}_{\mu\nu}, \bar{J}_\mu^\pm, \bar{J}_\mu^{\mathcal{Z}}\right)\,, \tag{2.33}$$

where the gauge fields satisfy the equations of motion

$$\begin{aligned} -\bar{d}_\mu\bar{\mathcal{W}}_{\mu\nu}^\pm = {} & -M_W^2\bar{\mathcal{Y}}_\nu^\pm \pm i(\bar{\mathcal{Z}}_{\mu\nu}+\bar{B}_{\mu\nu})\bar{\mathcal{W}}_\mu^\pm \mp i\bar{\mathcal{W}}_{\mu\nu}^\pm\bar{\mathcal{Z}}_\mu \mp i(\partial_\mu\bar{\mathcal{Z}}_\mu)\bar{\mathcal{W}}_\nu^\pm \pm i(\partial_\mu\bar{\mathcal{Z}}_\nu)\bar{\mathcal{W}}_\mu^\pm \\ & \pm i\bar{\mathcal{Z}}_\nu\bar{d}_\mu\bar{\mathcal{W}}_\mu^\pm \mp i\bar{\mathcal{Z}}_\mu\bar{d}_\mu\bar{\mathcal{W}}_\nu^\pm - (\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\mu)\bar{\mathcal{W}}_\nu^\pm + (\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\nu)\bar{\mathcal{W}}_\mu^\pm \pm 2\bar{\mathcal{W}}_\mu^\pm(\bar{\mathcal{W}}_\mu^+\bar{\mathcal{W}}_\nu^- - \bar{\mathcal{W}}_\nu^+\bar{\mathcal{W}}_\mu^-)\,, \end{aligned} \tag{2.34}$$

$$\begin{aligned} -\partial_\mu(\bar{\mathcal{Z}}_{\mu\nu}+\bar{B}_{\mu\nu}) = {} & -\bar{c}^2M_Z^2\bar{\mathcal{Y}}_\nu^{\mathcal{Z}} + 2\bar{\mathcal{Z}}_\mu(\bar{\mathcal{W}}_\mu^+\bar{\mathcal{W}}_\nu^- + \bar{\mathcal{W}}_\nu^+\bar{\mathcal{W}}_\mu^-) - 4\bar{\mathcal{Z}}_\nu\bar{\mathcal{W}}_\mu^+\bar{\mathcal{W}}_\mu^- + 2i(\bar{\mathcal{W}}_{\mu\nu}^+\bar{\mathcal{W}}_\mu^- - \bar{\mathcal{W}}_{\mu\nu}^-\bar{\mathcal{W}}_\mu^+) \\ & -2i(\bar{d}_\mu\bar{\mathcal{W}}_\mu^+\bar{\mathcal{W}}_\nu^- - \bar{d}_\mu\bar{\mathcal{W}}_\mu^-\bar{\mathcal{W}}_\nu^+ - \bar{d}_\mu\bar{\mathcal{W}}_\nu^+\bar{\mathcal{W}}_\mu^- + \bar{d}_\mu\bar{\mathcal{W}}_\nu^-\bar{\mathcal{W}}_\mu^+)\,, \end{aligned} \tag{2.35}$$

$$-\partial_\mu\bar{B}_{\mu\nu} = \bar{s}^2M_Z^2\mathrm{PT}_{\nu\mu}\bar{\mathcal{Y}}_\mu^{\mathcal{Z}} - \frac{\bar{e}^2}{\bar{c}^2}\partial_\mu\bar{K}_{\mu\nu}\,. \tag{2.36}$$

Using relation (2.12) the equations of motion for the massive gauge field $\bar{\mathcal{Z}}_\mu$ and the photon field $\bar{\mathcal{A}}_\mu$ can be obtained. The constraints are given by

$$\bar{d}_\mu\bar{\mathcal{Y}}_\mu^\pm = \pm i\bar{\mathcal{Z}}_\mu\bar{\mathcal{Y}}_\mu^\pm \mp i\bar{\rho}\bar{\mathcal{Y}}_\mu^{\mathcal{Z}}\bar{\mathcal{W}}_\mu^\pm\,, \tag{2.37}$$

$$\partial_\mu\bar{\mathcal{Y}}_\mu^{\mathcal{Z}} = 8i\frac{1}{\bar{\rho}}(\bar{\mathcal{W}}_\mu^+\bar{j}_\mu^- - \bar{\mathcal{W}}_\mu^-\bar{j}_\mu^+)\,. \tag{2.38}$$

They are obtained by varying the effective Lagrangian $\mathcal{L}_2$ with respect to the Goldstone boson field $\bar{U}$. In Eqs. (2.34)–(2.38) we have introduced the quantities

$$\bar{\mathcal{W}}_{\mu\nu}^\pm = \bar{d}_\mu\bar{\mathcal{W}}_\nu^\pm - \bar{d}_\nu\bar{\mathcal{W}}_\mu^\pm\,, \tag{2.39}$$

$$\bar{\mathcal{Z}}_{\mu\nu} = \partial_\mu\bar{\mathcal{Z}}_\nu - \partial_\nu\bar{\mathcal{Z}}_\mu\,, \tag{2.40}$$

$$\bar{\mathcal{Y}}_\mu^\pm = \bar{\mathcal{W}}_\mu^\pm + 4\bar{j}_\mu^\pm\,,\ \bar{\mathcal{Y}}_\mu^{\mathcal{Z}} = \bar{\mathcal{Z}}_\mu + 4\frac{1}{\bar{\rho}}\bar{J}_\mu^{\mathcal{Z}}\,, \tag{2.41}$$

$$\mathrm{PT}_{\mu\nu} = \delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\,. \tag{2.42}$$

The covariant derivatives in $\bar{d}_\mu\bar{\mathcal{W}}_{\mu\nu}^\pm$ and $\bar{d}_\mu\bar{\mathcal{Y}}_\mu^\pm$ are defined in the same way as in Eq. (2.20).

Several things about the equations of motion (2.34)–(2.38) are worth notice. As discussed in Ref. [16] the equations of motion uniquely determine only the physical degrees of freedom since we did not fix a gauge. The equations of motion can be rewritten in a form which only involves fully $SU(2)_L \times U(1)_Y$ gauge-invariant fields. Solutions for the massive gauge boson fields $\bar{\varphi}^{\mp}\bar{\mathcal{W}}^{\pm}_{\mu}$ follow from Eq. (2.34). Suitable linear combinations of Eqs. (2.35) and (2.36) determine the gauge boson field $\bar{\mathcal{Z}}_{\mu}$ and the transverse component of the massless photon field $\bar{\mathcal{A}}^T_{\mu} = \mathrm{PT}_{\mu\nu}\bar{\mathcal{A}}_{\nu}$. Note that the equations of motion do not determine the longitudinal component of the photon field and the phase of the gauge boson fields $\bar{\mathcal{W}}^{\pm}_{\mu}$ which correspond to the $U(1)_Y$ gauge degree of freedom. Even more they do not determine the classical Goldstone boson field $\bar{U}$ either, since it corresponds to the $SU(2)_L$ gauge degrees of freedom. Thus, gauge invariance implies that these equations have a whole class of solutions in terms of the original fields $\bar{U}, \bar{W}^a_{\mu}, \bar{B}_{\mu}$. Every two representatives are related to each other by a gauge transformation. Nevertheless, the physical degrees of freedom are uniquely determined by these equations of motion. Moreover, since the action is gauge-invariant, the generating functional in Eq. (2.33) is uniquely determined for the given set of source terms.

The most important point is the fact that the classical Goldstone boson field $\bar{U}$ represents the $SU(2)_L$ gauge degrees of freedom. Thus, no Goldstone bosons are propagating at the classical level of the theory. All gauge-invariant sources emit physical modes only. Moreover, Eqs. (2.37) and (2.38), which follow from the requirement that the variation of the Lagrangian with respect to the Goldstone boson field $\bar{U}$ vanishes, are not equations of motion, but constraints expressing the fact that the gauge fields $\bar{\varphi}^{\mp}\bar{\mathcal{W}}^{\pm}_{\mu}$, $\bar{\mathcal{Z}}_{\mu}$, and $\bar{\mathcal{A}}_{\mu}$ couple to conserved currents. They can also be obtained by taking the derivative of the equations of motion for the gauge fields. Note that we have already used the constraints to bring these equations of motion into the form given in Eqs. (2.34)–(2.36).

We note that the equations of motion can be solved in powers of the external sources, see Ref. [16].

2. The generating functional at order p^4

The one-loop contribution to the generating functional can be evaluated with the saddle-point method. If we write the fluctuations $\bar{y}$ around the classical fields $\bar{\mathcal{F}}^{cl}$ as $\bar{\mathcal{F}} = \bar{\mathcal{F}}^{cl} + \bar{y}$, we obtain the following representation for the one-loop approximation to the generating functional:

$$e^{-W_{eff}[\bar{K}_{\mu\nu},\bar{J}^{\pm}_{\mu},\bar{J}^{\mathcal{Z}}_{\mu}]} = e^{-\int \mathrm{d}^d x \mathcal{L}^{cl}_{eff}} \int \mathrm{d}\mu[\bar{y}] e^{-(1/2)\int \mathrm{d}^d x \bar{y}^T \tilde{\bar{D}} \bar{y}} \ . \tag{2.43}$$

Gauge invariance implies that the operator $\tilde{\bar{D}}$ has zero eigenvalues corresponding to fluctuations $\bar{y}$ which are equivalent to infinitesimal gauge transformations. Indeed, if $\bar{\mathcal{F}}^{cl,i}$ is a solution of the equation of motion, i.e., a stationary point of the classical action,

$$\left.\frac{\delta S_{eff}}{\delta \bar{\mathcal{F}}^i}\right|_{\bar{\mathcal{F}}=\bar{\mathcal{F}^{cl}}} = 0 \ , \tag{2.44}$$

then any gauge transformation yields another equivalent solution. The index i in $\bar{\mathcal{F}}^{cl,i}$ labels the different fields. Thus, differentiating equation (2.44) with respect to the gauge parameters ω^A one obtains

$$\left.\frac{\delta^2 S_{eff}}{\delta \bar{\mathcal{F}}^i \delta \bar{\mathcal{F}}^j} \frac{\delta \bar{\mathcal{F}}^j}{\delta \omega^A}\right|_{\bar{\mathcal{F}}=\bar{\mathcal{F}^{cl}}} = 0 \ . \tag{2.45}$$

The quadratic form which appears in Eq. (2.45) is identical to the differential operator $\tilde{\bar{D}}$. If these zero modes are treated properly [22,16], one can evaluate the path-integral representation for the generating functional at the one-loop level without the need to fix a gauge and without introducing ghost fields. Up to an irrelevant infinite constant one obtains the following result for the generating functional of the effective field theory at order p^4:

$$(W_2 + W_4)\,[\bar{K}_{\mu\nu}, \bar{J}^{\pm}_{\mu}, \bar{J}^{\mathcal{Z}}_{\mu}] = \int d^d x\,(\mathcal{L}_2 + \mathcal{L}_4) + \frac{1}{2}\ln\det{}'\tilde{\bar{D}} - \frac{1}{2}\ln\det \bar{P}^T \bar{P} \,, \tag{2.46}$$

where $\mathcal{L}_4$ is the effective Lagrangian of order p^4. The first term on the right-hand side represents the classical action which describes the tree-level contributions of order p^2 and p^4 to the generating functional. The two determinants on the right-hand side of this equation represent one-loop contributions to the generating functional. The first determinant describes all one-loop contributions with vertices from the Lagrangian $\mathcal{L}_2$ where $\det{}'\tilde{\bar{D}}$ is defined as the product of all non-zero eigenvalues of the operator $\tilde{\bar{D}}$. The second determinant originates from the path integral measure. The operator $\bar{P}$ satisfies the relation $\bar{P}^T\tilde{\bar{D}} = \tilde{\bar{D}}\bar{P} = 0$. The fields in Eq. (2.46) satisfy the equations of motion. At order p^4 the contributions from $\mathcal{L}_4$ to these equations of motion are not relevant. Hence, they are given by Eqs. (2.34)–(2.38). The explicit form of the differential operators $\tilde{\bar{D}}$ and $\bar{P}$ for the case $\bar{\rho} \neq 1$ is very complicated and we will not write it down here. We note that the results for $\tilde{\bar{D}}$ and $\bar{P}$ for $\bar{\rho} = 1$ can be inferred from the corresponding differential operators in the standard model, see the discussion after Eq. (4.12) below.

The most general effective Lagrangian at order p^4 is given by

$$\mathcal{L}_4 = \mathcal{L}_4^0 + \mathcal{L}_4^s \,. \tag{2.47}$$

The first term can be written in the form

$$\mathcal{L}_4^0 = \sum_{i=1}^{18} l_i \mathcal{O}_i \,, \tag{2.48}$$

where the operators $\mathcal{O}_i$ are given by

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\mu^-)(\bar{\mathcal{W}}_\nu^+ \bar{\mathcal{W}}_\nu^-)\,, \\
\mathcal{O}_2 &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)(\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)\,, \\
\mathcal{O}_3 &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu)(\bar{\mathcal{W}}_\nu^+ \bar{\mathcal{W}}_\nu^-)\,, \\
\mathcal{O}_4 &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\nu)(\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)\,, \\
\mathcal{O}_5 &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu)(\bar{\mathcal{Z}}_\nu \bar{\mathcal{Z}}_\nu)\,, \\
\mathcal{O}_6 &= \epsilon_{\mu\nu\rho\sigma} \bar{\mathcal{Z}}_\sigma (\bar{\mathcal{W}}_\rho^- \bar{\mathcal{W}}_{\mu\nu}^+ + \bar{\mathcal{W}}_\rho^+ \bar{\mathcal{W}}_{\mu\nu}^-)\,, \\
\mathcal{O}_7 &= i\bar{\mathcal{Z}}_{\mu\nu} (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^- - \bar{\mathcal{W}}_\nu^+ \bar{\mathcal{W}}_\mu^-)\,, \\
\mathcal{O}_8 &= i\bar{B}_{\mu\nu} (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^- - \bar{\mathcal{W}}_\nu^+ \bar{\mathcal{W}}_\mu^-)\,, \\
\mathcal{O}_9 &= i\bar{\mathcal{Z}}_\mu (\bar{d}_\mu \bar{\mathcal{W}}_\nu^+ \bar{\mathcal{W}}_\nu^- - \bar{d}_\mu \bar{\mathcal{W}}_\nu^- \bar{\mathcal{W}}_\nu^+)\,, \\
\mathcal{O}_{10} &= i\bar{\mathcal{Z}}_\nu (\bar{d}_\mu \bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^- - \bar{d}_\mu \bar{\mathcal{W}}_\mu^- \bar{\mathcal{W}}_\nu^+)\,, \\
\mathcal{O}_{11} &= \bar{\mathcal{Z}}_{\mu\nu} \bar{\mathcal{Z}}_{\mu\nu}\,, \\
\mathcal{O}_{12} &= \bar{B}_{\mu\nu} \bar{\mathcal{Z}}_{\mu\nu}\,, \\
\mathcal{O}_{13} &= (\bar{d}_\mu \bar{\mathcal{W}}_\mu^+)(\bar{d}_\nu \bar{\mathcal{W}}_\nu^-)\,, \\
\mathcal{O}_{14} &= (\partial_\mu \bar{\mathcal{Z}}_\mu)(\partial_\nu \bar{\mathcal{Z}}_\nu)\,, \\
\mathcal{O}_{15} &= M_W^2 \left(\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\mu^- + \frac{1}{4} \bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu \right)\,, \\
\mathcal{O}_{16} &= M_Z^2 \bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu\,, \\
\mathcal{O}_{17} &= \bar{\mathcal{W}}_{\mu\nu}^a \bar{\mathcal{W}}_{\mu\nu}^a\,, \\
\mathcal{O}_{18} &= \bar{B}_{\mu\nu} \bar{B}_{\mu\nu}\,.
\end{aligned} \tag{2.49}$$

We recall that we count the gauge fields $\bar{\mathcal{W}}_\mu^\pm, \bar{\mathcal{Z}}_\mu$ and the masses M_W, M_Z as order p in the low-energy expansion, therefore the custodial symmetry breaking term $\mathcal{O}_{16}$ is of the order p^4. The second term in Eq. (2.47) contains all contributions involving external sources:

$$\mathcal{L}_4^s = \sum_{i=1}^{76} l_i^s \mathcal{O}_i^s \,. \tag{2.50}$$

The operators $\mathcal{O}_i^s$ are listed in Appendix A. Note, that we consider CP-even terms only. The low-energy constants l_i and l_i^s are quantities of order p^0.

It is important to note, that the most general effective Lagrangian at this order is given as a linear combination of a maximal set of gauge-invariant terms of order p^4. One can then eliminate redundant terms by using algebraic relations of the form

$$\int d^d x (\bar{d}_\mu \bar{\mathcal{W}}_\nu^+)(\bar{d}_\nu \bar{\mathcal{W}}_\mu^-) = \int d^d x \left(\frac{1}{2}\mathcal{O}_8 + \mathcal{O}_{13} \right)\,, \tag{2.51}$$

which are readily verified by partial integration. On the other hand, the Lagrangian $\mathcal{L}_4$ contributes only at the classical level. Hence, the equations of motion (2.34)–(2.36) as well as the constraints (2.37) and (2.38) can also be used to eliminate further redundant terms [13,17]. Equivalently, one can also remove terms in the effective Lagrangian by performing appropriate reparametrizations of the fields and external sources in the path integral [18]. Note that we have already eliminated all algebraically dependent terms from the lists given in Eq. (2.49) and in Appendix A. Thus, we only need to employ the equations of motion and the constraints to eliminate further redundant terms. Note that in our gauge-invariant approach no gauge artifacts can enter through this procedure.

The constraints (2.37) and (2.38) yield the following relations between the operators in the Lagrangian $\mathcal{L}_4$:

$$\mathcal{O}_{10} = -2(1-\bar{\rho})\mathcal{O}_4 + 4\mathcal{O}^s_4 - 4\mathcal{O}^s_6 - 4\mathcal{O}^s_{46}\,, \tag{2.52}$$

$$\begin{aligned}\mathcal{O}_{13} = {} & (1-\bar{\rho})^2\mathcal{O}_4 - 4(1-\bar{\rho})\mathcal{O}^s_4 + 4(1-\bar{\rho})\mathcal{O}^s_6 + 16\mathcal{O}^s_{14} - 16\mathcal{O}^s_{17} \\ & + 16\mathcal{O}^s_{19} + 4(1-\bar{\rho})\mathcal{O}^s_{46} - 16\mathcal{O}^s_{51} + 16\mathcal{O}^s_{53} + 16\mathcal{O}^s_{74}\,,\end{aligned} \tag{2.53}$$

$$\mathcal{O}_{14} = \frac{64}{\bar{\rho}^2}(2\mathcal{O}^s_{10} - \mathcal{O}^s_{12}) + \frac{64}{\bar{\rho}^2}(\mathcal{O}^s_{49} - \mathcal{O}^s_{52}) + \frac{16}{\bar{\rho}^2}\mathcal{O}^s_{76}\,, \tag{2.54}$$

$$\mathcal{O}^s_{41} = -(1-\bar{\rho})\mathcal{O}^s_4 + 8\mathcal{O}^s_{14} - 4\mathcal{O}^s_{17} - 4\mathcal{O}^s_{51}\,, \tag{2.55}$$

$$\mathcal{O}^s_{43} = -(1-\bar{\rho})\mathcal{O}^s_6 + 4\mathcal{O}^s_{17} - 8\mathcal{O}^s_{19} - 4\mathcal{O}^s_{53}\,, \tag{2.56}$$

$$\mathcal{O}^s_{47} = \frac{8}{\bar{\rho}}(2\mathcal{O}^s_{10} - \mathcal{O}^s_{12}) + \frac{4}{\bar{\rho}}(\mathcal{O}^s_{49} - \mathcal{O}^s_{52})\,, \tag{2.57}$$

$$\mathcal{O}^s_{48} = -(1-\bar{\rho})\mathcal{O}^s_{16} + 4\mathcal{O}^s_{25} - 4\mathcal{O}^s_{27} - 4\mathcal{O}^s_{55}\,, \tag{2.58}$$

$$\mathcal{O}^s_{73} = -(1-\bar{\rho})\mathcal{O}^s_{46} + 4\mathcal{O}^s_{51} - 4\mathcal{O}^s_{53} - 8\mathcal{O}^s_{74}\,, \tag{2.59}$$

$$\mathcal{O}^s_{75} = -\frac{8}{\bar{\rho}}(\mathcal{O}^s_{49} - \mathcal{O}^s_{52}) - \frac{4}{\bar{\rho}}\mathcal{O}^s_{76}\,. \tag{2.60}$$

The equations of motion for $\bar{\mathcal{W}}^{\pm}_{\mu}$, Eq. (2.34), and $\bar{\mathcal{W}}^3_{\mu}$, Eq. (2.35), yield

$$\begin{aligned}\mathcal{O}_{11} = {} & -8\mathcal{O}_1 + 8\mathcal{O}_2 - 16\mathcal{O}_3 + 16\bar{\rho}\mathcal{O}_4 + 8\mathcal{O}_7 - 8\mathcal{O}_9 - 8\mathcal{O}_{15} + 2\bar{c}^2\left(\frac{1}{\bar{\rho}} - 2\right)\mathcal{O}_{16} - \mathcal{O}_{17} + \mathcal{O}_{18} \\ & +32\mathcal{O}^s_4 - 32\mathcal{O}^s_6 - 32\mathcal{O}^s_{46} - 16\mathcal{O}^s_{64} - 16\frac{\bar{c}^2}{\bar{\rho}}\mathcal{O}^s_{66}\,,\end{aligned} \tag{2.61}$$

$$\begin{aligned}\mathcal{O}_{12} = {} & 8\mathcal{O}_1 - 8\mathcal{O}_2 + 8\mathcal{O}_3 - 8\bar{\rho}\mathcal{O}_4 - 4\mathcal{O}_7 + 4\mathcal{O}_9 + 8\mathcal{O}_{15} - 2\bar{c}^2\left(\frac{1}{\bar{\rho}} - 1\right)\mathcal{O}_{16} + \mathcal{O}_{17} - \mathcal{O}_{18} \\ & -16\mathcal{O}^s_4 + 16\mathcal{O}^s_6 + 16\mathcal{O}^s_{46} + 16\mathcal{O}^s_{64} + 8\frac{\bar{c}^2}{\bar{\rho}}\mathcal{O}^s_{66}\,,\end{aligned} \tag{2.62}$$

$$\begin{aligned}\mathcal{O}^s_{68} = {} & -4\mathcal{O}^s_1 + 4\mathcal{O}^s_2 - 2\mathcal{O}^s_5 + 2\bar{\rho}\mathcal{O}^s_6 - \frac{32}{\bar{\rho}}\mathcal{O}^s_{10} + \frac{16}{\bar{\rho}}\mathcal{O}^s_{12} + 8\mathcal{O}^s_{17} - 16\mathcal{O}^s_{19} + 2\mathcal{O}^s_{35} + \mathcal{O}^s_{36} \\ & -4\mathcal{O}^s_{44} + 2\mathcal{O}^s_{46} - \frac{8}{\bar{\rho}}(\mathcal{O}^s_{49} - \mathcal{O}^s_{52}) - 8\mathcal{O}^s_{53} - 2\mathcal{O}^s_{64} - 16\mathcal{O}^s_{65}\,,\end{aligned} \tag{2.63}$$

$$\mathcal{O}^s_{70} = -8\mathcal{O}^s_3 + 4\bar{\rho}\mathcal{O}^s_4 + 32\mathcal{O}^s_{14} - 16\mathcal{O}^s_{17} + 4\mathcal{O}^s_{34} - 4\mathcal{O}^s_{42} - 16\mathcal{O}^s_{51} - 2\bar{c}^2\mathcal{O}^s_{66} - 8\frac{\bar{c}^2}{\bar{\rho}}\mathcal{O}^s_{67} - \mathcal{O}^s_{71}\,. \tag{2.64}$$

Note that we have frequently employed partial integrations to derive the Eqs. (2.52)–(2.64). Furthermore, we have already replaced all dependent terms on the right-hand side of Eqs. (2.61)–(2.64). Equation (2.62) can be derived by observing the identities

$$\bar{\mathcal{W}}^+_{\mu\nu}\bar{\mathcal{W}}^-_{\mu\nu} = -2\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_3 + 2\mathcal{O}_4 + 2\mathcal{O}_7 + \mathcal{O}_8 - 2\mathcal{O}_9 + 2\mathcal{O}_{10} - \frac{1}{4}\mathcal{O}_{11} - \frac{1}{2}\mathcal{O}_{12} + \frac{1}{4}\mathcal{O}_{17} - \frac{1}{4}\mathcal{O}_{18}\,, \tag{2.65}$$

and

$$\bar{\mathcal{W}}^+_{\mu\nu}\bar{\mathcal{W}}^-_{\mu\nu} = -\bar{\mathcal{W}}^+_{\nu}\bar{d}_\mu\bar{\mathcal{W}}^-_{\mu\nu} - \bar{\mathcal{W}}^-_{\nu}\bar{d}_\mu\bar{\mathcal{W}}^+_{\mu\nu}\,, \tag{2.66}$$

which are valid up to partial integrations. Afterwards one can employ the equation of motion (2.34) to substitute the expression for $\bar{d}_\mu\bar{\mathcal{W}}^{\pm}_{\mu\nu}$ in Eq. (2.66). In the same way one can obtain the relation (2.63) for $\mathcal{O}^s_{68}$. Similarly, performing partial integrations in $(\mathcal{O}_{11}+\mathcal{O}_{12})$ and $(\mathcal{O}^s_{70}+\mathcal{O}^s_{71})$ lead to $\partial_\mu(\bar{B}_{\mu\nu}+\bar{\mathcal{Z}}_{\mu\nu})$ where the equation of motion (2.35) can be applied in order to obtain Eqs. (2.61) and (2.64). Using the relations (2.52)–(2.64) one can eliminate the terms on the left-hand side of the corresponding equations from the set of terms in the Lagrangian $\mathcal{L}_4$. This reduces the number of low-energy constants by 13. Note that one has to adjust the values of the low-energy constants of the remaining terms accordingly. We will denote the modified low-energy constants by l_i' and $l_i^{s\prime}$ in order to distinguish them from the old ones.

Finally, there are terms in the Lagrangian $\mathcal{L}_4$ which are proportional to corresponding terms in the lowest order Lagrangian $\mathcal{L}_2$. These are the operators $\mathcal{O}_{15}$, $\mathcal{O}_{16}$, $\mathcal{O}_{17}$, $\mathcal{O}_{18}$, $\mathcal{O}^s_{64}$, $\mathcal{O}^s_{65}$, $\mathcal{O}^s_{66}$, $\mathcal{O}^s_{67}$, and $\mathcal{O}^s_{71}$. Following the interpretation given in Refs. [9,11] these terms lead to a renormalization of the low-energy constants and sources at order p^2 according to

$$\bar{v}^2 \to \bar{v}^2_{eff} = \bar{v}^2 \left(1 + 2l_{15}\frac{M_W^2}{\bar{v}^2}\right) , \tag{2.67}$$

$$\bar{\rho} \to \bar{\rho}_{eff} = \bar{\rho} - 2(\bar{\rho}-1)l_{15}\frac{M_W^2}{\bar{v}^2} + 8l_{16}\frac{M_Z^2}{\bar{v}^2} , \tag{2.68}$$

$$\bar{g}^2 \to \bar{g}^2_{eff} = \bar{g}^2\left(1 - 4l_{17}\bar{g}^2\right) , \tag{2.69}$$

$$\bar{g}'^2 \to \bar{g}'^2_{eff} = \bar{g}'^2\left(1 - 4l_{18}\bar{g}'^2\right) , \tag{2.70}$$

$$\bar{K}_{\mu\nu} \to \bar{K}_{\mu\nu;eff} = \bar{K}_{\mu\nu} - 2l^s_{71}\bar{J}^{\mathcal{Z}}_{\mu\nu} , \tag{2.71}$$

$$\bar{J}^{\pm}_\mu \to \bar{J}^{\pm}_{\mu;eff} = \bar{J}^{\pm}_\mu\left(1 + \left(\frac{1}{2}l^s_{64} - 2l_{15}\right)\frac{M_W^2}{\bar{v}^2}\right) , \tag{2.72}$$

$$\bar{J}^{\mathcal{Z}}_\mu \to \bar{J}^{\mathcal{Z}}_{\mu;eff} = \bar{J}^{\mathcal{Z}}_\mu\left(1 + l^s_{66}\frac{M_Z^2}{\bar{v}^2} - 2l_{15}\frac{M_W^2}{\bar{v}^2}\right) , \tag{2.73}$$

$$\bar{c}_{\mathcal{W}} \to \bar{c}_{\mathcal{W};eff} = \bar{c}_{\mathcal{W}}\left(1 + (2l_{15} - l^s_{64})\frac{M_W^2}{\bar{v}^2}\right) + \frac{1}{4}l^s_{65}\frac{M_W^2}{\bar{v}^2} , \tag{2.74}$$

$$\bar{c}_{\mathcal{Z}} \to \bar{c}_{\mathcal{Z};eff} = \bar{c}_{\mathcal{Z}}\left(1 + 2l_{15}\frac{M_W^2}{\bar{v}^2} - 2l^s_{66}\frac{M_Z^2}{\bar{v}^2}\right) + l^s_{67}\frac{M_Z^2}{\bar{v}^2} . \tag{2.75}$$

Hence, we end up with the following set of independent operators at order p^4:

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{\mathcal{W}}^+_\mu\bar{\mathcal{W}}^-_\mu)(\bar{\mathcal{W}}^+_\nu\bar{\mathcal{W}}^-_\nu) ,\\
\mathcal{O}_2 &= (\bar{\mathcal{W}}^+_\mu\bar{\mathcal{W}}^-_\nu)(\bar{\mathcal{W}}^+_\mu\bar{\mathcal{W}}^-_\nu) ,\\
\mathcal{O}_3 &= (\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\mu)(\bar{\mathcal{W}}^+_\nu\bar{\mathcal{W}}^-_\nu) ,\\
\mathcal{O}_4 &= (\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\nu)(\bar{\mathcal{W}}^+_\mu\bar{\mathcal{W}}^-_\nu) ,\\
\mathcal{O}_5 &= (\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\mu)(\bar{\mathcal{Z}}_\nu\bar{\mathcal{Z}}_\nu) ,\\
\mathcal{O}_6 &= \epsilon_{\mu\nu\rho\sigma}\bar{\mathcal{Z}}_\sigma(\bar{\mathcal{W}}^-_\rho\bar{\mathcal{W}}^+_{\mu\nu} + \bar{\mathcal{W}}^+_\rho\bar{\mathcal{W}}^-_{\mu\nu}) ,\\
\mathcal{O}_7 &= i\bar{\mathcal{Z}}_{\mu\nu}(\bar{\mathcal{W}}^+_\mu\bar{\mathcal{W}}^-_\nu - \bar{\mathcal{W}}^+_\nu\bar{\mathcal{W}}^-_\mu) ,\\
\mathcal{O}_8 &= i\bar{B}_{\mu\nu}(\bar{\mathcal{W}}^+_\mu\bar{\mathcal{W}}^-_\nu - \bar{\mathcal{W}}^+_\nu\bar{\mathcal{W}}^-_\mu) ,\\
\mathcal{O}_9 &= i\bar{\mathcal{Z}}_\mu(\bar{d}_\mu\bar{\mathcal{W}}^+_\nu\bar{\mathcal{W}}^-_\nu - \bar{d}_\mu\bar{\mathcal{W}}^-_\nu\bar{\mathcal{W}}^+_\nu) ,
\end{aligned} \tag{2.76}$$

and

$$\mathcal{O}^s_1, \ldots, \mathcal{O}^s_{40}, \mathcal{O}^s_{42}, \mathcal{O}^s_{44}, \mathcal{O}^s_{45}, \mathcal{O}^s_{46}, \mathcal{O}^s_{49}, \ldots, \mathcal{O}^s_{63}, \mathcal{O}^s_{69}, \mathcal{O}^s_{72}, \mathcal{O}^s_{74}, \mathcal{O}^s_{76} . \tag{2.77}$$

Thus, we obtain $9 + 63 = 72$ independent low-energy constants which we denote by l_i' and $l_i^{s\prime}$.

As discussed above, since $\bar{\rho} - 1$ is tiny, some people set $\bar{\rho} = 1$ and instead add the operator $M_Z^2\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\mu$ to the basis at order p^4. In order to facilitate the comparison with the literature, we cover this case by including the term

$$\mathcal{O}_0 \doteq M_Z^2\bar{\mathcal{Z}}_\mu\bar{\mathcal{Z}}_\mu \equiv \mathcal{O}_{16} , \tag{2.78}$$

with the corresponding low-energy constant l_0' into the basis from Eq. (2.76). The total number of independent low-energy constants in $\mathcal{L}_2 + \mathcal{L}_4$ remains the same, if we trade $\bar{\rho} - 1$ for l_0'. The momentum counting, however, is different, see the discussion after Eq. (2.32).

Note that one cannot obtain additional relations between the operators in $\mathcal{L}_4$ from the equation of motion for $\bar{B}_\mu$, Eq. (2.36), since it contains non-local terms involving the projection operator $\mathrm{PT}_{\mu\nu}$, cf. Eq. (2.42). Let us consider this equation in greater detail.

The presence of non-local terms in Eq. (2.36) results from our coupling sources to the non-local charged gauge-boson fields in Eq. (2.28). Indeed, switching off the sources $\bar{J}^{\pm}_{\mu}$ yields

$$\bar{\mathcal{W}}^{\pm}_{\mu} = 0\,, \tag{2.79}$$

$$\partial_{\mu}\bar{\mathcal{Y}}^{\mathcal{Z}}_{\mu} = 0\ . \tag{2.80}$$

Hence, Eq. (2.36) simplifies to

$$-\,\partial_{\mu}\bar{B}_{\mu\nu} = \bar{s}^2 M_{\mathcal{Z}}^2 \bar{\mathcal{Y}}^{\mathcal{Z}}_{\nu} - \frac{\bar{e}^2}{\bar{c}^2}\partial_{\mu}\bar{K}_{\mu\nu}\ . \tag{2.81}$$

Multiplying this equation by $\bar{\mathcal{Z}}_{\nu}$ one obtains by partial integration

$$\mathcal{O}_{12} = 2\bar{s}^2\mathcal{O}_{16} + 8\frac{\bar{s}^2}{\bar{\rho}}\mathcal{O}^{s}_{66} + \frac{\bar{e}^2}{\bar{c}^2}\bar{\mathcal{Z}}_{\mu\nu}\bar{K}_{\mu\nu}\ . \tag{2.82}$$

This relation involves the new operator

$$\bar{\mathcal{Z}}_{\mu\nu}\bar{K}_{\mu\nu}\ , \tag{2.83}$$

which we did not consider because it is physically irrelevant. In the case of the standard model the source $K_{\mu\nu}$ enters the Lagrangian as in Eq. (3.12) below. As will be shown in Sec. IV, this in turn implies that the corresponding effective field theory involves the source $\bar{K}_{\mu\nu}$ only through the single source term introduced in Eq. (2.28). As long as the field B_{μ} describes a weakly interacting $U(1)_Y$ gauge field, this is in fact true for any underlying theory. Hence, operators as the one shown in Eq. (2.83) need not be considered and Eq. (2.82) cannot be used to eliminate further redundant terms.

If the source $\bar{K}_{\mu\nu}$ is switched off as well, Eq. (2.82) simplifies to

$$\mathcal{O}_{12} = 2\bar{s}^2\mathcal{O}_{16} + 8\frac{\bar{s}^2}{\bar{\rho}}\mathcal{O}^{s}_{66}\ . \tag{2.84}$$

This relation can also be derived from Eqs. (2.61) and (2.62) since the equations of motion now have the solutions

$$\bar{\mathcal{W}}^{\pm}_{\mu} = 0 \tag{2.85}$$

$$\bar{\mathcal{A}}_{\mu} = 0\,, \tag{2.86}$$

implying $\bar{B}_{\mu} = -\bar{s}^2\bar{\mathcal{Z}}_{\mu}$ and $\bar{\mathcal{W}}^{3}_{\mu} = \bar{c}^2\bar{\mathcal{Z}}_{\mu}$. Equations (2.61), (2.62) and (2.84) do, in fact, require Eq. (2.86) to be satisfied. This result shows clearly, that one should be careful in using equations of motion to eliminate operators in the effective Lagrangian, if (some of) their solutions vanish. In doing so, one may accidentally remove terms that are not redundant at all.

In the remainder of this section we will compare our results with those obtained in the literature [10,11]. Since no source terms have been considered in these references we will switch off all the sources for the moment. Furthermore, we have to take into account that in Ref. [11] the low-energy constant $\bar{\rho}-1$ is treated as a quantity of order p^2. Thus, we will compare our 10 low-energy constants

$$l_1', \ldots, l_9' \text{ and } \bar{\rho}-1 \ \ (\text{or equivalently } l_0')\ , \tag{2.87}$$

with those obtained in the literature. The expression for the effective Lagrangian $\mathcal{L}^0_2$ in the notation which is usually used in the literature and the relation between our set of operators in $\mathcal{L}^0_4$ and the usual basis can be found in Appendix B. In Refs. [10,11] all operators in $\mathcal{L}^0_4$ that are proportional to terms in the lowest order Lagrangian $\mathcal{L}^0_2$ have been discarded right at the beginning. Hence, the authors start with 15 CP-even terms corresponding to the terms $\mathcal{O}_1, \ldots, \mathcal{O}_{14}$ and $\mathcal{O}_{16}$ in Eq. (2.49), see also Eq. (B4).

By making use of the equations of motion, $\mathrm{tr}(\hat{D}_{\mu}\hat{V}_{\mu}) = 0$ (for notations see Appendix B), corresponding to our constraints (2.37) and (2.38), the number of terms was reduced from 15 to 12 in these references. In fact, the three relations

$$L_{11} = 0\,, \tag{2.88}$$

$$L_{12} = 0\,, \tag{2.89}$$

$$L_{13} = \frac{1}{4}\bar{B}_{\mu\nu}\bar{B}_{\mu\nu} + L_1 + L_4 - L_5 - L_6 + L_7 + L_8\,, \tag{2.90}$$

given in Ref. [11][4] correspond to Eqs. (2.52)–(2.54), if we set all sources to zero and assume $\bar{\rho} = 1$ at leading order, i.e. to

$$\mathcal{O}_{10} = 0\,, \quad \mathcal{O}_{13} = 0\,, \quad \mathcal{O}_{14} = 0\,. \tag{2.91}$$

Note especially that Eq. (2.90) corresponds to $\mathcal{O}_{14} = 0$ in our basis, cf. the relation between the two sets of operators which is given in Eq. (B6).

In addition to the constraints we furthermore use the equations of motion for the gauge fields (2.34) and (2.35) to reduce the number of low-energy constants from 12 to 10. Since this step was not taken in Refs. [10,11] the set of low-energy constants used in these references is redundant.

This is an important result and we would like to add some comments. First of all, we stress again that we are studying for the moment a purely bosonic effective field theory which describes any underlying theory with the same symmetry breaking pattern as the standard model, i.e. no fermions have been included in the effective Lagrangian. In order to really compare our findings with Refs. [10,11] one has to consider the fermions in the analysis, which was implicitly done in these references, see also Ref. [21]. We will come back to this point below.

Using Eqs. (2.61) and (2.62) we have *chosen* to remove the operators $\mathcal{O}_{11}$ and $\mathcal{O}_{12}$ from the effective Lagrangian in Eq. (2.49). These operators contribute to the self-energies of the gauge bosons which are not observable anyway. In the basis which is usually used in the literature this corresponds to removing the operators L_1 and L_8 from the basis, see Appendix B. Sometimes the corresponding low-energy constants a_1 and a_8 are identified with the oblique correction parameters S and U [2]. Furthermore, the parameter T is identified with the low-energy constant a_0 which corresponds to $\bar{\rho}-1$, or, depending on the momentum counting, to the low-energy constant l_0' in our basis. Before any conclusions about the oblique parameters can be drawn, however, one has to study the inclusion of fermions in the effective field theory. This will be done below where we will compare our results with the experimentally determined values for the oblique parameters S, T, and U.

Of course, within our functional approach the source terms have to be considered as well. Even in this case, however, only the 10 low-energy constants $l_1', \ldots, l_9'$ and $\bar{\rho} - 1$ (or equivalently l_0') will contribute to physical quantities, like S-matrix elements, masses and decay constants of gauge bosons. The first group of source terms which will obviously not contribute to physical quantities are the contact terms $\mathcal{O}^s_{65}$, $\mathcal{O}^s_{67}$, $\mathcal{O}^s_{69}$, $\mathcal{O}^s_{72}$, $\mathcal{O}^s_{74}$, and $\mathcal{O}^s_{76}$ with two powers of the external sources, cf. Eq. (A3), and all terms in $\mathcal{L}^s_4$ with three or four powers of the fields and sources which contain at least one factor with an external source, i.e. the operators $\mathcal{O}^s_1, \ldots, \mathcal{O}^s_{63}$ in Eqs. (A1) and (A2). This is due to the fact that in physical S-matrix elements all external lines are amputated from the Green's functions. The corresponding low-energy constants are thus similar to the constants h_i in the ordinary chiral Lagrangian [13]. Furthermore, with the help of Eqs. (2.63), (2.64), (2.59), and (2.60), one can remove the operators $\mathcal{O}^s_{68}, \mathcal{O}^s_{70}, \mathcal{O}^s_{73}$, and $\mathcal{O}^s_{75}$ from the basis. Finally, the operators $\mathcal{O}^s_{64}, \mathcal{O}^s_{66}$ and $\mathcal{O}^s_{71}$ lead only to a renormalization of the sources $\bar{J}^{\pm}_\mu$, $\bar{J}^{\mathcal{Z}}_\mu$, and $\bar{K}_{\mu\nu}$ in the lowest order effective Lagrangian in Eq. (2.28), cf. Eqs. (2.71)–(2.73).

In summary, in a purely bosonic effective field theory with the same symmetry breaking pattern as the standard model, there are only 10 instead of 12 physically relevant low-energy constants at order p^4 in the electroweak chiral Lagrangian. In particular, one can choose to remove two low-energy constants l_{11} and l_{12} which contribute to the self-energies of the gauge bosons. An additional number of 63 low-energy constants contributes to the off-shell behavior of our gauge-invariant Green's functions. The latter low-energy constants, however, do not enter physical quantities.

The situation is more involved, however, if fermions are included in the analysis, since in that case the sources $\bar{J}^{\pm}_\mu$ and $\bar{J}^{\mathcal{Z}}_\mu$ also contain fermionic currents. We will now comment on this point.

B. On the inclusion of fermions

The fermionic part of the effective Lagrangian is of the form

$$\mathcal{L}^f_{eff} = \mathcal{L}^f_{eff}\left(\Psi^k_L, u^k_R, d^k_R, \bar{U}, D_\mu\Psi^k_L, D_\mu u^k_R, D_\mu d^k_R, \bar{D}_\mu\bar{U}, \ldots; M^k_L, N^k_L, M^k_R, N^k_R\right)\,, \tag{2.92}$$

where Ψ^k_L denotes the left-handed iso-doublet fields while d^k_R and u^k_R represent right-handed up- and down-type fermion fields comprising leptons and quarks. Note that all our fermion fields are weak eigenstates. The quantities $M^k_{L,R}$ and $N^k_{L,R}$ denote external sources coupling to these fermion fields. As discussed for the bosonic part, the effective

[4] We obtain a different sign of the terms L_4 and L_5 in Eq. (2.90) compared to Ref. [11].

Lagrangian is a sum of terms with an increasing number of derivatives and powers of fields and sources corresponding to an expansion of the generating functional in powers of the momenta and the masses. In addition to the counting rules discussed above we require that fermion fields are treated as quantities of order $\sqrt{p}$ and fermion masses, denoted by m_f^k, as of order p. This ensures that the low-energy expansion is carried out at a fixed ratio m_f^k/p.

The left-handed iso-doublet fields transform under $SU(2)_L$ gauge transformations in the following way:

$$\Psi_L^k \to \mathcal{V}\Psi_L^k\,, \quad \mathcal{V} \in SU(2)\,, \tag{2.93}$$

and under $U(1)_Y$ gauge transformations as follows:

$$\Psi_L^k \to e^{-iY(\Psi_L^k)\omega/2}\ \Psi_L^k. \tag{2.94}$$

The iso-singlets transform under $U(1)_Y$ gauge transformations in the following way:

$$\begin{aligned} u_R^k &\to e^{-iY(u_R^k)\omega/2}\ u_R^k\,, \\ d_R^k &\to e^{-iY(d_R^k)\omega/2}\ d_R^k\,. \end{aligned} \tag{2.95}$$

The hypercharges for lepton fields are $Y(\Psi_L^k) = -1$, $Y(u_R^k) = 0$ and $Y(d_R^k) = -2$ while those for quark fields are $Y(\Psi_L^k) = \frac{1}{3}$, $Y(u_R^k) = \frac{4}{3}$ and $Y(d_R^k) = -\frac{2}{3}$. The covariant derivatives for the fermion fields in Eq. (2.92) are given by

$$D_\mu \Psi_L^k = \left(\partial_\mu - i\frac{\tau^a}{2}\bar{W}_\mu^a - i\frac{Y(\Psi_L^k)}{2}\bar{B}_\mu\right)\Psi_L^k\,, \tag{2.96}$$

$$D_\mu f_R^k = \left(\partial_\mu - i\frac{Y(f_R^k)}{2}\bar{B}_\mu\right) f_R^k\,, \qquad f = u, d\,. \tag{2.97}$$

Following our approach to the bosonic sector, we can rewrite the effective Lagrangian (2.92) in terms of $SU(2)_L$ invariant fields, which are defined as [32]

$$u_L^k = \tilde{\bar{U}}^\dagger \Psi_L^k\,, \tag{2.98}$$

$$d_L^k = \bar{U}^\dagger \Psi_L^k\,. \tag{2.99}$$

They transform under $U(1)_Y$ gauge transformations as

$$\begin{aligned} u_L^k &\to e^{-iY(u_L^k)\omega/2}\ u_L^k\,, \\ d_L^k &\to e^{-iY(d_L^k)\omega/2}\ d_L^k\,, \end{aligned} \tag{2.100}$$

where $Y(u_L^k) = Y(u_R^k)$ and $Y(d_L^k) = Y(d_R^k)$.

At order p^2 the fermionic part of the effective Lagrangian contains several terms

$$\mathcal{L}_2^f = \mathcal{L}_2^{f,kin} + \mathcal{L}_2^{f,Y} + \mathcal{L}_2^{f,CC} + \mathcal{L}_2^{f,NC} + \mathcal{L}_2^{f,4F} + \mathcal{L}_2^{f,s}\,. \tag{2.101}$$

They denote the kinetic part of the Lagrangian, the Yukawa couplings, the coupling to charged and neutral currents, four-fermion interactions and source terms. The first four terms can readily be inferred from the corresponding terms in the fermionic sector of the standard model [32]

$$\mathcal{L}_2^{f,kin} = \sum_k \left(\bar{d}_L^k i\not{D} d_L^k + \bar{u}_L^k i\not{D} u_L^k + \bar{d}_R^k i\not{D} d_R^k + \bar{u}_R^k i\not{D} u_R^k\right)\,, \tag{2.102}$$

$$\mathcal{L}_2^{f,Y} = \bar{v}\sum_{ij}\left(\bar{g}_{ij}\bar{d}_L^i d_R^j + \bar{g}_{ji}^*\bar{d}_R^i d_L^j + \bar{h}_{ij}\bar{u}_L^i u_R^j + \bar{h}_{ji}^*\bar{u}_R^i u_L^j\right)\,, \tag{2.103}$$

$$\mathcal{L}_2^{f,CC} = \sum_{ij} c_{CC}^{ij,L}\left(\bar{\mathcal{W}}_\mu^+ j_\mu^{L,ij-} + \bar{\mathcal{W}}_\mu^- j_\mu^{L,ij+}\right) + c_{CC}^{ij,R}\left(\bar{\mathcal{W}}_\mu^+ j_\mu^{R,ij-} + \bar{\mathcal{W}}_\mu^- j_\mu^{R,ij+}\right)\,, \tag{2.104}$$

$$\mathcal{L}_2^{f,NC} = \sum_{ij} c_{NC}^{ij,L}\bar{\mathcal{Z}}_\mu J_\mu^{L,ij3} + \sum_{ij} c_{NC}^{ij,R}\bar{\mathcal{Z}}_\mu J_\mu^{R,ij3} - \bar{s}^2 J_\mu^{f,Q}\bar{\mathcal{Z}}_\mu\,, \tag{2.105}$$

where

$$D_\mu f^k_{L,R} = \left(\partial_\mu - iQ_{f^k}\bar{\mathcal{A}}_\mu\right) f^k_{L,R}\,, \tag{2.106}$$
$$j_\mu^{L/R,ij+} = \bar{d}^i_{L/R}\gamma_\mu u^j_{L/R}\,, \tag{2.107}$$
$$j_\mu^{L/R,ij-} = \bar{u}^i_{L/R}\gamma_\mu d^j_{L/R}\,, \tag{2.108}$$
$$J_\mu^{L/R,ij3} = \frac{1}{2}\left(\bar{u}^i_{L/R}\gamma_\mu u^j_{L/R} - \bar{d}^i_{L/R}\gamma_\mu d^j_{L/R}\right)\,, \tag{2.109}$$
$$J_\mu^{f,Q} = \sum_k \left(Q_{u^k}\bar{u}^k_L\gamma_\mu u^k_L + Q_{d^k}\bar{d}^k_L\gamma_\mu d^k_L + Q_{u^k}\bar{u}^k_R\gamma_\mu u^k_R + Q_{d^k}\bar{d}^k_R\gamma_\mu d^k_R\right)\,. \tag{2.110}$$

The electromagnetic charges are given by the quantities $Q_{f^k} = \frac{1}{2}Y(f^k)$. The Yukawa coupling constants $\bar{g}_{ij}$ and $\bar{h}_{ij}$ count as quantities of order p in the low-energy expansion. This ensures that fermion masses m^k_f are treated as of order p as well. The constants c^{ij}_{CC} and c^{ij}_{NC} are of order p^0. Gauge-invariant sources for fermions are readily constructed. We do not need to discuss this point here and refer the interested reader to Ref. [32].

A general effective Lagrangian analysis involves, *a priori*, all possible couplings between the fermions and the gauge bosons. Invariance under $U(1)$ gauge transformations completely determines only the coupling between fermions and the photon. The coupling between fermions and the massive gauge bosons, on the other hand, is only restricted such that the constants c^{ij}_{CC} and c^{ij}_{NC} vanish if the electromagnetic charge is not conserved at the vertex. However, from experiment one knows that many of these low-energy constants are very small, e.g. the couplings of the massive gauge bosons to right-handed fermions or those couplings which induce flavor-changing neutral currents or lepton-number violation. Therefore, in analogy to the low-energy constant $\bar{\rho}-1$ in the bosonic sector, one might set these low-energy constants in $\mathcal{L}^f_2$ equal to zero and consider them only at order p^4 in the effective Lagrangian. In general, however, these coupling constants are already present at order p^2.

It is interesting to note that the coupling to charged and neutral currents can readily be derived from Eq. (2.28) by substituting

$$\bar{v}^2\bar{j}^+_\mu \to \bar{v}^2\bar{j}^+_\mu + \sum_{ij} c^{ij,L}_{CC} j^{L,ij+}_\mu + \sum_{ij} c^{ij,R}_{CC} j^{R,ij+}_\mu\,, \tag{2.111}$$
$$\bar{v}^2\bar{J}^{\mathcal{Z}}_\mu \to \bar{v}^2\bar{J}^{\mathcal{Z}}_\mu + \sum_{ij} c^{ij,L}_{NC} J^{L,ij3}_\mu + \sum_{ij} c^{ij,R}_{NC} J^{R,ij3}_\mu - \bar{s}^2 J^{f,Q}_\mu\,. \tag{2.112}$$

For the case of four-fermion interactions this is also true. In substituting

$$\bar{v}^2\bar{j}^+_\mu \to \bar{v}^2\bar{j}^+_\mu + \sum_{ij} d^{ij,L}_{CC} j^{L,ij+}_\mu + \sum_{ij} d^{ij,R}_{CC} j^{R,ij+}_\mu\,, \tag{2.113}$$
$$\bar{v}^2\bar{J}^{\mathcal{Z}}_\mu \to \bar{v}^2\bar{J}^{\mathcal{Z}}_\mu + \sum_{ij} d^{ij,L}_{NC} J^{L,ij3}_\mu + \sum_{ij} d^{ij,R}_{NC} J^{R,ij3}_\mu - \bar{s}^2 J^{f,Q}_\mu\,, \tag{2.114}$$

all four-fermion interactions of the current-current type can be generated from the last two terms in Eq. (2.28). One should note, however, that there are other four-fermion interactions, which are not of this type and which cannot be generated in this way. The same procedure works at order p^4. Using our source terms given in Appendix A one can generate a host of terms involving the interaction of fermionic currents. Again, a considerable number of the corresponding low-energy constants is, however, either irrelevant to the current experimental situation or is very small. All terms involving four powers of currents and / or gauge fields, for example, contribute to eight-fermion processes only.

One should also note, that terms of order p^4 are already of next-to-next-to-leading order if fermions are present. This is due to the fact that fermionic fields count as order $\sqrt{p}$. Hence, the effective Lagrangian also contains terms of order p^3, for example

$$\bar{d}^i_L i\not{D} d^i_L \bar{u}^j_L u^j_L\,,\ldots\,. \tag{2.115}$$

This is well known from the effective Lagrangian analysis of pion-nucleon physics [40].

Now we are in the position to resume the comparison of our findings for the number of independent low-energy constants in the electroweak chiral Lagrangian with the results found in Refs. [10,11,21]. Furthermore, we want to clarify the role of the oblique correction parameters S, T, and U [2] within our effective field theory analysis.

Obviously the analysis presented in the preceding subsection is not affected by the presence of the fermions. One can use the equations of motion to eliminate the same operators. The only difference is that these equations now depend on a linear combination of external and fermionic currents. In particular, one can again remove the low-energy constants l_{11} and l_{12}. This will renormalize the external currents $\bar{J}^{\mathcal{Z}}_\mu$ and $\bar{J}^{\pm}_\mu$ as well as the coupling constants c^{ij}_{CC} and c^{ij}_{NC} in Eqs. (2.104) and (2.105) among other quantities. Hence, the complete low-energy analysis of a strongly interacting electroweak symmetry breaking sector does not involve the low-energy constants l_{11} and l_{12}, or equivalently, the low-energy constants a_1 and a_8 in the usual basis. These constants contribute to the self-energies of the gauge bosons which are not observable anyway. Note, that the situation here is similar to the one described in the purely bosonic effective field theory. The low-energy constants $\bar{v}^2$ and $\bar{\rho}-1$ in $\mathcal{L}_2$, Eq. (2.27), are of order p^0, however, there are terms in $\mathcal{L}_4$ which renormalize these low-energy constants as described in Eqs. (2.67) and (2.68). In the same way, removing l_{11} and l_{12} modifies two of the coupling constants c^{ij}_{CC} and c^{ij}_{NC} at order p^2. Therefore, it is not possible to remove two of the parameters c^{ij}_{CC} and c^{ij}_{NC} instead of l_{11} and l_{12}. It should be noted, however, that the reduction of the number of operators does not affect the result for any physical quantity evaluated by employing the effective Lagrangian.

As already mentioned in the previous subsection, the step to remove the two low-energy constants a_1 and a_8 from the basis was not taken in Refs. [10,11]. These authors were interested to parametrize the electroweak symmetry breaking sector by means of an effective chiral Lagrangian involving only the bosonic degrees of freedom (without the usual Higgs boson). The couplings of the fermions to the gauge bosons were assumed to have their standard model values. In this respect, no complete effective Lagrangian analysis was attempted in these references. The constraint equations then relate $\mathrm{tr}(\hat{D}_\mu \hat{V}_\mu)$ to a four-fermion term which can be transformed further by employing the equations of motion for the fermions. The quantity $\mathrm{tr}(\hat{D}_\mu \hat{V}_\mu)$ is then proportional to the square of the fermion masses which are small for external light fermions. Only in this approximate sense the terms L_{11}, L_{12}, and L_{13} have been removed from the basis in Refs. [10,11]. The application of the equations of motion for the gauge fields, on the other hand, leads to fermionic operators which would modify the usual couplings of the fermions to the gauge bosons. Therefore, no reduction of the number of independent terms can be achieved in this framework. This interplay of bosonic and fermionic operators when employing the equations of motion was also noted in Ref. [21]. In that paper a heavy Higgs boson is integrated out of the standard model including the fermions. However, no complete effective field theory analysis including the most general couplings of the fermions to the gauge bosons was given in that reference. Furthermore, only the constraint equations, not the equations of motion for the gauge fields, have been used to reduce the number of operators in the basis.

The low-energy constants a_1 and a_8 are sometimes identified with the oblique correction parameters S and U [2]. What is the relation of the above findings to the experimentally determined values for the oblique parameters[5] S, T, and U quoted by the particle data group [39] ?

From our point of view it is not possible to directly identify the low-energy constants l_{11}, l_{12}, and l_{16}, or equivalently, a_0, a_1, and a_8 with the oblique correction parameters S, T, and U. The reason is the following: the definition of the oblique parameters by Peskin and Takeuchi [2] is intended to parametrize the effects of heavy new physics *beyond* the standard model on the self-energies of the gauge bosons. In particular, it is assumed that there exists an elementary Higgs boson and that the full Lagrangian can be decomposed in the form $\mathcal{L}_{full} = \mathcal{L}_{SM} + \mathcal{L}_{new}$. This is also reflected by the fact that one has always to specify a reference value for the Higgs boson mass when quoting results for S, T, and U. In contrast to that, the parametrization of new physics by means of the electroweak chiral Lagrangian assumes that the electroweak symmetry breaking is mediated by a strongly interacting theory. This might either be the standard model with a heavy Higgs boson or another, genuinely strongly interacting model like technicolor where no Higgs particle exists at all. In order to make contact between the two descriptions one could try to mimic any strongly interacting symmetry breaking sector by studying the large Higgs boson mass limit. Note, however, that one cannot completely remove the Higgs particle from the theory in this way, since for $M_H \to \infty$, the Higgs sector becomes strongly interacting and non-perturbatively. The decoupling theorem [6] does not apply in this case.

Let us go back to Eqs. (2.23) and (2.24) and assume that fermions are included and that redundant terms have *not* yet been removed. The low-energy constants have the following form:

$$l_i^{(k)} = \delta_i^{(k)} \Lambda_\epsilon + l_i^{(k),r}(\mu) \,. \tag{2.116}$$

They contain a pole term $\delta_i^{(k)}\Lambda_\epsilon$, with $\Lambda_\varepsilon \doteq (\mu^{d-4}/16\pi^2)\left(1/(d-4) - \frac{1}{2}[\ln(4\pi) + \Gamma'(1) + 1]\right)$, and a renormalized low-energy constant $l_i^{(k),r}(\mu)$. Apart from redundancy the constants $\delta_i^{(k)}$ are universal, i.e. independent of the underlying theory. We now assume that the finite, renormalized low-energy constants can be decomposed as follows:

---

[5]The oblique parameter T is often identified with the low-energy constant a_0 which corresponds to $\bar{\rho}-1$, or, depending on the momentum counting, to the low-energy constant $l'_0 \equiv l_{16}$ in our basis.

$$l_i^{(k),r}(\mu) = l_i^{(k),SM}(\mu) + l_i^{(k),new}(\mu)\,, \tag{2.117}$$

where the first terms describe the contributions for the standard model with a heavy Higgs boson, i.e. the results given below for the bosonic sector up to order p^4, and the second terms describe new physics effects. In general, for $k \geq 4$ the contributions $l_i^{(k),SM}(\mu)$ diverge for $M_H \to \infty$, indicating that one enters the strongly interacting regime where the perturbative analysis breaks down.

The definition of S, T, and U given by Peskin and Takeuchi [2] now amounts to setting $l_i^{(k),new}(\mu) = 0$ for all i and k except for $k = 4$ and $i = 11, 12$ and 16. This introduces three finite parameters independent of each other to describe new physics effects. At this point the effective Lagrangian still involves a redundant set of operators $\mathcal{O}_i^{(k)}$ which can be reduced by employing the equations of motion. Hence, one can again remove the operators $\mathcal{O}_{11}$ and $\mathcal{O}_{12}$. In the present situation, however, this does not reduce the number of independent parameters. It merely moves them to some other operators.

To close this section we note that appropriate source terms for the fermions are given in Ref. [32]. They are gauge-invariant and yield local equations of motion for the fermion fields. These equations can then be used to eliminate additional terms in the effective Lagrangian at order p^3 and at order p^4. A complete analysis including the fermions and the corresponding source terms is, however, beyond the scope of the present work.

III. A MANIFESTLY GAUGE-INVARIANT APPROACH TO THE STANDARD MODEL

A. The Lagrangian and the gauge-invariant generating functional

The standard model with a heavy Higgs boson can be described by an effective Lagrangian as introduced in the previous section. For this specific case, the corresponding low-energy constants can be calculated explicitly in perturbation theory if the coupling constant of the Higgs boson is not too large. The effective Lagrangian can be evaluated by matching the standard model and the effective theory at low energies. In this section we will briefly introduce our gauge-invariant approach to the bosonic sector of the standard model, following the discussion in Ref. [16] to which we refer for more details. The matching calculation will be presented in Sec. IV.

The Lagrangian of the standard model without fermions is of the form

$$\mathcal{L} = \frac{1}{2} D_\mu \Phi^\dagger D_\mu \Phi - \frac{1}{2} m^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \frac{1}{4g^2} W^a_{\mu\nu} W^a_{\mu\nu} + \frac{1}{4g'^2} B_{\mu\nu} B_{\mu\nu}\,, \tag{3.1}$$

where $\Phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ denotes the Higgs boson doublet which is coupled to the $SU(2)_L$ gauge fields W^a_μ $(a = 1, 2, 3)$ and the $U(1)_Y$ gauge field B_μ through the covariant derivative

$$D_\mu \Phi = \left(\partial_\mu - i \frac{\tau^a}{2} W^a_\mu - i \frac{1}{2} B_\mu \right) \Phi\,. \tag{3.2}$$

We have again absorbed the coupling constants g and g' into the gauge fields W^a_μ and B_μ, respectively. The field strengths are defined analogously to Eqs. (2.4) and (2.5). The Higgs field Φ transforms under $SU(2)_L$ gauge transformations in the following way:

$$\Phi \to \mathcal{V} \Phi\,, \quad \mathcal{V} \in SU(2)\,, \tag{3.3}$$

and under $U(1)_Y$ gauge transformations as follows:

$$\Phi \to e^{-i\omega/2}\, \Phi\,. \tag{3.4}$$

For $m^2 > 0$ the classical potential has its minimum at a nonzero value $\Phi^\dagger \Phi = m^2/\lambda$ and the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken down to $U(1)_{\text{em}}$. Accordingly, the field Φ describes one massive mode, the Higgs particle, and three Goldstone bosons which render the gauge fields W and Z massive. Finally, the spectrum contains the massless photon. At tree level, the masses and the electric coupling constant e are given by the relations

$$M_H^2 = 2m^2\,,\; M_W^2 = \frac{m^2 g^2}{4\lambda}\,,\; M_Z^2 = \frac{m^2 (g^2 + g'^2)}{4\lambda}\,, e^2 = \frac{g^2 g'^2}{g^2 + g'^2}\,. \tag{3.5}$$

We will use the same definition of the weak mixing angle as in the effective field theory, cf. Eq. (2.14).

In order to have nontrivial solutions of the equations of motion, we furthermore couple external sources to the gauge fields and the Higgs boson. As in the preceding section we will couple sources only to gauge-invariant operators. Again we introduce another set of fields for the dynamical degrees of freedom which are already invariant under the non-Abelian group $SU(2)_L$ and, in parts, under the Abelian group $U(1)_Y$ as well. It is convenient to use a polar representation for the Higgs doublet field

$$\Phi = \frac{m}{\sqrt{\lambda}} R U \,, \tag{3.6}$$

where the unitary field U, satisfying $U^\dagger U = 1$, describes the three Goldstone bosons, while the radial component R represents the Higgs boson. Furthermore, we define the Y-charge conjugate doublet

$$\widetilde{\Phi} = i\tau_2 \Phi^* \,. \tag{3.7}$$

We introduce the following operators:

$$\begin{aligned} V_\mu^1 &= i\widetilde{\Phi}^\dagger D_\mu \Phi + i\Phi^\dagger D_\mu \widetilde{\Phi} = \frac{m^2}{\lambda} R^2 \mathcal{W}_\mu^1 \,, \\ V_\mu^2 &= -\widetilde{\Phi}^\dagger D_\mu \Phi + \Phi^\dagger D_\mu \widetilde{\Phi} = \frac{m^2}{\lambda} R^2 \mathcal{W}_\mu^2 \,, \\ V_\mu^3 &= i\widetilde{\Phi}^\dagger D_\mu \widetilde{\Phi} - i\Phi^\dagger D_\mu \Phi = \frac{m^2}{\lambda} R^2 \mathcal{Z}_\mu \,, \end{aligned} \tag{3.8}$$

and

$$V_\mu^\pm = \frac{1}{2}(V_\mu^1 \mp i V_\mu^2) \,, \tag{3.9}$$

where the $SU(2)_L$ gauge-invariant fields $\mathcal{W}_\mu^a$ and $\mathcal{Z}_\mu$ are defined analogously to Eqs. (2.9)–(2.11). Up to a constant factor the operators V_μ^i in Eq. (3.8) correspond to the currents of the global symmetry $SU(2)_R$.

In terms of these composite fields the Lagrangian from Eq. (3.1) reads

$$\mathcal{L}_{\mathrm{SM}}^0 = \frac{1}{2}\frac{m^2}{\lambda}\left[\partial_\mu R \partial_\mu R - m^2 R^2 + \frac{m^2}{2} R^4 + R^2\left(\mathcal{W}_\mu^+ \mathcal{W}_\mu^- + \frac{1}{4}\mathcal{Z}_\mu \mathcal{Z}_\mu\right)\right] + \frac{1}{4g^2}\mathcal{W}_{\mu\nu}^a \mathcal{W}_{\mu\nu}^a + \frac{1}{4g'^2} B_{\mu\nu} B_{\mu\nu} \,, \tag{3.10}$$

where $\mathcal{W}_{\mu\nu}^a$ is defined similarly to Eq. (2.29).

In order to calculate Green's functions from which we then can extract physical masses, coupling constants and S-matrix elements, we have to introduce external sources which emit one-particle states of the Higgs field and the gauge bosons. In analogy to the Abelian case [27] we couple sources to the $SU(2)_L \times U(1)_Y$ gauge-invariant operator $\Phi^\dagger \Phi$ and the field strength $B_{\mu\nu}$. As discussed in the previous section, for the massive gauge bosons the situation is more involved. Compensating the residual gauge dependence of the currents $V_\mu^\pm$ under the $U(1)_Y$ gauge transformations from Eqs. (2.7) and (3.4)

$$V_\mu^\pm \to e^{\mp i\omega} V_\mu^\pm \,, \tag{3.11}$$

by a phase factor [36,37,32], we can write appropriate $SU(2)_L \times U(1)_Y$ gauge-invariant source terms for all the fields as follows:

$$\mathcal{L}_{source}^1 = -\frac{1}{2} h \Phi^\dagger \Phi - \frac{1}{2} K_{\mu\nu} B_{\mu\nu} + J_\mu^a \varphi^{ab} V_\mu^b \,, \tag{3.12}$$

with external sources $h, K_{\mu\nu}$, and $J_\mu^a (a = 1, 2, 3)$. The phase factor in Eq. (3.12) is defined by

$$\varphi(x) = \exp\left(T \int d^d y\, \mathcal{G}_0(x-y)\, \partial_\mu B_\mu(y)\right) \,, \tag{3.13}$$

with

$$T = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \,, \tag{3.14}$$

and $\mathcal{G}_0(x-y)$ is given by Eq. (2.18). Since the vacuum in the spontaneously broken phase corresponds to the value $R=1$, Green's functions of the field $\Phi^\dagger\Phi$ contain one-particle poles of the Higgs boson, whereas those of $\varphi^{ab}V^b_\mu$ have one-particle poles of the gauge bosons W and Z.

In Ref. [32] it was shown to all orders in perturbation theory that a phase factor φ which is defined analogously to Eq. (3.13) does not spoil the renormalizability of QED. Since the proof did not rely on any particular feature of QED, the same should be true for the present case as well. This is due to the fact that the phase factor only contains the Abelian gauge degree of freedom which does not affect the dynamics of the theory. Since the operator $\Phi^\dagger\Phi$ and the currents V^a_μ from Eq. (3.8) have dimension less than four, source terms involving these operators do not spoil the renormalizability either. The reader should note, however, that we do not have a formal proof of renormalizability to all orders in perturbation theory for the present case. As was shown in Ref. [16], at the one-loop level everything works fine and on physical grounds we expect this to happen at all orders.

Green's functions of the operators in Eq. (3.12) are, however, more singular at short distances than (gauge-dependent) Green's functions of the fields Φ, W^a_μ, and B_μ themselves. Time ordering of these operators gives rise to ambiguities, and the corresponding Green's functions are only unique up to contact terms. In order to make the theory finite, these contact terms of dimension four need to be added to the Lagrangian which is then given by

$$\mathcal{L}_{\rm SM} = \mathcal{L}^0_{\rm SM} + \widehat{\mathcal{L}}^1_{source} + \mathcal{L}^2_{source}\,. \tag{3.15}$$

The first term in Eq. (3.15) is defined in Eq. (3.10). The second term is given by

$$\widehat{\mathcal{L}}^1_{source} = -\frac{1}{2}\widehat{h}\Phi^\dagger\Phi - \frac{1}{2}\widehat{K}_{\mu\nu}B_{\mu\nu} + J^a_\mu\varphi^{ab}V^b_\mu\,, \tag{3.16}$$

where

$$\widehat{h} = h + 4v_{jj}J^+_\mu J^-_\mu + c_{jj}J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu + 4J^a_\mu J^a_\mu\,, \tag{3.17}$$

$$\widehat{K}_{\mu\nu} = K_{\mu\nu} + c_{Bj}(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu) - 2ic_{Bjj}(J^+_\mu J^-_\nu - J^-_\mu J^+_\nu)\,. \tag{3.18}$$

The last term in Eq. (3.15) is defined by

$$\begin{aligned}\mathcal{L}^2_{source} = &-v_{djj}J^{\mathcal{Z}}_\nu[i(d_\mu j^+_\nu - d_\nu j^+_\mu)j^-_\mu - i(d_\mu j^-_\nu - d_\nu j^-_\mu)j^+_\mu] + v_{dj}(d_\mu j^+_\nu - d_\nu j^+_\mu)(d_\mu j^-_\nu - d_\nu j^-_\mu)\\ &-\frac{i}{2}c_{djj}(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu)(J^+_\mu J^-_\nu - J^-_\mu J^+_\nu) + \frac{1}{4}c_{dj}(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu)(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu)\\ &+16v_{JJ2}(J^+_\mu J^-_\mu)^2 + 4v_{JJJJ}(J^+_\mu J^-_\nu + J^-_\mu J^+_\nu)^2 + c_{JJ2}(J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu)^2\\ &+4v_{J2ZZ}J^+_\mu J^-_\mu J^{\mathcal{Z}}_\nu J^{\mathcal{Z}}_\nu + 2v_{JJZZ}(J^+_\mu J^-_\nu + J^-_\mu J^+_\nu)J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\nu\\ &+c_{hh}h^2 + c_{mh}m^2h + 4c_{hJJ}hJ^+_\mu J^-_\mu + 4c_{mJJ}m^2J^+_\mu J^-_\mu + c_{hZZ}hJ^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu + c_{mZZ}m^2J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu\,,\end{aligned} \tag{3.19}$$

where we introduced the quantities

$$J^\pm_\mu = \frac{1}{2}(J^1_\mu \mp iJ^2_\mu)\,, \quad J^{\mathcal{Z}}_\mu \equiv J^3_\mu\,. \tag{3.20}$$

The quantities $d_\mu j^\pm_\nu$ and $j^\pm_\mu$ are defined analogously to Eqs. (2.20) and (2.31). The contact terms in $\mathcal{L}^2_{source}$ will not contribute to any physical S-matrix elements.

For later use we introduce the quantities

$$\underline{V}^a_\mu = \varphi^{ab}V^b_\mu\,, \tag{3.21}$$

$$\mathcal{Y}^\pm_\mu = \mathcal{W}^\pm_\mu + 4j^\pm_\mu\,,\ \mathcal{Y}^{\mathcal{Z}}_\mu = \mathcal{Z}_\mu + 4J^{\mathcal{Z}}_\mu\,. \tag{3.22}$$

The generating functional $W_{\rm SM}[h, K_{\mu\nu}, J^a_\mu]$ for the gauge-invariant Green's functions is defined by the path integral

$$e^{-W_{\rm SM}[h,K_{\mu\nu},J^a_\mu]} = \int \mathrm{d}\mu[\Phi, W^a_\mu, B_\mu]e^{-\int \mathrm{d}^dx\mathcal{L}_{\rm SM}}\,. \tag{3.23}$$

Note that we still integrate over the original fields Φ, W^a_μ, and B_μ in Eq. (3.23). Furthermore, we have absorbed an appropriate normalization factor into the measure $\mathrm{d}\mu[\Phi, W^a_\mu, B_\mu]$. Derivatives of this functional with respect to the field h generate Green's functions of the scalar density $\Phi^\dagger\Phi$, derivatives with respect to the source $K_{\mu\nu}$ generate

Green's functions of the field strength $B_{\mu\nu}$, while derivatives with respect to J_μ^a generate Green's functions for the currents $\underline{V}_\mu^a$.

In the spontaneously broken phase, these Green's functions have one-particle poles from the Higgs boson as well as the gauge bosons. Thus, one can extract S-matrix elements for the physical degrees of freedom from the generating functional in Eq. (3.23). Due to the equivalence theorem [41] these S-matrix elements will be identical to the ones obtained from those Green's functions which are used in the usually employed formalism. The presence of the contact terms in $\mathcal{L}^2_{source}$ in Eq. (3.19) reflects the fact that the off-shell continuation of the S-matrix is not unambiguously defined. Note that this is a general feature of any field theory and not particular to those involving a gauged symmetry. The continuation we choose has the virtue of being gauge-invariant.

As was pointed out in Refs. [22,32,16] it is possible to evaluate the path integral in Eq. (3.23) without the need to fix a gauge as will be shown below.

B. Tree level

At tree level, the generating functional for the bosonic sector of the standard model is given by

$$W_{\rm SM}[h, K_{\mu\nu}, J_\mu^a] = \int \mathrm{d}^d x\, \mathcal{L}_{\rm SM}(R^{cl}, \mathcal{W}_\mu^{cl,\pm}, \mathcal{Z}_\mu^{cl}, \mathcal{A}_\mu^{cl})\ , \tag{3.24}$$

where $R^{cl}, \mathcal{W}_\mu^{cl,\pm}, \mathcal{Z}_\mu^{cl}$, and $\mathcal{A}_\mu^{cl}$ are determined by the equations of motion

$$-\Box R = -\left[m^2(R^2-1) + \mathcal{Y}_\mu^+\mathcal{Y}_\mu^- + \frac{1}{4}\mathcal{Y}_\mu^{\mathcal{Z}}\mathcal{Y}_\mu^{\mathcal{Z}} - \widehat{h}\right] R\,, \tag{3.25}$$

$$\begin{aligned}-d_\mu\mathcal{W}_{\mu\nu}^\pm = &-M_W^2R^2\mathcal{Y}_\nu^\pm \pm i(\mathcal{Z}_{\mu\nu}+B_{\mu\nu})\mathcal{W}_\mu^\pm \mp i\mathcal{W}_{\mu\nu}^\pm\mathcal{Z}_\mu \mp i(\partial_\mu\mathcal{Z}_\mu)\mathcal{W}_\nu^\pm \pm i(\partial_\mu\mathcal{Z}_\nu)\mathcal{W}_\mu^\pm \\ &\pm i\mathcal{Z}_\nu d_\mu\mathcal{W}_\mu^\pm \mp i\mathcal{Z}_\mu d_\mu\mathcal{W}_\nu^\pm - (\mathcal{Z}_\mu\mathcal{Z}_\mu)\mathcal{W}_\nu^\pm + (\mathcal{Z}_\mu\mathcal{Z}_\nu)\mathcal{W}_\mu^\pm \pm 2\mathcal{W}_\mu^\pm(\mathcal{W}_\mu^+\mathcal{W}_\nu^- - \mathcal{W}_\nu^+\mathcal{W}_\mu^-)\,,\end{aligned} \tag{3.26}$$

$$-\partial_\mu\mathcal{Z}_{\mu\nu} = \mathrm{PT}_{\nu\mu}\left(-M_Z^2R^2\mathcal{Y}_\mu^{\mathcal{Z}} + T_\mu\right) + \frac{e^2}{c^2}\partial_\mu\widehat{K}_{\mu\nu} + \frac{e^2}{c^2}\mathrm{PT}_{\nu\mu}S_\mu\,, \tag{3.27}$$

$$-\partial_\mu\mathcal{A}_{\mu\nu} = s^2\mathrm{PT}_{\nu\mu}T_\mu - e^2\partial_\mu\widehat{K}_{\mu\nu} - e^2\mathrm{PT}_{\nu\mu}S_\mu\ . \tag{3.28}$$

Furthermore, the equations for the Goldstone boson field U correspond to

$$d_\mu\mathcal{Y}_\mu^\pm = -2\frac{\partial_\mu R}{R}\mathcal{Y}_\mu^\pm \pm i\mathcal{Z}_\mu\mathcal{Y}_\mu^\pm \mp i\mathcal{Y}_\mu^{\mathcal{Z}}\mathcal{W}_\mu^\pm\,, \tag{3.29}$$

$$\partial_\mu\mathcal{Y}_\mu^{\mathcal{Z}} = -2\frac{\partial_\mu R}{R}\mathcal{Y}_\mu^{\mathcal{Z}} - 8i(j_\mu^+\mathcal{W}_\mu^- - j_\mu^-\mathcal{W}_\mu^+)\ . \tag{3.30}$$

In order to simplify the notation we have omitted the prescription "cl" in the equations above. In Eqs. (3.25)–(3.30) we have introduced the quantities

$$\mathcal{A}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu\,, \tag{3.31}$$

$$\begin{aligned}T_\mu = &\,2\mathcal{Z}_\rho(\mathcal{W}_\rho^+\mathcal{W}_\mu^- + \mathcal{W}_\mu^+\mathcal{W}_\rho^-) - 4\mathcal{Z}_\mu\mathcal{W}_\rho^+\mathcal{W}_\rho^- + 2i(\mathcal{W}_{\rho\mu}^+\mathcal{W}_\rho^- - \mathcal{W}_{\rho\mu}^-\mathcal{W}_\rho^+) \\ &- 2i(d_\rho\mathcal{W}_\rho^+\mathcal{W}_\mu^- - d_\rho\mathcal{W}_\rho^-\mathcal{W}_\mu^+ - d_\rho\mathcal{W}_\mu^+\mathcal{W}_\rho^- + d_\rho\mathcal{W}_\mu^-\mathcal{W}_\rho^+)\,,\end{aligned} \tag{3.32}$$

$$\begin{aligned}S_\mu = &-v_{djj}J_\rho^{\mathcal{Z}}(J_\rho^+J_\mu^- + J_\rho^-J_\mu^+) + 2v_{djj}J_\mu^{\mathcal{Z}}J_\rho^+J_\rho^- \\ &- 2v_{dj}[i(d_\rho j_\mu^+ - d_\mu j_\rho^+)j_\rho^- - i(d_\rho j_\mu^- - d_\mu j_\rho^-)j_\rho^+]\,.\end{aligned} \tag{3.33}$$

The projector $\mathrm{PT}_{\mu\nu}$ has been defined in Eq. (2.42). The quantities $\mathcal{W}_{\mu\nu}^\pm$ and $\mathcal{Z}_{\mu\nu}$ are defined analogously to Eqs. (2.39) and (2.40). The covariant derivatives in $d_\mu\mathcal{W}_\mu^\pm, d_\mu j_\nu^\pm, d_\mu\mathcal{Y}_\nu^\pm$, and $d_\mu\mathcal{W}_{\mu\nu}^\pm$ are defined in the same way as in Eq. (2.20).

The equations of motion (3.25)–(3.30) have similar properties as those in the effective field theory, see the discussion after Eq. (2.42) above. We only note here that the radial variable R which is related to the massive Higgs boson is determined by Eq. (3.25). Solutions for the massive gauge boson fields $\varphi^\mp\mathcal{W}_\mu^\pm$ and $\mathcal{Z}_\mu$ follow from Eqs. (3.26) and (3.27). Finally, Eq. (3.28) determines the transverse component of the massless photon field $\mathcal{A}_\mu^T = \mathrm{PT}_{\mu\nu}\mathcal{A}_\nu$. The solutions of the equations of motion for the physical degrees of freedom in powers of the external sources can be found in Ref. [16].

C. One-loop level

The one-loop contribution to the generating functional can be evaluated with the saddle-point method. If we write the fluctuations y around the classical fields $\mathcal{F}^{cl}$ as $\mathcal{F} = \mathcal{F}^{cl} + y$, we obtain the following representation for the one-loop approximation to the generating functional:

$$e^{-W_{\rm SM}[h,K_{\mu\nu},J^a_\mu]} = e^{-\int {\rm d}^d x \mathcal{L}^{cl}_{\rm SM}} \int {\rm d}\mu[y] e^{-(1/2)\int {\rm d}^d x y^T \widetilde{D} y} \; . \tag{3.34}$$

Gauge invariance implies that the operator $\widetilde{D}$ has zero eigenvalues corresponding to fluctuations y which are equivalent to infinitesimal gauge transformations. Treating these zero modes appropriately [22,16], see also Sec. II A 2 above, one can evaluate the path-integral representation for the generating functional at the one-loop level without the need to fix a gauge and without introducing ghost fields. Up to an irrelevant infinite constant one obtains the following result for the one-loop generating functional from Eq. (3.34):

$$W_{\rm SM}[h,K_{\mu\nu},J^a_\mu] = \int {\rm d}^d x \mathcal{L}_{\rm SM} + \frac{1}{2} \ln \det{}' \widetilde{D} - \frac{1}{2} \ln \det P^T P \; . \tag{3.35}$$

The first term on the right-hand side represents the classical action which describes the tree-level contributions to the generating functional. In the second term, the determinant $\det' \widetilde{D}$ is defined as the product of all non-zero eigenvalues of the operator $\widetilde{D}$. The last term originates from the path integral measure. The sum of the last two terms in Eq. (3.35) corresponds to the one-loop contributions to the generating functional. The operator P satisfies the relation $P^T \widetilde{D} = \widetilde{D} P = 0$.

For the explicit evaluation of the one-loop contributions to the generating functional in Eq. (3.34) it is very important to choose an appropriate parametrization of the physical modes and their quantum fluctuations. Otherwise the expression for the differential operator becomes too complicated. We introduce fluctuations f, η^a, w^a_μ, and b_μ around the Higgs field R, the Goldstone boson field U, the three $SU(2)_L$ gauge fields W^a_μ and the $U(1)_Y$ gauge field B_μ, respectively. Furthermore, we collect the fluctuations of the gauge fields in a vector $q^A_\mu \doteq \left(w^a_\mu, b_\mu\right)$. Following the steps described in Ref [16], the generating functional at the one-loop level can then be written in the form

$$W_{\rm SM}[h,K_{\mu\nu},J^a_\mu] = \int {\rm d}^d x \mathcal{L}_{\rm SM} + \frac{1}{2} \ln \det \left(\widetilde{D} + PP^T + \delta_P\right) - \ln \det P^T P \, , \tag{3.36}$$

where the solutions of the equations of motion (3.25)–(3.28) have to be inserted. Eq. (3.36) represents all tree-level and one-loop contributions of the bosonic sector of the standard model. Note that in order to obtain Eq. (3.36) we have used the identity

$$\ln \det{}' \widetilde{D} = \ln \det \left(\widetilde{D} + PP^T + \delta_P\right) - \ln \det (P^T P) \; , \tag{3.37}$$

to rewrite the determinant $\det' \widetilde{D}$, i.e. the product of all non-zero eigenvalues of the differential operator $\widetilde{D}$, which appears in Eq. (3.35). Equation (3.37), which is valid up to an irrelevant infinite constant, follows from the fact that zero and non-zero eigenvectors are orthogonal to each other.

The explicit expressions for the components of the differential operator $\widetilde{D} + PP^T + \delta_P$, which we parametrize by

$$\widetilde{D} + PP^T + \delta_P \doteq \begin{pmatrix} d & \delta & \delta_\nu \\ \delta^T & D & \Delta_\nu \\ \delta^T_\mu & \Delta^T_\mu & D_{\mu\nu} \end{pmatrix} , \tag{3.38}$$

can be found in Eqs. (C1)–(C9) in Appendix C. The operators PP^T, $P^T P$, and δ_P are listed in Eqs. (C20)–(C22). The 3×3-matrix of the differential operator $\widetilde{D} + PP^T + \delta_P$ from Eq. (3.38) is acting on the 3-dimensional space of fluctuations $y = \left(f, \eta^a, q^A_\mu\right)$.

We would like to stress an important point here. At the classical level only physical modes propagate. The classical Goldstone boson field U^{cl} represents the $SU(2)_L$ gauge degrees of freedom. At the quantum level, however, the situation is different. Quantum fluctuations around the classical field U^{cl}, denoted by η^a, imply virtual Goldstone boson modes propagating within loops. Note that these modes are absent in any gauge-dependent approach based on the unitary gauge. They are, however, necessary in order to ensure a decent high-energy behavior of the theory.

In order to separate the heavy Higgs boson mode from the light modes of the Goldstone and the gauge bosons it is useful to diagonalize the differential operator $\widetilde{D} + PP^T + \delta_P$. First, we introduce some additional quantities

$$\mathcal{D}_{\mu\nu} = D_{\mu\nu} - \delta_\mu^T d^{-1} \delta_\nu - \vartheta_\mu^T \Theta^{-1} \vartheta_\nu \,, \tag{3.39}$$

$$\Theta = D - \delta^T d^{-1} \delta \,, \tag{3.40}$$

$$\vartheta_\nu = \Delta_\nu - \delta^T d^{-1} \delta_\nu \ . \tag{3.41}$$

Using the identity

$$\mathcal{T}^T \left(\widetilde{D} + PP^T + \delta_P \right) \mathcal{T} = \mathrm{diag}\,(\mathrm{d},\, \Theta,\, \mathcal{D}_{\mu\nu}) \ , \tag{3.42}$$

where

$$\mathcal{T} = \begin{pmatrix} 1 & -d^{-1}\delta & -d^{-1}\delta_\nu + d^{-1}\delta\Theta^{-1}\vartheta_\nu \\ 0 & 1 & -\Theta^{-1}\vartheta_\nu \\ 0 & 0 & \delta_{\mu\nu} \end{pmatrix} , \tag{3.43}$$

and the fact that the transformation matrix $\mathcal{T}$ has unit determinant, one obtains the following result for the generating functional:

$$W_{\mathrm{SM}}[h, K_{\mu\nu}, J_\mu^a] = \int \mathrm{d}^d x \mathcal{L}_{\mathrm{SM}} + \frac{1}{2} \ln\det d + \frac{1}{2} \ln\det \Theta + \frac{1}{2} \ln\det \mathcal{D} - \ln\det P^T P \ . \tag{3.44}$$

Equation (3.36) and the equivalent form in Eq. (3.44) represent our result for the generating functional $W_{\mathrm{SM}}[h, K_{\mu\nu}, J_\mu^a]$ for the gauge-invariant Green's functions for the bosonic sector of the standard model. These formulae encode the full tree-level and one-loop effects of the theory. If one expands the generating functional up to a given order in powers of the external sources one can extract any n-point Green's functions for the gauge-invariant operators $\Phi^\dagger\Phi$, $B_{\mu\nu}$, and $\underline{V}_\mu^a$.

As noted before, the generating functional $W_{\mathrm{SM}}[h, K_{\mu\nu}, J_\mu^a]$ from Eq. (3.36) or Eq. (3.44) can be renormalized by an appropriate choice of renormalization prescriptions for the fields, the mass parameter m^2, the coupling constants, and the sources. The full list can be found in Appendix B of Ref. [16]. The relations between bare and renormalized fields, masses and coupling constants which will be needed in Sec. V are given by

$$W_\mu^a = W_\mu^{a,r} \,, \tag{3.45}$$

$$B_\mu = B_\mu^r \,, \tag{3.46}$$

$$\phi = Z_\phi^{1/2} \phi_r \,, \tag{3.47}$$

$$Z_\phi = 1 - (6g_r^2 + 2{g'_r}^2)[\Lambda_\varepsilon(2m_r^2) + \delta z] \,, \tag{3.48}$$

$$m^2 = m_r^2 \left[1 - \frac{1}{2}(24\lambda_r + 3g_r^2 + {g'_r}^2)[\Lambda_\varepsilon(2m_r^2) + \delta m^2] - (Z_\phi - 1) \right] , \tag{3.49}$$

$$\lambda = \lambda_r \left[1 - \left(24\lambda_r + 3g_r^2 + {g'_r}^2 + \frac{3}{8} \frac{(g_r^2 + {g'_r}^2)^2 + 2g_r^4}{\lambda_r} \right) [\Lambda_\varepsilon(2m_r^2) + \delta\lambda] - 2(Z_\phi - 1) \right] , \tag{3.50}$$

$$g^2 = g_r^2 \left[1 + \frac{43}{3} g_r^2 [\Lambda_\varepsilon(2m_r^2) + \delta g^2] \right] , \tag{3.51}$$

$$g'^2 = {g'_r}^2 \left[1 - \frac{1}{3} {g'_r}^2 [\Lambda_\varepsilon(2m_r^2) + \delta g'^2] \right] , \tag{3.52}$$

where we denoted the pole term by

$$\Lambda_\varepsilon(2m_r^2) \doteq \frac{\mu^{d-4}}{16\pi^2} \left(\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + \Gamma'(1) + 1] \right) + \frac{1}{32\pi^2} \ln\left(\frac{2m_r^2}{\mu^2} \right) \ . \tag{3.53}$$

The finite renormalization constants $\delta m^2, \ldots, \delta g'^2$ which appear in the Eqs. (3.49)–(3.52) are determined by the renormalization scheme, cf. Ref. [16].

With the renormalization conditions from Eqs. (3.45)–(3.52) and the corresponding relations for the sources [16], the generating functional for the standard model, $W_{\mathrm{SM}}[h, K_{\mu\nu}, J_\mu^a]$, can be renormalized at the one-loop level. In this way we have completely defined our theory at the one-loop level. The expression (3.44) for the generating functional will be used as the starting point of the matching calculation for the case of the standard model with a heavy Higgs boson, which will be discussed in the next section.

IV. MATCHING

A. Evaluating the matching relation for the case of a heavy Higgs boson

The effective Lagrangian for the case of a heavy Higgs boson is determined by requiring that both the full and the effective theory yield the same Green's functions in the low-energy region, i.e. by the matching relation:

$$W_{eff}[\bar{h}, \bar{K}_{\mu\nu}, \bar{J}^a_\mu] = W_{\rm SM}[h, K_{\mu\nu}, J^a_\mu]\,. \tag{4.1}$$

Note that Eq. (4.1) should not be understood as an identity but rather as an asymptotic equality in the low-energy region. See Refs. [27,22] for a more detailed discussion of this point. Furthermore, we note that in the standard model we have introduced a source h coupled to the scalar density $\Phi^\dagger\Phi$, cf. Eq. (3.12). Therefore, in this specific case the effective Lagrangian will also contain terms involving a source $\bar{h}$, cf. Ref. [27]. As mentioned before, we will consider only Green's functions of gauge-invariant operators in the matching relation (4.1). At low energies, these Green's functions have non-local contributions involving only the vector bosons, which are the light particles in the theory. These contributions drop out of the matching relation. The remaining contributions involve the propagator of the heavy Higgs boson and allow a systematic low-energy expansion. In order to evaluate this expansion one has to understand the counting of loops in the full theory and of the low-energy expansion in the effective theory, cf. Ref. [22].

The loop expansion in the full theory generates a power series in the coupling constants λ, g^2, and g'^2, while the low-energy expansion produces powers of the momenta and the gauge boson masses M_W and M_Z. It is, however, not possible to treat these six quantities as independent of each other, since the gauge boson masses depend on the coupling constants through the relations (3.5). These expressions also indicate that it will not be very transparent to count mass factors in terms of the quantities λ, g^2, and g'^2. The loop expansion in the full theory generates positive powers of the coupling λ, while the low-energy expansion produces negative powers thereof. It is possible, however, to discard the coupling constants g and g' from the counting scheme. This is a consequence of the definition of the vector fields W^a_μ and B_μ in Eq. (3.2), which are scaled such that the coupling constants do not explicitly occur in the covariant derivative. As a result, these coupling constants naturally enter all loop corrections only through the gauge boson masses M_W and M_Z as well as through the weak mixing angle $\sin\theta_W$. Regarding the one-loop contributions to the generating functional, this can readily be inferred from the results for the differential operators listed in Appendix C. With this bookkeeping powers of λ count the number of loops in the full theory.

In order to evaluate the low-energy expansion at a given loop-level, we treat the covariant derivative D_μ, the gauge boson masses M_W and M_Z, the momenta and the external source J^a_μ as in the effective theory, i.e. as quantities of order p. The external source h is of order p^2, while the scalar field Φ, the mass parameter m, the coupling constant λ, and the external source $K_{\mu\nu}$ are quantities of order p^0.

If the coupling constant λ of the Higgs field is not too strong, the low-energy constants l_i from Eq. (2.24) admit an expansion in powers of the parameter λ,

$$l_i = \frac{1}{\lambda} l_i^{tree} + l_i^{1-loop} + \lambda l_i^{2-loop} + \cdots\,, \tag{4.2}$$

corresponding to the loop expansion in the full theory. In this case the accuracy of the effective field theory description is controlled by the order of both the momentum and the coupling constant λ. For values of λ close to the strong coupling region, one may consider higher orders in the expansion (4.2). Large values of the momentum or the gauge boson masses may require including higher orders in Eq. (2.23). In the following, we will determine the effective Lagrangian up to order p^4, and the low-energy constants up to order λ^0, i.e. at the one-loop level.

In order to evaluate the low-energy constants, one can calculate the generating functional in both the full and the effective theory, and solve the matching relation (4.1). It turns out, however, that the evaluation of the one-loop contributions to the generating functional in the effective theory for the case of a general coefficient $\bar{\rho} \neq 1$ in $\mathcal{L}_2^0$ in Eq. (2.27) is quite involved. Therefore, we proceed in a similar way as in the Abelian case [22] and make use of the fact that powers of the constant λ count the number of loops in the full theory. At leading order in λ, i.e. λ^{-1}, we get contributions to the parameters l_i^{tree} in the terms $\mathcal{L}_2$ and $\mathcal{L}_4$. Only the parameters l_i^{tree} in $\mathcal{L}_2$ will, however, be relevant to evaluate the one-loop contribution to the generating functional of the effective theory up to order λ^0.

The leading contributions in λ to the effective Lagrangian can be read off from the low-energy expansion of the classical action of the full theory, i.e., from

$$\int {\rm d}^d x \mathcal{L}_{\rm SM} = \int {\rm d}^d x \Bigg(-\frac{m^4}{4\lambda} R^4 + \frac{1}{4g^2} \mathcal{W}^a_{\mu\nu}\mathcal{W}^a_{\mu\nu} + \frac{1}{4g'^2} B_{\mu\nu}B_{\mu\nu} - \frac{1}{2}\widehat{K}_{\mu\nu}B_{\mu\nu} + \mathcal{L}^2_{source} \Bigg)\,. \tag{4.3}$$

The Lagrangian $\mathcal{L}^2_{source}$ was defined in Eq. (3.19). For slowly varying external fields, the behavior of the massive mode R is under control and the equation of motion (3.25) can be solved algebraically. The result is a series of local terms with increasing order in p^2:

$$R = 1 + r_2 + r_4 + \cdots \,, \quad r_n = \mathcal{O}(p^n)\,, \tag{4.4}$$

$$r_2 = \frac{1}{2m^2}\left(-\frac{1}{4}\mathcal{Y}^a_\mu\mathcal{Y}^a_\mu + \widehat{h}\right)\,, \tag{4.5}$$

$$r_4 = -\frac{1}{8m^4}\left(-\frac{1}{4}\mathcal{Y}^a_\mu\mathcal{Y}^a_\mu + \widehat{h}\right)^2 + \frac{1}{2m^2}\Box r_2\,. \tag{4.6}$$

Inserting the solution for R into the classical action Eq. (4.3) we obtain the following tree level contributions to the effective Lagrangian up to order p^4:

$$\begin{aligned}\mathcal{L}^{tree}_2 &= -\frac{m^2}{2\lambda}\left(-\frac{1}{4}\mathcal{Y}^a_\mu\mathcal{Y}^a_\mu + \widehat{h}\right) + \frac{1}{4g^2}\mathcal{W}^a_{\mu\nu}\mathcal{W}^a_{\mu\nu} + \frac{1}{4g'^2}B_{\mu\nu}B_{\mu\nu} - \frac{1}{2}K_{\mu\nu}B_{\mu\nu}\\ &\quad + c_{mh}m^2 h + 4c_{mJJ}m^2 J^+_\mu J^-_\mu + c_{mZZ}m^2 J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu\,, \end{aligned}\tag{4.7}$$

$$\begin{aligned}\mathcal{L}^{tree}_4 &= -\frac{1}{4\lambda}\left(-\frac{1}{4}\mathcal{Y}^a_\mu\mathcal{Y}^a_\mu + \widehat{h}\right)^2 - \frac{1}{2}B_{\mu\nu}\left[c_{Bj}(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu) - 2ic_{Bjj}(J^+_\mu J^-_\nu - J^-_\mu J^+_\nu)\right]\\ &\quad - v_{djj}J^{\mathcal{Z}}_\nu[i(d_\mu j^+_\nu - d_\nu j^+_\mu)j^-_\mu - i(d_\mu j^-_\nu - d_\nu j^-_\mu)j^+_\mu] + v_{dj}(d_\mu j^+_\nu - d_\nu j^+_\mu)(d_\mu j^-_\nu - d_\nu j^-_\mu)\\ &\quad - \frac{i}{2}c_{djj}(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu)(J^+_\mu J^-_\nu - J^-_\mu J^+_\nu) + \frac{1}{4}c_{dj}(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu)(\partial_\mu J^{\mathcal{Z}}_\nu - \partial_\nu J^{\mathcal{Z}}_\mu)\\ &\quad + 16v_{JJ2}(J^+_\mu J^-_\mu)^2 + 4v_{JJJJ}(J^+_\mu J^-_\nu + J^-_\mu J^+_\nu)^2 + c_{JJ2}(J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu)^2 + 4v_{J2ZZ}J^+_\mu J^-_\mu J^{\mathcal{Z}}_\nu J^{\mathcal{Z}}_\nu\\ &\quad + 2v_{JJZZ}(J^+_\mu J^-_\nu + J^-_\mu J^+_\nu)J^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\nu + c_{hh}h^2 + 4c_{hJJ}hJ^+_\mu J^-_\mu + c_{hZZ}hJ^{\mathcal{Z}}_\mu J^{\mathcal{Z}}_\mu\,. \end{aligned}\tag{4.8}$$

Hence, at leading order in λ the parameters and low-energy constants in $\mathcal{L}_2$ are given by

$$\bar{v}^2 = \frac{m^2}{\lambda}\,,\ \bar{\rho} = 1\,,\ \bar{g} = g\,,\ \bar{g}' = g'\,, \tag{4.9}$$

and

$$\begin{aligned}&\bar{J}^\pm_\mu = J^\pm_\mu\,, \quad \bar{J}^{\mathcal{Z}}_\mu = J^{\mathcal{Z}}_\mu\,, \quad \bar{K}_{\mu\nu} = K_{\mu\nu}\,, \quad \bar{h} = h\,,\\ &\bar{c}_h = -\frac{1}{2} + c_{mh}\lambda\,, \quad \bar{c}_{\mathcal{W}} = -\frac{1}{2}v_{jj} + c_{mJJ}\lambda\,, \quad \bar{c}_{\mathcal{Z}} = -\frac{1}{2}c_{jj} + c_{mZZ}\lambda\,,\end{aligned}\tag{4.10}$$

where $\bar{c}_h$ denotes the coefficient of $\bar{v}^2\bar{h}$ in $\mathcal{L}_2$. Since there are no custodial symmetry breaking effects in the standard model at tree level we get $\bar{\rho} = 1$. Note that the matching condition (4.1) determines the low-energy constants and the sources in the effective theory.

From $\mathcal{L}^{tree}_4$ in Eq. (4.8) we obtain the following tree-level contributions to the low-energy constants l_i in $\mathcal{L}^0_4$ in Eq. (2.48):

$$l^{tree}_1 = -\frac{1}{4\lambda}\,, \quad l^{tree}_3 = -\frac{1}{8\lambda}\,, \quad l^{tree}_5 = -\frac{1}{64\lambda}\,. \tag{4.11}$$

All other low-energy constants l_i in $\mathcal{L}^0_4$ vanish at tree level. From Eq. (4.8) we can also read off the tree-level contributions to the low-energy constants of the source terms at order p^4. Only some of the 76 terms which appear in the general expression $\mathcal{L}^s_4$ in Eq. (2.50) are non-zero at tree level for the present case. It will not be necessary later on to list these contributions here explicitly.

Now one can evaluate the one-loop contribution to the generating functional in the effective theory using the technique described in Sec. II A 2. At order λ^0, the matching relation (4.1) is of the form [cf. Eq. (3.44)]:

$$\begin{aligned}&\int \mathrm{d}^d x(\mathcal{L}_2 + \mathcal{L}_4) + \frac{1}{2}\ln\det\bar{D} + \frac{1}{2}\ln\det\bar{\mathcal{D}} - \ln\det\bar{P}^T\bar{P}\\ &= \int \mathrm{d}^d x\mathcal{L}_{\rm SM} + \frac{1}{2}\ln\det d + \frac{1}{2}\ln\det\Theta + \frac{1}{2}\ln\det\mathcal{D} - \ln\det P^T P\,.\end{aligned}\tag{4.12}$$

The first terms on both sides of Eq. (4.12) represent the tree level contributions in the effective and full theory, respectively. The differential operators on the right-hand side, describing the one-loop contributions in the full theory, are defined in Eqs. (C1)–(C9), (3.39)–(3.41), and (C21). The differential operators on the left-hand side, indicated with a bar, represent the one-loop contributions in the effective theory. Using the iterative matching procedure described above, these differential operators can be inferred from the corresponding operators in the full theory by taking the limit $R \to 1$ and by disregarding all operators which involve the fluctuations f for the radial component R of the Higgs field. Furthermore, we make the identifications $\bar{v}^2 = m^2/\lambda$ and $\bar{\rho} = 1$, cf. Eq. (4.9).

Note that the quantities on the left-hand side of the matching relation (4.12) involve the solutions of the equations of motion in the effective theory, while those on the right-hand side depend on the solutions of the equations of motion in the full theory. At the stationary point, however, the corresponding corrections are of second order in the shift of the fields and beyond the present accuracy. Thus, our notation will not distinguish between the two solutions from now on.

The last three terms on the right-hand side of Eq. (4.12) contain non-local contributions from loops which involve only the light degrees of freedom. They are, however, canceled by the corresponding contributions in the effective theory on the left-hand side of the matching condition.

The fact that all the infrared effects of the massless and light particles cancel out of the matching relation (4.12) is a considerable advantage of the matching of Green's functions. In contrast to that, matching S-matrix elements in the full and the effective theory involves the evaluation of all infrared effects.

For completeness sake, we list below all one-loop corrections to the generating functional of the full theory which will contribute to the effective Lagrangian up to the order p^4.

One obtains the following terms from the first determinant on the right-hand side of Eq. (4.12) which involve only the propagator of the massive Higgs mode:

$$\frac{1}{2}\ln\det d = \frac{1}{2}\ln\det d_m + \frac{1}{2}\mathrm{Tr}\left(d_m^{-1}\sigma_m\right) - \frac{1}{4}\mathrm{Tr}\left((d_m^{-1}\sigma_m)^2\right) . \tag{4.13}$$

Here we used the decomposition $d = d_m + \sigma_m, d_m = -\Box + 2m^2$. The explicit form of σ_m can be inferred from Eq. (C1). The second term in Eq. (4.13), a tadpole graph, is of order p^2, whereas the third term is of order p^4.

Mixed loops, which contain Higgs and Goldstone boson propagators, are given by

$$\begin{aligned}\frac{1}{2}\ln\det\Theta &\equiv \frac{1}{2}\ln\det\left(D - \delta^T d^{-1}\delta\right) \\ &= \frac{1}{2}\ln\det D - \frac{1}{2}\mathrm{Tr}\left(\delta D^{-1}\delta^T d_m^{-1}\right) + \frac{1}{2}\mathrm{Tr}\left(\delta D^{-1}\delta^T d_m^{-1}\sigma_m d_m^{-1}\right) - \frac{1}{4}\mathrm{Tr}\left((\delta D^{-1}\delta^T d_m^{-1})^2\right) . \end{aligned}\tag{4.14}$$

As noted above, the term $\frac{1}{2}\ln\det D$ on the right-hand side cancels against the corresponding contribution in the effective theory. The next term is of order p^2, whereas the last two terms lead to contributions of order p^4.

Finally, the following terms involve the gauge boson propagators:

$$\frac{1}{2}\ln\det\mathcal{D} = \frac{1}{2}\ln\det\bar{\mathcal{D}} + \frac{1}{2}\mathrm{Tr}\left(\bar{\mathcal{D}}^{-1}\delta\mathcal{D}\right) , \tag{4.15}$$

where we used the decomposition $\mathcal{D}_{\mu\nu} = \bar{\mathcal{D}}_{\mu\nu} + \delta\mathcal{D}_{\mu\nu}, \delta\mathcal{D}_{\mu\nu} = \mathcal{O}(p^4)$. Again the first term on the right-hand side of Eq. (4.15) cancels against the corresponding contribution in the effective theory. The second term is of order p^4.

Finally we note that the difference between the contribution from the path integral measure in the full theory, $\ln\det P^T P$, and in the effective theory, $\ln\det\bar{P}^T\bar{P}$, in the matching relation (4.12) is of order p^6.

Techniques to evaluate the low-energy expansion of the traces in Eqs. (4.13), (4.14), and (4.15) are discussed in detail in Ref. [27]. The results for the terms (4.13) and (4.14) can be inferred from the expressions given there. The evaluation of the second term in Eq. (4.15), involving the gauge bosons, proceeds in the same way with the result

$$\frac{1}{2}\mathrm{Tr}\left(\bar{\mathcal{D}}^{-1}\delta\mathcal{D}\right) = \int d^d x \left(\Lambda_\varepsilon(2m^2) M_W^2 \mathcal{Y}_\mu^a \mathcal{Y}_\mu^a + \left(\frac{3}{4}\Lambda_\varepsilon(2m^2) + \frac{1}{16}\frac{1}{16\pi^2}\right)(M_Z^2 - M_W^2)\mathcal{Y}_\mu^3\mathcal{Y}_\mu^3\right) + \mathcal{O}(p^6) , \tag{4.16}$$

with

$$\Lambda_\varepsilon(2m^2) \doteq \frac{\mu^{d-4}}{16\pi^2}\left(\frac{1}{d-4} - \frac{1}{2}(\ln(4\pi) + \Gamma'(1) + 1)\right) + \frac{1}{32\pi^2}\ln\left(\frac{2m^2}{\mu^2}\right) . \tag{4.17}$$

B. The bare effective Lagrangian

Collecting all contributions we obtain the following result for the bare effective Lagrangian for the standard model with a heavy Higgs boson, up to order p^4 and up to λ^0, i.e. at the one loop level:

$$\mathcal{L}_2 = \left(\frac{1}{4\lambda} - 3\Lambda_\varepsilon(2m^2) + \frac{1}{4}\frac{1}{16\pi^2}\right)(2m^2)\left(\mathcal{W}^+_\mu \mathcal{W}^-_\mu + \frac{1}{4}\mathcal{Z}_\mu \mathcal{Z}_\mu\right) + \frac{1}{4g^2}\mathcal{W}^a_{\mu\nu}\mathcal{W}^a_{\mu\nu} + \frac{1}{4g'^2}B_{\mu\nu}B_{\mu\nu} + \mathcal{L}^s_2\,, \tag{4.18}$$

$$\mathcal{L}_4 = \sum_{i=1}^{18} l^b_i \mathcal{O}_i + \mathcal{L}^s_4\,, \tag{4.19}$$

with the following results for the bare low-energy constants l^b_i:

$$\begin{aligned}
l^b_1 &= -\frac{1}{4\lambda} + 5\Lambda_\varepsilon(2m^2) + \frac{19}{12}\frac{1}{16\pi^2}\,,\\
l^b_2 &= 0\,,\\
l^b_3 &= -\frac{1}{8\lambda} + \frac{5}{2}\Lambda_\varepsilon(2m^2) + \frac{19}{24}\frac{1}{16\pi^2}\,,\\
l^b_4 &= 0\,,\\
l^b_5 &= \frac{1}{16}\left(-\frac{1}{4\lambda} + 5\Lambda_\varepsilon(2m^2) + \frac{19}{12}\frac{1}{16\pi^2}\right)\,,\\
l^b_6 &= 0\,,\\
l^b_7 &= \frac{1}{6}\Lambda_\varepsilon(2m^2) - \frac{11}{72}\frac{1}{16\pi^2}\,,\\
l^b_8 &= -\frac{1}{6}\frac{1}{16\pi^2}\,,\\
l^b_9 &= -\frac{1}{6}\Lambda_\varepsilon(2m^2) + \frac{11}{72}\frac{1}{16\pi^2}\,,\\
l^b_{10} &= \frac{1}{6}\Lambda_\varepsilon(2m^2) - \frac{11}{72}\frac{1}{16\pi^2}\,,\\
l^b_{11} &= 0\,,\\
l^b_{12} &= \frac{1}{12}\Lambda_\varepsilon(2m^2) + \frac{1}{144}\frac{1}{16\pi^2}\,,\\
l^b_{13} &= -\frac{1}{12}\frac{1}{16\pi^2}\,,\\
l^b_{14} &= -\frac{1}{48}\frac{1}{16\pi^2}\,,\\
l^b_{15} &= 3\Lambda_\varepsilon(2m^2) + \frac{1}{4}\frac{1}{16\pi^2}\,,\\
l^b_{16} &= s^2\left(\frac{3}{4}\Lambda_\varepsilon(2m^2) + \frac{1}{16}\frac{1}{16\pi^2}\right)\,,\\
l^b_{17} &= -\frac{1}{24}\Lambda_\varepsilon(2m^2) - \frac{1}{288}\frac{1}{16\pi^2}\,,\\
l^b_{18} &= \frac{1}{24}\Lambda_\varepsilon(2m^2) + \frac{1}{288}\frac{1}{16\pi^2}\,.
\end{aligned} \tag{4.20}$$

Note that only bare quantities (coupling constants, masses, fields) appear in the result for the effective Lagrangian in Eqs. (4.18)–(4.20).

In order to simplify the expressions for the effective Lagrangian and to compare our results with other calculations in the literature we have not explicitly written down the contributions from the source terms $\mathcal{L}^s_2$ and $\mathcal{L}^s_4$ in Eqs. (4.18) and (4.19) respectively. The contributions including the sources at tree-level are given in Eqs. (4.7) and (4.8). All contributions from the source terms at the one-loop level can be calculated from Eqs. (4.13), (4.14), and (4.16), if one inserts the explicit expressions for the differential operators given in Appendix C. Note that we have not yet used the equations of motion to reduce the number of terms in the basis of $\mathcal{L}_4$.

The result for the bare electroweak chiral Lagrangian in the usually employed notation and the corresponding bare low-energy constants a^b_i in the usual basis at order p^4 can be found in Appendix B, Eqs. (B12) and (B13). Following the conventions used in chiral perturbation theory [13] we have included some additional, finite terms in our definition of the pole term $\Lambda_\varepsilon(2m^2)$, Eq. (4.17), compared to the conventions used in Refs. [19–21]. Taking this into account the results for the bare low-energy constants $a_0^b, \ldots, a_{14}^b$ agree with those obtained in Ref. [19].

The results for the bare low-energy constants l_{15}^b, l_{17}^b, and l_{18}^b, or equivalently, the low-energy constants a_{15}^b, a_{16}^b, and a_{17}^b in Eq. (B13), which correspond to operators in $\mathcal{L}_4^0$ that are proportional to terms in $\mathcal{L}_2^0$, agree with the results obtained in Ref. [21].

In the following section we are going to express the bare effective Lagrangian from Eqs. (4.18)–(4.20) in terms of physical quantities.

V. RENORMALIZATION

A. Physical input parameters from gauge-invariant Green's functions

In this section we want to express the bare parameters which appear in the effective Lagrangian (4.18)–(4.20) through physical quantities. As physical input parameters we choose the masses of the Higgs and the W- and Z-bosons, and the electric charge (on-shell scheme). The physical mass of the Higgs boson, which we denote by $M_{H,\text{pole}}^2$, is determined by the pole position of the two-point function

$$\langle 0|T(\Phi^\dagger\Phi)(x)(\Phi^\dagger\Phi)(y)|0\rangle \, . \tag{5.1}$$

The physical masses of the W-boson, $M_{W,\text{pole}}^2$, and the Z-boson, $M_{Z,\text{pole}}^2$, are defined by the pole positions of the two-point function

$$\langle 0|T(\underline{V}_\mu^a)(x)(\underline{V}_\nu^b)(y)|0\rangle \, . \tag{5.2}$$

As discussed in Ref. [16] one can define a renormalized electric charge as the residue at the photon pole of the two-point function

$$\langle 0|TB_{\mu\nu}(x)B_{\rho\sigma}(y)|0\rangle \, . \tag{5.3}$$

We will denote the corresponding coupling constant by e_{res}^2. As was shown in Ref. [16] by an explicit one-loop calculation, the coupling constant e_{res}^2 agrees with the usual result for the electric charge in the Thompson limit. We note that the residue of the two-point function of the field strength $B_{\mu\nu}$ in Eq. (5.3) differs from unity and that it is uniquely determined. This can be traced back to our normalization of the gauge field B_μ in the covariant derivative in Eq. (3.2). Gauge invariance requires that this field is not renormalized, cf. Eq. (3.46). The same statement holds for the gauge field W_μ^a, cf. Eq. (3.45).

For the determination of the two-point functions in Eqs. (5.1)–(5.3) we need the generating functional $W_{\text{SM}}[h, K_{\mu\nu}, J_\mu^a]$ up to second order in the external sources. The calculation of the physical masses and the coupling constant e_{res}^2 was performed in Ref. [16] at the one-loop level. Below we will use the relations between the bare and physical masses and electric charge which were obtained in that reference. Because we are interested here in expressing the bare effective Lagrangian from Eqs. (4.18)–(4.20) in terms of physical quantities we will only write down the low-energy expansion of the physical quantities.

In order to determine the effective Lagrangian up to order p^4 we need the physical Higgs boson mass $M_{H,\text{pole}}^2$ up to order p^0

$$M_{H,\text{pole}}^2 = M_H^2 \left(1 + \lambda \delta M_{H,0}^2 + \mathcal{O}(p^2)\right) \, , \tag{5.4}$$

$$\delta M_{H,0}^2 = 12\Lambda_\varepsilon(M_H^2) - \frac{1}{16\pi^2}(12 - 3\sqrt{3}\pi) \, . \tag{5.5}$$

On the right-hand side of the equations only bare quantities appear. Furthermore, we have introduced the abbreviations

$$M_H^2 \equiv 2m^2 \ , \ \lambda \equiv \frac{1}{8}\frac{e^2}{s^2}\frac{M_H^2}{M_W^2} \ , \ c^2 \equiv \frac{M_W^2}{M_Z^2} \ . \tag{5.6}$$

For the physical masses of the gauge bosons, $M_{W,\text{pole}}^2$ and $M_{Z,\text{pole}}^2$, we need the low-energy expansion up to order p^4. For the W-boson mass we get

$$M^2_{W,\text{pole}} = M^2_W \left(1 + \lambda\delta M^2_{W,0} + \lambda\frac{M^2_W}{M^2_H}\delta M^2_{W,2} + \mathcal{O}(p^4)\right) , \tag{5.7}$$

$$\delta M^2_{W,0} = -12\Lambda_\varepsilon(M^2_H) + \frac{1}{16\pi^2} , \tag{5.8}$$

$$\delta M^2_{W,2} = c^W_1 \Lambda_\varepsilon(M^2_H) + c^W_2 \ln(\frac{M^2_W}{M^2_H}) + c^W_3 \ln(\frac{M^2_Z}{M^2_H}) + c^W_4 + c^W_5 \left[\sigma \ln\left(\frac{1+\sigma}{1-\sigma}\right)\right] , \tag{5.9}$$

where

$$\sigma = \sqrt{1 - 4M^2_W/M^2_Z} , \tag{5.10}$$

and

$$\begin{aligned}
c^W_1 &= \frac{1}{c^2}\left(-\frac{272}{3}c^2 + 12\right) , \\
c^W_2 &= \frac{1}{\pi^2 c^6}\left(-\frac{13}{4}c^6 + \frac{17}{8}c^4 - \frac{7}{24}c^2 - \frac{1}{48}\right) , \\
c^W_3 &= \frac{1}{\pi^2 c^6}\left(-\frac{7}{4}c^4 + \frac{7}{24}c^2 + \frac{1}{48}\right) , \\
c^W_4 &= \frac{1}{\pi^2 c^6}\left(\frac{461}{72}c^6 - \frac{7}{12}c^4 - \frac{1}{24}c^2\right) , \\
c^W_5 &= \frac{1}{\pi^2 c^6}\left(c^6 + \frac{17}{12}c^4 - \frac{1}{3}c^2 - \frac{1}{48}\right) .
\end{aligned} \tag{5.11}$$

For the Z-boson mass we obtain the expression

$$M^2_{Z,\text{pole}} = M^2_Z \left(1 + \lambda\delta M^2_{Z,0} + \lambda\frac{M^2_Z}{M^2_H}\delta M^2_{Z,2} + \mathcal{O}(p^4)\right) , \tag{5.12}$$

$$\delta M^2_{Z,0} = -12\Lambda_\varepsilon(M^2_H) + \frac{1}{16\pi^2} , \tag{5.13}$$

$$\delta M^2_{Z,2} = c^Z_1 \Lambda_\varepsilon(M^2_H) + c^Z_2 \ln\left(\frac{M^2_W}{M^2_H}\right) + c^Z_3 + c^Z_4 \left[\sigma \ln\left(\frac{\sigma-1}{\sigma+1}\right)\right] , \tag{5.14}$$

with

$$\begin{aligned}
c^Z_1 &= -112c^4 + \frac{56}{3}c^2 + \frac{44}{3} , \\
c^Z_2 &= \frac{1}{\pi^2}\left(-\frac{7}{2}c^4 + \frac{7}{12}c^2 + \frac{1}{24}\right) , \\
c^Z_3 &= \frac{1}{\pi^2}\left(4c^6 + \frac{13}{6}c^4 - \frac{7}{18}c^2\right) , \\
c^Z_4 &= \frac{1}{\pi^2}\left(2c^6 + \frac{17}{6}c^4 - \frac{2}{3}c^2 - \frac{1}{24}\right) .
\end{aligned} \tag{5.15}$$

Note that the low-energy expansion for the physical gauge boson masses starts at order p^2 since $M^2_W, M^2_Z = O(p^2)$. Furthermore, the factors δM^2 in Eqs. (5.4)–(5.14) count as quantities of order p^0 in the low-energy expansion. The p^2-weighted prefactors have been extracted explicitly.

Finally, we get the following relation between the physical coupling constant e^2_{res} and the bare coupling constant e^2:

$$e^2_{\text{res}} = e^2\left(1 + e^2\delta e^2_2 + \mathcal{O}(p^4)\right) , \tag{5.16}$$

$$\delta e^2_2 = -14\left[\Lambda_\varepsilon(M^2_H) + \frac{1}{32\pi^2}\ln\left(\frac{M^2_W}{M^2_H}\right)\right] - \frac{19}{3}\frac{1}{16\pi^2} . \tag{5.17}$$

We recall that the coupling constant e^2 is a quantity of order p^2 according to our momentum counting rules. The factor δe^2_2 counts as order p^0 in the low-energy expansion. As noted above the result for e^2_{res} agrees with the usual definition of the electric charge in the Thompson limit [42] in the absence of fermion contributions.

The expressions for the physical masses, Eqs. (5.4), (5.7), (5.12) and the coupling constant $e^2_{\rm res}$, Eq. (5.16), are finite if we insert the renormalization prescriptions (3.45)–(3.52) for the bare quantities on the right-hand side. Of course, this is true for the complete results for the masses, not only for the expressions after the low-energy expansion has been carried out. Furthermore, in the limit $g' \to 0$, which implies $c^2 \to 1$, we get $M^2_{W,\rm pole} \equiv M^2_{Z,\rm pole}$ as expected.

B. The effective Lagrangian

We are now in the position to express the bare parameters which appear in the effective Lagrangian in Eqs. (4.18)–(4.20) in terms of physical quantities using the relations from Eqs. (5.4)–(5.17). Note that the gauge fields $\mathcal{W}^\pm_\mu, \mathcal{Z}_\mu$ and B_μ are not renormalized due to gauge invariance, cf. Eqs. (3.45) and (3.46). At the one-loop level and up to order p^4 in the low-energy expansion we obtain the following expression for the effective Lagrangian for the standard model with a heavy Higgs boson:

$$\mathcal{L}_2 = \left(2M^2_{W,\rm pole}\frac{s_p^2}{e^2_{\rm res}}\right)\left(\mathcal{W}^+_\mu\mathcal{W}^-_\mu + \frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) + \frac{s_p^2}{4e^2_{\rm res}}\mathcal{W}^a_{\mu\nu}\mathcal{W}^a_{\mu\nu} + \frac{c_p^2}{4e^2_{\rm res}}B_{\mu\nu}B_{\mu\nu} + \mathcal{L}_2^s\,, \tag{5.18}$$

$$\mathcal{L}_4 = \sum_{i=1}^{18} l_i\mathcal{O}_i + \mathcal{L}_4^s\,, \tag{5.19}$$

with

$$l_1 = -\Lambda_\varepsilon - \frac{2s_p^2M^2_{W,\rm pole}}{e^2_{\rm res}M^2_{H,\rm pole}} - \frac{1}{2}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{1}{16\pi^2}\frac{58-9\sqrt{3}\pi}{12}\,,$$
$$l_2 = 0\,,$$
$$l_3 = -\frac{1}{2}\Lambda_\varepsilon - \frac{s_p^2M^2_{W,\rm pole}}{e^2_{\rm res}M^2_{H,\rm pole}} - \frac{1}{4}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{1}{16\pi^2}\frac{58-9\sqrt{3}\pi}{24}\,,$$
$$l_4 = 0\,,$$
$$l_5 = -\frac{1}{16}\Lambda_\varepsilon - \frac{s_p^2M^2_{W,\rm pole}}{8e^2_{\rm res}M^2_{H,\rm pole}} - \frac{1}{32}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{1}{16\pi^2}\frac{58-9\sqrt{3}\pi}{192}\,,$$
$$l_6 = 0\,,$$
$$l_7 = \frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) - \frac{11}{72}\frac{1}{16\pi^2}\,,$$
$$l_8 = -\frac{1}{6}\frac{1}{16\pi^2}\,,$$
$$l_9 = -\frac{1}{6}\Lambda_\varepsilon - \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{11}{72}\frac{1}{16\pi^2}\,,$$
$$l_{10} = \frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) - \frac{11}{72}\frac{1}{16\pi^2}\,,$$
$$l_{11} = 0\,,$$
$$l_{12} = \frac{1}{12}\Lambda_\varepsilon + \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{1}{144}\frac{1}{16\pi^2}\,,$$
$$l_{13} = -\frac{1}{12}\frac{1}{16\pi^2}\,,$$
$$l_{14} = -\frac{1}{48}\frac{1}{16\pi^2}\,,$$
$$l_{15} = 3\Lambda_\varepsilon + \frac{3}{2}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{1}{4}\frac{1}{16\pi^2} - \frac{1}{4}(1-\frac{c_p^2}{s_p^2})\delta M^2_{W,2} - \frac{1}{4s_p^2}\delta M^2_{Z,2} + 2s_p^2\delta e_2^2\,,$$
$$l_{16} = s_p^2\left(\frac{3}{4}\Lambda_\varepsilon + \frac{3}{8}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\rm pole}}{\mu^2}\right) + \frac{1}{16}\frac{1}{16\pi^2}\right)\,,$$

$$l_{17} = -\frac{1}{24}\Lambda_\varepsilon - \frac{1}{48}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{1}{288}\frac{1}{16\pi^2} + \frac{c_p^2}{32s_p^2}\delta M^2_{W,2} - \frac{1}{32s_p^2}\delta M^2_{Z,2} + \frac{s_p^2}{4}\delta e_2^2\,,$$

$$l_{18} = \frac{1}{24}\Lambda_\varepsilon + \frac{1}{48}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) + \frac{1}{288}\frac{1}{16\pi^2} - \frac{c_p^2}{32s_p^2}\delta M^2_{W,2} + \frac{1}{32s_p^2}\delta M^2_{Z,2} + \frac{c_p^2}{4}\delta e_2^2\,. \tag{5.20}$$

The results for the low-energy constants l_1, l_3, and l_5 are obtained by expressing the bare coupling constant λ which appears in Eq. (4.20) through physical quantities. In order to obtain l_{15}, l_{17}, and l_{18} one has to express the bare quantities $m^2/\lambda, g^2$, and g'^2 in Eq. (4.18) through physical quantities. The quantities $\delta M^2_{W,2}, \delta M^2_{Z,2}$, and δe_2^2 are defined in Eqs. (5.9), (5.14), and (5.17), respectively. We use the on-shell definition for the weak mixing angle

$$c_p^2 \doteq \frac{M^2_{W,\text{pole}}}{M^2_{Z,\text{pole}}}\,, \quad s_p^2 \doteq 1 - c_p^2\,. \tag{5.21}$$

The pole term in $d = 4$ dimensions is given by

$$\Lambda_\varepsilon \doteq \frac{\mu^{d-4}}{16\pi^2}\left(\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + \Gamma'(1) + 1]\right)\,. \tag{5.22}$$

In order to simplify the expressions we have not explicitly written down the results for the source terms $\mathcal{L}_2^s$ and $\mathcal{L}_4^s$ in Eqs. (5.18) and (5.19), respectively.

As discussed in Sec. II A 2 we can reduce the number of terms in the effective Lagrangian $\mathcal{L}_4$ by making use of the equations of motion in the effective field theory and by renormalizing the parameters and low-energy constants in the lowest order Lagrangian $\mathcal{L}_2$. The source terms in $\mathcal{L}_4^s$ will thereby not affect the terms $\mathcal{L}_4^0$ without sources. Switching off the sources altogether, we then obtain the following result for the effective Lagrangian:

$$\mathcal{L}_2 = \frac{\bar{v}^2_{eff}}{2}\left(\mathcal{W}^+_\mu\mathcal{W}^-_\mu + \frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) + \frac{1}{4\bar{g}^2_{eff}}\mathcal{W}^a_{\mu\nu}\mathcal{W}^a_{\mu\nu} + \frac{1}{4\bar{g}'^2_{eff}}B_{\mu\nu}B_{\mu\nu}\,, \tag{5.23}$$

with

$$\begin{aligned}\bar{v}^2_{eff} = 4M^2_{W,\text{pole}}\frac{s_p^2}{e^2_{\text{res}}}\Bigg(1 + \frac{e^2_{\text{res}}}{s_p^2}\Bigg[\frac{11}{6}\Lambda_\varepsilon + \frac{11}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) + \frac{11}{72}\frac{1}{16\pi^2}\\ - \frac{1}{8}\left(1 - \frac{c_p^2}{s_p^2}\right)\delta M^2_{W,2} - \frac{1}{8s_p^2}\delta M^2_{Z,2} + s_p^2\delta e_2^2\Bigg]\Bigg)\,,\end{aligned} \tag{5.24}$$

$$\bar{g}^2_{eff} = \frac{e^2_{\text{res}}}{s_p^2}\left(1 + \frac{e^2_{\text{res}}}{s_p^2}\left[-\frac{1}{6}\Lambda_\varepsilon - \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{1}{72}\frac{1}{16\pi^2} - \frac{c_p^2}{8s_p^2}\delta M^2_{W,2} + \frac{1}{8s_p^2}\delta M^2_{Z,2} - s_p^2\delta e_2^2\right]\right)\,, \tag{5.25}$$

$$\bar{g}'^2_{eff} = \frac{e^2_{\text{res}}}{c_p^2}\left(1 + \frac{e^2_{\text{res}}}{c_p^2}\left[\frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) + \frac{1}{72}\frac{1}{16\pi^2} + \frac{c_p^2}{8s_p^2}\delta M^2_{W,2} - \frac{1}{8s_p^2}\delta M^2_{Z,2} - c_p^2\delta e_2^2\right]\right)\,. \tag{5.26}$$

At order p^4 we obtain the result

$$\mathcal{L}_4 = \sum_{i=0}^{9} l_i'\mathcal{O}_i\,, \tag{5.27}$$

where the low-energy constants l_i' corresponding to the independent terms in the Lagrangian $\mathcal{L}_4$ are given by

$$l_0' = s_p^2\left(\frac{3}{4}\Lambda_\varepsilon + \frac{3}{8}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) + \frac{1}{16}\frac{1}{16\pi^2}\right)\,,$$

$$l_1' = -\frac{1}{3}\Lambda_\varepsilon - \frac{2s_p^2M^2_{W,\text{pole}}}{e^2_{\text{res}}M^2_{H,\text{pole}}} - \frac{1}{6}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) + \frac{1}{16\pi^2}\frac{176 - 27\sqrt{3}\pi}{36}\,,$$

$$l_2' = -\frac{2}{3}\Lambda_\varepsilon - \frac{1}{3}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{1}{18}\frac{1}{16\pi^2}\,,$$

$$
\begin{aligned}
l_3' &= \frac{1}{6}\Lambda_\varepsilon - \frac{s_p^2 M_{W,\text{pole}}^2}{e_{\text{res}}^2 M_{H,\text{pole}}^2} + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\text{pole}}^2}{\mu^2}\right) + \frac{1}{16\pi^2}\frac{178-27\sqrt{3}\pi}{72}\,,\\
l_4' &= -\frac{2}{3}\Lambda_\varepsilon - \frac{1}{3}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\text{pole}}^2}{\mu^2}\right) - \frac{1}{18}\frac{1}{16\pi^2}\,,\\
l_5' &= -\frac{1}{16}\Lambda_\varepsilon - \frac{s_p^2 M_{W,\text{pole}}^2}{8e_{\text{res}}^2 M_{H,\text{pole}}^2} - \frac{1}{32}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\text{pole}}^2}{\mu^2}\right) + \frac{1}{16\pi^2}\frac{58-9\sqrt{3}\pi}{192}\,,\\
l_6' &= 0\,,\\
l_7' &= -\frac{1}{6}\Lambda_\varepsilon - \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\text{pole}}^2}{\mu^2}\right) - \frac{13}{72}\frac{1}{16\pi^2}\,,\\
l_8' &= -\frac{1}{6}\frac{1}{16\pi^2}\,,\\
l_9' &= \frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\text{pole}}^2}{\mu^2}\right) + \frac{13}{72}\frac{1}{16\pi^2}\,.
\end{aligned}
\tag{5.28}
$$

C. Discussion

Equations (5.23)–(5.28) represent our final result for the effective Lagrangian for the standard model with a heavy Higgs boson, expressed through the physical masses of the Higgs boson, the W- and the Z-boson, as well as the electric charge. The effective Lagrangian includes all contributions at one-loop in the standard model and up to order p^4 in the low-energy expansion.

Let us first discuss the lowest order Lagrangian $\mathcal{L}_2$ in Eq. (5.23) and the corresponding low-energy constants in Eqs. (5.24)–(5.26). Comparing with the general effective Lagrangian in Eq. (2.27) we note that $\bar{\rho}=1$. This is due to the fact that in the standard model with a heavy Higgs boson the custodial symmetry violating effects in $\Delta\rho$ are proportional to ${g'}^2$, i.e. they are of higher order in the momentum expansion. We recall that $g' = \mathcal{O}(p)$ according to the counting rules discussed in Sec. II A.

The additional terms proportional to e_{res}^2/s_p^2 in $\bar{v}_{eff}^2, \bar{g}_{eff}^2$, Eqs. (5.24), (5.25), and the additional terms proportional to e_{res}^2/c_p^2 in $\bar{g'}_{eff}^2$, Eq. (5.26), deserve some comments. Employing our counting rules these terms will contribute only at order p^4. They originate from the low-energy constants l_{15}, l_{17}, and l_{18} in Eq. (5.20) before removing redundant terms from the effective Lagrangian. These low-energy constants are not independently observable and only renormalize the low-energy constants $\bar{v}_{eff}^2, \bar{g}_{eff}^2$, and $\bar{g'}_{eff}^2$ in the lowest order Lagrangian $\mathcal{L}_2$, nevertheless their contributions have to be kept in order to fully describe all effects for the standard model with a heavy Higgs boson up to order p^4.

For convenience, we have included these contributions into the low-energy constants $\bar{v}_{eff}^2, \bar{g}_{eff}^2$, and $\bar{g'}_{eff}^2$. Thus, $\bar{v}_{eff}^2$ contains terms of order p^0 and p^2, while $\bar{g}_{eff}^2$ and $\bar{g'}_{eff}^2$ contain terms of order p^2 and p^4. Since we have chosen the on-shell scheme, low-energy physics enters the effective Lagrangian after the renormalization through the input parameters $M_{W,\text{pole}}, M_{Z,\text{pole}}$, and e_{res}, leading to these nonanalytic terms. We note that the same happens in ordinary chiral perturbation theory for low-energy QCD. The relations between the parameters F and M in the effective Lagrangian and the physical pion decay constant F_π and the physical pion mass M_π both contain a nonanalytic chiral logarithm $\ln(M_\pi^2)$, see Ref. [13].

Thus, it is important to distinguish between the general, local effective Lagrangian with arbitrary bare low-energy constants that have to be determined from experiment from the explicit result for the effective Lagrangian for a given underlying theory, here the standard model with a heavy Higgs boson, evaluated in a given regularization and renormalization scheme.

Next we turn to the result for the effective Lagrangian $\mathcal{L}_4$ in Eq. (5.27) and the corresponding low-energy constants l_i' in Eq. (5.28). Since $\bar{\rho}=1$ we have 10 independent low-energy constants in $\mathcal{L}_4$. Only the low-energy constants $l_0' \equiv l_{16}, l_5', l_6'$, and l_8' in Eq. (5.28) are equal to their counterparts l_i in Eq. (5.20) before the elimination of redundant terms.

The low-energy constants in Eqs. (5.20) and (5.28) have the following general form:

$$
\begin{aligned}
l_i &= \delta_i \Lambda_\varepsilon + l_i^r(\mu)\,,\\
l_i' &= \delta_i' \Lambda_\varepsilon + {l_i'}^r(\mu)\,,
\end{aligned}
\tag{5.29}
$$

i.e. they contain a pole term proportional to Λ_ε, cf. Eq. (5.22), and a scale dependent part. We denote the coupling constants $l_i^r(\mu)$ and $l_i'^r(\mu)$ as renormalized low-energy constants. The renormalization group running of the coupling constants $l_i^r(\mu)$ and $l_i'^r(\mu)$ is determined by the coefficient δ_i and δ_i' of the respective pole term. These coefficients are determined by the one-loop divergences of the gauged nonlinear sigma model described by $\mathcal{L}_2^0$ and have been calculated long time ago [8,9]. They are universal, i.e. independent of any underlying strongly interacting model with the same symmetry breaking pattern as the standard model. Note that we obtain these universal pole terms only after the renormalization has been carried out in the standard model. The pole terms of the low-energy constants l_1, l_3, and l_5, which receive a tree-level contribution in the standard model with a heavy Higgs boson, have changed compared to the results for the bare low-energy constants in Eq. (4.20) which contain a term $1/\lambda$, where λ is the bare, divergent scalar coupling constant.

The effective Lagrangian given in Eqs. (5.23)–(5.28) can now be used to calculate physical quantities like scattering amplitudes up to order p^4, by adding tree-level diagrams from $\mathcal{L}_2 + \mathcal{L}_4$ and contributions from one-loop graphs with the Lagrangian $\mathcal{L}_2$. Note that the contributions from the source terms and from the path integral measure have to be taken into account as well. As discussed above, the renormalization has, however, been carried out already. In particular, there is no need to calculate once more the masses of the light particles, like the W or the Z-boson, in the effective field theory. Note that the effective Lagrangian $\mathcal{L}_4$ in Eqs. (5.27) and (5.28) contains pole terms Λ_ε even after the renormalization. This fact is well known from chiral perturbation theory [12,13]. One-loop graphs with vertices from $\mathcal{L}_2$ generate divergences which are canceled by the corresponding pole terms in the low-energy constants from $\mathcal{L}_4$. In this way, physical quantities will be finite.

We would like to add a few comments about the size of the renormalized low-energy constants $l_i'^r(\mu)$ in Eq. (5.28). First of all we note that due to the Veltman screening theorem [43], there are only logarithmic non-decoupling terms of the form $\ln(M_{H,\text{pole}}^2)$ in the low-energy constants $l_i'^r(\mu)$ in Eq. (5.28) at the one-loop level. In addition, the low-energy constants $l_1'^r(\mu), l_3'^r(\mu)$, and $l_5'^r(\mu)$ contain a tree-level contribution proportional to $1/M_{H,\text{pole}}^2$. Even though we assume that the Higgs boson is heavy, we cannot simply take $M_{H,\text{pole}} \to \infty$ and drop these terms. This would be equivalent to the assumption that the one-loop terms dominate over the tree-level contributions. Since our matching calculation was done by using perturbation theory this is certainly not permissible. The renormalized low-energy constants $l_i'^r(\mu)$ depend on a reference scale μ. We will vary this scale between the mass of the Z-boson, M_Z, and a value of 2 TeV, which lies in the resonance region of a truly strongly interacting symmetry breaking sector, e.g. this scale corresponds roughly to the mass of a technirho in technicolor models [31]. We thus follow the conventions usually adopted in chiral perturbation theory [13] for QCD where the setting $\mu = M_\rho$ is used to quote values for the renormalized low-energy constants. The Higgs boson mass is varied between M_Z and 2 TeV as well, although for Higgs boson masses above 1 TeV the applicability of perturbation theory is certainly questionable. We then find that the values of those renormalized low-energy constants $l_i'^r(\mu)$ which receive only contributions from loops are of the size which one would expect from using naive dimensional analysis [44], i.e. they are of the order of $1/(16\pi^2)$. On the other hand, as mentioned above, the low-energy constants $l_1'^r(\mu), l_3'^r(\mu)$, and $l_5'^r(\mu)$ contain a tree-level contribution proportional to $1/M_{H,\text{pole}}^2$. For all values of μ in the range between M_Z and 2 TeV this term dominates for Higgs boson masses below 1 TeV. In fact, in the low-energy constant $l_1'^r(\mu)$ the tree level and the one-loop term are of the same order of magnitude only for Higgs boson masses of the order of 2.5 TeV, due to an accidental cancellation in the one-loop contribution.

Some phenomenological consequences of the analysis presented here for models of a strongly interacting electroweak symmetry breaking sector can be found in Ref. [45]. In particular, we compare in that paper the results for the reduced set of independent low-energy constants $l_i'^r(\mu)$ for the standard model with a heavy Higgs boson with those for a simple technicolor model.

Finally, we would like to compare our result for the effective Lagrangian from Eqs. (5.23)–(5.28) for the standard model with a heavy Higgs boson after the renormalization with those obtained in the literature [19–21]. As noted above, the result for the bare effective Lagrangian in Eqs. (4.18)–(4.20) agreed with the literature. In order to facilitate the comparison we will use the usual notation for the electroweak chiral Lagrangian and discuss the low-energy constants a_i expressed through physical quantities as given in Appendix B, in Eq. (B16), and the low-energy constants a_i' after the elimination of redundant terms as given in Eq. (B17). First of all, the expression of the lowest order effective Lagrangian $\mathcal{L}_2$, Eq. (B14), agrees with Refs. [19–21], i.e. we have $\bar{\rho} = 1$. At order p^4 our result for the pole terms and the finite parts of the low-energy constants $a_i, i = 0, \ldots, 14$, given in Eq. (B16), agrees with the results obtained in Refs. [19]. Note that we have included some finite parts in the definition of the pole term Λ_ε, cf. Eq. (5.22), compared to the conventions used in that reference.

Reducing the number of terms as outlined in Sec. II A 2 leads to the results for the low-energy constants a_i' as given in Eq. (B17). Only the value of the low-energy constant a_3' has changed compared to a_3 in Eq. (B16). Note, however, that a_1 and a_8 have disappeared from the list of independent low-energy constants. In this respect our result differs from the literature since this further elimination of redundant terms was not carried out in Refs. [19–21].

Furthermore, the expressions for $\bar{v}_{eff}^2, \bar{g}_{eff}^2$, and $\bar{g'}_{eff}^2$ in $\mathcal{L}_2$ as given in Eqs. (5.24), (5.25), and (5.26), respectively, differ from the results obtained in Refs. [19–21]. This is due to the fact that we went one step further in the low-energy expansion of the mass for the Higgs boson in Eq. (5.4), of the masses for the gauge bosons in Eqs. (5.7), (5.12), and of the electric charge in Eq. (5.16). As mentioned above this is necessary in order to obtain all contributions in the effective field theory up to order p^4, if the low-energy constants in the effective Lagrangian are expressed through these physical input parameters.

As was noted already in Ref. [19] the results for the low-energy constants agree with those obtained in the ungauged $O(4)$-linear sigma model [13,27], in all cases where such a comparison is possible. Note that there are more low-energy constants in the present case, since the symmetry is $SU(2)_L \times U(1)_Y$ instead of $SU(2)_L \times SU(2)_R$ for the case of the sigma model. Employing a functional approach this agreement can easily be inferred from the matching relation (4.12). After the diagonalization of the differential operator in the full theory, those loops which contain gauge bosons are separated from the loops involving the Higgs and the Goldstone bosons. A similar observation was made in Ref. [21]. Since we count powers of g^2 and g'^2 as quantities of order p^2, any correction from gauge-boson loops to the low-energy constants in $\mathcal{L}_4$ must be of order p^6 in the effective field theory. Therefore within the standard model with a heavy Higgs boson, the effects from gauge-boson loops are suppressed compared to the contributions from the Higgs and the Goldstone bosons.

VI. SUMMARY AND DISCUSSION

In this article we have reanalyzed the electroweak chiral Lagrangian which describes the low-energy structure of a strongly interacting electroweak symmetry breaking sector. We have employed a manifestly gauge-invariant functional approach that was introduced recently [16]. It is well suited to analyze two issues related to gauge invariance where there are some subtleties involved, because one has to deal with off-shell quantities. First, we determined the number of independent low-energy constants in the electroweak chiral Lagrangian. By employing the equations of motion we found that the set of parameters currently used in the literature [10,11] is redundant. The second topic of this paper was the evaluation of the low-energy constants in the effective Lagrangian by matching the full and effective theory at low energies. As an example we studied the standard model with a heavy Higgs boson[6] where the calculation can be performed by using perturbative methods.

We first introduced the effective field theory for the bosonic part of a strongly interacting electroweak symmetry breaking sector under the assumption that $p^2, M_W^2, M_Z^2 \ll M^2$, where p is a typical momentum and M is the mass scale for heavy particles in the underlying theory, e.g. a heavy Higgs boson in the standard model or a technirho in some technicolor model [31]. In order to preserve the gauge symmetry we employed the gauge-invariant functional approach presented in Ref. [16]. Its essential feature is to consider Green's functions of gauge-invariant operators which excite one-particle states of the photon, the W-, and the Z-boson, respectively. The effective field theory is then described by an effective Lagrangian which is gauge-invariant and depends on the Goldstone boson field $\bar{U}$, the vector fields $\bar{W}_\mu^a, \bar{B}_\mu$, and external sources.

We have constructed the effective Lagrangian including appropriate source terms up to order p^4 in the low-energy expansion. The lowest order effective Lagrangian $\mathcal{L}_2$ involves the four physical parameters $\bar{e}, M_W, M_Z$, and $\bar{\rho}$, corresponding to the electric charge, the masses of the gauge bosons and the $\bar{\rho}$-parameter in the effective field theory, respectively. Furthermore, there are two additional low-energy constants from the source terms. At order p^4 the effective Lagrangian is given as a linear combination of a maximal set of gauge-invariant terms. One can then eliminate redundant terms by using algebraic relations which follow by partial integration. Since the Lagrangian $\mathcal{L}_4$ contributes only at the classical level one can also use the equations of motion to eliminate further redundant terms [13,17]. We note that in our gauge-invariant approach no gauge artifacts can enter through this procedure, because there is no gauge-fixing term and the sources respect the gauge symmetry. Finally, there are terms in the Lagrangian $\mathcal{L}_4$ which are proportional to corresponding terms in the lowest order Lagrangian $\mathcal{L}_2$. These terms lead to a renormalization of the low-energy constants and sources at order p^2 and therefore have no observable effect.

In this way we find that if one considers a purely bosonic effective field theory with the same symmetry breaking pattern as the standard model there are 10 physically relevant low-energy constants at order p^4 in the electroweak chiral Lagrangian. In particular, by employing the equations of motion of the gauge fields, one can choose to remove two low-energy constants, usually denoted by a_1 and a_8 [10], which contribute to the self-energies of the gauge bosons.

---

[6]Since all recent fits to electroweak precision data prefer a light Higgs boson [28], we regard the standard model with a heavy Higgs boson only as a testing ground for our method of matching.

This is in contrast to the number of 12 low-energy constants which is quoted in the literature [10,11]. An additional number of 63 low-energy constants contributes to the off-shell behavior of our gauge-invariant Green's functions. The latter low-energy constants, however, do not enter physical quantities.

If fermions are included the situation changes as follows. There are many more terms present in the effective Lagrangian, including sources coupled to the fermions. Therefore, a host of additional low-energy constants enters the effective Lagrangian. Many of them are, however, strongly bounded by experiments or irrelevant to the current experimental situation. A complete effective field theory analysis including the fermions was beyond the scope of the present work. Nevertheless, even when fermions are included, it is possible to eliminate the same two terms in the effective Lagrangian at order p^4 which contribute to the self-energies of the gauge bosons. This will only lead to a renormalization of the external sources as well as the couplings of the gauge fields to the fermions. Hence, even in the presence of fermions, the complete low-energy analysis of a strongly interacting symmetry breaking sector does not involve the low-energy constants a_1 and a_8.

These two low-energy constants are often identified with the oblique parameters S and U [2]. As discussed in Sec. II B this identification is not possible. The oblique parameters S, T, and U describe new physics *beyond* the standard model with an elementary Higgs boson, whereas the low-energy constants in the electroweak chiral Lagrangian describe any strongly interacting symmetry breaking sector, even if there is no Higgs boson at all. From the point of view of an effective Lagrangian analysis the parametrization of new physics effects by Peskin and Takeuchi amounts to setting all low-energy constants to their standard model values (assuming a heavy Higgs boson), except for three parameters contributing to gauge-boson self-energies. Employing the equations of motion one can still remove the terms corresponding to a_1 and a_8, however, two other low-energy constants will then differ from their values in the standard model and the total number of parameters to describe new physics remains three.

In the second part of the paper we have investigated the issue of evaluating the effective Lagrangian for a given underlying theory. The effective field theory can be defined by requiring, for instance, that corresponding Green's functions in the full and in the effective theory have the same low-energy structure. In order to make sure that no gauge artifacts can enter in this matching procedure, we propose to match gauge-invariant Green's functions. As an example we have considered the standard model with a heavy Higgs boson where the low-energy constants can explicitly be calculated using perturbative methods, if the scalar coupling constant is not too large. We briefly recapitulated the main results from our manifestly gauge-invariant approach [16] to the bosonic sector of the standard model. We then evaluated the matching condition at the one-loop level and at order p^4 in the low-energy expansion, employing functional techniques that have been discussed in detail in Ref. [27]. In this way we obtained the effective Lagrangian expressed through bare quantities. The results agree with the literature [19–21].

We then expressed the low-energy constants in the effective Lagrangian through physical quantities. As physical input parameters we chose the mass of the Higgs boson, the masses of the W- and Z-boson, and the electric charge (on-shell scheme) which have been extracted from two-point functions of appropriately chosen gauge-invariant operators in Ref. [16]. We went one step further in the low-energy expansion of the physical masses for the Higgs boson and the gauge bosons and the electric charge compared to Refs. [19–21]. In this way we obtained explicit expressions for the effective low-energy constants $\bar{v}^2_{eff}, \bar{g}^2_{eff}$, and $\bar{g}'^2_{eff}$ which appear in $\mathcal{L}_2$. As discussed in Sec. V C this is necessary in order to obtain all contributions in the effective field theory up to order p^4, if the low-energy constants in the effective Lagrangian are expressed through these physical input parameters. Furthermore, we removed the redundant terms in the effective Lagrangian by the procedure outlined in Sec. II A 2.

The effective Lagrangian given in Eqs. (5.23)–(5.28) can now be used to calculate physical quantities like scattering amplitudes up to order p^4, by adding tree-level diagrams from $\mathcal{L}_2 + \mathcal{L}_4$ and contributions from one-loop graphs with the Lagrangian $\mathcal{L}_2$. Note that the contributions from the source terms and from the path integral measure have to be taken into account as well. The renormalization has, however, been carried out already. In particular, there is no need to calculate once more the masses of the light particles, like the W or the Z-boson, in the effective field theory.

As was noted in Ref. [19] the results for the low-energy constants at order p^4 agree with those obtained in the ungauged $O(4)$-linear sigma model [13,27], in all cases where such a comparison is possible. This can easily be understood within our functional framework from the matching relation and the counting of powers of g^2 and g'^2 as quantities of order p^2. We note that this counting rule is needed for the consistency of the effective field theory. Therefore within the standard model with a heavy Higgs boson, the effects from gauge-boson loops are suppressed compared to the contributions from the Higgs and the Goldstone bosons. The situation is, however, different, if higher orders in the momentum expansion or in the loop expansion are evaluated or if other theories are considered. A well defined matching procedure which deals only with gauge-invariant quantities as proposed in this paper is mandatory in such cases.

Some phenomenological consequences of the analysis presented in this article for models of a strongly interacting electroweak symmetry breaking sector are discussed in Ref. [45]. In particular, we compare in that paper the results for the reduced set of independent low-energy constants l_i' (in the bosonic sector) for the standard model with a heavy Higgs boson with those for a simple technicolor model. The low-energy constants for the technicolor model have been estimated assuming that the exchange of the lowest lying resonances dominates the numerical values of the renormalized low-energy constants in the resonance region. This assumption works reasonably well for the coefficients in the ordinary chiral Lagrangian for QCD [13,46] and can be justified using large-N_c arguments and constraints from sum rules [47]. Since the pattern of the low-energy constants is very different in these two models it may be misleading to mimic any strongly interacting symmetry breaking sector by a heavy Higgs boson as done in Ref. [39]. From our investigation we conclude, in accordance with Ref. [14], that current electroweak precision data do not really rule out such strongly interacting models.

ACKNOWLEDGMENTS

We are grateful to F. Jegerlehner and V. Ravindran for enlightening discussions, a careful reading of the manuscript and suggestions for improvements. We are furthermore indebted to J. Gasser, M. Knecht, H. Leutwyler, E. de Rafael, R. Sommer, J. Stern, O. Veretin, and A. Vicini for useful discussions. A.N. is grateful to the members of the Yale Physics department for their kind hospitality during the early stages of this project. He also acknowledges financial support by Schweizerischer Nationalfonds during that period.

APPENDIX A: SOURCE TERMS AT ORDER p^4

In this Appendix we list all algebraically independent CP-even source terms which appear at order p^4 in the electroweak chiral Lagrangian in Eq. (2.50). We have not yet used the equations of motion to reduce the number of terms. The terms are grouped according to the total number of fields and sources.

Terms with four powers of fields and external sources:

$$
\begin{aligned}
\mathcal{O}_1^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\mu^-)(\bar{\mathcal{W}}_\nu^+ \bar{j}_\nu^- + \bar{\mathcal{W}}_\nu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_2^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)(\bar{\mathcal{W}}_\mu^+ \bar{j}_\nu^- + \bar{\mathcal{W}}_\nu^- \bar{j}_\mu^+)\,, \\
\mathcal{O}_3^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\mu^-)(\bar{\mathcal{Z}}_\nu \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_4^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^- + \bar{\mathcal{W}}_\mu^- \bar{\mathcal{W}}_\nu^+)(\bar{\mathcal{Z}}_\mu \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_5^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu)(\bar{\mathcal{W}}_\nu^+ \bar{j}_\nu^- + \bar{\mathcal{W}}_\nu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_6^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\nu)(\bar{\mathcal{W}}_\mu^+ \bar{j}_\nu^- + \bar{\mathcal{W}}_\mu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_7^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu)(\bar{\mathcal{Z}}_\nu \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_8^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\mu^-)(\bar{j}_\nu^+ \bar{j}_\nu^-)\,, \\
\mathcal{O}_9^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)(\bar{j}_\mu^+ \bar{j}_\nu^-)\,, \\
\mathcal{O}_{10}^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)(\bar{j}_\mu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_{11}^s &= (\bar{\mathcal{W}}_\mu^+ \bar{j}_\nu^-)(\bar{\mathcal{W}}_\mu^+ \bar{j}_\nu^-) + (\bar{\mathcal{W}}_\mu^- \bar{j}_\nu^+)(\bar{\mathcal{W}}_\mu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_{12}^s &= (\bar{\mathcal{W}}_\mu^+ \bar{j}_\mu^-)(\bar{\mathcal{W}}_\nu^+ \bar{j}_\nu^-) + (\bar{\mathcal{W}}_\mu^- \bar{j}_\mu^+)(\bar{\mathcal{W}}_\nu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_{13}^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\mu^-)(\bar{J}_\nu^{\mathcal{Z}} \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_{14}^s &= (\bar{\mathcal{W}}_\mu^+ \bar{\mathcal{W}}_\nu^-)(\bar{J}_\mu^{\mathcal{Z}} \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_{15}^s &= (\bar{\mathcal{Z}}_\mu \bar{J}_\mu^{\mathcal{Z}})(\bar{\mathcal{W}}_\nu^+ \bar{j}_\nu^- + \bar{\mathcal{W}}_\nu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_{16}^s &= (\bar{\mathcal{Z}}_\mu \bar{J}_\nu^{\mathcal{Z}})(\bar{\mathcal{W}}_\mu^+ \bar{j}_\nu^- + \bar{\mathcal{W}}_\mu^- \bar{j}_\nu^+)\,, \\
\mathcal{O}_{17}^s &= (\bar{\mathcal{Z}}_\mu \bar{J}_\nu^{\mathcal{Z}})(\bar{\mathcal{W}}_\nu^+ \bar{j}_\mu^- + \bar{\mathcal{W}}_\nu^- \bar{j}_\mu^+)\,, \\
\mathcal{O}_{18}^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu)(\bar{j}_\nu^+ \bar{j}_\nu^-)\,, \\
\mathcal{O}_{19}^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\nu)(\bar{j}_\mu^+ \bar{j}_\nu^-)\,, \\
\mathcal{O}_{20}^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\mu)(\bar{J}_\nu^{\mathcal{Z}} \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_{21}^s &= (\bar{\mathcal{Z}}_\mu \bar{\mathcal{Z}}_\nu)(\bar{J}_\mu^{\mathcal{Z}} \bar{J}_\nu^{\mathcal{Z}})\,, \\
\mathcal{O}_{22}^s &= (\bar{j}_\mu^+ \bar{j}_\mu^-)(\bar{j}_\nu^+ \bar{\mathcal{W}}_\nu^- + \bar{j}_\nu^- \bar{\mathcal{W}}_\nu^+)\,, \\
\mathcal{O}_{23}^s &= (\bar{j}_\mu^+ \bar{j}_\nu^-)(\bar{j}_\mu^+ \bar{\mathcal{W}}_\nu^- + \bar{j}_\nu^- \bar{\mathcal{W}}_\mu^+)\,,
\end{aligned}
$$

$$\begin{aligned}
\mathcal{O}^s_{24} &= (\bar{J}^{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu)(\bar{j}^+_\nu \bar{\mathcal{W}}^-_\nu + \bar{j}^-_\nu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{25} &= (\bar{J}^{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\nu)(\bar{j}^+_\mu \bar{\mathcal{W}}^-_\nu + \bar{j}^-_\mu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{26} &= (\bar{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu)(\bar{j}^+_\nu \bar{j}^-_\nu)\,,\\
\mathcal{O}^s_{27} &= (\bar{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\nu)(\bar{j}^+_\mu \bar{j}^-_\nu + \bar{j}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{28} &= (\bar{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu)(\bar{J}^{\mathcal{Z}}_\nu \bar{J}^{\mathcal{Z}}_\nu)\,,\\
\mathcal{O}^s_{29} &= (\bar{j}^+_\mu \bar{j}^-_\mu)(\bar{j}^+_\nu \bar{j}^-_\nu)\,,\\
\mathcal{O}^s_{30} &= (\bar{j}^+_\mu \bar{j}^-_\nu)(\bar{j}^+_\mu \bar{j}^-_\nu)\,,\\
\mathcal{O}^s_{31} &= (\bar{J}^{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu)(\bar{j}^+_\nu \bar{j}^-_\nu)\,,\\
\mathcal{O}^s_{32} &= (\bar{J}^{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\nu)(\bar{j}^+_\mu \bar{j}^-_\nu)\,,\\
\mathcal{O}^s_{33} &= (\bar{J}^{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu)(\bar{J}^{\mathcal{Z}}_\nu \bar{J}^{\mathcal{Z}}_\nu)\,.
\end{aligned} \tag{A1}$$

Terms with three powers of fields and external sources:

$$\begin{aligned}
\mathcal{O}^s_{34} &= i\bar{J}^{\mathcal{Z}}_{\mu\nu}(\bar{\mathcal{W}}^+_\mu \bar{\mathcal{W}}^-_\nu - \bar{\mathcal{W}}^+_\nu \bar{\mathcal{W}}^-_\mu)\,,\\
\mathcal{O}^s_{35} &= i\bar{\mathcal{Z}}_{\mu\nu}(\bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{\mathcal{W}}^+_\nu \bar{j}^-_\mu - \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu + \bar{\mathcal{W}}^-_\nu \bar{j}^+_\mu)\,,\\
\mathcal{O}^s_{36} &= i\bar{B}_{\mu\nu}(\bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{\mathcal{W}}^+_\nu \bar{j}^-_\mu - \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu + \bar{\mathcal{W}}^-_\nu \bar{j}^+_\mu)\,,\\
\mathcal{O}^s_{37} &= i\bar{J}^{\mathcal{Z}}_{\mu\nu}(\bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{\mathcal{W}}^+_\nu \bar{j}^-_\mu - \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu + \bar{\mathcal{W}}^-_\nu \bar{j}^+_\mu)\,,\\
\mathcal{O}^s_{38} &= i\bar{\mathcal{Z}}_{\mu\nu}(\bar{j}^+_\mu \bar{j}^-_\nu - \bar{j}^+_\nu \bar{j}^-_\mu)\,,\\
\mathcal{O}^s_{39} &= i\bar{B}_{\mu\nu}(\bar{j}^+_\mu \bar{j}^-_\nu - \bar{j}^+_\nu \bar{j}^-_\mu)\,,\\
\mathcal{O}^s_{40} &= i\bar{J}^{\mathcal{Z}}_{\mu\nu}(\bar{j}^+_\mu \bar{j}^-_\nu - \bar{j}^+_\nu \bar{j}^-_\mu)\,,\\
\mathcal{O}^s_{41} &= i\bar{J}^{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{\mathcal{W}}^+_\mu \bar{\mathcal{W}}^-_\nu - \bar{d}_\mu \bar{\mathcal{W}}^-_\mu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{42} &= i\bar{J}^{\mathcal{Z}}_\mu(\bar{d}_\mu \bar{\mathcal{W}}^+_\nu \bar{\mathcal{W}}^-_\nu - \bar{d}_\mu \bar{\mathcal{W}}^-_\nu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{43} &= i\bar{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{d}_\mu \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{44} &= i\bar{\mathcal{Z}}_\mu(\bar{d}_\mu \bar{\mathcal{W}}^+_\nu \bar{j}^-_\nu - \bar{d}_\mu \bar{\mathcal{W}}^-_\nu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{45} &= i\bar{\mathcal{Z}}_\mu(\bar{d}_\nu \bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{d}_\nu \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{46} &= i\bar{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{j}^+_\mu \bar{\mathcal{W}}^-_\nu - \bar{d}_\mu \bar{j}^-_\mu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{47} &= i(\partial_\mu \bar{\mathcal{Z}}_\mu)(\bar{\mathcal{W}}^+_\nu \bar{j}^-_\nu - \bar{\mathcal{W}}^-_\nu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{48} &= i\bar{J}^{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{d}_\mu \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{49} &= i\bar{J}^{\mathcal{Z}}_\mu(\bar{d}_\mu \bar{\mathcal{W}}^+_\nu \bar{j}^-_\nu - \bar{d}_\mu \bar{\mathcal{W}}^-_\nu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{50} &= i\bar{J}^{\mathcal{Z}}_\mu(\bar{d}_\nu \bar{\mathcal{W}}^+_\mu \bar{j}^-_\nu - \bar{d}_\nu \bar{\mathcal{W}}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{51} &= i\bar{J}^{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{j}^+_\mu \bar{\mathcal{W}}^-_\nu - \bar{d}_\mu \bar{j}^-_\mu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{52} &= i\bar{J}^{\mathcal{Z}}_\mu(\bar{d}_\mu \bar{j}^+_\nu \bar{\mathcal{W}}^-_\nu - \bar{d}_\mu \bar{j}^-_\nu \bar{\mathcal{W}}^+_\nu)\,,\\
\mathcal{O}^s_{53} &= i\bar{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{j}^+_\mu \bar{j}^-_\nu - \bar{d}_\mu \bar{j}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{54} &= i\bar{\mathcal{Z}}_\mu(\bar{d}_\mu \bar{j}^+_\nu \bar{j}^-_\nu - \bar{d}_\mu \bar{j}^-_\nu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{55} &= i\bar{J}^{\mathcal{Z}}_\nu(\bar{d}_\mu \bar{j}^+_\mu \bar{j}^-_\nu - \bar{d}_\mu \bar{j}^-_\mu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{56} &= i\bar{J}^{\mathcal{Z}}_\mu(\bar{d}_\mu \bar{j}^+_\nu \bar{j}^-_\nu - \bar{d}_\mu \bar{j}^-_\nu \bar{j}^+_\nu)\,,\\
\mathcal{O}^s_{57} &= \epsilon_{\mu\nu\rho\sigma}\bar{J}^{\mathcal{Z}}_\sigma(\bar{\mathcal{W}}^-_\rho \bar{\mathcal{W}}^+_{\mu\nu} + \bar{\mathcal{W}}^+_\rho \bar{\mathcal{W}}^-_{\mu\nu})\,,\\
\mathcal{O}^s_{58} &= \epsilon_{\mu\nu\rho\sigma}\bar{\mathcal{Z}}_\sigma(\bar{j}^-_\rho \bar{\mathcal{W}}^+_{\mu\nu} + \bar{j}^+_\rho \bar{\mathcal{W}}^-_{\mu\nu})\,,\\
\mathcal{O}^s_{59} &= \epsilon_{\mu\nu\rho\sigma}\bar{\mathcal{Z}}_\sigma(\bar{\mathcal{W}}^-_\rho \bar{j}^+_{\mu\nu} + \bar{\mathcal{W}}^+_\rho \bar{j}^-_{\mu\nu})\,,\\
\mathcal{O}^s_{60} &= \epsilon_{\mu\nu\rho\sigma}\bar{J}^{\mathcal{Z}}_\sigma(\bar{j}^-_\rho \bar{\mathcal{W}}^+_{\mu\nu} + \bar{j}^+_\rho \bar{\mathcal{W}}^-_{\mu\nu})\,,\\
\mathcal{O}^s_{61} &= \epsilon_{\mu\nu\rho\sigma}\bar{J}^{\mathcal{Z}}_\sigma(\bar{\mathcal{W}}^-_\rho \bar{j}^+_{\mu\nu} + \bar{\mathcal{W}}^+_\rho \bar{j}^-_{\mu\nu})\,,\\
\mathcal{O}^s_{62} &= \epsilon_{\mu\nu\rho\sigma}\bar{\mathcal{Z}}_\sigma(\bar{j}^-_\rho \bar{j}^+_{\mu\nu} + \bar{j}^+_\rho \bar{j}^-_{\mu\nu})\,,
\end{aligned}$$

$$\mathcal{O}^s_{63} = \epsilon_{\mu\nu\rho\sigma} \bar{J}^{\mathcal{Z}}_\sigma (\bar{j}^-_\rho \bar{j}^+_{\mu\nu} + \bar{j}^+_\rho \bar{j}^-_{\mu\nu}) \,. \tag{A2}$$

Terms with two powers of fields and external sources:

$$\begin{aligned}
\mathcal{O}^s_{64} &= M_W^2 (\bar{\mathcal{W}}^+_\mu \bar{j}^-_\mu + \bar{\mathcal{W}}^-_\mu \bar{j}^+_\mu) \,, \\
\mathcal{O}^s_{65} &= M_W^2 \bar{j}^+_\mu \bar{j}^-_\mu \,, \\
\mathcal{O}^s_{66} &= M_Z^2 \bar{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu \,, \\
\mathcal{O}^s_{67} &= M_Z^2 \bar{J}^{\mathcal{Z}}_\mu \bar{J}^{\mathcal{Z}}_\mu \,, \\
\mathcal{O}^s_{68} &= \bar{\mathcal{W}}^+_{\mu\nu} \bar{j}^-_{\mu\nu} + \bar{\mathcal{W}}^-_{\mu\nu} \bar{j}^+_{\mu\nu} \,, \\
\mathcal{O}^s_{69} &= \bar{j}^+_{\mu\nu} \bar{j}^-_{\mu\nu} \,, \\
\mathcal{O}^s_{70} &= \bar{\mathcal{Z}}_{\mu\nu} \bar{J}^{\mathcal{Z}}_{\mu\nu} \,, \\
\mathcal{O}^s_{71} &= \bar{B}_{\mu\nu} \bar{J}^{\mathcal{Z}}_{\mu\nu} \,, \\
\mathcal{O}^s_{72} &= \bar{J}^{\mathcal{Z}}_{\mu\nu} \bar{J}^{\mathcal{Z}}_{\mu\nu} \,, \\
\mathcal{O}^s_{73} &= (\bar{d}_\mu \bar{\mathcal{W}}^+_\mu)(\bar{d}_\nu \bar{j}^-_\nu) + (\bar{d}_\mu \bar{\mathcal{W}}^-_\mu)(\bar{d}_\nu \bar{j}^+_\nu) \,, \\
\mathcal{O}^s_{74} &= (\bar{d}_\mu \bar{j}^+_\mu)(\bar{d}_\nu \bar{j}^-_\nu) \,, \\
\mathcal{O}^s_{75} &= (\partial_\mu \bar{\mathcal{Z}}_\mu)(\partial_\nu \bar{J}^{\mathcal{Z}}_\nu) \,, \\
\mathcal{O}^s_{76} &= (\partial_\mu \bar{J}^{\mathcal{Z}}_\mu)(\partial_\nu \bar{J}^{\mathcal{Z}}_\nu) \,,
\end{aligned} \tag{A3}$$

where we introduced the quantities

$$\bar{j}^\pm_{\mu\nu} = \bar{d}_\mu \bar{j}^\pm_\nu - \bar{d}_\nu \bar{j}^\pm_\mu \,, \tag{A4}$$

$$\bar{J}^{\mathcal{Z}}_{\mu\nu} = \partial_\mu \bar{J}^{\mathcal{Z}}_\nu - \partial_\nu \bar{J}^{\mathcal{Z}}_\mu \,. \tag{A5}$$

APPENDIX B: THE ELECTROWEAK CHIRAL LAGRANGIAN

It became customary in the literature to describe the low-energy effective field theory of the bosonic sector of strongly interacting models of electroweak symmetry breaking in terms of the so called electroweak chiral Lagrangian, introduced in Refs. [8–11]. Before we write down the effective Lagrangian in the notation employed in these references, we would like to add some comments. Following the first paper of Ref. [9] and Ref. [10] we include a custodial symmetry breaking term proportional to $\bar{\rho}-1$ already at order p^2 in the low-energy expansion, cf. Eq. (2.27). This is in contrast to the recent literature which follows mostly the conventions used in the second paper of Ref. [9] or those of the second paper of Ref. [19]. These conventions may be recovered in our approach by setting $\bar{\rho} = 1$. Furthermore, we include in the list of operators at order p^4 the four terms $\mathcal{O}_{15}, \mathcal{O}_{16}, \mathcal{O}_{17}$, and $\mathcal{O}_{18}$, cf. Eq. (2.49), which are proportional to corresponding terms in $\mathcal{L}_2$. The use of the equations of motion and the renormalization of the low-energy constants in $\mathcal{L}_2$ in order to reduce the number of terms in $\mathcal{L}_4$ will be discussed later. Finally, no external sources have been introduced in Refs. [8–11,19–21]. We therefore list here only the terms $\mathcal{L}^0_2$, Eq. (2.27), and $\mathcal{L}^0_4$, Eq. (2.48), which do not contain external sources.

Following Refs. [8–11] we introduce a $SU(2)$ matrix notation for the Goldstone bosons and the gauge fields:

$$\begin{aligned}
\hat{U} &= \exp\left(i \frac{\tau^a \pi^a}{\bar{v}} \right) \in SU(2) \,, \\
\hat{W} &= W^a_\mu \frac{\tau^a}{2} \,,\ \hat{B} = B_\mu \frac{\tau^3}{2} \,, \\
\hat{D}_\mu \hat{U} &= \partial_\mu \hat{U} - i \hat{W}_\mu \hat{U} + i \hat{U} \hat{B}_\mu \,, \\
\hat{W}_{\mu\nu} &= \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\nu - i[\hat{W}_\mu, \hat{W}_\nu] \,.
\end{aligned} \tag{B1}$$

The effective Lagrangian can then be written in the following way:

$$\mathcal{L}^0_2 = \frac{1}{4} \bar{v}^2 \mathrm{tr}(\hat{D}_\mu \hat{U}^\dagger \hat{D}_\mu \hat{U}) - (\bar{\rho} - 1) \frac{\bar{v}^2}{8} \left[\mathrm{tr}(\hat{T} \hat{V}_\mu) \right]^2 + \frac{1}{2\bar{g}^2} \mathrm{tr}(\hat{W}_{\mu\nu} \hat{W}_{\mu\nu}) + \frac{1}{2\bar{g}'^2} \mathrm{tr}(\hat{B}_{\mu\nu} \hat{B}_{\mu\nu}) \,, \tag{B2}$$

$$\mathcal{L}^0_4 = \sum_{i=0}^{17} a_i L_i \,, \tag{B3}$$

with the basic set of operators (CP-even terms only):

$$
\begin{aligned}
L_0 &= \frac{\bar{v}^2}{4}\bar{g}'^2 \left[\mathrm{tr}(\hat{T}\hat{V}_\mu)\right]^2 \,, \\
L_1 &= -\frac{1}{2}B_{\mu\nu}\mathrm{tr}(\hat{T}\hat{W}_{\mu\nu})\,, \\
L_2 &= i\frac{1}{2}B_{\mu\nu}\mathrm{tr}(\hat{T}[\hat{V}_\mu,\hat{V}_\nu])\,, \\
L_3 &= -i\mathrm{tr}(\hat{W}_{\mu\nu}[\hat{V}_\mu,\hat{V}_\nu])\,, \\
L_4 &= -\left[\mathrm{tr}(\hat{V}_\mu\hat{V}_\nu)\right]^2 \,, \\
L_5 &= -\left[\mathrm{tr}(\hat{V}_\mu\hat{V}_\mu)\right]^2 \,, \\
L_6 &= -\mathrm{tr}(\hat{V}_\mu\hat{V}_\nu)\mathrm{tr}(\hat{T}\hat{V}_\mu)\mathrm{tr}(\hat{T}\hat{V}_\nu)\,, \\
L_7 &= -\mathrm{tr}(\hat{V}_\mu\hat{V}_\mu)\left[\mathrm{tr}(\hat{T}\hat{V}_\nu)\right]^2 \,, \\
L_8 &= \frac{1}{4}\left[\mathrm{tr}(\hat{T}\hat{W}_{\mu\nu})\right]^2 \,, \\
L_9 &= -i\frac{1}{2}\mathrm{tr}(\hat{T}\hat{W}_{\mu\nu})\mathrm{tr}(\hat{T}[\hat{V}_\mu,\hat{V}_\nu])\,, \\
L_{10} &= -\left[\mathrm{tr}(\hat{T}\hat{V}_\mu)\mathrm{tr}(\hat{T}\hat{V}_\mu)\right]^2 \,, \\
L_{11} &= -\mathrm{tr}((\hat{D}_\mu\hat{V}_\mu)^2)\,, \\
L_{12} &= -\mathrm{tr}(\hat{T}\hat{D}_\mu\hat{D}_\nu\hat{V}_\nu)\mathrm{tr}(\hat{T}\hat{V}_\mu)\,, \\
L_{13} &= -\frac{1}{2}\left[\mathrm{tr}(\hat{T}\hat{D}_\mu\hat{V}_\nu)\right]^2 \,, \\
L_{14} &= \epsilon_{\mu\nu\rho\sigma}\mathrm{tr}(\hat{W}_{\mu\nu}\hat{V}_\rho)\mathrm{tr}(\hat{T}\hat{V}_\sigma)\,, \\
L_{15} &= M_W^2\mathrm{tr}(\hat{D}_\mu\hat{U}^+\hat{D}_\mu\hat{U})\,, \\
L_{16} &= \mathrm{tr}(\hat{W}_{\mu\nu}\hat{W}_{\mu\nu})\,, \\
L_{17} &= \mathrm{tr}(\hat{B}_{\mu\nu}\hat{B}_{\mu\nu})\,.
\end{aligned}
\qquad \text{(B4)}
$$

In Eqs. (B2)–(B4) we used the building blocks

$$
\begin{aligned}
\hat{T} &= \hat{U}\tau^3\hat{U}^\dagger \,,\ \hat{V}_\mu = (\hat{D}_\mu\hat{U})\hat{U}^\dagger\,, \\
\hat{D}_\mu\hat{V}_\nu &= \partial_\mu\hat{V}_\nu - i[\hat{W}_\mu,\hat{V}_\nu]\,.
\end{aligned}
\qquad \text{(B5)}
$$

We recall that we count the gauge coupling constants g, g' as order p in the low-energy expansion, therefore the custodial symmetry breaking term L_0 is of the order p^4. Note that we have used in Eqs. (B2)–(B5) a different convention for the signs of the gauge coupling constants compared to the literature. Specifically, we have $g \to -\bar{g}$ and $g' \to -\bar{g}'$ compared to Ref. [19]. Furthermore, we have again absorbed the gauge coupling constants into the gauge fields, cf. Eq. (2.3). Note that $\bar{v}$ corresponds to the pion decay constant F_π in chiral perturbation theory.

The relations between the two sets of operators, $\mathcal{O}_i$ from Eq. (2.49) and the L_i, read

$$
\begin{aligned}
L_0 &= -\bar{s}^2\mathcal{O}_{16}\,, \\
L_1 &= \mathcal{O}_8 - \frac{1}{2}\mathcal{O}_{12} - \frac{1}{2}\mathcal{O}_{18}\,, \\
L_2 &= -\mathcal{O}_8\,, \\
L_3 &= -4\mathcal{O}_1 + 4\mathcal{O}_2 - 4\mathcal{O}_3 + 4\mathcal{O}_4 + 2\mathcal{O}_7 + \mathcal{O}_8 - 2\mathcal{O}_9 + 2\mathcal{O}_{10}\,, \\
L_4 &= -2\mathcal{O}_1 - 2\mathcal{O}_2 - 2\mathcal{O}_4 - \frac{1}{4}\mathcal{O}_5\,, \\
L_5 &= -4\mathcal{O}_1 - 2\mathcal{O}_3 - \frac{1}{4}\mathcal{O}_5\,, \\
L_6 &= -2\mathcal{O}_4 - \frac{1}{2}\mathcal{O}_5\,,
\end{aligned}
$$

$$
\begin{aligned}
L_7 &= -2\mathcal{O}_3 - \frac{1}{2}\mathcal{O}_5\,, \\
L_8 &= 2\mathcal{O}_1 - 2\mathcal{O}_2 - \mathcal{O}_7 - \mathcal{O}_8 + \frac{1}{4}\mathcal{O}_{11} + \frac{1}{2}\mathcal{O}_{12} + \frac{1}{4}\mathcal{O}_{18}\,, \\
L_9 &= -4\mathcal{O}_1 + 4\mathcal{O}_2 + \mathcal{O}_7 + \mathcal{O}_8\,, \\
L_{10} &= -\mathcal{O}_5\,, \\
L_{11} &= 2\mathcal{O}_{13} + \frac{1}{2}\mathcal{O}_{14}\,, \\
L_{12} &= 2\mathcal{O}_{10} - \mathcal{O}_{14}\,, \\
L_{13} &= 4\mathcal{O}_1 - 4\mathcal{O}_2 - \mathcal{O}_7 + \frac{1}{4}\mathcal{O}_{11} + \frac{1}{2}\mathcal{O}_{14}\,, \\
L_{14} &= i\mathcal{O}_6\,, \\
L_{15} &= 2\mathcal{O}_{15}\,, \\
L_{16} &= \frac{1}{2}\mathcal{O}_{17}\,, \\
L_{17} &= \frac{1}{2}\mathcal{O}_{18}\,, \qquad \text{(B6)}
\end{aligned}
$$

which are valid up to partial integrations.

As discussed in Sec. II A 2 the equations of motion in the effective field theory lead to relations between the operators L_i in $\mathcal{L}_4$, cf. the relations in Eqs. (2.52)–(2.64) between the operators $\mathcal{O}_i$. From the constraint equations (2.37) and (2.38), which are equivalent to $\mathrm{tr}(\hat{D}_\mu \hat{V}_\mu) = 0$ in the usually employed notation, we obtain the following relations:

$$L_{11} = 0\,, \qquad \text{(B7)}$$

$$L_{12} = 0\,, \qquad \text{(B8)}$$

$$L_{13} = \frac{\bar{c}^2}{2\bar{s}^2}L_0 + L_3 + L_4 - L_5 - L_6 + L_7 - L_9 - L_{15} - \frac{1}{2}L_{16} + \frac{1}{2}L_{17}\,. \qquad \text{(B9)}$$

The equations of motion for the gauge fields in Eqs. (2.34) and (2.35) lead to the relations

$$L_1 = L_3 - 2L_{15} - L_{16}\,, \qquad \text{(B10)}$$

$$L_8 = \frac{\bar{c}^2}{2\bar{s}^2}L_0 - L_9 + L_{15} + \frac{1}{2}L_{16}\,. \qquad \text{(B11)}$$

For simplicity we have set $\bar{\rho} = 1$ and switched off the external sources. The general relations can be inferred from Eqs. (2.52)–(2.64) by making use of Eq. (B6) to convert the basis with the operators $\mathcal{O}_i$ into the basis with the operators L_i. Note that Eq. (B9) has changed compared to Eq. (2.90) because we have replaced above the operators L_1 and L_8 on the right-hand side of the equation. Furthermore, we note that we get a different sign of the terms L_4 and L_5 in Eq. (B9) compared to Ref. [11].

With the help of Eqs. (B7)–(B11) we can remove the operators L_1, L_8, L_{11}, L_{12}, and L_{13} from the basis. Furthermore, we can remove the terms L_{15}, L_{16}, and L_{17}, which are proportional to terms in the Lagrangian $\mathcal{L}_2^0$, by renormalizing the parameters and low-energy constants in the lowest order Lagrangian, cf. Eqs. (2.67)–(2.70).

1. The effective Lagrangian for the standard model with a heavy Higgs boson

The result for the bare effective Lagrangian from Eqs. (4.18)–(4.20) for the standard model with a heavy Higgs boson translates into the following expression for the bare low-energy constant $\bar{v}_b^2$ in $\mathcal{L}_2$, Eq. (B2),

$$\bar{v}_b^2 = \frac{m^2}{\lambda}\left[1 + \lambda\left(-12\Lambda_\varepsilon(2m^2) + \frac{1}{16\pi^2}\right)\right]\,. \qquad \text{(B12)}$$

Furthermore we obtain $\bar{\rho} = 1$ in Eq. (B2). The bare low-energy constants a_i^b in $\mathcal{L}_4$, Eq. (B3), are given by

$$
\begin{aligned}
a_0^b &= -\frac{3}{4}\Lambda_\varepsilon(2m^2) - \frac{1}{16}\frac{1}{16\pi^2}\,, \\
a_1^b &= -\frac{1}{6}\Lambda_\varepsilon(2m^2) - \frac{1}{72}\frac{1}{16\pi^2}\,,
\end{aligned}
$$

$$
\begin{aligned}
a_2^b &= -\frac{1}{12}\Lambda_\varepsilon(2m^2) + \frac{11}{144}\frac{1}{16\pi^2}\,,\\
a_3^b &= \frac{1}{12}\Lambda_\varepsilon(2m^2) - \frac{11}{144}\frac{1}{16\pi^2}\,,\\
a_4^b &= \frac{1}{6}\Lambda_\varepsilon(2m^2) - \frac{11}{72}\frac{1}{16\pi^2}\,,\\
a_5^b &= \frac{1}{16\lambda} - \frac{17}{12}\Lambda_\varepsilon(2m^2) - \frac{35}{144}\frac{1}{16\pi^2}\,,\\
a_{11}^b &= -\frac{1}{24}\frac{1}{16\pi^2}\,,\\
a_{15}^b &= \frac{3}{2}\Lambda_\varepsilon(2m^2) + \frac{1}{8}\frac{1}{16\pi^2}\,,\\
a_{16}^b &= -\frac{1}{12}\Lambda_\varepsilon(2m^2) - \frac{1}{144}\frac{1}{16\pi^2}\,,\\
a_{17}^b &= -\frac{1}{12}\Lambda_\varepsilon(2m^2) - \frac{1}{144}\frac{1}{16\pi^2}\,.
\end{aligned} \tag{B13}
$$

All other bare low-energy constants a_i^b vanish. Note that we have included some additional, finite terms into our definition of the pole term $\Lambda_\varepsilon(2m^2)$, cf. Eq. (4.17), compared to the conventions used in Refs. [19–21].

Inserting the physical masses and coupling constants from Sec. V, the effective Lagrangian reads

$$
\mathcal{L}_2 = \left(M_{W,\mathrm{pole}}^2 \frac{s_p^2}{e_{\mathrm{res}}^2}\right) \mathrm{tr}(\hat{D}_\mu \hat{U}^+ \hat{D}_\mu \hat{U}) + \frac{s_p^2}{2e_{\mathrm{res}}^2}\mathrm{tr}(\hat{W}_{\mu\nu}\hat{W}_{\mu\nu}) + \frac{c_p^2}{2e_{\mathrm{res}}^2}\mathrm{tr}(\hat{B}_{\mu\nu}\hat{B}_{\mu\nu})\,, \tag{B14}
$$

$$
\mathcal{L}_4 = \sum_{i=0}^{17} a_i L_i\,, \tag{B15}
$$

with the non-vanishing low-energy constants

$$
\begin{aligned}
a_0 &= -\frac{3}{4}\Lambda_\varepsilon - \frac{3}{8}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{1}{16}\frac{1}{16\pi^2}\,,\\
a_1 &= -\frac{1}{6}\Lambda_\varepsilon - \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{1}{72}\frac{1}{16\pi^2}\,,\\
a_2 &= -\frac{1}{12}\Lambda_\varepsilon - \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) + \frac{11}{144}\frac{1}{16\pi^2}\,,\\
a_3 &= \frac{1}{12}\Lambda_\varepsilon + \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{11}{144}\frac{1}{16\pi^2}\,,\\
a_4 &= \frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{11}{72}\frac{1}{16\pi^2}\,,\\
a_5 &= \frac{1}{12}\Lambda_\varepsilon + \frac{s_p^2 M_{W,\mathrm{pole}}^2}{2e_{\mathrm{res}}^2 M_{H,\mathrm{pole}}^2} + \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{1}{16\pi^2}\frac{152-27\sqrt{3}\pi}{144}\,,\\
a_{11} &= -\frac{1}{24}\frac{1}{16\pi^2}\,,\\
a_{15} &= \frac{3}{2}\Lambda_\varepsilon + \frac{3}{4}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) + \frac{1}{8}\frac{1}{16\pi^2} - \frac{1}{8}(1-\frac{c_p^2}{s_p^2})\delta M_{W,2}^2 - \frac{1}{8s_p^2}\delta M_{Z,2}^2 + s_p^2\delta e_2^2\,,\\
a_{16} &= -\frac{1}{12}\Lambda_\varepsilon - \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{1}{144}\frac{1}{16\pi^2} + \frac{c_p^2}{16s_p^2}\delta M_{W,2}^2 - \frac{1}{16s_p^2}\delta M_{Z,2}^2 + \frac{1}{2}s_p^2\delta e_2^2\,,\\
a_{17} &= -\frac{1}{12}\Lambda_\varepsilon - \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,\mathrm{pole}}^2}{\mu^2}\right) - \frac{1}{144}\frac{1}{16\pi^2} - \frac{c_p^2}{16s_p^2}\delta M_{W,2}^2 + \frac{1}{16s_p^2}\delta M_{Z,2}^2 + \frac{1}{2}c_p^2\delta e_2^2\,.
\end{aligned} \tag{B16}
$$

The pole term in $d = 4$ dimensions, Λ_ε, is defined in Eq. (5.22). We denoted the pole-masses of the Higgs boson, the W- and the Z-boson by $M_{H,\text{pole}}$, $M_{W,\text{pole}}$, and $M_{Z,\text{pole}}$, respectively. The electric charge is denoted by e_{res}. The quantities $\delta M^2_{W,2}, \delta M^2_{Z,2}$, and δe^2_2 are defined in Eqs. (5.9), (5.14), and (5.17), respectively. Furthermore, we use the on-shell definition for the weak mixing angle c^2_p, s^2_p, cf. Eq. (5.21).

Finally, we can remove the redundant terms L_1, L_8, L_{11}, L_{12}, and L_{13} from the basis by employing the Eqs. (B7)–(B11) and the terms L_{15}, L_{16}, and L_{17} by renormalizing the parameters in the lowest order Lagrangian $\mathcal{L}^0_2$. In this way we obtain the expression for the Lagrangian $\mathcal{L}_2$ as given in Eqs. (5.23)–(5.26) and the following results for the 10 low-energy constants corresponding to independent terms in the Lagrangian $\mathcal{L}^0_4$:

$$
\begin{aligned}
a'_0 &= -\frac{3}{4}\Lambda_\varepsilon - \frac{3}{8}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{1}{16}\frac{1}{16\pi^2}\,, \\
a'_2 &= -\frac{1}{12}\Lambda_\varepsilon - \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) + \frac{11}{144}\frac{1}{16\pi^2}\,, \\
a'_3 &= -\frac{1}{12}\Lambda_\varepsilon - \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{13}{144}\frac{1}{16\pi^2}\,, \\
a'_4 &= \frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{11}{72}\frac{1}{16\pi^2}\,, \\
a'_5 &= \frac{1}{12}\Lambda_\varepsilon + \frac{s^2_p M^2_{W,\text{pole}}}{2e^2_{\text{res}} M^2_{H,\text{pole}}} + \frac{1}{24}\frac{1}{16\pi^2}\ln\left(\frac{M^2_{H,\text{pole}}}{\mu^2}\right) - \frac{1}{16\pi^2}\frac{152 - 27\sqrt{3}\pi}{144}\,, \\
a'_6 &= 0\,, \\
a'_7 &= 0\,, \\
a'_9 &= 0\,, \\
a'_{10} &= 0\,, \\
a'_{14} &= 0\,.
\end{aligned}
\tag{B17}
$$

We have denoted the modified low-energy constants by a'_i in order to distinguish them from the old ones. Only the low-energy constant a'_3 has changed in comparison with the values given in Eq. (B16). Note, however, that a_1 and a_8 have disappeared from the list of independent low-energy constants.

The low-energy constants in Eqs (B16) and (B17) have the following general form:

$$
\begin{aligned}
a_i &= \Delta_i \Lambda_\varepsilon + a^r_i(\mu)\,, \\
a'_i &= \Delta'_i \Lambda_\varepsilon + a'^{\,r}_i(\mu)\,,
\end{aligned}
\tag{B18}
$$

i.e. they contain a pole term proportional to Λ_ε and a scale dependent part. We denote the coupling constants $a^r_i(\mu)$ and $a'^{\,r}_i(\mu)$ as renormalized low-energy constants.

APPENDIX C: DIFFERENTIAL OPERATORS IN THE STANDARD MODEL

The explicit results for the differential operators $\widetilde{D} + PP^T + \delta_P$ and $P^T P$ which appear in Eq. (3.36) in Sec. III are given below. In the following, upper case Latin indices $A, B, \ldots$ run from 1 to 4, lower case Latin indices $a, b, \ldots$ run from 1 to 3, and Greek indices $\alpha, \beta, \ldots$ label the components $1, 2$.

The components of the differential operator $\widetilde{D} + PP^T + \delta_P$ in Eq. (3.38) are given by

$$d = -\Box + 2m^2 + 3m^2(R^2 - 1) + \frac{1}{4}\mathcal{Y}^a_\mu \mathcal{Y}^a_\mu - \widehat{h}\,, \tag{C1}$$

$$\delta^b = -\mathcal{Y}^a_\rho \widehat{\mathcal{D}}^{ab}_\rho - \frac{1}{2}(\widehat{\mathcal{D}}_\rho \mathcal{Y}_\rho)^b\,, \tag{C2}$$

$$\delta^{T\,a} = \mathcal{Y}^a_\rho \partial_\rho + \frac{1}{2}(\widehat{\mathcal{D}}_\rho \mathcal{Y}_\rho)^a\,, \tag{C3}$$

$$D^{ab} = -(\widehat{\mathcal{D}}_\rho \widehat{\mathcal{D}}_\rho)^{ab} + \delta^{ab}\left(m^2(R^2 - 1) - \widehat{h}\right) + M^2_W R^2 \delta^{ab} + \frac{1}{4}\mathcal{Y}^a_\rho \mathcal{Y}^b_\rho\,, \tag{C4}$$

$$\delta_\nu^B = M_W R \widetilde{\mathcal{Y}}_\mu^A \widetilde{\mathrm{PT}}_{\mu\nu}^{AB} \,, \tag{C5}$$
$$\delta_\mu^{T,A} = M_W \widetilde{\mathrm{PT}}_{\mu\nu}^{AB} R \widetilde{\mathcal{Y}}_\nu^B \,, \tag{C6}$$
$$\Delta_\nu^{aB} = f^{aBc} M_W R \mathcal{Y}_\nu^c + 2M_W(\partial_\nu R)\delta^{aB} - sM_Z \delta^{4B}\left(2\delta^{a3}(\partial_\mu R) + RT^{ac}\mathcal{W}_\mu^c\right)\mathrm{PT}_{\mu\nu} \,, \tag{C7}$$
$$\Delta_\mu^{T,Ab} = -f^{Abc} M_W R \mathcal{Y}_\mu^c + 2M_W(\partial_\mu R)\delta^{Ab} + sM_Z \delta^{A4}\mathrm{PT}_{\mu\nu}\left(R\mathcal{W}_\nu^c T^{cb} - 2(\partial_\nu R)\delta^{3b}\right) \,, \tag{C8}$$
$$D_{\mu\nu}^{AB} = -\delta_{\mu\nu}(\widetilde{\mathcal{D}}_\rho \widetilde{\mathcal{D}}_\rho)^{AB} + 2f^{ABc}\mathcal{W}_{\mu\nu}^c + (\widetilde{M}^2)^{AB}\mathrm{PT}_{\mu\nu} + M_W^2 \delta^{AB}\mathrm{PL}_{\mu\nu} + \widetilde{\mathrm{PT}}_{\mu\alpha}^{AC}(\widetilde{M}^2)^{CD}(R^2-1)\widetilde{\mathrm{PT}}_{\alpha\nu}^{DB} + \delta^{A4}\delta^{B4}\mathrm{PT}_{\mu\rho}\widehat{J}_{\rho\sigma}\mathrm{PT}_{\sigma\nu} \,, \tag{C9}$$

where we introduced the quantities

$$\widehat{\mathcal{D}}_\mu^{ab} = \mathcal{D}_\mu^{ab} + \frac{1}{2}\varepsilon^{abc}\mathcal{Y}_\mu^c \,, \tag{C10}$$
$$\mathcal{D}_\mu^{ab} = \partial_\mu \delta^{ab} - \varepsilon^{abc}\mathcal{W}_\mu^c \,, \tag{C11}$$
$$\widetilde{\mathcal{D}}_\mu^{AB} = \delta^{AB}\partial_\mu - f^{ABc}\mathcal{W}_\mu^c \,, \tag{C12}$$
$$f^{ABc} = \begin{cases} \varepsilon^{abc} & , \; A = a, B = b, \\ 0 & , \; A = 4 \text{ and / or } B = 4, \end{cases} \tag{C13}$$
$$\widetilde{\mathcal{Y}}_\mu^A = \begin{pmatrix} \mathcal{Y}_\mu^a \\ -\frac{s}{c}\mathcal{Y}_\mu^3 \end{pmatrix} \,, \tag{C14}$$
$$\widetilde{\mathrm{PT}}_{\mu\nu} = \mathrm{diag}\left(\delta_{\mu\nu}, \delta_{\mu\nu}, \delta_{\mu\nu}, \mathrm{PT}_{\mu\nu}\right) \,, \tag{C15}$$
$$\mathrm{PT}_{\mu\nu} = \delta_{\mu\nu} - \mathrm{PL}_{\mu\nu} \,, \quad \mathrm{PL}_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\Box} \,, \tag{C16}$$
$$\widetilde{M}^2 = \begin{pmatrix} M_W^2 & 0 & 0 & 0 \\ 0 & M_W^2 & 0 & 0 \\ 0 & 0 & c^2 M_Z^2 & -csM_Z^2 \\ 0 & 0 & -csM_Z^2 & s^2 M_Z^2 \end{pmatrix} \,, \tag{C17}$$
$$\widehat{J}_{\mu\nu} = {g'}^2 v_{dj}\left(\delta_{\mu\nu} J_\kappa^\alpha J_\kappa^\alpha - J_\mu^\alpha J_\nu^\alpha\right) \,. \tag{C18}$$

In the basis (f, η^a, q_μ^A) the differential operator P which creates zero modes can be written as follows:

$$\begin{pmatrix} 0 \\ M_W R \delta^{aB} \\ \widetilde{\mathcal{D}}_\mu^{AB} \end{pmatrix} \alpha^B \equiv P\alpha \,, \tag{C19}$$

where α^B are four arbitrary scalar functions. From this expression we obtain the following results for the differential operators PP^T and $P^T P$ which appear in Eq. (3.36):

$$PP^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_W^2 R^2 \delta^{ab} & -M_W R \widetilde{\mathcal{D}}_\nu^{aB} \\ 0 & M_W \widetilde{\mathcal{D}}_\mu^{Ab} R & -(\widetilde{\mathcal{D}}_\mu \widetilde{\mathcal{D}}_\nu)^{AB} \end{pmatrix} \,, \tag{C20}$$
$$P^T P = \begin{pmatrix} -\mathcal{D}_\mu^{ac}\mathcal{D}_\mu^{cb} + M_W^2 R^2 \delta^{ab} & 0 \\ 0 & -\Box \end{pmatrix} \,. \tag{C21}$$

Furthermore, the operator δ_P is defined by

$$\delta_P = \mathrm{diag}\left(0, 0, \delta^{A4}\delta^{4B} M_W^2 \mathrm{PL}_{\mu\nu}\right) \,. \tag{C22}$$

Since we perform a saddle-point approximation in the path integral, the fields which appear in the list of differential operators in Eqs. (C1)–(C9) obey the equations of motion (3.25)–(3.30). We have used this fact to simplify the expressions of those operators which correspond to the fluctuations η^a of the Goldstone bosons. Furthermore, it is important to ensure that the full differential operator $\widetilde{D} + PP^T + \delta_P$ is Hermitian, i.e. satisfies the relation $(y, [\widetilde{D} + PP^T + \delta_P]y') = (y', [\widetilde{D} + PP^T + \delta_P]y)$ for arbitrary fluctuation vectors y, y'.

[1] For reviews on the use of effective Lagrangians in electroweak physics, see F. Feruglio, Int. J. Mod. Phys. A **8**, 4937 (1993); J. Wudka, *ibid.* **9**, 2301 (1994).
[2] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).
[3] G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991); G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. **B369**, 3 (1992); **B376**, 444(E) (1992).
[4] K.J.F. Gaemers and G.J. Gounaris, Z. Phys. C **1**, 259 (1979); K. Hagiwara, R.D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. **B282**, 253 (1987).
[5] M. Kuroda, J. Maalampi, K.H. Schwarzer, and D. Schildknecht, Nucl. Phys. **B284**, 271 (1987); M. Kuroda, F.M. Renard, and D. Schildknecht, Phys. Lett. B **183**, 366 (1987); G. Bélanger and F. Boudjema, *ibid.* **288**, 201 (1992).
[6] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975).
[7] W. Buchmüller and D. Wyler, Nucl. Phys. **B268**, 621 (1986).
[8] T. Appelquist and C. Bernard, Phys. Rev. D **22**, 200 (1980); T. Appelquist, in *Gauge Theories and Experiments at High Energies,* edited by K. C. Bowler and D. G. Sutherland (Scottish University Summer School in Physics, St. Andrews, 1980).
[9] A. C. Longhitano, Phys. Rev. D **22**, 1166 (1980); Nucl. Phys. **B188**, 118 (1981).
[10] T. Appelquist and G.-H. Wu, Phys. Rev. D **48**, 3235 (1993); **51**, 240 (1995).
[11] See the first paper in Ref. [1].
[12] S. Weinberg, Physica A **96**, 327 (1979).
[13] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).
[14] J.A. Bagger, A.F. Falk, and M. Swartz, Phys. Rev. Lett. **84**, 1385 (2000).
[15] H. Leutwyler, Ann. Phys. (N.Y.) **235**, 165 (1994).
[16] A. Nyffeler and A. Schenk, Phys. Rev. D **62**, 036002 (2000).
[17] J. Bijnens, G. Colangelo, and G. Ecker, J. High Energy Phys. **02**, 020 (1999).
[18] S. Scherer and H. W. Fearing, Phys. Rev. D **52**, 6445 (1995), and references therein.
[19] M. J. Herrero and E. R. Morales, Nucl. Phys. **B418**, 431 (1994); **B437**, 319 (1995).
[20] D. Espriu and J. Matias, Phys. Lett. B **341**, 332 (1995).
[21] S. Dittmaier and C. Grosse-Knetter, Nucl. Phys. **B459**, 497 (1996).
[22] A. Nyffeler and A. Schenk, Phys. Rev. D **53**, 1494 (1996).
[23] J. Matias, Nucl. Phys. **B478**, 90 (1996).
[24] V. Borodulin and G. Jikia, Nucl. Phys. **B520**, 31 (1998).
[25] P. Ciafaloni and D. Espriu, Phys. Rev. D **56**, 1752 (1997).
[26] A. Dobado, M. J. Herrero, and S. Peñaranda, Eur. Phys. J. C **7**, 313 (1999); **12**, 673 (2000).
[27] A. Nyffeler and A. Schenk, Ann. Phys. (N.Y.) **241**, 301 (1995).
[28] A. Straessner, talk at the XXXVth Rencontres de Moriond on Electroweak Interactions and Unified Theories, Les Arcs, France, 2000.
[29] K. Lane, in *Proceedings of the 28th International Conference on High-Energy Physics (ICHEP 96),* Warsaw, Poland, 1996, edited by Z. Ajduk and A.K. Wroblewski (World Scientific, Singapore, 1997), hep-ph/9610463.
[30] T. Appelquist, M. J. Bowick, E. Cohler, and A. I. Hauser, Phys. Rev. D **31**, 1676 (1985); E. Bagan, D. Espriu, and J. Manzano, *ibid.* **60**, 114035 (1999).
[31] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, *ibid.* **20**, 2619 (1979); E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981).
[32] A. Schenk, Karlsruhe University Report No. TTP-96-47, 1996, hep-ph/9610279.
[33] T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979).
[34] G. 't Hooft, in *Recent Developments in Gauge Theories,* Cargèse, France, 1979, edited by G. 't Hooft *et al.* (Plenum, New York, 1980), Lecture II, p. 117; for a recent account, see G. 't Hooft, in *International School of Subnuclear Physics, 36th Course: From the Planck Length to the Hubble Radius,* Erice, Italy, 1998, edited by A. Zichichi (World Scientific, Singapore, 2000), hep-th/9812204.
[35] F. Jegerlehner and J. Fleischer, Phys. Lett. **151B**, 65 (1985); Acta Phys. Pol. B **17**, 709 (1986), and references therein; M. Chanowitz, M. Golden, and H. Georgi, Phys. Rev. D **36**, 1490 (1987).
[36] O. Steinmann, Ann. Phys. (N.Y.) **157**, 232 (1984), and references therein.
[37] For a recent review, see R. Horan, M. Lavelle, and D. McMullan, Pramana, J. Phys. **51**, 317 (1998), hep-th/9810089, and references therein.
[38] R. Urech, Nucl. Phys. **B433**, 234 (1995).
[39] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[40] J. Gasser, M. E. Sainio, and A. Švarc, Nucl. Phys. **B307**, 779 (1988).

[41] R. Haag, Phys. Rev. **112**, 669 (1958).
[42] For a calculation in a general R_ξ-gauge, see J. Fleischer and F. Jegerlehner, Phys. Rev. D **23**, 2001 (1981).
[43] M. Veltman, Acta Phys. Pol. B **8**, 475 (1977).
[44] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[45] A. Nyffeler, Linear Collider Note, LC-TH-1999-018, hep-ph/9912472, to appear in the Proceedings of the 2nd ECFA/DESY Study on *Physics Studies for a Future Linear Collider*, DESY report 123F, edited by R. Heuer, F. Richard, and P. Zerwas.
[46] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321**, 311 (1989); G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B **223**, 425 (1989).
[47] S. Peris, M. Perrottet, and E. de Rafael, J. High Energy Phys. **05**, 011 (1998).